Age-Population Demographics and Asset Return in the US

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Jack Wen-Hao Liu

Thesis Supervisor : Associate Professor Cao Jing

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Abstract

This paper studies the linkage between age-population demographics and asset return in the United States. It first explores an over-lapping generation model introduced by Geanakoplos, Magill, and Quinzii (GMQ, 2004). Then, it confirms previous studies' finding on how a specific demographic measure – the MY ratio – covaries with the US equity market. Next, the paper replicates analyses done in US equities to US Treasuries and US housing. For US Treasuries, statistical relationship between the MY ratio and bond returns tends to strengthen as bond returns become less uncertain. For US housing, which is not a pure financial asset whose sole purpose is to regulate intertemporal consumption choices, only very weak relationships are found. Lastly, based on the economics of the original GMQ model, alternative demographic measures are constructed, and statistical analyses reveal largely similar results. In the conclusion, the paper suggests the MY ratio's previously strong explanatory power over long-run asset returns likely will not repeat in the future. The future return environment for financial assets in the US may depend heavily on the productivity of its senior citizens.

Key words: age population demographics; asset pricing; asset returns; population aging

Contents

1. Introduction

Studies trying to address the link between asset return and age-population demographics appear quite frequently in the investment industry[1], but in the academic world, this topic is perhaps not considered mainstream. One possible reason is it is hard to fit demographics into the 'risk framework' of modern asset pricing. Demographics, by its nature, moves very slowly and can be predicted with rather high precision. For instance, in all likelihood, the ratio of people aged 60 to people aged 40 ten years from now will be very close the ratio of people aged 50 to people aged 30 at the present moment in a populous country like the US[2]. According to efficient market hypothesis, since future demographic information is easily forcastable, its effect on financial securities, if any, should already be factored in by current security prices. What's going to affect returns should be 'surprises' in demographic shifts that people have not previously considered. This paper will show, both in model setting and with empirical test, that demographics has a strong link with financial asset returns, but not through any surprises. Yet, in the model to be introduced, all individuals still behave perfectly rational. A simple intuition is laid out in the next paragraph.

Suppose, in a world without risk, the function of "market rate of return" in an economy is to balance consumption demand and production supply. When production next period is greater than production this period, market return will be high to promote consumption next period and demote consumption this period. Conversely, when production next period is less than production this period, market return will be low to encourage consumption this period and discourage consumption next period. In other words, equilibrium return acts like an invisible hand that allocates consumption across time periods such that consumption demand and production supply are balanced for each period.

In their original paper, Geanakoplos, Magill, and Quinzii (GMQ, 2004) reported strong empirical link between the middle-young age cohort ratio (MY ratio) and one-year S&P 500 returns. Later authors like Favero and Tamoni (FT, 2010) and Favero, Gozluklu, and Tamoni (FGT, 2011) found even stronger associations using long-run return data of the S&P 500. This paper confirms those previous findings in US equities, and incorporates US Treasuries and US housing to the study. Furthermore, it creates additional demographic measures based on the economics of the original GMQ model, and present to the reader their statistical relationships against equities, Treasuries, and housing market in the US. The reader shall find:

1. After the baby-boom generation entered the US economy (1952-2012), variation of the MY ratio highly coincides with variation of long-run stock market returns. Before the baby-boom generation (1900-1951), however, the relationship is not there.
2. For S&P 500, 'shocks' or 'surprises' in demographic trends bear no relationship with returns.
3. For US Treasuries, covariance between the MY ratio and returns is also strong, but data is only available for the post-baby-boom era: 1952-2012.
4. For US Treasuries, statistical linkage between MY and bond return tends to intensify when the bond's return becomes less uncertain.
5. For US housing, although MY's correlation with home price return strengthens during the post-baby-boom era, the relationship is very weak. Nevertheless, strong relationship should only be expected for financial assets, which housing is not.
6. All of the above findings remain valid when replacing the MY ratio by other demographic measures[3], albeit statistical outputs may be slightly weaker.
7. Most probably, the MY ratio is not going to be a meaningful explanatory factor for future asset returns. Measures that take into account senior citizens' relative productivity and growing population will likely do a better job.

[1] For example, the CFA Journal and popular authors like Dent.

[2] In equation format, $\frac{Pop\ of\ 60\ year\ olds\ at\ time\ t+10}{Pop\ of\ 40\ year\ olds\ at\ time\ t+10} \approx \frac{Pop\ of\ 50\ year\ olds\ at\ time\ t}{Pop\ of\ 30\ year\ olds\ at\ time\ t}$

[3] More specifically, demographic measures that reflect excess demand for saving in the economy.

This paper is organized as follows: Section 2 summarizes the theoretical foundation (the GMQ model) for the linkage between age-population demographics and asset returns. Section 3, 4, 5 examines the statistical relationships between the MY ratio and the US equity market, the US Treasury market, and the US housing market, respectively. Section 6 conducts the same statistical analyses on alternative measures of age-population demographics. Section 7 concludes.

All age-population demographic data are obtained online through the US Census Bureau, Population Division. Data from 1900 to 2012 are actual estimates of US population categorized by age. Data from 2013 to 2060 are forecasts made by the Census Bureau. For US equity, returns include both dividend payouts and capital gains. Data source is Robert Shiller and Kenneth French's[4] websites. For US Treasury, returns are based on artificially constructed zero-coupon bonds. Data source is Center for Research in Security Prices (CRSP, www.crsp.com). For US housing, returns include only capital gains. Data source is Robert Shiller's website. All data frequencies are annual. All financial time-series return data are in real terms, i.e., adjusted for inflation, and are natural-log transformed (continuously compounded). Transaction cost or bid-ask spreads are not considered in all return analyses in this paper.

2. The Basic GMQ Model

The GMQ model, built by Geanakoplos, Magill, and Quinzii (2004), provides a simple, easy-to-understand framework to connect age-population demographics and asset return. For simplicity, the author will present only the most basic version of the model. Interested readers can refer to the original paper for extended versions with added features and less restrictive assumptions. Regardless of which version of the model is applied, primary results are identical (GMQ, 2004).

2. 1 Set-up

Assume in an overlapping generation exchange economy, agents' economic life has only three periods: {young, middles-age, retired}, abbreviated as {y, m, r}. All agents share identical preferences and endowments. These agents only differ by the period at which they enter the economic scene. Their utility function is given by:

$$U(\boldsymbol{c}) = E\left[u(c_y) + \delta u(c_m) + \delta^2 u(c_r)\right], \quad \delta > 0$$
$$u(x) = \frac{1}{1-\alpha} x^{1-\alpha}, \ \alpha > 0 \qquad ..(2.1)$$

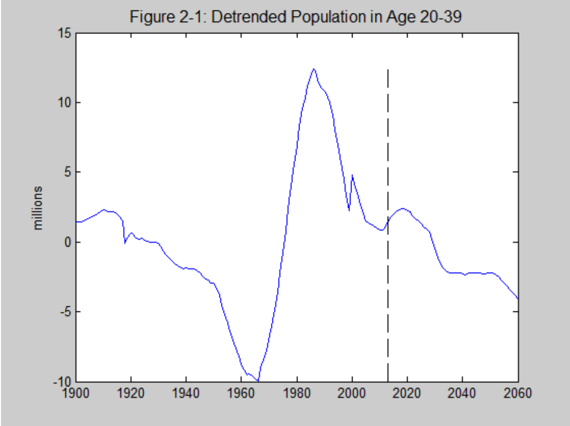
The utility function in each life stage is standard power utility. **c** = (c_y, c_m, c_r) is the consumption stream that takes place in periods {y, m, r}, respectively. Agents seek to optimize lifetime utility U(**c**)[5] by choosing how much to consume in each period. δ is the inter-period subjective discount factor. α is the intertemporal elasticity of substitution found in standard economic theory.

Each agent has endowment w = (w_y, w_m, w_r), where w_i can be seen as labor income earned in period i. There are only two financial instruments: risk-free bond and equity. The risk-free bond pays one unit of income over the next period and its price in the current period is denoted by q. Equity pays dividend amount D in each period and its price is denoted by p. In the basic model, dividend and wage income are deterministic, so in absence of uncertainty, bonds and equity are perfect substitutes. It is assumed that

[4] Robert Shiller: http://www.econ.yale.edu/~shiller/data.htm
Kenneth French: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.htm
[5] Technically, in the basic GMQ model, risk does not exist. The expectation sign around utilities u(c) can be removed. The author preserves the expectation sign to maintain consistency with the original paper.

population is "detrended" to factor out the effects of overall population growth. In each period, a new cohort of young agents enter the economy while retired agents from previous period exit the scene. The new cohort's population always alternates between large and small, denoted by {N, n}. Thus, in each period, the adult population-age structure is either (N,n,N) (call this odd period) or (n,N,n) (call this even period). Odd period and even periods are the only two states in this basic economy, and shift in demographic structure is the sole source of fluctuation in the model.

Figure 2-1 on the right illustrates the {N, n} alternation of new agents coming to an economy is not a bad assumption from after World War II to present time (2013). By considering each period to be approximately 22 years, we see the number of young agents in the economy is very low between 1952-1973, very high between 1974-1995, and drops low again after 1995. To make the model more tractable, GMQ calibrated the endowment (wage) vector to (2,3,0), reflecting the fact that agents become more productive and generate higher income during middle age and live off previous savings during retirement.



Let q_t be the time t price of a bond that pays 1 in time t+1. Since the economy has only two states in odd and even periods due to demographic shifts, in equilibrium, q_t would also alternate between the two states. Thus, $q_t = q_1$ when t is odd, $q_t = q_2$ when t is even. In a deterministic economy, equity and bonds are perfect substitutes, so equity price p also alternates between even and odd periods. The following equation characterizes what just said:

$$\frac{D+p_2}{p_1} = 1 + r_1 = \frac{1}{q_1}\,,\ \frac{D+p_1}{p_2} = 1 + r_2 = \frac{1}{q_2}$$
$$\Rightarrow \frac{p_1}{D} = \frac{q_1 q_2 + q_1}{1 - q_1 q_2}\,,\ \frac{p_2}{D} = \frac{q_1 q_2 + q_2}{1 - q_1 q_2} \qquad ..(2.2)$$

Thus, $\frac{p_1}{D} > \frac{p_2}{D}$ if and only if $q_1 > q_2$. This result will not change if we define equity as sum of discounted dividends:

$$p_1 = D(q_1 + q_1 q_2 + q_1 q_2 q_1 + q_1 q_2 q_1 q_2 + \cdots)$$
$$p_2 = D(q_2 + q_1 q_2 + q_2 q_1 q_2 + q_1 q_2 q_1 q_2 + \cdots)$$

Agents use bond/equity to transfer income across time, so they face the constraint that present value of consumption stream must equal present value of income stream. Agents who are young in odd periods choose consumption stream (c_{1y}, c_{1m}, c_{1r}) to maximize utility function (2.1) subject to the budget constraint:

$$c_1^y + q_1 c_1^m + q_1 q_2 c_1^r = w^y + q_1 w^m + q_1 q_2 w^r = 2 + 3q_1 \qquad ..(2.3)$$

Likewise, agents who are young in even periods choose consumption stream (c_{2y}, c_{2m}, c_{2r}) to maximize (2.1) subject to the budget constraint:

$$c_2^y + q_2 c_2^m + q_1 q_2 c_2^r = w^y + q_2 w^m + q_1 q_2 w^r = 2 + 3q_2 \qquad ..(2.4)$$

Each agent takes market discount rate in odd (q_1) and even (q_2) periods as given, and make consumption-investment-borrow decisions in accordance with q1, q2, wages, subjective discount factor (δ), and intertemporal elasticity of substitution (α). Taking the first order condition of the representative agent who is young in odd periods, we get:

$$u'(c_{1,y}) - \lambda_1 = 0$$
$$\delta u'(c_{1,m}) - \lambda_1 q_1 = 0$$
$$\delta^2 u'(c_{1,r}) - \lambda_1 q_1 q_2 = 0$$
$$c_{1,y} + q_1 c_{1,m} + q_1 q_2 c_{1,r} = 2 + 3q_1 \qquad ..(2.5)$$

Solve equation 2-5 would yield:

$$c_{1,y} = \frac{2+3q_1}{1+q_1^{(1-1/\alpha)}\delta^{1/\alpha}+(q_1q_2)^{(1-1/\alpha)}\delta^{2/\alpha}}$$
$$c_{1,m} = c_{1,y}(\frac{\delta}{q_1})^{(1/\alpha)}$$
$$c_{1,r} = c_{1,y}(\frac{\delta^2}{q_1q_2})^{(1/\alpha)} \qquad ..(2.6)$$

In equilibrium, q_1 and q_2 have to adjust in order to satisfy the collective market clearing condition in each period:

$$Nc_{1,y} + nc_{2,m} + Nc_{1,r} = 2N + 3n + D$$
$$nc_{2,y} + Nc_{1,m} + nc_{2,r} = 2n + 3N + D \qquad ..(2.7)$$

Market clearing condition (2.7) marks the central thesis of the GMQ model: asset returns serve to balance demand and supply in an economy. Market discount rates q_1 and q_2 are derived from relative risk preference (α), subjective discount factor (δ), and population parameters n and N.[6] Equation 2.7 implies that even without stochastic shocks, demographic shifts alone would cause returns to adjust. Market returns q_1 and q_2 serve to balance consumption demand (left-hand side of Eq. 2.7) and production supply (right-hand side of Eq. 2.7) in an economy. Let's now turn to see calibrated examples.

2. 2 Calibrated Numerical Examples

Between 1925 and 1944, there were approximately 52 million births given in the US; between 1945 and 1964, the number was 79 million; between 1965 and 1984, 69 million[7]. Dividend D is taken to be 19% of the average of aggregate wages between odd and even periods. Assume each period occupies 20 years, an annual subjective discount factor of 0.966 would translate to δ = 0.5. Take {N, n} to be {79, 52}, D would be around 62 = 19% x (2 x 79 + 3 x 52 + 2 x 52 + 3 x 79)/2. If market discount factors $q_{1,2}$ were equal to subjective discount factors δ = 0.5, then the first order condition in (2.5) would ensure agents to smooth their consumption stream to [2, 2, 2]. Thus, in odd period, consumption demand is [2, 2, 2]*[79, 52, 79]' = 420, but output in the economy is 2x79 + 3x52 + D = 376, generating excess demand for consumption. Similarly, in even period, consumption demand is [2, 2, 2]*[52, 79, 52]' = 366, but output is 2x52 + 3x79 + D = 403, generating excess supply of production. To clear the market, during odd period (with excess consumption demand), return needs to be high to discourage consumption and encourage saving; during even period (with excess saving demand), return needs to be low to promote consumption and demote savings. With the given parameters (α = 4), this is exactly the case. In odd period, market clearing condition requires q_1 = 0.29 → r_{annual} = 6.46%, price dividend ratio P_1/D = 0.84; in even period, q_2 = 1.05 → r_{annual} = -0.25%, price dividend ratio P_2/D = 1.93, significantly higher than P_1/D.

Table 2-1 lists the derived price-dividend ratios and annualized market rate of return against different values of α and {N, n}:

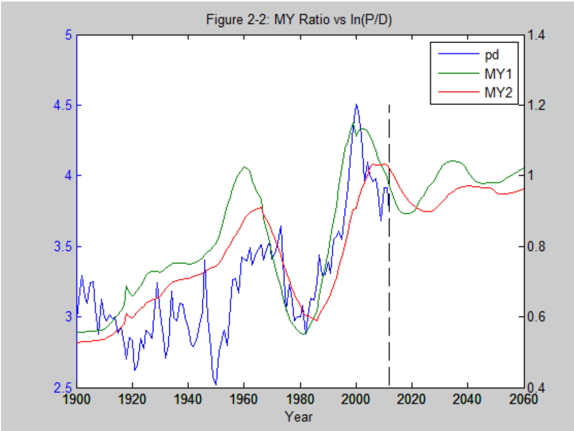
[6] Explicit solution to Eq. 2.7 cannot be found. Numerical solutions, however, do exist. Closed form solution was also not given in the original GMQ (2004) paper.

[7] The calibrated numbers are taken straight out of GMQ (2004).

Table 2-1: Equilibrium Market Rate of Return in Different Scenarios

{N, n}	{79, 52}				{79, 69}			
Period	Odd: (N,n,N)		Even: (n,N,n)		Odd: (N,n,N)		Even: (n,N,n)	
	P/D	r_{annual}	P/D	r_{annual}	P/D	r_{annual}	P/D	r_{annual}
α = 2	0.88	5.29%	1.46	1.26%	0.97	4.05%	1.15	2.73%
α = 4	0.84	6.46%	1.93	-0.25%	0.95	4.44%	1.26	2.21%
α = 6	0.82	7.31%	2.37	-1.31%	0.93	4.72%	1.34	1.84%

From Table 2-1, we see that the greater is the difference between N and n, the greater is the difference between: a) price-dividend ratios b) rate of return in alternating periods. GMQ devised a measure, the MY ratio, to capture the population ratio of middle aged to young agents. Thus, when MY is small (big), the period corresponds to odd (even). From Table 2-1, odd (even) periods are associated with lower (higher) price-dividend ratios. The MY ratio is predicted to co-move positively with price-dividend ratio, especially when the value of MY is particularly low or high (meaning that the absolute value |N-n| is large). Or, based on the assumption of oscillating demographic structure, MY is small in odd periods because it will rise in next period. Likewise, MY is big in even periods because it will fall in next period. Therefore, incorporating dynamics into account, when future change in MY is positive, return is high; when future change in MY is negative, return is low.



There are numerous ways to compute the MY ratio. Figure 2-2 plots two types of MY ratios versus logged price-dividend ratio of the S&P500 index. In the figure, MY1 is calculated as population in the 40-49 age group divided by population in the 20-29 age group; MY2 is calculated as population in the 40-59 group divided by population in the 20-39 group. While MY2 more accurately computes the ratio of middle aged cohorts over young cohorts, MY1 offers a slightly better fit for asset return data. Regressing the logged price-dividend ratio pd separately on MY1 and MY2, the results are almost identical: R^2 = (0.5054, 0.4949), t = (10.65, 10,43), respectively. Because GMQ and later authors adopted MY1 in their computation of MY, I will use this definition from now on. Notwithstanding the technicalities of how MY is computed, the bottom line is clear: when MY undertakes large swings, its covariance with price-dividend ratio is high, closely matching prediction from the basic GMQ model.

We now turn to analyze the empirical linkage between the MY ratio and asset returns. First, let's start from equities.

3. The US Equity Market

3. 1 Preliminary Study of Real S&P Index Return and Demographics

In earlier empirical studies of demography and asset returns, Poterba (2001, 2004), Ang and Maddaloni (2003) studied asset returns' link with various classes of demographic variables[8]. Using logged real S&P 500 index total returns (including both dividend payouts and capital gains) as the dependent variable (*r*), their

[8] Poterba focused on 1-year returns. Ang and Maddaloni extended investment horizons to 1, 2, and 5 year periods. MY was not used as an explanatory variable in their study.

findings generally agree with the following regression:

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_1 MY_t + \varepsilon_{t+h} \qquad h = 1, \dots, 5 \qquad ..(3.1)$$

Note the time subscript t is based on annual frequency; *h* stands for investment horizon in years. Table 3-1 shows the regression output. Two types of standard errors are reported. t1 refers to t-statistics based on ordinary least squares (OLS) standard error; t2 refers to t-statistics based on Newey-West standard error. For t2, the band-width chosen in estimating the spectral density matrix is $n^{1/3}$, where n is number of observations. Range of sample period is between 1900 to 2012. p1 and p2 correspond to the two-tail p-value of t1 and t2, respectively. "Prob > F" is the p-value for OLS F-statistic. R^2_adj is the adjusted R^2 statistic. "rmse" corresponds to square root of mean squared error under OLS regression. "rmse/√h" is the annualized rmse. T-stats and F-stats with p-value less than 5% are highlighted in bold. Note for 1-tail test, significance threshold (bold font) becomes 2.5%.

Table 3-1, Overlapping Regression Output for Equation 3.1

h	B1	t1	t2	p1	p2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.04	0.43	0.52	0.66	0.60	0.00	-0.01	0.66	0.183
2	0.06	0.41	0.38	0.68	0.71	0.00	-0.01	0.68	0.187
3	0.04	0.27	0.20	0.79	0.84	0.00	-0.01	0.79	0.178
4	-0.01	-0.05	-0.03	0.96	0.97	0.00	-0.01	0.96	0.178
5	-0.09	-0.42	-0.27	0.67	0.78	0.00	-0.01	0.67	0.174

From the table, we can see that demographic information available at time t, by itself, has zero explanatory power over future S&P500 returns. T-statistics are all very low, and overall model statistics all indicate Equation 3.1 is a poorly specified model. Similar results were well documented in Poterba, Ang and Maddaloni's studies[9]. However, at this point, it would be too early to reject the GMQ model. Let's consider some possibilities why the simple regression (3-1) failed. Possibility one, demographic time series are very smooth, whereas short-horizon return series of financial assets display much more zigzag (or noise). As investment horizon increases, the random component in stock returns tend to cancel out, allowing the trend component to emerge. To remind the reader, assumed time horizon in the set-up of the basic GMQ model is across generations. Possibility two, in the GMQ model, a critical assumption is that economy oscillates between two states: odd ({N,n,N}) and even ({n,N,n}). MY ratio is small (n/N) in odd periods, and turns big (N/n) in even periods. Eq. 3.1 does not take MY ratio's dynamics into account, which may be the more relevant variable. Possibility three, noted by Poterba (2004), employing demographic variables alone in asset return regressions may suffer omitted variable bias. If we take all or some of the above possibilities into account, i.e. (1) extend investment horizon beyond 5 years, (2) form time t expectation of MY_{t+h} with current population ratio of people in age group [(40:49)-h] to people in age group [(20:29)-h], (3) include a business-cycle control variable such as logged dividend yield, then better results should surface. Section 3.2 examines these possibilities.

3. 2 Explaining Real S&P Equity Index Returns

Table 3-2 lists pairwise correlation between h-year S&P500 logged returns $\sum_{j=1}^{h} r_{t+j}$ and three variables: MY ratio (MY_t), expected change in MY ratio ($E_t(MY_{t+h}-MY_t)$), and dividend yield (dp_t). Recall from Table 2-1, in odd periods (small MY), future equilibrium return is high; in even periods (big MY), future equilibrium return is low. Thus, negative correlation is expected between MY and returns. The exact same scenario can

[9] To be exact, Poterba's study did find population share between 40-65 had statistically significant coefficient in explaining annual returns of S&P500 during 1947-2003. While Ang and Maddaloni did not report statistically significant coefficients for demographic variables in the US, they reported statistically significant coefficients for most international countries in their data. Interestingly, countries with higher social security benefits had a stronger link between demographic variables and asset returns.

be interpreted from a different angle. We saw when MY grew from small to big (odd period to even period), the economy had high equilibrium return; when MY grew from big to small (even period to odd period), the economy had low equilibrium return. Hence, a positive correlation is expected between future change in MY and returns. dp should be positively correlated with future S&P500 returns. For a more detailed treatment of the relationship between return and dividend yield, consult Appendix A1. Two types of correlations are reported in Table 3-2: overlapping and non-overlapping time intervals. In overlapping time-interval, sample size is 113 – h[10]. In non-overlapping time intervals, sample size is reduced from 113 – h to $\lfloor(113-h)/h\rfloor$. The trade-off is sample size for independence of observations. ΔMY is a shorthand for $E_t(MY_{t+h}\text{-}MY_t)$.

Table 3-2, Correlation Between S&P500 Return and [MY, ΔMY, dp]

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.04	0.04	0.03	0.00	-0.04	-0.06	-0.07	-0.10	-0.12	-0.15
	ΔMY	0.18	0.28	0.37	0.44	0.51	0.55	0.57	0.58	0.59	0.59
	dp	0.17	0.20	0.26	0.30	0.34	0.37	0.40	0.42	0.43	0.44
non-overlap	MY	0.04	0.04	0.04	0.02	0.04	-0.02	-0.16	-0.04	0.03	0.10
	ΔMY	0.18	0.23	0.40	0.37	0.46	0.57	0.62	0.63	0.61	0.46
	dp	0.17	0.17	0.25	0.27	0.25	0.31	0.48	0.36	0.44	0.40

Between MY and returns, under overlapping time intervals, correlation tends to decrease as return horizon *h* increases, and eventually becomes negative when $h \geq 5$. But for non-overlapping time intervals, the trend disappears. Between ΔMY and returns, correlation signs are all positive, completely match with the GMQ model. Between dp and returns, correlation signs are also positive as expected, and they tend to increase along with *h*. Surprisingly, the magnitude of correlations between <ΔMY, *r*> is even greater than the magnitude of correlations between <dp, *r*>.

Let us now examine in regression setting the connection between ΔMY and returns.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_2 E_t(MY_{t+h} - MY_t) + \varepsilon_{t+h} \qquad h = 1, \ldots, 10 \qquad ..(3.2)$$

Table 3-3, Regression Output for Equation 3.2

	Overlapping Time Interval							Non-Overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	1.59	1.97	**3.18**	0.03	0.03	0.05	0.180	1.59	1.97	**3.18**	0.03	0.03	0.05	0.180
2	1.82	**3.00**	**4.01**	0.08	0.07	**0.00**	0.180	1.57	1.76	**3.26**	0.05	0.04	0.08	0.190
3	1.92	**4.20**	**4.71**	0.14	0.13	**0.00**	0.165	1.96	**2.61**	**5.42**	0.16	0.14	**0.01**	0.157
4	1.97	**5.14**	**5.09**	0.20	0.19	**0.00**	0.159	1.95	2.03	**4.51**	0.14	0.10	0.05	0.200
5	2.00	**6.09**	**5.22**	0.26	0.25	**0.00**	0.150	1.67	**2.29**	**4.69**	0.21	0.17	**0.03**	0.152
6	1.94	**6.80**	**5.28**	0.31	0.30	**0.00**	0.141	2.03	**2.74**	**7.37**	0.32	0.28	**0.01**	0.151
7	1.87	**7.14**	**5.47**	0.33	0.32	**0.00**	0.138	2.35	**2.95**	**6.76**	0.38	0.34	**0.01**	0.161
8	1.81	**7.20**	**5.61**	0.33	0.33	**0.00**	0.139	1.60	**2.80**	**5.69**	0.40	0.35	**0.02**	0.114
9	1.75	**7.33**	**5.74**	0.35	0.34	**0.00**	0.137	1.35	**2.43**	**4.20**	0.37	0.31	**0.04**	0.111
10	1.69	**7.40**	**5.84**	0.35	0.35	**0.00**	0.136	0.95	1.55	**3.19**	0.21	0.12	0.16	0.122

Table 3-3 shows strong statistical outputs for the model specified in Eq. 3.2. We see that as *h* increases, t-stats and R^2s generally increase, and annualized RMSE generally decreases. Certainly, compared with Table 3-1, which reflects earlier negative empirical findings, the results given by Table 3-3 are very different. Also, all coefficient signs fit with the calibrated numerical example seen in Table 2-1: ΔMY covaries positively with equilibrium market return. Let us now add MY and dividend yield dp to the regression equation.

[10] Recall sample period is 1900-2012.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_1 MY_t + B_2 E_t(MY_{t+h} - MY_t) + B_3 dp_t + \varepsilon_{t+h} \qquad h = 1, \dots, 10 \qquad ..(3.3)$$

Table 3-4a, Overlapping Regression Output for Equation 3.3

h	B1	t1	t2	B2	t1	t2	B3	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.30	**2.26**	**3.57**	1.10	1.36	**2.42**	0.15	**2.65**	**4.10**	0.10	0.07	**0.01**	0.176
2	0.48	**2.65**	**3.43**	1.46	**2.39**	**3.58**	0.24	**2.94**	**3.60**	0.15	0.13	**0.00**	0.174
3	0.66	**3.28**	**3.53**	1.62	**3.60**	**4.88**	0.32	**3.63**	**3.98**	0.24	0.22	**0.00**	0.156
4	0.78	**3.52**	**3.48**	1.74	**4.61**	**5.85**	0.39	**3.93**	**4.42**	0.31	0.29	**0.00**	0.149
5	0.88	**3.83**	**3.24**	1.84	**5.73**	**6.55**	0.44	**4.33**	**4.56**	0.38	0.36	**0.00**	0.139
6	1.04	**4.46**	**3.20**	1.86	**6.80**	**7.29**	0.50	**4.91**	**4.50**	0.45	0.43	**0.00**	0.127
7	1.22	**5.08**	**3.27**	1.86	**7.50**	**7.97**	0.58	**5.55**	**4.80**	0.50	0.49	**0.00**	0.120
8	1.37	**5.29**	**3.29**	1.86	**7.84**	**8.44**	0.65	**5.89**	**5.28**	0.52	0.51	**0.00**	0.119
9	1.50	**5.40**	**3.22**	1.88	**8.27**	**8.64**	0.69	**6.03**	**5.42**	0.54	0.52	**0.00**	0.116
10	1.63	**5.51**	**3.13**	1.90	**8.60**	**8.41**	0.73	**6.16**	**5.35**	0.55	0.54	**0.00**	0.114

Table 3-4b, Non-Overlapping Regression Output for Equation 3.3

h	B1	t1	t2	B2	t1	t2	B3	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.30	**2.26**	**3.57**	1.10	1.36	**2.42**	0.15	**2.65**	**4.10**	0.10	0.07	**0.01**	0.176
2	0.46	1.66	**3.07**	1.24	1.35	**2.96**	0.22	1.80	**2.98**	0.11	0.06	0.09	0.188
3	0.66	1.96	**3.70**	1.68	**2.21**	**5.53**	0.31	2.02	**4.08**	0.27	0.20	**0.02**	0.152
4	1.17	1.99	**3.45**	1.72	1.81	**3.80**	0.54	**2.11**	**3.69**	0.28	0.19	**0.04**	0.190
5	0.96	1.77	**2.73**	1.70	**2.28**	**4.55**	0.37	1.63	**3.48**	0.34	0.23	0.06	0.146
6	1.40	2.04	**3.37**	2.12	**2.90**	**6.22**	0.54	1.94	**3.37**	0.49	0.38	**0.02**	0.140
7	1.47	1.95	**2.33**	2.21	**2.79**	**6.97**	0.78	**2.30**	**3.02**	0.58	0.48	**0.01**	0.144
8	1.56	**2.56**	**2.77**	1.78	**3.26**	**6.63**	0.61	2.18	**3.23**	0.64	0.53	**0.01**	0.096
9	1.71	**3.47**	**3.30**	1.68	**3.85**	**8.20**	0.67	**2.98**	**4.05**	0.77	0.68	**0.01**	0.075
10	1.94	**2.82**	**5.49**	1.26	2.29	**5.76**	0.59	**2.40**	**5.22**	0.65	0.51	**0.05**	0.091

From Table 3-4, we also see very strong numbers: t-stats and F-stats are mostly significant, R^2s increase along with *h* and are astonishingly high, F-stats are all favorable, annualized RMSE decrease along with *h*. However, a caveat must be presented to the reader at this point. By itself, a straight line cannot explain S&P500 returns at all. But, it occurs that if one regresses returns on dividend yield and a straight line, output statistics will improve on all fronts, as shown in Figure 3-1 and Table 3-5.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_3 dp_t + B_4 t + \varepsilon_{t+h} \qquad h = 1, \dots, 10 \qquad ..(3.4)$$

Figure 3-1: Fitted vs Actual Returns for Eq. 3.4

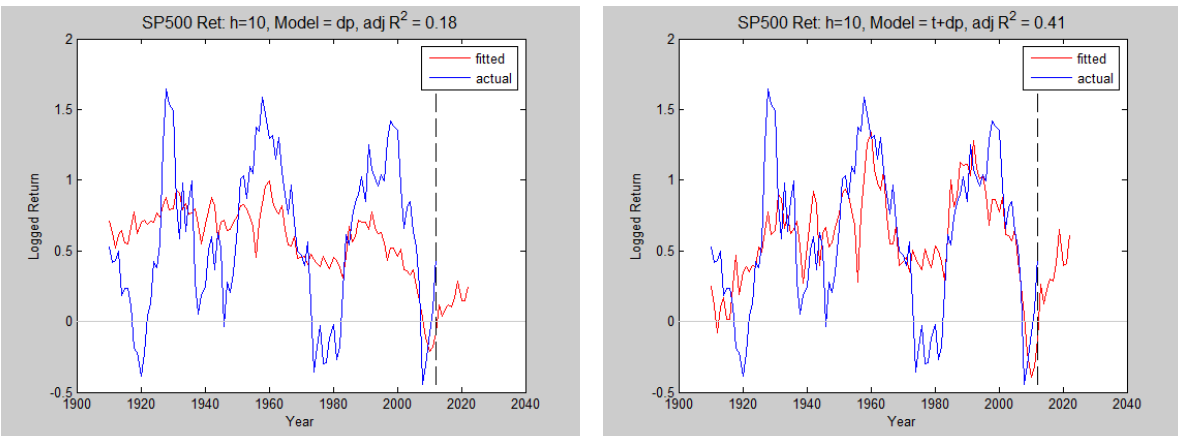


Figure 3-1 shows fitted value versus actual value for 10-year S&P500 returns. The left figure uses only dividend yield and a constant to produce fits. The right figure uses time (a straight line), dividend yield, and a constant to produce fits. By simply including a straight line[11] in the linear regression, adjusted R^2 goes

[11] The exact series is [1:113]*0.01.

from 0.18 in left figure to 0.41 in the right figure. Similarly, the adjusted R^2 for the straight line only model is 0.00, with no significant t-stats. By simply adding dp to the linear regression, adjusted R^2 shoots up to 0.41, with t-stats for B_4 (coefficient for time) all become significant:

Table 3-5a Overlapping Regression Output for Equation 3.4

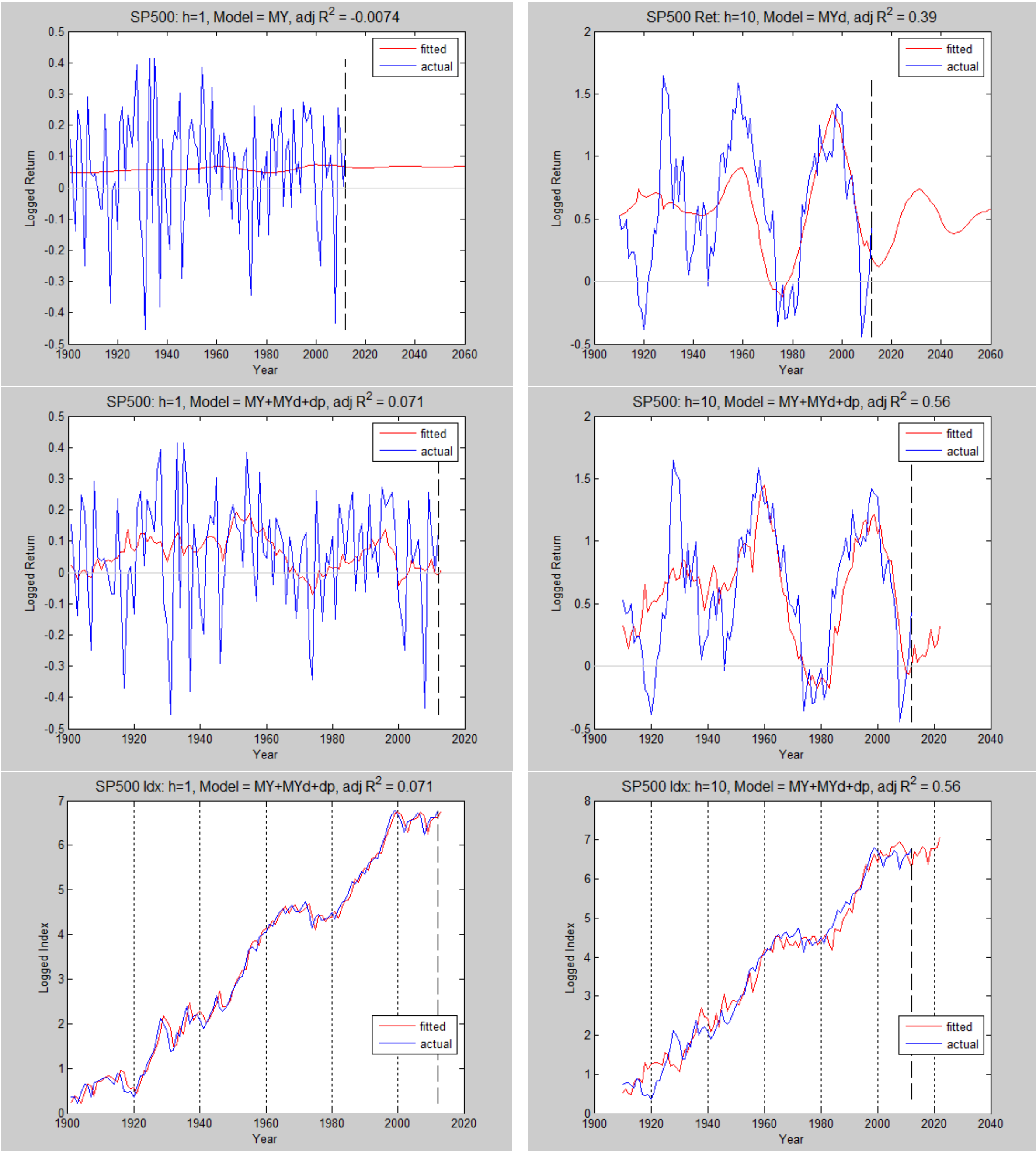
h	B3	t1	t2	B4	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.16	**2.79**	**4.30**	0.16	**2.13**	**2.78**	0.07	0.05	**0.02**	0.178
2	0.28	**3.40**	**4.36**	0.28	**2.60**	**2.62**	0.10	0.08	**0.00**	0.179
3	0.41	**4.46**	**4.79**	0.40	**3.39**	**2.71**	0.16	0.14	**0.00**	0.164
4	0.52	**5.08**	**5.28**	0.51	**3.76**	**2.78**	0.20	0.18	**0.00**	0.160
5	0.63	**5.74**	**5.42**	0.60	**4.13**	**2.90**	0.24	0.22	**0.00**	0.153
6	0.74	**6.57**	**5.72**	0.72	**4.84**	**3.20**	0.29	0.28	**0.00**	0.143
7	0.87	**7.49**	**6.56**	0.86	**5.59**	**3.57**	0.35	0.34	**0.00**	0.136
8	0.98	**7.95**	**7.12**	0.95	**5.88**	**3.71**	0.39	0.37	**0.00**	0.134
9	1.07	**8.22**	**7.29**	1.03	**6.10**	**3.97**	0.40	0.39	**0.00**	0.132
10	1.15	**8.46**	**7.20**	1.10	**6.31**	**4.34**	0.42	0.41	**0.00**	0.129

Table 3-5b Non-overlapping Regression Output for Equation 3.4

h	B3	t1	t2	B4	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	0.16	**2.79**	**4.30**	0.16	**2.13**	**2.78**	0.07	0.05	**0.02**	0.178
2	0.24	**2.04**	**3.61**	0.25	1.57	**2.32**	0.07	0.04	0.13	0.190
3	0.39	**2.50**	**4.57**	0.38	1.93	**3.64**	0.16	0.11	0.05	0.160
4	0.67	**2.49**	**3.72**	0.70	1.98	**2.77**	0.20	0.13	0.06	0.196
5	0.53	**2.10**	**5.13**	0.61	1.76	**3.92**	0.19	0.11	0.13	0.157
6	0.61	1.97	**3.57**	0.63	1.46	**3.10**	0.21	0.10	0.18	0.169
7	1.30	**3.83**	**7.11**	1.22	**2.86**	**5.81**	0.53	0.46	**0.01**	0.146
8	1.04	**2.99**	**4.57**	1.08	**2.53**	**3.84**	0.45	0.35	**0.04**	0.113
9	1.12	**3.59**	**3.85**	1.12	**2.99**	**4.67**	0.60	0.51	**0.02**	0.094
10	0.66	**2.33**	**3.62**	0.79	1.88	**4.12**	0.42	0.27	0.11	0.111

Although statistical output looks great in Table 3-5, essentially Eq. 3.4 is a poorly specified model, because the relationship cannot possibly extend beyond mere statistical appearance. What's worrisome is that this caveat is not restricted to a straight line. If we replace the straight line t in Eq. 3.4 with almost any arbitrary smooth and none-volatile series, strong output would still emerge. Thus, if results improve after including dividend yield alongside a slow-moving demographic measure, there is no confirmation that such demographic measure can explain asset returns, unless the measure can already explain a non-trivial portion of asset return variations before dividend yield is included, and coefficient signs conform with a-priori theoretical expectation. This suggests Table 3-2 and 3-3 are of more importance than Table 3-4, and the author will focus on examining correlation and simple regressions rather than presenting models with the best statistics. Figure 3-2 summarizes this paper's main empirical discoveries up to this point.

Figure 3-2: Actual vs Fitted Returns and Index



While previous studies showed that regression models with demographic variables do not fit well with return data at short horizons, as represented by the upper left figure, the picture completely changes in the upper right figure. By replacing MY with ΔMY and also extending return horizon to 10 years, the model is able to account for much of the post-World-War-II return cycles for S&P500 after noises aggregate over the long-run. Furthermore, after controlling for valuation, i.e. including dp as another exogenous variable, MY's explanatory power starts to emerge even for short horizon returns, and long horizon fit also improves (middle figures). But this improvement of model fitness should not be taken seriously unless a decent fit is already there before dp is added, like in the upper right figure. Furthermore, the reader should be aware that in spite of the remarkable statistics seen in Table 3-4, 10-year returns during several time periods still cannot be explained by demographics and dividend yields. Those returns include the highly volatile swings in the first half of the 20th century and the unusual drawdown during the recent Great Recession. Lastly, although Eq. 3.3 can explain over 50% of the variations of 10-year S&P 500 returns, the magnitude of error would still be too large if the goal is to profit from return predictions. Just either compare the scale of y-axis in Figure 3-2's right and left panels or compare the margin of error in the bottom two plots.

3. 3 Is MY Forecast Error ($MY_{t+h} - E_t(MY_{t+h})$) a Risk Factor?

Every year, the US Census Bureau publishes updated figures of population forecast. According to Tammany Mulder (2002), researcher of the US Census Bureau, while population forecast benefited from improvement in data quality and methodology, forecasters still, on occasions, failed to foresee turning points in population trends. Population forecast has been reliable since the 1990s, but is perhaps mostly due to stabilization of components of population change. While this is not an encouraging comment if high precision for population forecast is needed, this sub-section will show that forecast errors for MY are not meaningfully large. Table 3-6 reports the R^2 statistics from regressing MY_{t+h} on $E_t(MY_{t+h})$[12]:

Table 3-6, Non-Overlapping Regression R^2 for Fitting MY_{t+h}

h	1	2	3	4	5	6	7	8	9	10
R^2	0.9984	0.9955	0.9952	0.9930	0.9902	0.9885	0.9856	0.9816	0.9839	0.9770

Table 3-6 tells us the margin of error for forecasting MY_{t+h} is quite slim. Notwithstanding this fact, the forecasting errors may still be a source of risk for asset returns, as unexpected population shift can disrupt the consumption-production equilibrium established in the basic GMQ framework. Table 3-7 runs (overlapping interval) return regression against 'surprises' in population shift. To save space, only coefficient for B_4 is reported.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_1 MY_t + B_2 E_t(MY_{t+h} - MY_t) + B_3\, dp_t + B_4(MY_{t+h} - E_t(MY_{t+h})) + \varepsilon_{t+h}$$
$$h = 1, \dots, 10 \qquad ..(3.5)$$

Table 3-7, Overlapping Regression Output for Equation 3.5

h	B4	t1	t2	p1	p2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.81	0.41	0.77	0.68	0.44	0.10	0.06	0.03	0.176
2	1.13	0.59	0.56	0.56	0.58	0.15	0.12	0.00	0.175
3	1.12	0.66	0.51	0.51	0.61	0.25	0.22	0.00	0.157
4	0.98	0.62	0.58	0.54	0.57	0.31	0.29	0.00	0.150
5	0.76	0.51	0.47	0.61	0.64	0.38	0.36	0.00	0.139
6	0.51	0.37	0.31	0.71	0.76	0.45	0.43	0.00	0.127
7	0.92	0.70	0.54	0.49	0.59	0.50	0.48	0.00	0.120
8	1.39	1.04	0.75	0.30	0.46	0.53	0.51	0.00	0.119
9	2.46	1.86	1.20	0.07	0.23	0.55	0.54	0.00	0.115
10	3.44	**2.57**	1.61	0.01	0.11	0.58	0.56	0.00	0.111

Compare Table 3-7 with Table 3-4a, adding the 'surprise' factor ($MY_{t+h} - E_t(MY_{t+h})$) only adds negligible explanatory power after investment horizon expands beyond 8 years. Out of the 20 reported t-statistics for B_4, 19 are not significant at the 5% level, 18 are not significant at the 10% level. Thus, it is safe to conclude that empirical evidence suggests S&P500 return carries no loading on 'surprises' in MY forecasts. Also, when taking into account mortality rate and migration, forecast of ΔMY should be even more accurate than the simple estimate given here. This is a topic beyond the scope of this paper, though. To save space, similar analysis will not be repeated in other asset classes. Until Section 6, the author will only use information available at time t to explain returns between time *t* and time *t+h*.

[12] Recall $E_t(MY_{t+h})$ is defined as [number of people in age group (40-49-*h*)] divided by [number of people in age group (20-29-*h*)].

3.4 Broad US Equity Market

After using real S&P500 returns as the dependent variable, we now turn to the broad US equity market. The time series for US broad equity market can be found in Kenneth French's online data library, listed as the R_m-R_f factor under "Fama/French Factors". For ease of notation, the R_m-R_f series will be abbreviated as R_m. R_m is one of the most widely used financial time series in American academia. It is the "excess market return factor", defined as the value weighted excess market return (market return minus risk free rate) of all CRSP firms incorporated in the US and listed on the three major American stock exchanges: NYSE, AMEX, or NASDAQ[13]. The risk free rate used to compute excess return is from US Treasury Bill rate. Compared to the S&P500 index, the R_m series is much more broadly based, but the sample range is shorter. The series' start date is 1927, generating only 86 years of observations up to 2012. Because R_m is quoted as an excess return, inflation is automatically taken out of the series. The regression model will still be based on Eq. 3.2, except r_t is now the R_m return series instead of the S&P500 series. Like S&P500 returns, R_m is also based on total returns and is natural-log transformed.

Table 3-8 reports pairwise correlations between <R_m, MY> and between <R_m, ΔMY>. Expected correlation sign is negative between MY and all asset returns, and is positive between ΔMY and all asset returns.

Table 3-8, Correlation Between R_m and [MY, ΔMY]

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.03	0.03	0.00	-0.04	-0.11	-0.17	-0.21	-0.26	-0.30	-0.33
	ΔMY	0.14	0.23	0.33	0.41	0.49	0.56	0.61	0.63	0.64	0.63
non-overlap	MY	0.03	0.04	0.03	0.03	0.03	-0.01	-0.19	-0.25	-0.21	-0.14
	ΔMY	0.14	0.18	0.36	0.32	0.34	0.52	0.66	0.62	0.62	0.37

For the large part, correlation signs conform with expectations. Here, due to exclusion of the 1900-1926 time period, the negative correlations between <MY, R_m> are more pronounced than between <MY, S&P500>. The positive correlation between <ΔMY, R_m> are slightly higher than between <ΔMY, S&P500>. Table 3-9 reports regression results for Eq. 3.2.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + \beta_2 E_t(MY_{t+h} - MY_t) + \varepsilon_{t+h} \qquad h = 1, \dots, 10 \qquad ..(3.2)$$

Table 3-9: Regression Output for US Broad Equity (Eq. 3.2)

	Overlapping Time Interval							Non-Overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	1.24	1.33	**2.17**	0.02	0.01	0.19	0.206	1.24	1.33	**2.17**	0.02	0.01	0.19	0.206
2	1.55	**2.18**	**3.06**	0.05	0.04	**0.03**	0.208	1.27	1.15	**2.25**	0.03	0.01	0.26	0.231
3	1.72	**3.12**	**3.92**	0.11	0.10	**0.00**	0.195	1.80	1.97	**4.46**	0.13	0.10	0.06	0.187
4	1.79	**3.98**	**4.47**	0.17	0.16	**0.00**	0.183	1.73	1.46	**4.49**	0.10	0.05	0.16	0.241
5	1.80	**5.00**	**4.71**	0.24	0.23	**0.00**	0.162	1.37	1.38	**4.33**	0.11	0.05	0.19	0.201
6	1.73	**6.02**	**4.79**	0.32	0.31	**0.00**	0.140	1.71	2.09	**5.04**	0.27	0.21	0.06	0.164
7	1.69	**6.73**	**4.90**	0.37	0.36	**0.00**	0.130	2.05	**2.75**	**6.23**	0.43	0.37	**0.02**	0.147
8	1.65	**7.14**	**4.92**	0.40	0.39	**0.00**	0.126	1.39	2.23	**3.29**	0.38	0.31	0.06	0.122
9	1.62	**7.18**	**4.96**	0.41	0.40	**0.00**	0.128	1.33	2.08	2.08	0.38	0.29	0.08	0.125
10	1.58	**6.98**	**5.01**	0.40	0.39	**0.00**	0.133	0.74	0.96	1.72	0.13	-0.01	0.37	0.149

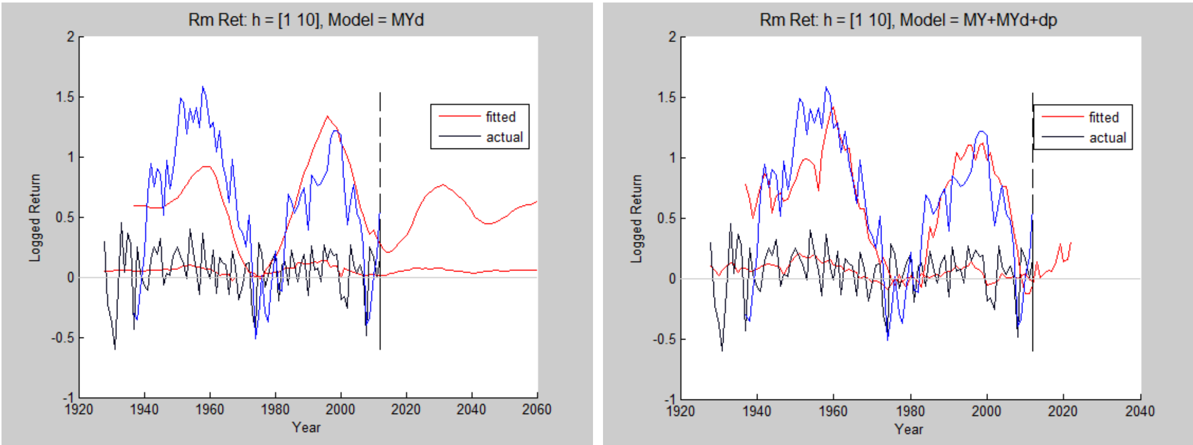
Compare the left panel of Table 3-9 to the left panel of Table 3-3, the overlapping time intervals, the results are nearly identical: positive coefficient signs, highly significant t-stats and F-stats, high R^2s, and annualized RMSE decrease as return horizon increases. To an extent, the same is also true for the non-overlapping window regressions, although the adjusted R^2s and F-stats reported in Table 3-9's right panel are more

[13] For a full description of the data series, please refer to Kenneth French's online data library.

unstable than in Table 3-3.

Below, the left graph of Figure 3-3 presents a visual inspection for the goodness of fit of Eq. 3.2. The right graph additionally incorporates dividend yield and MY as regressors. Black lines represent 1-year returns; blue lines represent 10-year returns. 10-year return patterns of the R_m series match well with ΔMY (left figure). After dividend yield is included in the model, the fit is even tighter (right figure). Although actual 10-year returns from 2008-2009 deviated wildly from fitted returns, the decline of long-run returns starting from late 1990s is consistent with changes in the MY ratio.

Figure 3-3: Actual vs Fitted Returns



3. 5 Pre-Baby-Boom vs Post-Baby-Boom

From Figure 2-2, the MY ratio only experienced mild swings between 1900-1951. It was arrival of the baby-boom generation that brought a wave of demographic changes in the US and around many parts of the world. Without large oscillation, MY's effectiveness in explaining equity returns can be compromised. This sub-section divides the 1900-2012 sample period of S&P 500 returns into two sub-periods: the pre-baby-boom era (1900-1951) and the post-baby-boom era (1952-2012)[14]. Table 3-10 and Table 3-11 give us the pre-baby-boom era correlations and regression outputs (Eq. 3.2). Many of the correlation signs in Table 3-10 run contrary to a-priori theoretical predictions.

Table 3-10: Correlation Between S&P500 and [MY, ΔMY] during 1900-1951

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.10	0.13	0.15	0.14	0.15	0.19	0.24	0.25	0.28	0.31
	ΔMY	0.16	0.15	0.14	0.14	0.15	0.12	0.06	-0.03	-0.11	-0.19
non-overlap	MY	0.10	0.15	0.14	0.14	0.19	0.11	0.35	0.20	0.55	0.71
	ΔMY	0.16	0.14	0.18	0.31	0.24	0.08	-0.17	0.14	-0.17	-0.58

[14] The author chooses 1952 as the dividing year in order to allow direct comparison between equity and Treasury, whose data starts from 1952.

Table 3-11: Regression Output for S&P500 and ΔMY (Eq. 3.2, 1900-1951)

	Overlapping Time Interval							Non-overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	6.28	1.10	1.73	0.02	0.00	0.27	0.204	6.28	1.10	1.73	0.02	0.00	0.27	0.204
2	4.21	1.03	1.12	0.02	0.00	0.31	0.212	4.06	0.68	1.31	0.02	-0.02	0.50	0.224
3	2.82	0.95	0.82	0.02	0.00	0.35	0.200	3.56	0.73	1.40	0.03	-0.03	0.48	0.197
4	2.37	0.97	0.74	0.02	0.00	0.34	0.196	4.95	1.02	1.42	0.09	0.00	0.33	0.196
5	2.06	1.00	0.65	0.02	0.00	0.32	0.183	3.41	0.68	1.55	0.06	-0.06	0.51	0.216
6	1.42	0.80	0.46	0.01	-0.01	0.43	0.169	0.87	0.20	0.35	0.01	-0.16	0.85	0.184
7	0.62	0.37	0.21	0.00	-0.02	0.71	0.164	-1.46	-0.38	-0.59	0.03	-0.17	0.72	0.138
8	-0.26	-0.16	-0.10	0.00	-0.02	0.87	0.162	1.58	0.29	0.46	0.02	-0.22	0.79	0.208
9	-1.06	-0.72	-0.47	0.01	-0.01	0.48	0.155	-1.11	-0.30	-0.55	0.03	-0.29	0.78	0.130
10	-1.63	-1.20	-0.88	0.03	0.01	0.24	0.146	-5.61	-1.22	-2.84	0.33	0.11	0.31	0.142

In Table 3-11, all statistics point to lack of relationship between ΔMY and S&P500 return before the baby-boom generation arrived. The contrast is startling when compared with statistics from the post-baby-boom era in Tables 3-12 and 3-13.

Table 3-12: Correlation Between S&P500 and [MY, ΔMY] during 1952-2012

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.01	-0.03	-0.08	-0.14	-0.21	-0.26	-0.31	-0.36	-0.41	-0.47
	ΔMY	0.26	0.41	0.55	0.64	0.70	0.75	0.78	0.79	0.79	0.80
non-overlap	MY	0.01	-0.01	-0.07	-0.08	-0.10	-0.17	-0.45	-0.25	-0.24	-0.15
	ΔMY	0.26	0.31	0.64	0.63	0.62	0.81	0.86	0.80	0.84	0.64

Table 3-13: Regression Output for S&P500 and ΔMY (Eq. 3.2, 1952-2012)

	Overlapping Time Interval							Non-overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	1.48	**2.03**	**2.94**	0.07	0.05	**0.05**	0.159	1.48	**2.03**	**2.94**	0.07	0.05	**0.05**	0.159
2	1.83	**3.43**	**3.86**	0.17	0.16	**0.00**	0.152	1.52	1.73	**3.09**	0.10	0.06	0.10	0.181
3	1.89	**4.95**	**4.67**	0.30	0.29	**0.00**	0.132	1.98	**3.58**	**6.27**	0.42	0.38	**0.00**	0.112
4	1.90	**6.13**	**5.26**	0.41	0.40	**0.00**	0.122	1.86	**2.95**	**5.01**	0.40	0.35	**0.01**	0.127
5	1.90	**7.21**	**5.69**	0.49	0.48	**0.00**	0.114	1.57	**2.47**	**4.24**	0.38	0.32	**0.03**	0.128
6	1.87	**8.22**	**5.98**	0.56	0.55	**0.00**	0.107	2.00	**3.92**	**6.93**	0.66	0.62	**0.00**	0.101
7	1.82	**8.99**	**6.44**	0.61	0.60	**0.00**	0.101	2.08	**4.12**	**16.4**	0.74	0.70	**0.01**	0.094
8	1.78	**9.12**	**6.82**	0.62	0.61	**0.00**	0.102	1.52	**2.94**	**8.52**	0.63	0.56	**0.03**	0.097
9	1.74	**9.16**	**7.19**	0.63	0.62	**0.00**	0.104	1.25	**3.16**	**3.67**	0.71	0.64	**0.03**	0.073
10	1.69	**9.22**	**7.54**	0.63	0.63	**0.00**	0.104	1.12	1.67	**9.07**	0.41	0.27	0.17	0.130

It appears that for equities returns, MY's relevance as an explanatory variable is intimately tied with the emergence of the baby-boom generation which brought major transformation to the demographic landscape. If this is the case, then MY may become irrelevant again, as the ratio is only projected to oscillate mildly between 2013-2060[15]. This issue will be treated and discussed in Section 6 and Section 7.

4.US Treasury Market

Section 4 repeats those analyses performed in the equity market to the Treasury Market. Data source will be the Fama-Bliss dataset published by the Center for Research in Security Prices (CRSP). Besides CRSP, the dataset can also be obtained through the Wharton Research Data Service (WRDS). The Fama-Bliss dataset contains artificial prices of zero coupon bonds expiring 1, 2, 3, 4, and 5 years out made from US Treasury

[15] See Figure 2-2 and Appendix A.2.

Bills. The bond prices are constructed to reflect the general price level of T-Bills. Full description of the dataset is given by the appendix of Fama and Bliss (1987). Price data of zero coupon bonds covers 1952 – 2012, the post-baby-boom era.

For zero coupon bonds, real return generated by holding a bond to maturity differs from nominal yield-to-maturity (abbreviated as *y*) by amount of inflation. *M*-year return for a *m*-year maturity bond, therefore, can still be uncertain. Since this section's focus is on the empirical relationship between **real** zero coupon bond returns and MY ratios, for a zero-coupon bond maturing in 5 years, regressions for return horizons 1, 2, 3, 4, 5 years will be reported. Similarly, for a 4 year maturity zero-coupon bond, regressions for return horizons 1, 2, 3, 4 years will be reported, and so on.

4. 1 Correlation Analysis

Table 4-1 shows the correlation matrix of one-year real logged returns across bonds of maturity 1 to 5. Rows and columns correspond to maturity in years. Yearly returns across bonds of different maturities are strongly correlated, especially when maturities are close to each other.

Table 4-1: One-Year Return Correlation among Fama-Bliss Zero Coupon Bonds

1.0000	0.9090	0.8029	0.7242	0.6477
0.9090	1.0000	0.9725	0.9319	0.8826
0.8029	0.9725	1.0000	0.9879	0.9623
0.7242	0.9319	0.9879	1.0000	0.9899
0.6477	0.8826	0.9623	0.9899	1.0000

On the other hand, between returns of the broad equity market index (R_m) and zero coupon bonds, the correlations are rather low.

Table 4-2: Return Correlation between R_m and Fama-Bliss Zero Coupon Bonds

*m**h*	1	2	3	4	5
1	0.2609				
2	0.1777	0.2242			
3	0.1362	0.1437	0.1984		
4	0.1003	0.1001	0.1165	0.2322	
5	0.0731	0.0905	0.0897	0.1722	0.3051

Lastly, between returns of zero coupon bonds and [MY, ΔMY, *y*], the analogous of Table 3-2 looks like:

Table 4-3, Correlation Between Fama-Bliss Bond Returns and [MY, ΔMY, *y*]

		Overlapping time-interval					Non-overlap time-interval				
m	var*h*	1	2	3	4	5	1	2	3	4	5
1	MY	-0.18					-0.18				
	ΔMY	0.39					0.39				
	y	0.49					0.49				
2	MY	-0.12	-0.19				-0.12	-0.19			
	ΔMY	0.34	0.44				0.34	0.50			
	y	0.44	0.54				0.44	0.54			
3	MY	-0.04	-0.11	-0.17			-0.04	-0.11	-0.07		
	ΔMY	0.29	0.41	0.47			0.29	0.44	0.61		
	y	0.37	0.50	0.58			0.37	0.50	0.51		
4	MY	-0.01	-0.05	-0.11	-0.18		-0.01	-0.04	-0.02	-0.21	
	ΔMY	0.26	0.38	0.45	0.52		0.26	0.40	0.55	0.59	
	y	0.34	0.46	0.56	0.62		0.34	0.45	0.51	0.67	
5	MY	0.01	-0.02	-0.07	-0.13	-0.19	0.01	-0.01	0.03	-0.17	-0.17
	ΔMY	0.23	0.36	0.44	0.52	0.58	0.23	0.36	0.51	0.57	0.61
	y	0.29	0.41	0.51	0.59	0.63	0.29	0.41	0.44	0.64	0.50

Recall that expected correlation sign between asset returns and MY is negative; between asset returns and ΔMY is positive. Table 4-3 shows all correlation signs conform with a-priori expectations. It is worthy to highlight that despite the low return correlations between equity and Treasury, the two asset classes both share high correlations with ΔMY. Moreover, fixing *h*, return correlation with MY becomes more negative as bond's maturity gets shorter, i.e. as return becomes less risky or uncertain. Similarly, fixing *m*, correlation with MY also becomes more negative as horizon extends longer and return becomes less uncertain. The exact same phenomenon also applies to correlation with ΔMY, except correlation becomes more positive as return becomes less uncertain. This seems to suggest that demographics' impact is stronger for less risky assets, which is consistent with the risk-free setting of the basic GMQ model.

4. 2 Regression Results

Regressions will be run for logged real returns of zero coupon bonds of maturity 1, 2, 3, 4, and 5 years.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_2 E_t(MY_{t+h} - MY_t) + \varepsilon_{t+h} \qquad h = 1, \ldots, m \qquad ..(4.1)$$

Table 4-4: Regression Output for Treasuries (Eq. 4.1)

		Overlapping Time Interval							Non-overlapping Time Interval						
m	*h*	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1	0.33	**3.23**	**2.93**	0.15	0.14	**0.00**	0.022	0.33	**3.23**	**2.93**	0.15	0.14	**0.00**	0.022
2	1	0.41	**2.74**	**3.56**	0.11	0.10	**0.01**	0.033	0.41	**2.74**	**3.56**	0.11	0.10	**0.01**	0.033
2	2	0.42	**3.72**	**3.57**	0.20	0.18	**0.00**	0.032	0.45	**3.03**	**3.92**	0.25	0.22	**0.01**	0.031
3	1	0.48	**2.31**	**3.30**	0.08	0.07	**0.02**	0.045	0.48	**2.31**	**3.30**	0.08	0.07	**0.02**	0.045
3	2	0.51	**3.35**	**3.71**	0.16	0.15	**0.00**	0.043	0.54	**2.59**	**3.98**	0.19	0.16	**0.02**	0.043
3	3	0.46	**4.03**	**3.74**	0.22	0.21	**0.00**	0.040	0.53	**3.29**	**3.76**	0.37	0.34	**0.00**	0.033
4	1	0.52	**2.05**	**2.99**	0.07	0.05	**0.04**	0.056	0.52	**2.05**	**2.99**	0.07	0.05	**0.04**	0.056
4	2	0.58	**3.09**	**3.52**	0.14	0.13	**0.00**	0.054	0.60	**2.29**	**3.61**	0.16	0.13	**0.03**	0.054
4	3	0.55	**3.80**	**3.70**	0.21	0.19	**0.00**	0.050	0.59	**2.81**	**3.45**	0.31	0.27	**0.01**	0.042
4	4	0.52	**4.53**	**4.02**	0.27	0.26	**0.00**	0.045	0.64	**2.62**	**3.16**	0.34	0.29	**0.02**	0.050
5	1	0.54	1.80	**2.67**	0.05	0.04	0.08	0.065	0.54	1.80	**2.67**	0.05	0.04	0.08	0.065
5	2	0.63	**2.88**	**3.28**	0.13	0.11	**0.01**	0.063	0.62	2.04	**3.15**	0.13	0.10	0.05	0.063
5	3	0.63	**3.66**	**3.54**	0.19	0.18	**0.00**	0.059	0.62	**2.49**	**3.11**	0.26	0.21	**0.02**	0.050
5	4	0.61	**4.49**	**3.97**	0.27	0.25	**0.00**	0.054	0.72	**2.53**	**3.14**	0.33	0.28	**0.03**	0.057
5	5	0.58	**5.26**	**4.39**	0.34	0.33	**0.00**	0.048	0.56	**2.47**	**2.91**	0.38	0.32	**0.03**	0.046

For all maturities 1, 2, 3, 4, 5 years, ΔMY can explain a significant portion of real US Treasury returns during the post-baby-boom era. Whether overlapping or non-overlapping windows regressions are implemented, conclusion remains the same. The t-stats and adjusted R^2 statistics for non-overlapping time interval are very strong. Figure 4-1 plots real (inflation adjusted) yield-to-maturity against ΔMY for 3-year and 5-year bonds in non-overlapping time intervals. There is a close match between real Treasury return and ΔMY.

Figure 4-1: Real Return-to-Maturity vs ΔMY

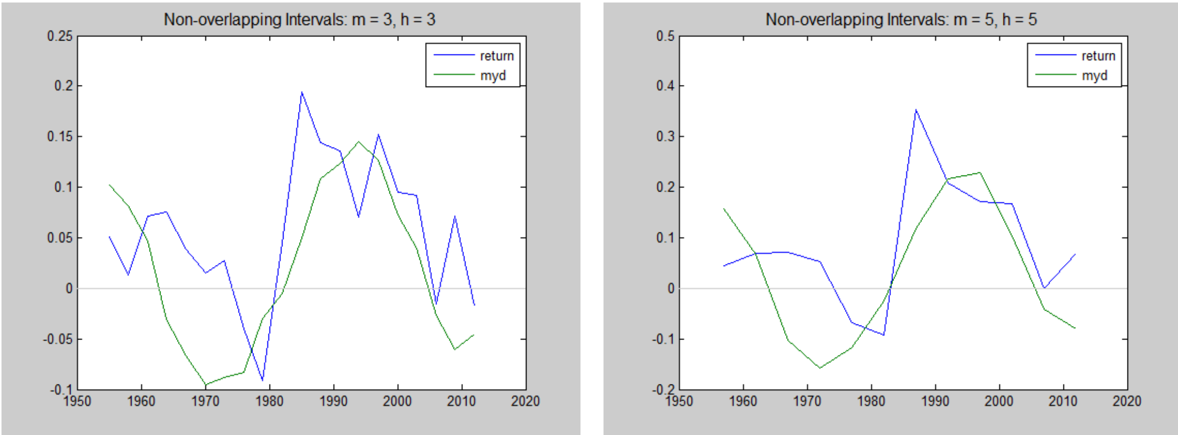
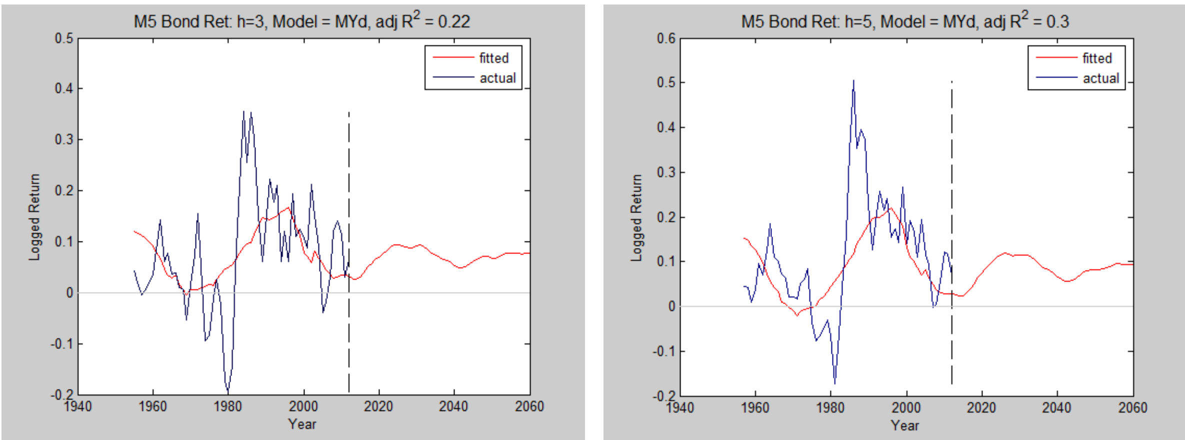


Figure 4-2 plots actual returns against fitted returns of 5-year maturity bonds on overlapping time intervals. Left graph shows 3-year investment horizon. Right graph shows 5-year investment horizon.

Figure 4-2: Fitted vs Actual Returns



Thus far, we have seen that when MY undergoes turbulent swings (the hallmark of post-baby-boom era), expected change in MY fits the backbone of return patterns in **both** equity and Treasury markets. Signs of coefficients match with theory and are mostly significant. These facts offer empirical validation for the GMQ model. In Section 5, the author explores the linkage between age-population demographics and another major asset market in the US – the housing market.

5. US Housing Market

The financial return time series r_t used in this section is the logged return of inflation adjusted home price index. The index, developed by Karl Case, Robert Shiller, and Allan Weiss, is a composite of single-family home prices at the national level in the US. It is used in Professor Robert Shiller's famous book, *Irrational*

Exuberance (2005), and is available from Shiller's online data library[16]. Unlike equities, the index covers only capital gains. Sample period ranges between 1900 - 2012.

5. 1 Correlation Analysis

Table 5-1 gives the correlation between housing market return and broad equity market return (R_m). Sample period is 1927-2012.

Table 5-1: Return Correlation between Housing Return and R_m

h	1	2	3	4	5	6	7	8	9	10
ρ	0.2728	0.2965	0.2950	0.2645	0.2198	0.1725	0.1387	0.1238	0.1511	0.1837

Table 5-2 gives the correlation between housing market return and Fama-Bliss zero coupon bond return. Sample period is 1952-2012.

Table 5-2: Return Correlation between Housing and Fama-Bliss Zero Coupon Bonds

m\h	1	2	3	4	5
1	0.0335				
2	-0.0174	0.0469			
3	-0.0499	-0.0028	0.0764		
4	-0.0699	-0.0311	0.0278	0.1195	
5	-0.0747	-0.0369	0.0047	0.0745	0.1529

Correlations between housing and equity market are rather low, and they are even lower between housing and US Treasuries. Next, I will examine the correlation between housing returns and the MY ratio. Two sample periods are reported: 1900-2012 and 1952-2012.

Table 5-3a, Correlation Between Housing Returns and [MY, ΔMY] (1900-2012)

time interval	h	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.06	0.06	0.07	0.08	0.10	0.13	0.19	0.25	0.29	0.32
	ΔMY	0.10	0.13	0.13	0.13	0.12	0.10	0.08	0.06	0.05	0.05
non-overlap	MY	0.06	0.07	0.04	0.10	0.12	0.07	0.12	0.10	0.05	0.04
	ΔMY	0.10	0.12	0.14	0.15	0.09	0.17	0.18	0.09	0.31	0.34

Table 5-3b, Correlation Between Housing Returns and [MY, ΔMY] (1952-2012)

time interval	h	1	2	3	4	5	6	7	8	9	10
overlap	MY	0.04	0.03	0.03	0.03	0.03	0.05	0.09	0.14	0.16	0.16
	ΔMY	0.18	0.20	0.20	0.19	0.19	0.19	0.18	0.17	0.19	0.21
non-overlap	MY	0.04	0.03	0.02	0.00	0.07	-0.04	0.03	-0.01	-0.20	-0.33
	ΔMY	0.18	0.20	0.23	0.25	0.22	0.30	0.26	0.17	0.51	0.70

Recall between MY and equilibrium return, expected correlation is negative; between $E_t(MY_{t+h}\text{-}MY_t)$ and equilibrium return, expected correlation is positive. For MY, the correlation signs do not match GMQ's prediction. For ΔMY, the correlation signs do match GMQ's prediction, albeit much weaker than the level of correlation found in equity and Treasuries during comparable periods. This weakening of relationship does not necessarily testify against the GMQ model, however. The author hypothesizes that when an average person makes choice between current consumption versus savings, risk-free return would be more important than equity return, which in turn would be more important than housing return. For most people,

[16] For full data description or data download, please consult http://www.econ.yale.edu/~shiller/data.htm.

housing is a more of a consumption item than an investment item. In the event that housing is an investment tool that regulates intertemporal consumption choices, it is proper to compute returns from both rent and capital gain (as in equities) rather than capital gains alone. The author reserves this conjecture as a candidate for future research.

5. 2 Regression Results

The same regression outputs reported for equities and Treasuries will be repeated for housing.

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_2 E_t(MY_{t+h} - MY_t) + \varepsilon_{t+h} \qquad h = 1, \ldots, 10 \qquad ..(5.1)$$

Table 5-4a: Regression Output for US Housing (1900-2012, Eq. 5.1)

	Overlapping Time Interval							Non-overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	0.31	1.05	1.08	0.01	0.00	0.30	0.066	0.31	1.05	1.08	0.01	0.00	0.30	0.066
2	0.33	1.41	1.11	0.02	0.01	0.16	0.069	0.29	0.87	1.01	0.01	0.00	0.39	0.070
3	0.29	1.40	1.04	0.02	0.01	0.16	0.074	0.33	0.85	1.16	0.02	-0.01	0.40	0.080
4	0.25	1.36	0.96	0.02	0.01	0.18	0.075	0.25	0.75	0.97	0.02	-0.02	0.46	0.070
5	0.21	1.28	0.90	0.02	0.01	0.20	0.075	0.15	0.39	0.69	0.01	-0.04	0.70	0.077
6	0.16	1.08	0.79	0.01	0.00	0.28	0.072	0.26	0.69	1.07	0.03	-0.03	0.50	0.078
7	0.10	0.80	0.65	0.01	0.00	0.43	0.068	0.31	0.67	1.12	0.03	-0.04	0.51	0.093
8	0.07	0.57	0.51	0.00	-0.01	0.57	0.064	0.13	0.33	0.49	0.01	-0.07	0.75	0.078
9	0.05	0.50	0.46	0.00	-0.01	0.62	0.061	0.32	1.02	1.24	0.09	0.00	0.33	0.062
10	0.05	0.53	0.48	0.00	-0.01	0.60	0.059	0.34	1.07	1.55	0.11	0.01	0.31	0.064

Table 5-4b: Regression Output for US Housing (1952-2012, Eq. 5.1)

	Overlapping Time Interval							Non-overlapping Time Interval						
h	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	0.34	1.38	1.11	0.03	0.02	0.17	0.053	0.34	1.38	1.11	0.03	0.02	0.17	0.053
2	0.36	1.53	1.14	0.04	0.02	0.13	0.066	0.36	1.06	1.15	0.04	0.00	0.30	0.069
3	0.32	1.49	1.09	0.04	0.02	0.14	0.074	0.38	1.00	1.28	0.05	0.00	0.33	0.077
4	0.29	1.45	1.05	0.04	0.02	0.15	0.078	0.33	0.91	1.13	0.06	-0.01	0.38	0.072
5	0.26	1.45	1.05	0.04	0.02	0.15	0.078	0.26	0.73	1.14	0.05	-0.04	0.48	0.072
6	0.22	1.37	1.02	0.03	0.02	0.18	0.075	0.41	0.88	1.33	0.09	-0.03	0.41	0.093
7	0.18	1.29	0.98	0.03	0.01	0.20	0.069	0.38	0.66	1.27	0.07	-0.09	0.53	0.107
8	0.15	1.26	0.98	0.03	0.01	0.21	0.063	0.21	0.39	0.78	0.03	-0.16	0.71	0.103
9	0.14	1.36	1.03	0.04	0.02	0.18	0.058	0.44	1.17	1.61	0.26	0.07	0.31	0.069
10	0.14	1.54	1.12	0.05	0.03	0.13	0.053	0.41	1.95	2.03	0.49	0.36	0.12	0.041

Output from Eq. 5.1 cannot reject the null hypothesis that MY bears no relationship with US housing returns. With the exception of 10-year non-overlapping returns, all adjusted R^2s are close to zero. Nevertheless, it is worthy to note that although not a single t-stats or F-stats can cross the threshold of significance, statistics from the post-baby-boom era are uniformly stronger than the overall period, .

So far, we have seen that the MY variable is able to explain future asset returns across equities and Treasuries, but not US national housing index. Can this remarkable match with financial securities' long-run return patterns be an outcome of data-mining? It is destined that out of countlessly many demographic measures, some measures will coincide with long-term return pattern of financial assets. This question motivates our next section. Drawing from economic principle of the GMQ model, the author tries to compose alternative measures of age-population demographics, and test those measures' empirical linkage with asset returns.

6. Beyond the MY Ratio

This section seeks to build demographic measures that aims to reflect the balance between aggregate savings demand and aggregate consumption demand in the US economy. In the basic GMQ model, it is assumed that young workers earn 2 units of currency and middle aged workers earn 3 units of currency. Each period, the economy's output amounts to $2N_y + 3N_m + D$, where $[N_y\ N_m]$ refer to the number of young and middle age workers, respectively, and D refers to dividends. The key determinant for the equilibrium rate of return in the GMQ model is the market clearing condition (2.7). If we assume market discount factor and subjective discount factors are identical, then it is optimal for agents in the model to consume equally across three stages of life. Table 6-0 takes the calibrated numerical example of Section 2.2 and computes output in odd and even periods.

Table 6-0: Output, Supply-Demand Balance, and Equilibrium Return in Basic GMQ Setting

{N, n}	{79, 52}						{79, 69}					
Period	Odd: (N,n,N)			Even: (n,N,n)			Odd: (N,n,N)			Even: (n,N,n)		
	Output	S/D	r_{annual}	Output	S/D	r_{annual}	Output	S/D	r_{annual}	Output	S/D	r_{annual}
α = 4	376	0.90	6.46%	403	1.10	-0.25%	435	0.96	4.44%	445	1.03	2.21%

In odd period, output is low and (future) return is high. In even period, output is high and (future) return is low. Looking from a different angle, when economy transits from odd to even period, output increases and return is high; when economy transits from even to odd period, output decreases and return is low. Thus, if an output indicator is formed, it should have negative correlation with asset returns, and (future) change in output should have positive correlation with returns. In the real world, output constantly grows, so we need to normalize output by consumption demand. Assume in each period, agents uniformly prefer to consume 2 units of currency. Dividing output by consumption demand, we arrive at the S/D figure. S/D < 1 means there are less supply than demand, so returns are high to demote current consumption. S/D > 1 means there are more supply than demand, so returns are low to promote current consumption. Alternatively, we can interpret when agents foresee S/D will increase in later periods, return rises to promote future consumption; when agents foresee S/D will decrease in later periods, return falls to demote future consumption.

In the rest of this section, the author will construct two "supply" measures, abbreviated as S_i, from demographic statistics and wage data. Essentially, each S_i aims to capture the degree of supply-demand imbalance in the economy, just like the S/D figure from Table 6-0. All S_i are derived out of Eq. 6.1:

$$S_i = \frac{\boldsymbol{w}'\boldsymbol{n}}{\boldsymbol{c}'\boldsymbol{n}} \qquad ..(6.1)$$

where ***w*** is a wage vector, ***n*** is a population vector, and ***c*** is a consumption vector. Each cell in a vector corresponds to a specific age group. Essentially, measures' difference arise from the ***w*** vector, the ***c*** vector, and specification of age groups. Exclusion of an age group carries the implicit assumption that its wage and consumption are both zero. All measures will be tested against broad equity market excess returns (1927-2012) and Treasury returns (1952-2012). Similar to testing performed for the MY ratio, for each measure, a correlation table and a regression output table will be presented for each asset class. Also like the MY ratio, the GMQ model predicts negative correlation between asset return and S_i, and positive correlation between asset return and future change in S_i (ΔS_i, for short). All regression outputs shall correspond to Equation 6.2 below:

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_1 Si_t + B_2\,(Si_{t+h} - Si_t) + \varepsilon_{t+h} \qquad ..(6.2)$$

From this point on, ΔS_i will denote actual changes instead of expected changes. This will introduce some look-ahead bias into regression estimates, but Section 3-4 has demonstrated the difference between MY_{t+h}

– MY_t and $E_t(MY_{t+h} - MY_t)$ is immaterial. In all likelihood, the look-ahead bias is not going to alter any conclusions.

6. 1 The Supply Measure, Part I

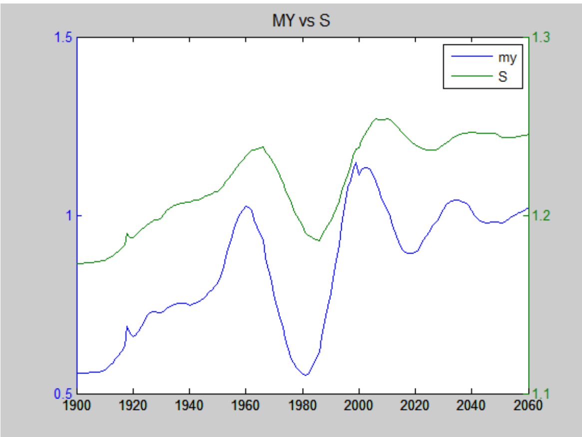
This sub-section directly takes age group specification and wage vector out of the GMQ model. Let ***n*** be $[n_y, n_m, n_r]$, where n_y represents population in age group [20:39], n_m represents population in age group [40:59], n_r represents population of age group 60 and above. Wage vector ***w*** is [2, 3, 0]. Consumption vector ***c*** is [2, 2, 2]. Take the S measure derived out of this setting and apply regression equation 6.2 with $r = R_m$, we get:

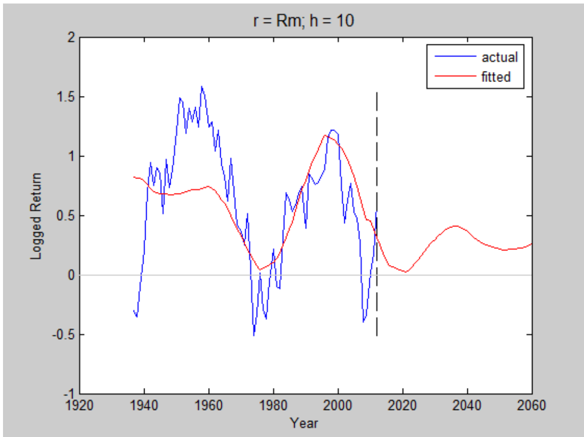
Table 6-1-1: Overlapping Interval Regression Output with No Income for Senior Citizens

h	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	-0.20	-0.32	-0.29	1.39	0.16	0.18	0.00	-0.02	0.92	0.210
2	-0.45	-0.50	-0.32	3.40	0.52	0.46	0.01	-0.02	0.70	0.214
3	-0.56	-0.52	-0.28	3.64	0.68	0.52	0.01	-0.01	0.59	0.207
4	-0.30	-0.25	-0.13	4.47	0.95	0.69	0.02	-0.01	0.52	0.200
5	0.57	0.44	0.24	6.28	1.56	1.07	0.03	0.01	0.30	0.184
6	1.46	1.11	0.64	7.04	**2.01**	1.31	0.05	0.03	0.14	0.166
7	2.18	1.56	0.91	7.58	**2.31**	1.49	0.07	0.04	0.07	0.159
8	3.09	**2.06**	1.19	8.42	**2.65**	1.74	0.09	0.07	**0.03**	0.156
9	3.86	**2.33**	1.27	9.03	**2.82**	1.88	0.11	0.08	**0.02**	0.158
10	4.57	**2.49**	1.27	9.59	**2.91**	1.95	0.11	0.09	**0.01**	0.162

Compare with output produced by the MY ratio, statistics from Table 6-1-1 are very weak. However, ***w*** = [2, 3, 0] is not a realistic scenario. People of age 60 or above do generate income. Generally speaking, measures that assume zero income for retirees would produce similar (bad) results. For simplicity's sake, let us ignore people of age 60 and above altogether in the rest of Section 6.1. Let ***n*** = $[n_y, n_m]$, ***w*** = [2, 3], ***c*** = [2, 2][17]. We will name the resulting supply measure "S1". Figure 6-1 plots S1 together with MY (upper left plot) and plots fitted values from regression 6.2 against actual returns in R_m (upper right), Treasury (lower left), and housing (lower right).

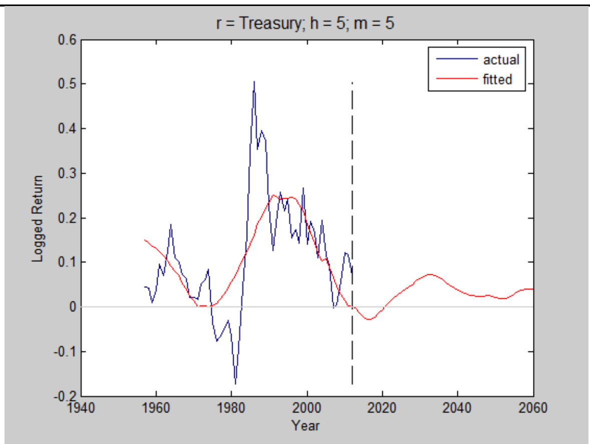
Figure 6-1: The S1 Measure

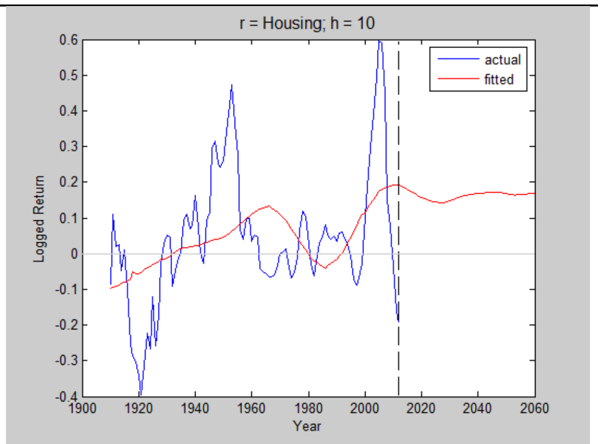




[17] If ***c*** is a constant vector, then it doesn't matter which particular value ***c*** takes. Correlations and regression results are unaffected.

Text





Not surprisingly, S1 appears to be a lagged version the MY ratio, and its fit with different asset returns appear similar to MY as well. Output for R_m and Treasury will be reported, but housing will be skipped.

6. 1. 1 Excess Equity Market Return

Table 6-1-2 gives the correlation between S1 and R_m. All coefficient signs conform with expectation. In Table 6-1-3, most overlapping regression t-stats are significant for B_2, the coefficient for ΔS1. For non-overlapping regressions, results are weaker, as reduced sample size for large *h* becomes problematic.

Table 6-1-2: Correlations Between S1 and R_m

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	S1	-0.03	-0.05	-0.09	-0.14	-0.24	-0.33	-0.38	-0.43	-0.46	-0.47
	ΔS1	0.14	0.20	0.25	0.29	0.35	0.41	0.45	0.48	0.49	0.49
non-overlap	S1	-0.03	-0.02	-0.05	-0.02	-0.04	-0.13	-0.28	-0.49	-0.45	-0.34
	ΔS1	0.14	0.15	0.25	0.21	0.29	0.36	0.44	0.56	0.58	0.47

Table 6-1-3a: Overlapping Interval Regression Output for S1 and R_m

h	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	-0.35	-0.29	-0.36	12.20	1.25	1.85	0.02	0.00	0.45	0.208
2	-0.73	-0.43	-0.40	12.80	1.80	**2.18**	0.04	0.02	0.19	0.211
3	-1.34	-0.65	-0.52	12.77	**2.27**	**2.49**	0.07	0.04	0.06	0.201
4	-2.42	-1.04	-0.80	12.20	**2.58**	**2.61**	0.10	0.07	**0.02**	0.191
5	-4.35	-1.80	-1.40	11.55	**2.99**	**2.75**	0.16	0.13	**0.00**	0.172
6	-5.77	**-2.39**	-1.79	10.86	**3.43**	**2.81**	0.22	0.20	**0.00**	0.150
7	-6.93	**-2.71**	-1.90	10.26	**3.64**	**2.81**	0.27	0.25	**0.00**	0.140
8	-8.24	**-2.95**	-1.95	9.70	**3.67**	**2.72**	0.31	0.29	**0.00**	0.136
9	-9.27	**-2.92**	-1.82	9.25	**3.50**	**2.54**	0.32	0.30	**0.00**	0.138
10	-10.14	**-2.74**	-1.64	8.74	**3.20**	**2.26**	0.32	0.30	**0.00**	0.142

Table 6-1-3b: Non-overlapping Interval Regression Output for S1 and R_m

h	B1	t1	t2	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	-0.35	-0.29	-0.36	12.20	1.25	1.85	0.02	0.00	0.45	0.208
2	-0.21	-0.08	-0.11	10.58	0.95	1.75	0.02	-0.03	0.64	0.235
3	-0.55	-0.16	-0.19	12.34	1.28	**3.16**	0.06	-0.01	0.44	0.198
4	0.20	0.03	0.05	11.85	0.93	**2.65**	0.05	-0.06	0.66	0.255
5	0.27	0.04	0.06	12.13	1.14	**2.72**	0.09	-0.04	0.53	0.211
6	-1.05	-0.15	-0.21	11.63	1.20	**3.61**	0.13	-0.03	0.46	0.186
7	-4.08	-0.52	-0.70	12.00	1.25	**3.28**	0.21	0.04	0.34	0.182
8	-7.44	-1.10	-1.75	10.27	1.45	1.80	0.41	0.25	0.15	0.127
9	-6.12	-0.75	-2.22	10.60	1.40	1.77	0.40	0.20	0.22	0.133
10	-4.03	-0.37	-0.97	8.28	0.91	1.21	0.24	-0.07	0.51	0.153

6. 1. 2 Fama-Bliss Zero Coupon Bonds

Again, between S1 and US Treasuries, all correlation signs conform with expectations. There is no material difference between output produced by S1 and output produced by MY.

Table 6-1-4: Correlations Between S1 and Treasuries

		Overlapping time-interval					Non-overlap time-interval				
m	var*h*	1	2	3	4	5	1	2	3	4	5
1	S1	-0.44					-0.44				
	ΔS1	0.24					0.24				
2	S1	-0.34	-0.45				-0.34	-0.49			
	ΔS1	0.22	0.29				0.22	0.35			
3	S1	-0.23	-0.35	-0.44			-0.23	-0.38	-0.43		
	ΔS1	0.21	0.29	0.35			0.21	0.34	0.56		
4	S1	-0.19	-0.27	-0.37	-0.46		-0.19	-0.29	-0.35	-0.54	
	ΔS1	0.20	0.30	0.36	0.41		0.20	0.33	0.53	0.48	
5	S1	-0.14	-0.23	-0.31	-0.41	-0.49	-0.14	-0.24	-0.27	-0.49	-0.48
	ΔS1	0.18	0.30	0.37	0.43	0.48	0.18	0.32	0.51	0.50	0.54

Holding *h* fixed, R^2s increase along with decrease in maturity *m*. Holding *m* fixed, R^2s increase along with increase in return horizon *h*. The phenomenon we saw in Table 4-4 is repeated again in Table 6-1-5, i.e. return variations conform with demographic trend more as they carry less loading from other risk factors.

Table 6-1-5a: Overlapping Interval Regression Output for S1 and Treasuries

m	*h*	B2	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1	-0.50	**-3.96**	**-2.62**	2.29	**2.34**	1.53	0.26	0.23	**0.00**	0.021
2	1	-0.58	**-2.93**	**-2.17**	2.99	1.97	1.58	0.17	0.14	**0.00**	0.032
2	2	-1.07	**-3.98**	**-2.45**	2.75	**2.65**	1.75	0.29	0.26	**0.00**	0.043
3	1	-0.54	-1.95	-1.55	3.83	1.80	1.62	0.10	0.07	**0.04**	0.045
3	2	-1.11	**-2.95**	-1.89	3.64	**2.51**	1.81	0.21	0.18	**0.00**	0.060
3	3	-1.59	**-3.76**	**-2.16**	3.19	**2.96**	1.93	0.30	0.28	**0.00**	0.066
4	1	-0.53	-1.56	-1.25	4.40	1.67	1.58	0.08	0.05	0.10	0.056
4	2	-1.08	**-2.27**	-1.45	4.56	**2.49**	1.89	0.17	0.14	**0.01**	0.076
4	3	-1.66	**-3.05**	-1.75	4.12	**2.97**	2.00	0.26	0.23	**0.00**	0.085
4	4	-2.18	**-3.81**	**-2.05**	3.62	**3.35**	**2.14**	0.35	0.32	**0.00**	0.087
5	1	-0.47	-1.17	-0.93	4.59	1.48	1.49	0.06	0.02	0.20	0.066
5	2	-1.03	-1.85	-1.16	5.18	**2.42**	1.92	0.14	0.11	**0.01**	0.089
5	3	-1.62	**-2.51**	-1.41	5.01	**3.04**	**2.11**	0.23	0.20	**0.00**	0.101
5	4	-2.27	**-3.33**	-1.75	4.52	**3.50**	**2.22**	0.32	0.30	**0.00**	0.104
5	5	-2.76	**-3.97**	-2.00	3.98	**3.83**	**2.37**	0.41	0.38	**0.00**	0.102

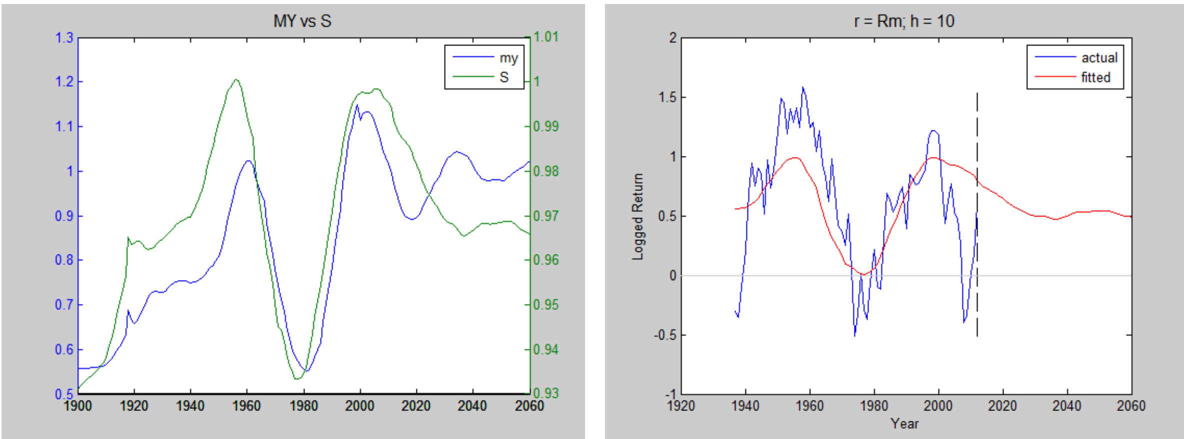
Table 6-1-5b: Non-overlapping Interval Regression Output for S1 and Treasuries

m	*h*	B2	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1	-0.50	**-3.96**	**-2.62**	2.29	**2.34**	1.53	0.26	0.23	**0.00**	0.021
2	1	-0.58	**-2.93**	**-2.17**	2.99	1.97	1.58	0.17	0.14	**0.00**	0.032
2	2	-1.14	**-3.23**	**-3.42**	3.17	**2.32**	2.03	0.37	0.32	**0.00**	0.040
3	1	-0.54	-1.95	-1.55	3.83	1.80	1.62	0.10	0.07	**0.04**	0.045
3	2	-1.19	**-2.31**	**-2.62**	4.05	2.03	1.96	0.26	0.20	**0.02**	0.059
3	3	-1.34	**-2.29**	**-2.75**	4.59	**3.02**	**3.87**	0.47	0.41	**0.00**	0.054
4	1	-0.53	-1.56	-1.25	4.40	1.67	1.58	0.08	0.05	0.10	0.056
4	2	-1.13	-1.70	-1.90	4.91	1.92	1.94	0.20	0.14	0.05	0.076
4	3	-1.30	-1.69	-1.90	5.40	**2.69**	**3.78**	0.38	0.31	**0.02**	0.071
4	4	-2.80	**-2.35**	**-2.73**	4.70	2.02	**2.40**	0.47	0.39	**0.02**	0.092
5	1	-0.47	-1.17	-0.93	4.59	1.48	1.49	0.06	0.02	0.20	0.066
5	2	-1.04	-1.34	-1.41	5.38	1.80	1.88	0.16	0.09	0.10	0.089
5	3	-1.12	-1.21	-1.27	6.03	**2.50**	**3.67**	0.32	0.24	**0.04**	0.085
5	4	-2.82	-2.01	**-2.22**	5.64	2.05	**2.58**	0.44	0.34	**0.03**	0.109
5	5	-2.32	-1.58	-2.14	4.27	1.87	**2.77**	0.45	0.32	0.07	0.101

6. 2 The Supply Measure, Part II

Both MY ratio and S1 are very crude measures of supply-demand balance in the economy. At any given point in time, expected earning across ages should have a hump-shaped curve. The US Census Bureau reports statistics on average income for six different age groups: {16-24, 25-34, 35-44, 45-54, 55-64, 65+}[18]. From 1974 to 2012, normalized by the second age group {25-34}, income averaged across this data period is {0.40, 1, 1.29, 1.35, 1.18, 0.72}. Income is normalized by the {25-34} age group because income and consumption are roughly balanced around age 25-26 in the US[19]. Let ***w*** be the normalized income vector, ***n*** be the population vector for the six age groups, and ***c*** be a constant vector of ones. We shall refer this supply measure as S2.

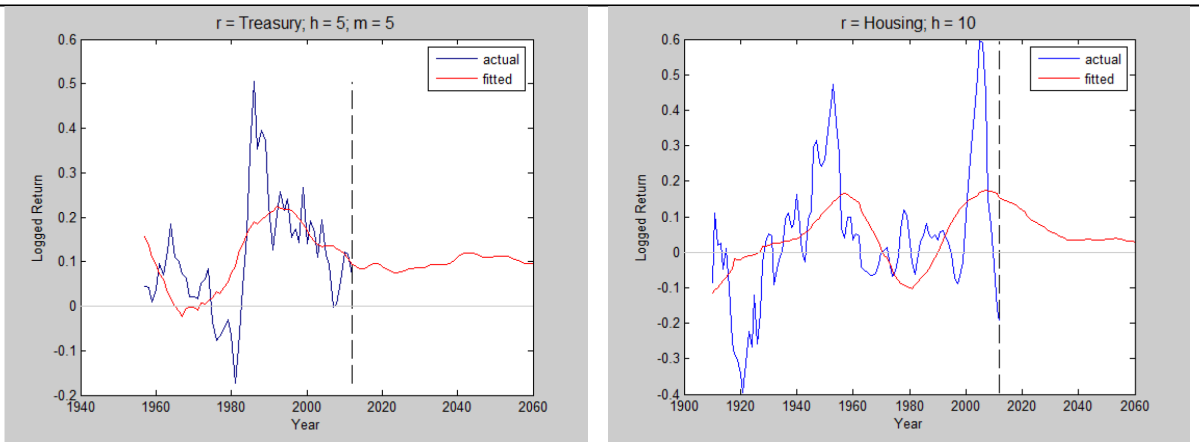
Figure 6-2: The S2 Measure



18 Data is available via http://www.census.gov/hhes/www/income/data/historical/people/. Table P-10: Age—People (Both Sexes Combined) by Median and Mean.

19 Source: National Transfer Accounts – Consumption and Labor Income Profile Release. USA 2003 data. website: http://www.ntaccounts.org/web/nta/show

Text



From the upper left plot, the S2 measure also behaves similarly with the MY ratio, but there is a meaningful divergence starting from year 2000. After the MY ratio left its peak in 2000, the S2 measure kept staying at its peak until 2006. Unlike the MY ratio, S2's decline sees no recovery. Implications for future returns between the two measures are markedly different. The MY ratio hints rising equity return between 2015-2040 (Figure 3-3) but the S2 ratio suggests a declining or tepid equity return environment all the way through 2060 (upper right corner of Figure 6-2).

6. 2. 1 Excess Equity Market Return

Looking at Table 6-2-1 below, although sign for the <ΔS2, R_m> correlation pair match with a-priori prediction, sign for the <S2, R_m> pair is not negative as expected. This does not necessarily contradict the GMQ model. In the model, S2 has to drop in the future in order for high S2 to be associated with low returns. The key driving force that alters the state of the economy is change in demographics (ΔS), not level of demographics (S).

Table 6-2-1: Correlations Between S2 and R_m

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	S2	0.11	0.15	0.18	0.18	0.17	0.17	0.17	0.15	0.13	0.10
	ΔS2	0.13	0.18	0.22	0.28	0.34	0.39	0.43	0.45	0.46	0.46
non-overlap	S2	0.11	0.15	0.21	0.21	0.24	0.25	0.15	0.22	0.27	0.26
	ΔS2	0.13	0.17	0.25	0.16	0.24	0.34	0.48	0.43	0.43	0.34

Table 6-2-2a: Overlapping Interval Regression Output for S2 and R_m

h	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1.19	1.10	1.55	10.33	1.19	**2.03**	0.03	0.01	0.29	0.207
2	2.45	1.57	1.71	11.46	1.81	**2.62**	0.06	0.04	0.08	0.209
3	3.69	**2.02**	1.84	11.93	**2.38**	**2.95**	0.10	0.07	**0.02**	0.198
4	4.73	**2.32**	1.90	12.88	**3.06**	**3.29**	0.14	0.11	**0.00**	0.187
5	5.48	**2.62**	1.88	13.52	**3.90**	**3.49**	0.19	0.17	**0.00**	0.168
6	6.56	**3.18**	1.99	13.69	**4.79**	**3.76**	0.25	0.23	**0.00**	0.148
7	7.85	**3.68**	**2.07**	14.02	**5.52**	**4.07**	0.31	0.29	**0.00**	0.137
8	9.16	**4.02**	**2.10**	14.52	**6.10**	**4.36**	0.35	0.33	**0.00**	0.132
9	10.69	**4.23**	**2.13**	15.10	**6.39**	**4.57**	0.37	0.35	**0.00**	0.133
10	12.35	**4.32**	**2.15**	15.70	**6.49**	**4.68**	0.37	0.35	**0.00**	0.136

Table 6-2-2b: Non-overlapping Interval Regression Output for S2 and R_m

h	B1	t1	t2	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	1.19	1.10	1.55	10.33	1.19	**2.03**	0.03	0.01	0.29	0.207
2	2.61	1.07	1.68	11.61	1.17	**2.23**	0.06	0.01	0.33	0.231
3	4.19	1.37	1.86	13.05	1.55	**3.58**	0.13	0.06	0.18	0.191
4	6.00	1.11	1.95	10.46	0.93	**2.48**	0.09	-0.01	0.44	0.249
5	7.20	1.31	**2.18**	12.12	1.33	**2.35**	0.16	0.04	0.29	0.202
6	8.57	1.49	2.05	14.14	1.74	**4.31**	0.27	0.13	0.18	0.171
7	9.11	1.39	1.58	17.72	2.22	**3.79**	0.37	0.23	0.13	0.163
8	9.59	1.56	1.55	13.01	1.99	**4.61**	0.39	0.22	0.17	0.129
9	12.91	1.88	1.93	14.45	2.19	**3.73**	0.48	0.31	0.14	0.123
10	13.46	1.51	1.95	12.75	1.64	**3.63**	0.39	0.15	0.29	0.137

Tables 6-2-1 and 6-2-2 offer the first corroborative evidence outside the MY ratio that asset returns are linked with demographics. The significant t-stats for B_2 (coefficient for ΔS2) across overlapping and non-overlapping time windows deserve attention.

6. 2. 2 S&P500: Pre-Baby-Boom vs Post-Baby-Boom

In the pre-baby-boom era (1900-1951), S2 has strong correlations with returns, but unfortunately, all signs show up contrary to a-priori expectation.

Table 6-2-3: Correlations Between S2 and S&P500 (1900-1951)

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	S2	0.09	0.13	0.17	0.18	0.21	0.28	0.36	0.38	0.42	0.44
	ΔS2	-0.12	-0.24	-0.34	-0.40	-0.48	-0.56	-0.59	-0.61	-0.64	-0.68
non-overlap	S2	0.09	0.15	0.15	0.20	0.25	0.18	0.37	0.34	0.58	0.73
	ΔS2	-0.12	-0.21	-0.37	-0.40	-0.21	-0.71	-0.34	-0.58	0.28	-0.35

In the post-baby-boom era (1952-2012), S2 has low correlation with S&P500 returns, but ΔS2 has high positive correlations, matching a-priori expectation. The higher positive correlation between S2 and equity during the pre-baby-boom era may be responsible for the positive correlation seen between S2 and R_m during the overall period: 1927-2012 (Table 6-2-1).

Table 6-2-4: Correlations Between S2 and S&P500 (1952-2012)

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	S2	0.11	0.14	0.15	0.11	0.07	0.05	0.02	-0.02	-0.07	-0.12
	ΔS2	0.20	0.28	0.35	0.41	0.45	0.47	0.49	0.51	0.52	0.53
non-overlap	S2	0.11	0.12	0.19	0.21	0.18	0.17	-0.13	0.05	0.06	0.25
	ΔS2	0.20	0.25	0.44	0.34	0.41	0.48	0.62	0.49	0.52	0.32

As for regression outputs, post-baby-boom statistics for S2 and ΔS2 are generally weaker than their ΔMY counterparts (Table 3-13), but strong enough to maintain decent R^2s and status of significance for B_2's positive t-stats.

Table 6-2-5a: Overlapping Interval Regression Output for S2 and S&P500 (1952-2012)

h	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.87	1.00	1.21	11.62	1.63	**2.17**	0.06	0.02	0.19	0.161
2	1.77	1.45	1.30	12.36	**2.42**	**2.60**	0.11	0.08	**0.04**	0.159
3	2.51	1.79	1.29	12.48	**3.18**	**2.75**	0.17	0.14	**0.01**	0.145
4	3.01	1.89	1.20	12.74	**3.77**	**2.83**	0.22	0.19	**0.00**	0.141
5	3.39	1.86	1.08	12.96	**4.18**	**2.87**	0.25	0.22	**0.00**	0.140
6	4.24	**2.08**	1.12	13.03	**4.52**	**2.97**	0.28	0.26	**0.00**	0.138
7	4.93	**2.20**	1.11	13.06	**4.79**	**3.02**	0.31	0.28	**0.00**	0.135
8	5.58	**2.19**	1.07	13.25	**4.88**	**3.06**	0.32	0.30	**0.00**	0.138
9	6.22	**2.15**	1.03	13.46	**4.90**	**3.06**	0.33	0.30	**0.00**	0.141
10	6.72	**2.08**	0.98	13.58	**4.91**	**2.99**	0.34	0.32	**0.00**	0.141

Table 6-2-5b: Non-overlapping Interval Regression Output for S2 and S&P500 (1952-2012)

h	B1	t1	t2	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	0.87	1.00	1.21	11.62	1.63	**2.17**	0.06	0.02	0.19	0.161
2	1.75	0.87	1.22	12.26	1.46	**2.33**	0.09	0.02	0.29	0.185
3	2.82	1.33	1.39	13.94	**2.32**	**3.89**	0.27	0.18	0.07	0.129
4	4.03	1.18	1.27	11.45	1.57	**2.47**	0.21	0.07	0.25	0.153
5	4.65	1.14	1.43	11.99	1.71	2.20	0.27	0.11	0.24	0.146
6	6.20	1.22	1.56	14.34	1.92	**6.07**	0.36	0.18	0.20	0.147
7	3.43	0.51	0.63	15.34	1.83	**3.58**	0.41	0.18	0.27	0.155
8	6.44	0.89	1.05	12.07	1.50	2.51	0.36	0.04	0.41	0.143
9	7.25	1.01	1.83	10.91	1.57	**4.51**	0.45	0.09	0.40	0.117
10	11.87	1.14	1.54	11.71	1.23	**4.45**	0.37	-0.04	0.49	0.155

6. 2. 3 Fama-Bliss Zero Coupon Bonds

For US Treasuries, bondholders carry only inflation risk when holding bonds to maturity, whereas bondholders additionally carry interest rate risk if they sell bonds before maturity. Interestingly, echoing the MY ratio and the S1 measure, Treasury returns' correlation with S2 also tends to decline as return becomes less uncertain. Likewise, correlation with ΔS2 also tends to rise as return becomes less risky. The author postulates that if an asset has less exposure to risky factors, its behavior is more likely to be governed by those mechanics outlined in the GMQ model. Note direct comparison can also be made between Treasury's outputs and S&P500's outputs for the post-baby-boom era (1952-2012).

Table 6-2-6: Correlations Between S2 and Treasuries

		Overlapping time-interval					Non-overlap time-interval				
m	var*h*	1	2	3	4	5	1	2	3	4	5
1	S2	-0.11					-0.11				
	ΔS2	0.40					0.40				
2	S2	-0.05	-0.09				-0.05	-0.08			
	ΔS2	0.39	0.44				0.39	0.52			
3	S2	0.03	-0.02	-0.06			0.03	-0.01	0.06		
	ΔS2	0.34	0.42	0.47			0.34	0.47	0.54		
4	S2	0.05	0.04	-0.01	-0.06		0.05	0.06	0.10	-0.09	
	ΔS2	0.33	0.41	0.48	0.52		0.33	0.44	0.53	0.58	
5	S2	0.08	0.07	0.04	-0.01	-0.05	0.08	0.09	0.15	-0.04	-0.02
	ΔS2	0.31	0.40	0.48	0.54	0.58	0.31	0.42	0.51	0.59	0.52

Similar to the MY ratio, regression outputs reported in Table 6-2-7 are quite strong. Holding *m* fixed, R^2s uniformly increase as maturity gets closer (bond becomes less risky). Moreover, for equal return horizons, R^2s are generally higher in Treasuries than in equities.

Table 6-2-7a: Overlapping Interval Regression Output for S2 and Treasuries

m	*h*	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1	-0.07	-0.61	-0.35	3.11	**3.21**	1.95	0.16	0.13	**0.01**	0.022
2	1	-0.02	-0.13	-0.09	4.49	**3.14**	**2.45**	0.15	0.12	**0.01**	0.033
2	2	-0.06	-0.24	-0.13	3.76	**3.60**	**2.30**	0.19	0.17	**0.00**	0.046
3	1	0.11	0.47	0.34	5.39	**2.75**	**2.46**	0.12	0.09	**0.03**	0.045
3	2	0.12	0.36	0.21	4.87	**3.49**	**2.65**	0.18	0.15	**0.00**	0.061
3	3	0.12	0.31	0.17	4.31	**3.98**	**2.66**	0.23	0.20	**0.00**	0.070
4	1	0.18	0.62	0.48	6.51	**2.71**	**2.60**	0.12	0.09	**0.03**	0.055
4	2	0.33	0.80	0.49	5.92	**3.44**	**2.83**	0.18	0.15	**0.00**	0.075
4	3	0.36	0.75	0.41	5.55	**4.14**	**3.10**	0.24	0.21	**0.00**	0.086
4	4	0.37	0.71	0.37	4.99	**4.55**	**3.07**	0.28	0.25	**0.00**	0.091
5	1	0.28	0.82	0.67	7.12	**2.53**	**2.61**	0.11	0.08	**0.04**	0.064
5	2	0.50	1.06	0.66	6.81	**3.45**	**3.01**	0.18	0.15	**0.00**	0.087
5	3	0.65	1.18	0.65	6.58	**4.25**	**3.38**	0.25	0.22	**0.00**	0.099
5	4	0.69	1.16	0.60	6.22	**4.95**	**3.56**	0.31	0.29	**0.00**	0.105
5	5	0.77	1.23	0.63	5.65	**5.35**	**3.57**	0.35	0.33	**0.00**	0.107

Table 6-2-7b: Non-overlapping Interval Regression Output for S2 and Treasuries

m	*h*	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	1	-0.07	-0.61	-0.35	3.11	**3.21**	1.95	0.16	0.13	**0.01**	0.022
2	1	-0.02	-0.13	-0.09	4.49	**3.14**	**2.45**	0.15	0.12	**0.01**	0.033
2	2	-0.02	-0.07	-0.06	4.31	**3.08**	**2.48**	0.27	0.21	**0.02**	0.044
3	1	0.11	0.47	0.34	5.39	**2.75**	**2.46**	0.12	0.09	**0.03**	0.045
3	2	0.16	0.36	0.32	5.45	**2.83**	**2.82**	0.23	0.17	**0.03**	0.060
3	3	0.50	0.87	0.90	4.64	**2.84**	**2.40**	0.32	0.25	**0.04**	0.061
4	1	0.18	0.62	0.48	6.51	**2.71**	**2.60**	0.12	0.09	**0.03**	0.055
4	2	0.40	0.69	0.66	6.32	**2.61**	**2.93**	0.20	0.15	**0.05**	0.075
4	3	0.74	1.05	1.16	5.66	**2.83**	**2.69**	0.33	0.25	**0.03**	0.074
4	4	0.31	0.27	0.29	6.11	**2.47**	**2.38**	0.34	0.23	0.08	0.103
5	1	0.28	0.82	0.67	7.12	**2.53**	**2.61**	0.11	0.08	**0.04**	0.064
5	2	0.56	0.84	0.84	6.96	**2.50**	**3.05**	0.19	0.13	0.05	0.087
5	3	1.03	1.27	1.46	6.26	**2.73**	**2.87**	0.32	0.24	**0.04**	0.085
5	4	0.63	0.49	0.54	7.24	**2.61**	**2.61**	0.36	0.26	0.07	0.116
5	5	0.75	0.53	0.73	4.76	1.94	2.24	0.29	0.14	0.21	0.114

6. 2. 4 Case-Shiller Housing Index

Like the MY ratio, the S2 measure does not offer good explanation for variation in home price returns. Correlation between housing and ΔS2 is weak. Like in equities, S2 is positively associated with returns, contrary to what theory suggests. Perhaps, for risky assets, excess saving demand is a positive driving force over real returns, but t-statistics for coefficient B_1 in Table 6-2-9 are too weak to support this hypothesis. Correlations and coefficients for ΔS2 remain positive as expected, nonetheless.

Table 6-2-8: Correlations Between S2 and Housing

time interval	*h*	1	2	3	4	5	6	7	8	9	10
overlap	S2	0.06	0.06	0.08	0.10	0.12	0.15	0.20	0.26	0.31	0.33
	ΔS2	0.00	0.02	0.03	0.03	0.05	0.05	0.06	0.07	0.08	0.09
non-overlap	S2	0.06	0.08	0.05	0.13	0.11	0.09	0.15	0.10	0.16	0.19
	ΔS2	0.00	0.01	0.03	-0.01	0.10	0.16	0.09	0.10	0.19	0.23

Table 6-2-9a: Overlapping Interval Regression Output for S2 and Housing

h	B1	t1	t2	B2	t1	t2	R^2	R^2_adj	Prob > F	rmse/√h
1	0.19	0.68	0.59	0.25	0.10	0.11	0.00	-0.01	0.79	0.066
2	0.28	0.65	0.44	0.64	0.32	0.29	0.00	-0.01	0.79	0.070
3	0.52	0.91	0.56	0.90	0.50	0.46	0.01	-0.01	0.63	0.075
4	0.76	1.12	0.63	1.00	0.61	0.53	0.01	-0.01	0.51	0.075
5	1.11	1.44	0.77	1.34	0.90	0.76	0.02	0.00	0.32	0.075
6	1.52	1.84	0.95	1.57	1.18	0.95	0.03	0.02	0.16	0.071
7	2.22	**2.60**	1.31	1.93	1.62	1.22	0.06	0.05	**0.03**	0.066
8	3.07	**3.54**	1.75	2.40	**2.25**	1.56	0.11	0.10	**0.00**	0.061
9	3.91	**4.39**	**2.12**	2.87	**2.93**	1.89	0.17	0.15	**0.00**	0.056
10	4.73	**5.16**	**2.43**	3.35	**3.66**	**2.23**	0.22	0.20	**0.00**	0.053

Table 6-2-9b: Non-overlapping Interval Regression Output for S2 and Housing

h	B1	t1	t2	B2	t1	t2	R2	R2_adj	Prob > F	rmse/√h
1	0.19	0.68	0.59	0.25	0.10	0.11	0.00	-0.01	0.79	0.066
2	0.36	0.59	0.67	0.47	0.17	0.21	0.01	-0.03	0.84	0.071
3	0.35	0.32	0.41	0.84	0.24	0.40	0.00	-0.05	0.94	0.082
4	0.83	0.66	1.04	0.38	0.12	0.19	0.02	-0.06	0.80	0.071
5	1.14	0.64	1.18	2.11	0.61	1.04	0.03	-0.07	0.75	0.079
6	1.42	0.63	1.26	3.09	0.84	2.07	0.05	-0.07	0.67	0.079
7	2.32	0.76	1.44	2.75	0.60	1.05	0.05	-0.10	0.72	0.096
8	1.66	0.55	1.00	2.21	0.56	1.33	0.04	-0.14	0.81	0.080
9	2.83	0.95	1.61	3.42	1.01	2.08	0.12	-0.07	0.55	0.064
10	3.89	1.18	1.97	4.42	1.25	1.97	0.19	-0.01	0.42	0.065

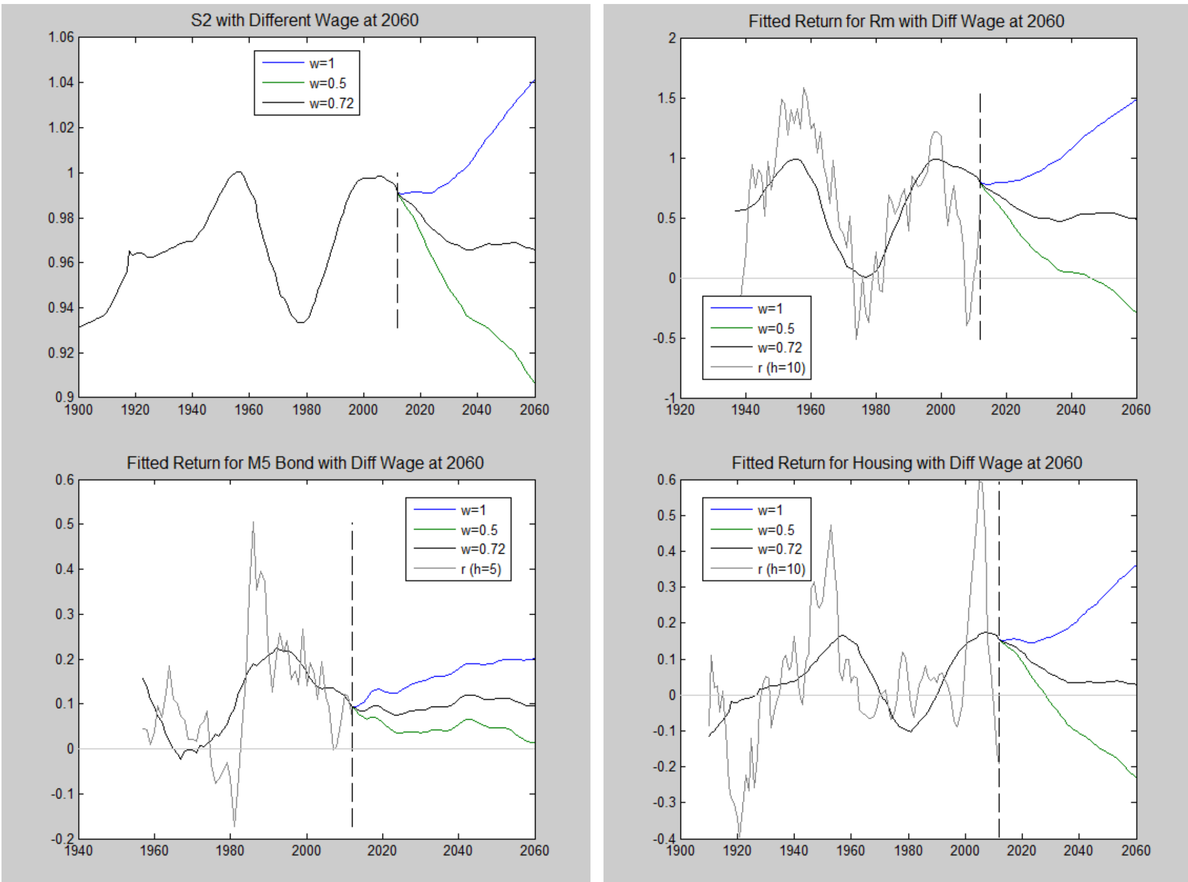
7. Conclusion

To sum up, aiming to explore the linkage between age-population demographics and asset returns, this paper is able to find more supportive evidence than otherwise, conditional on large demographic changes taking place. First, a formal theory was set out to guide the reader's understanding of the economics behind demographic forces and market returns. Then, the author tested the relationship between the MY ratio and three major asset classes in the US: broad equities, Treasuries, and housing, with strength of results following the same order. Beyond the MY ratio, the author composed measures aiming to reflect excess saving demand in an economy. Statistical outputs are mostly in-line with results produced by the MY ratio and they largely conform with a-priori predictions during the post-baby-boom era. For the most refined measure of excess savings demand: S2, strength of relationship with asset returns seems to correspond with the asset class' "riskiness" as a savings tool: with Treasuries having the strongest statistics, then equities, and housing having the weakest statistics. Among Treasuries, returns that carry less uncertainty also tend to have stronger statistics.

Although the S2 measure does not necessarily fit past data better than the MY ratio, the author deems for future outlook, S2 would be a better predictor than MY. In the US, average/median age and senior citizens' share of total population have been increasing consistently throughout the 20th Century, and they are projected to increase much more rapidly in the 21st Century (see Appendix A.2). The MY ratio does not take old-age population into account at all. In 2012, people older than 64 make up 13.8% of total US population. By 2030, the number will be 21.0%, just over one fifth of total population. Senior citizens will continue to matter more for the economy. On the one hand, if senior citizens will produce nothing in the future, then effectively they exit the economy and future supply will be low, and equilibrium return will be low to ensure people consume more now than in the future. On the other hand, if senior citizens will produce abundantly, then future supply will be high, and equilibrium return shall be high to ensure people will consume more in the future than at present. In order for the economy to realize its full growth potential, both public sector and private sector ought to promote productivity and to eliminate as much barriers to work as possible for senior citizens.

For the S2 measure, the author took average wage between 1974-2012 and assigned values [0.40, 1.00, 1.29, 1.35, 1.18, 0.72] to the wage vector ***w***. The last value (0.72) corresponds to wage earned by senior citizens (age ≥ 65) relative to the [25:34] age group. Suppose senior citizens' relative wage gradually grows from 0.72 in 2012 to 1.00 in 2060, then future equilibrium return would be very different than if their relative wage stays the same. The same holds true if senior citizens' relative wage gradually declines from 0.72 to 0.5 in 2060. Figure 7-1 plots out the three scenarios. In the high productivity scenario, represented by the blue line, rising relative wage leads to a high return environment. In the low productivity scenario, represented by the green line, declining relative wage leads to a low return environment. Upper left figure graphs the S2 measure. Upper right figure graphs the 10-year fitted return for R_m. Lower left figure graphs 5-year fitted return for 5-year maturity Fama-Bliss bond. Lower right figure graphs 10-year fitted return for Case-Shiller Housing Index.

Figure 7-1: S2 and Fitted Returns under Three Scenarios



The author ends this paper with the following note. From 1974 to 2012, relative income of senior citizens has in fact been consistently rising against the [25:34] age group. At the same time, according to the Consumer Expenditure Survey data from the Bureau of Labor Statistics, between 1988 to 2012, relative consumption of senior citizens has also been consistently rising against the [25:34] age group. For future research topic, interesting discovery may emerge by exploring the evolution of income-expenditure data of different age groups over time and studying their empirical linkage with asset returns.

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Appendix

A.1 Equity Return and Dividend Yield

In their "small structural model of demographics", Favero and Tamoni (FT), 2010, characterized stock market returns as dependent on dividend yield and MY. To begin, FT used the Campbell Shiller (1988) decomposition to approximate stock return:

$$1 = R_{t+1}^{-1}\frac{P_{t+1}+D_{t+1}}{P_t} \Rightarrow \frac{P_t}{D_t} = R_{t+1}^{-1}\left(1+\frac{P_{t+1}}{D_{t+1}}\right)\frac{D_{t+1}}{D_t} \qquad ..(A1.1)$$

Taking natural logs and then first-order Taylor approximation of $\log(1 + exp(p_{t+1} - d_{t+1}))$ around mean price-dividend ratio gives:

$$\begin{aligned} r_{t+1} &= d_t - p_t + \Delta d_{t+1} + \log(1+\exp(p_{t+1} - d_{t+1})) \\ &= \Delta d_{t+1} + k - \rho\left(dp_{t+1} - \overline{dp}\right) + (dp_t - \overline{dp}) \end{aligned} \qquad ..(A1.2)$$

Iterating (A1.2) h periods forward, a similar expression arises:

$$\sum_{j=1}^{h} \rho^{j-1} r_{t+j} = dp_t - \overline{dp} + \sum_{j=1}^{h} \rho^{j-1}\Delta d_{t+j} + \rho^h(dp_{t+h+1} - \overline{dp}) \qquad ..(A1.3)$$

Equation A1.2 and A1.3 tell us that future return is a function of current dp ratio, future dividend growth, and future dp ratio. As h becomes large, the importance of future dividend yield becomes smaller. Another way to express (A1.3) is found in John Cochrane (Chapter 20, 2005): move dpt in (A1.2) to the left-hand side, take expectation, and let h approach infinity:

$$pd_t = const + E_t\left[\sum_{j=1}^{\infty} \rho^{j-1}\,(\Delta d_{t+j} - r_{t+j})\right] \qquad ..(A1.4)$$

(A1.4) says pd is a weighted sum of expected future dividend growth and returns. If pd is persistent, then a high pd must be followed by low returns and/or high dividend growth. A high pd, or low dp, would imply higher probability of negative return to bring pd back to long-term average. Similarly, a low pd, or high dp, would imply higher probability of positive return. Thus, a positive sign is expected for B3 in regression model (A1.5).

$$\sum_{j=1}^{h} r_{t+j} = \beta_0 + B_3\, dp_t + \varepsilon_{t+h} \qquad h = 1, \ldots, 10 \qquad ..(A1.5)$$

Table A1-1 shows the test result for Equation A1.5. Annual S&P500 index return is the dependent variable.

Table A1-1, Regression Output for Equation A1.5

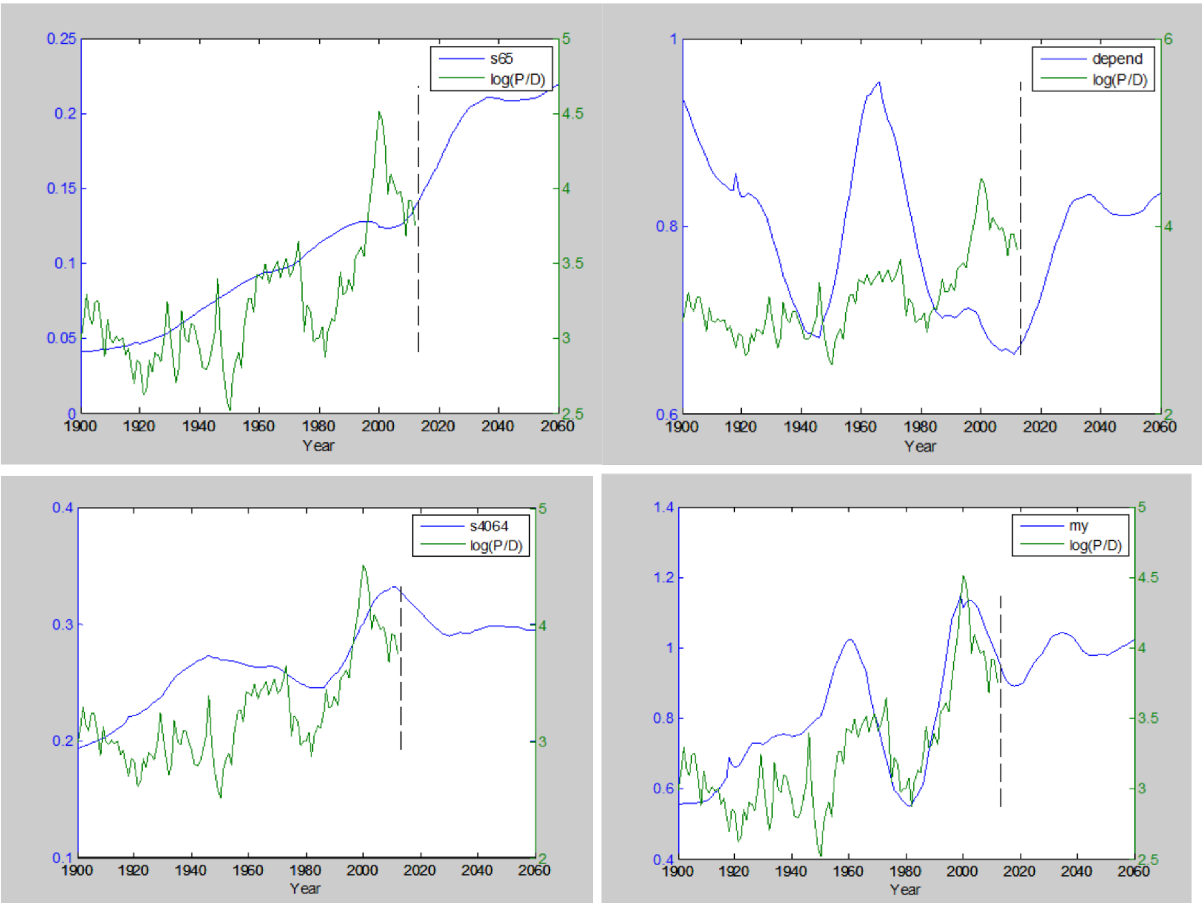
h	B3	t1	t2	R^2	R^2 adj	Prob > F	rmse/√h
1	0.07	1.79	**2.23**	0.03	0.02	0.08	0.180
2	0.13	**2.14**	**2.25**	0.04	0.03	**0.03**	0.183
3	0.19	**2.79**	**2.53**	0.07	0.06	**0.01**	0.172
4	0.25	**3.24**	**2.71**	0.09	0.08	**0.00**	0.170
5	0.32	**3.71**	**2.81**	0.12	0.11	**0.00**	0.164
6	0.37	**4.06**	**2.88**	0.14	0.13	**0.00**	0.158
7	0.44	**4.44**	**3.18**	0.16	0.15	**0.00**	0.154
8	0.51	**4.71**	**3.55**	0.18	0.17	**0.00**	0.154
9	0.56	**4.82**	**3.70**	0.19	0.18	**0.00**	0.153
10	0.61	**4.91**	**3.78**	0.19	0.18	**0.00**	0.151

From the table, B_3 is mostly positive and significant, supporting the hypothesis that pd is a persistent ratio, especially at return horizon longer than one year (h>1). Similar results can be found in many asset pricing textbooks[20].

A.2 Age-Population Demographics

Age-population demographic measures are indicators derived out of population statistics collected by the Bureau of Census. Figure A2-1 below tracks four classes of demographic variables, represented by the blue, relatively smooth line. Each blue line is accompanied by a green, zigged line, which represents the logged price-dividend ratio of the S&P500 index. Data for the logged price-dividend ratios, labeled as log(P/D), is available from 1900 to 2012. The vertical dashed line corresponds to year 2012, where the green line stops.

Figure A2-1, Demographic Variables vs Logged Price-Dividend Ratio



Source: US Census Bureau and Robert Shiller's website

The first class of age-population demographic variables (henceforth demographic variables, for short), seen in the upper left corner of Figure A2-1, is heavily influenced by the trend of increasing longevity of human beings. Members of this class include percentage of retirees versus total population, median age of total population, and average age of total population. This class of demographic variable has been consistently rising on a smooth trajectory, and is expected to keep on increasing until the end of the forecast period – 2060. This phenomenon of "aging society" is not particular to the United States, but is happening all over the globe. Compared with the European Union and Japan, the US actually has a younger population and is aging rather slowly. In the upper left graph, 's65' (blue line) represents the share of individuals aged 65 or higher against the total US population. Based on visual examination, the blue line and green line (pd ratio) seem to share a common positive trend. It turns out that the two series have a surprisingly high correlation of 0.69 between 1900 and 2012. Unless economic fundamentals are extremely strong, the correlation is

[20] Such as Cochrane (2005), chapter 20.

unlikely to persist over the next decades, nonetheless. It is rather difficult for S&P500 price-dividend ratio to increase further on the basis that there will be relatively more senior citizens in the US population.

The second class of demographic variables, on the upper right in Figure 6-1, is the dependence ratio: the percentage of people who are dependent on other people's income. One proxy for this ratio would be one minus the share of population between ages 20 to 65 (the blue line, labeled as 'depend'). The series exhibits greater fluctuation than the first class (upper right plot versus upper left plot), but it has a weaker, negative correlation of -0.25 with the pd time series. Nevertheless, unlike the first class, the dependence ratio may have an economic link with the pd ratio. It can be argued that the pd ratio shall decline as the economic system will be supported by proportionally less human capital. The projected increase in dependence ratio is discussed widely in both public and private sectors. As we can see from the upper right plot in Figure A2-1, however, dependence ratios had been much higher during the 1960s and early 1900s. During those times, pd was at its highest point of the 20th century. It was not until later in 1990s and early 2000s that pd broke its previous high while the nation had an ultra-low dependence ratio.

The third class of demographic variables is the percentage of population in their prime earning years. In the lower left corner of Figure A2-1, the blue line, labeled as 's4064', is share of population between ages 40-64. In this class of variables, we see more of a cyclical fluctuation rather than the monotonic movement that we saw in the first class. The economic meaning of this class of demographic variables is quite close to that of the second class. Basically, ratios from the second class should be close to the inverse of the ratios from the third class. However, the 's4064' series does have a stronger correlation, $\rho = 0.66$, with the pd ratio than the 'depend' series. If this correlation is going to persist into the future, then the high equity valuation of the 1990s and early 2000s will likely not appear again until well after 2060.

The forth class of demographic variable, the MY ratio, started receiving attention in academia since first introduced by GMQ in 2004. It is defined as the ratio of middle to young aged population, or number of people in their 40s divided by number of people in their 20s. In GMQ's overlapping generational model, equilibrium condition suggests positive co-movement between financial assets and MY. On the lower right corner of Figure A2-1, we see that this is indeed the case for the US stock market during the post-World War II (WWII) period (correlation = 0.69 during 1946-2012). From the earlier, pre-WWII period (1900-1945), MY had a negative correlation of -0.35 with the S&P500 pd ratio. This fact actually does not contradict with the GMQ model because a key assumption in GMQ's model is that there should be large population difference across successive generations[21], i.e. the MY ratio should see large swings across generational time periods. During 1900-1945, MY only grew from 0.56 to 0.77, this movement is considerably smaller than the subsequent swing seen in the post-WWII period, which swung between 0.55 and 1.17. With these said, the reader should notice the mild swing of MY ratio projected for 2013-2060. This can potentially limit the predictive power of the MY ratio in the decades ahead.

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[21] The exact assumption is individuals live only three periods: {young, middle aged, old}. The number of people across generations would always alternate between small (n) and big (N). Thus, at any given time, the population distribution would be either {n,N,n} or {N,n,N}.