EGZAMIN Z ANALIZY NUMERYCZNEJ (L)

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Pierwszy termin

Pracuj samodzielnie!!!

Imię i nazwisko: Kacper Bajkiewicz

Numer części: I   Numer zadania: 4

$$(f,g) = f(-3)g(-3) + f(-2)g(-2) + f(0)g(0) + g(2)f(2) + g(3)f(3)$$

Ciąg wielomianów ortogonalnych definiujemy w taki sposób:

$P_0 = 1$, $P_1 = x$, $P_k = (x - c_k)P_{k-1}(x) - d_k P_{k-2}(x)$ $(k = 2, 3, \dots, n)$

Przy $c_k = \dfrac{\langle xP_{k-1}, P_{k-1}\rangle}{\langle P_{k-1}, P_{k-1}\rangle}$, $d_k = \dfrac{\langle P_{k-1}, P_{k-1}\rangle}{\langle P_{k-2}, P_{k-2}\rangle}$.

Wielomian optymalny $w_2^* \in \Pi_2$ będzie dany wzorem:

$$w_2^*(x) = \sum_{k=0}^{2} a_k P_k(x), \text{ gdzie } a_k = \frac{\langle f, P_k\rangle}{\langle P_k, P_k\rangle}$$

Policzmy wielomiany ortogonalne względem tego iloczynu

$P_0 = 1$

$P_1 = x$.

P_2

$$c_2 = \frac{\langle xP_1, P_1\rangle}{\langle P_1, P_1\rangle} = \frac{-3\cdot(-3)\cdot(-3) + (-2)(-2)(-2) + 0 + 2\cdot2\cdot2 + 3\cdot3\cdot3}{\langle P_1, P_1\rangle} =$$

No ale licznik się zeruje, więc $c_2 = 0$.

$$d_2 = \frac{\langle P_1, P_1\rangle}{\langle P_0, P_0\rangle} = \frac{-3(-3) + -2(-2) + 0 + 2\cdot2 + 3\cdot3}{1\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1 + 1\cdot1} = \frac{26}{5}.$$

Czyli $P_2 = xP_{k-1}(x) - \frac{26}{5}P_{k-2}(x) = x\cdot x - \frac{26}{5} = x^2 - \frac{26}{5}$.

Pamiętaj o zasadach nadsyłania rozwiązań!

Teraz szukamy wielomianu optymalnego najlepiej dopasowanego do danych:

$P_1 = x$, $P_0 = 1$, $P_2 = x^2 - \frac{26}{5}$.

$f \Rightarrow$

x_k	-3	-2	0	2	3
y_k	4	1	2	1	4

$$W_2^*(x) = \frac{\langle f, P_0\rangle}{\langle P_0, P_0\rangle} P_0(x) + \frac{\langle f, P_1\rangle}{\langle P_1, P_1\rangle} P_1(x) + \frac{\langle f, P_2\rangle}{\langle P_2, P_2\rangle} P_2(x).$$

$$\langle f, P_0\rangle = 1\cdot f(-3) + 1\cdot f(-2) + 1\cdot f(0) + 1\cdot f(2) + 1\cdot f(3) = 4+1+2+1+4 = 12.$$

$$\langle f, P_1\rangle = f(-3)(-3) + f(-2)(-2) + f(0)\cdot 0 + f(2)\cdot 2 + f(3)\cdot 3 = -12-2+0+2+12 = 0.$$

$$\langle f, P_2\rangle = f(-3)\left((-3)^2 - \frac{26}{5}\right) + f(-2)\left((-2)^2 - \frac{26}{5}\right) + f(0)\left(0^2 - \frac{26}{5}\right) + f(2)\left(2^2 - \frac{26}{5}\right) + f(3)\left(3^2 - \frac{26}{5}\right) =$$

$$f(-3)\cdot(-3)^2 - f(-3)\cdot\frac{26}{5} + f(-2)(-2)^2 - f(-2)\cdot\frac{26}{5} + f(0)\cdot 0 - f(0)\cdot\frac{26}{5} + f(2)\cdot 2^2 - f(2)\cdot\frac{26}{5} + f(3)\cdot 3^2 - f(3)\cdot\frac{26}{5} = 36 - \frac{4\cdot 26}{5} + 4 - \frac{26}{5} + 0 - \frac{2\cdot 26}{5} + 4 - \frac{26}{5} + 36 - \frac{4\cdot 26}{5} = 80 - \frac{12\cdot 26}{5} = \frac{400}{5} - \frac{312}{5} = \frac{88}{5}$$

Czyli $\langle f, P_2\rangle = \frac{88}{5}$.

$$\left((-3)^2 - \frac{26}{5}\right)\left((-3)^2 - \frac{26}{5}\right) = \left((-3)^2 - \frac{26}{5}\right)^2 \qquad 8 = \frac{45}{5}$$

$$\langle P_0, P_0\rangle = 1+1+1+1+1 = 6.$$

$$\langle P_1, P_1\rangle = (-3)^2 + (-2)^2 + 2^2 + 3^2 = 26$$

$$\langle P_2, P_2\rangle = \left((-3)^2 - \frac{26}{5}\right)\left((-3)^2 - \frac{26}{5}\right) + \left((-2)^2 - \frac{26}{5}\right)\left((-2)^2 - \frac{26}{5}\right) + \left(0 - \frac{26}{5}\right)\left(0 - \frac{26}{5}\right) + \left(2^2 - \frac{26}{5}\right)\left(2^2 - \frac{26}{5}\right) + \left(3^2 - \frac{26}{5}\right)\left(3^2 - \frac{26}{5}\right) = 2\left(8 - \frac{26}{5}\right)^2 + 2\left(4 - \frac{26}{5}\right)^2 + \left(-\frac{26}{5}\right)^2 =$$

$$2\cdot\left(\frac{19}{5}\right)^2 + 2\left(\frac{6}{5}\right)^2 + \left(\frac{26}{5}\right)^2 = 2\cdot\frac{361}{25} + 2\cdot\frac{36}{25} + 2\cdot\frac{676}{25} = \frac{2146}{25}.$$

$$\frac{\langle f, P_2\rangle}{\langle P_2, P_2\rangle} = \frac{\frac{88}{5}}{\frac{2146}{25}} = \frac{88}{2146}\cdot\frac{25}{5} = \frac{440}{2146} \qquad \frac{\langle f, P_0\rangle}{\langle P_0, P_0\rangle} = \frac{12}{6} = 2 \qquad \frac{\langle f, P_1\rangle}{\langle P_1, P_1\rangle} = \frac{0}{26} = 0.$$

Zatem: $W_2^* = 2\cdot P_0(x) + 0\cdot P_1(x) + \frac{440}{2146} P_2(x) = 2 + \frac{440}{2146}\left(x^2 - \frac{26}{5}\right)$.