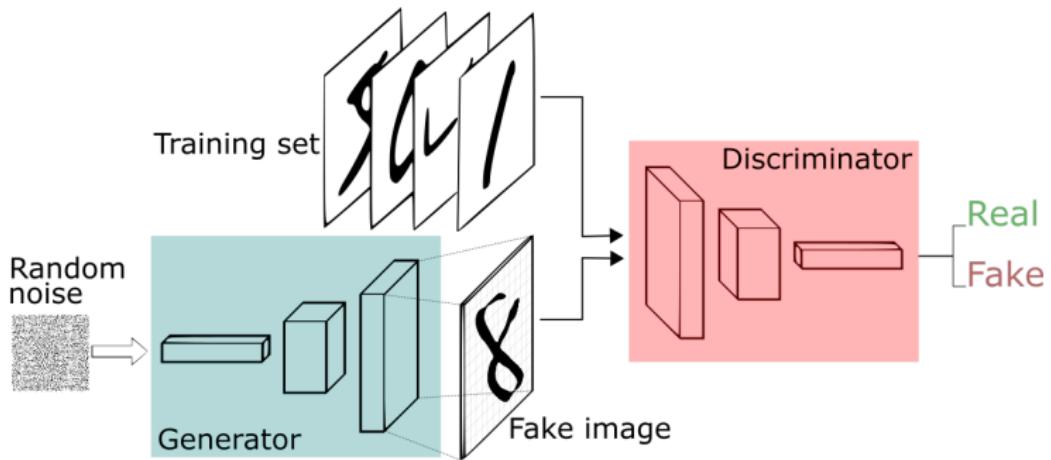
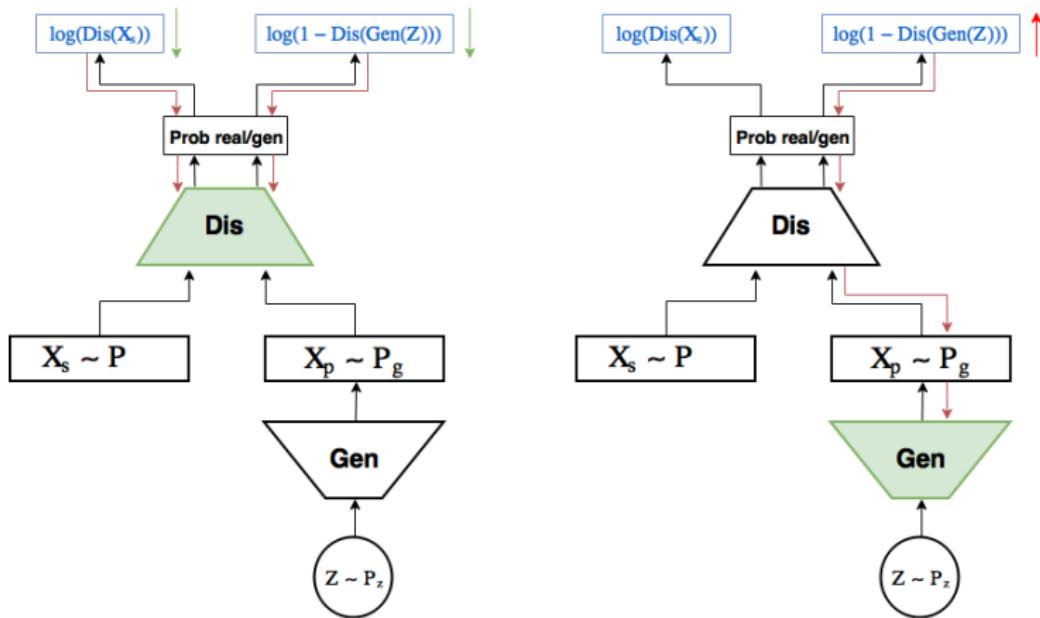


Spectral Normalization for GAN

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$$V(G, D) = E_{x \sim q_{data}} [\log D(x)] + E_{x' \sim p_G} [\log(1 - D(x'))]$$

$$\min_G \max_D V(G, D)$$

q_{data} -data distribution
 p_G -generator distribution.

Дискриминатор

$$D(x, \theta) = \mathcal{A}(f(x, \theta)) = \mathcal{A}(W^{L+1}a_L(W^L \dots a_1(W^1x) \dots))$$

$$\theta := \{W^1, \dots, W^{L+1}\}, W^l \in \mathbb{R}^{d_l \times d_{l-1}}, d_{L+1} = 1$$

При фиксированном генераторе:

$$D_G^*(x) := \frac{q_{data}(x)}{q_{data}(x) + p_G(x)}, f^*(x) = \log \frac{q_{data}(x)}{p_G(x)}$$

Какие проблемы?

$$\nabla_x f^*(x) = \frac{1}{q_{data}(x)} \nabla_x q_{data}(x) - \frac{1}{p_G(x)} \nabla_x p_G(x)$$

derivative can be:

- unbounded
- incomputable

Выход: введем условия регуляризации на функцию $f(X, \theta)$

$$\arg \max_{\|f\|_{Lip} \leq K} V(G, D)$$

Определения

Lipschitz norm:

$$\|g\|_{Lip} := \sup_h \sigma(\nabla g(h))$$

Спектральная норма матрицы:

$$\sigma(A) := \max_{h: h \neq 0} \frac{\|Ah\|_2}{\|h\|_2} = \max_{\|h\|_2 \leq 1} \|Ah\|_2 = \sigma_{\max}$$

Спектральная нормализация:

$$\bar{W}_{SN}(W) := \frac{W}{\sigma(W)}$$

Контроль константы Липшица

$$\|g\|_{Lip} = \sigma(W), \|a\|_{Lip} = 1, \sigma(\bar{W}_{SN}(W)) = 1$$

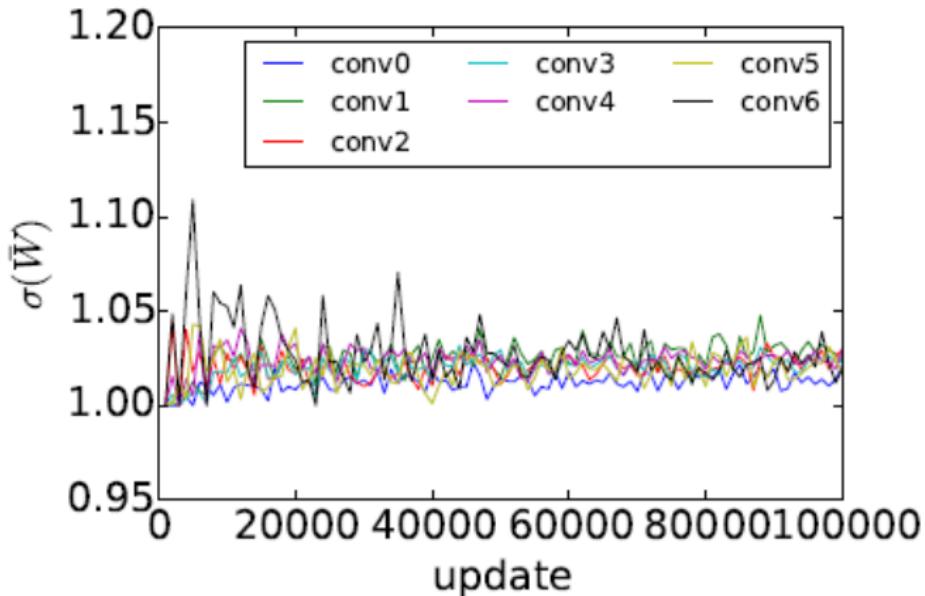
$$\|g_1 \circ g_2\|_{Lip} \leq \|g_1\|_{Lip} \cdot \|g_2\|_{Lip}$$

$$\|f\|_{Lip} \leq \prod_{l=1}^{L+1} \|(h_{l-1} \mapsto W^l h_{l-1})\|_{Lip} = \prod_{l=1}^{L+1} \sigma(W^l)$$

После спектральной нормализации:

$$\|f\|_{Lip} \leq 1$$

Спектральная норма каждого слоя



Ускоряем подсчет сингулярного числа

Algorithm 1 SGD with spectral normalization

- Initialize $\tilde{u}_l \in \mathcal{R}^{d_l}$ for $l = 1, \dots, L$ with a random vector (sampled from isotropic distribution).
- For each update and each layer l :
 1. Apply power iteration method to a unnormalized weight W^l :

$$\tilde{v}_l \leftarrow (W^l)^T \tilde{u}_l / \| (W^l)^T \tilde{u}_l \|_2 \quad (20)$$

$$\tilde{u}_l \leftarrow W^l \tilde{v}_l / \| W^l \tilde{v}_l \|_2 \quad (21)$$

2. Calculate \bar{W}_{SN} with the spectral norm:

$$\bar{W}_{\text{SN}}^l(W^l) = W^l / \sigma(W^l), \text{ where } \sigma(W^l) = \tilde{u}_l^T W^l \tilde{v}_l \quad (22)$$

3. Update W^l with SGD on mini-batch dataset \mathcal{D}_M with a learning rate α :

$$W^l \leftarrow W^l - \alpha \nabla_{W^l} \ell(\bar{W}_{\text{SN}}^l(W^l), \mathcal{D}_M) \quad (23)$$

Обучение

$$\frac{\partial \bar{W}_{SN}(W)}{\partial W_{ij}} = \frac{E_{ij}}{\sigma(W)} - \frac{W \partial \sigma(W)}{\sigma(W)^2 \partial W_{ij}} = \frac{E_{ij} - [u_1 v_1^T]_{ij} \bar{W}_{SN}}{\sigma(W)}$$

E_{ij} - матрица с 1 на (i, j) месте

u_1 - левый сингулярный вектор W

v_1 - правый сингулярный вектор W

В данном методе нормализации:

- Константа Липшица является единственным гиперпараметром.
- Простая реализация.
- Маленькие дополнительные вычислительные затраты.

Примеры работы



Сравнение с другими методами

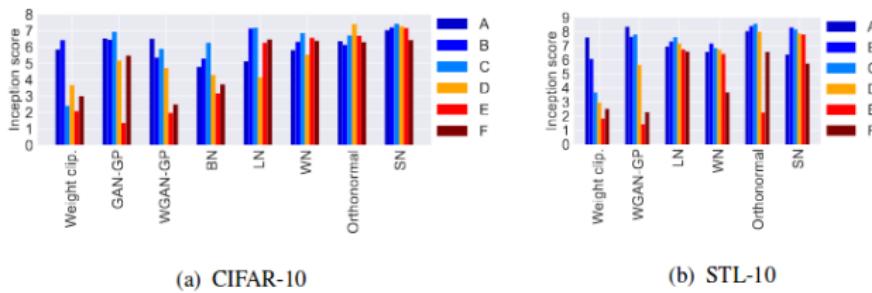


Figure 1: Inception scores on CIFAR-10 and STL-10 with different methods and hyperparameters (higher is better).

Сравнение с другими методами

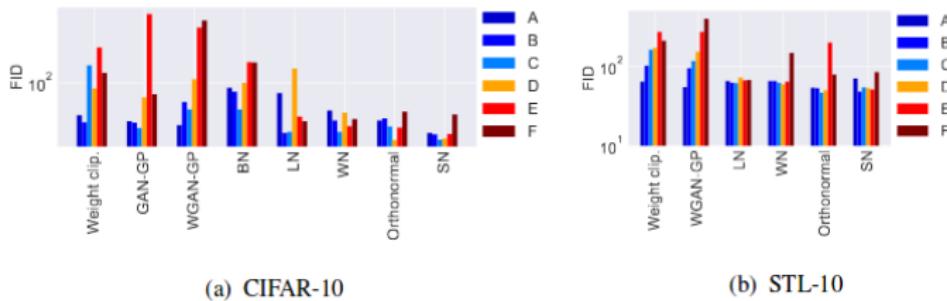


Figure 2: FIDs on CIFAR-10 and STL-10 with different methods and hyperparameters (lower is better).

Архитектуры

Table 3: Standard CNN models for CIFAR-10 and STL-10 used in our experiments on image Generation. The slopes of all IReLU functions in the networks are set to 0.1.

$z \in \mathbb{R}^{128} \sim \mathcal{N}(0, I)$	RGB image $x \in \mathbb{R}^{M \times M \times 3}$
dense $\rightarrow M_g \times M_g \times 512$	3×3 , stride=1 conv 64 IReLU
4×4 , stride=2 deconv. BN 256 ReLU	4×4 , stride=2 conv 64 IReLU
4×4 , stride=2 deconv. BN 128 ReLU	3×3 , stride=1 conv 128 IReLU
4×4 , stride=2 deconv. BN 64 ReLU	4×4 , stride=2 conv 128 IReLU
3×3 , stride=1 conv. 3 Tanh	3×3 , stride=1 conv 256 IReLU
(a) Generator, $M_g = 4$ for SVHN and CIFAR10, and $M_g = 6$ for STL-10	4×4 , stride=2 conv 256 IReLU
	3×3 , stride=1 conv. 512 IReLU
	dense $\rightarrow 1$
	(b) Discriminator, $M = 32$ for SVHN and CIFAR10, and $M = 48$ for STL-10

Архитектуры

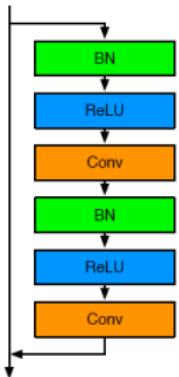


Figure 8: ResBlock architecture. For the discriminator we removed BN layers in ResBlock.

Table 4: ResNet architectures for CIFAR10 dataset. We use similar architectures to the ones used in Gulrajani et al. (2017).

$z \in \mathbb{R}^{128} \sim \mathcal{N}(0, I)$
dense, $4 \times 4 \times 256$
ResBlock up 256
ResBlock up 256
ResBlock up 256
BN, ReLU, 3×3 conv, 3 Tanh

(a) Generator

RGB image $x \in \mathbb{R}^{32 \times 32 \times 3}$
ResBlock down 128
ResBlock down 128
ResBlock 128
ResBlock 128
ReLU
Global sum pooling
dense $\rightarrow 1$

(b) Discriminator

Архитектуры

Table 5: ResNet architectures for STL-10 dataset.

$z \in \mathbb{R}^{128} \sim \mathcal{N}(0, I)$	RGB image $x \in \mathbb{R}^{48 \times 48 \times 3}$
dense, $6 \times 6 \times 512$	ResBlock down 64
ResBlock up 256	ResBlock down 128
ResBlock up 128	ResBlock down 256
ResBlock up 64	ResBlock down 512
BN, ReLU, 3×3 conv, 3 Tanh	ResBlock 1024
(a) Generator	ReLU
	Global sum pooling
	dense $\rightarrow 1$
	(b) Discriminator

- Takeru Miyato, Toshiki Kataoka, Masanori Koyama, Yuichi Yoshida. Spectral Normalization for Generative Adversarial Networks
- Generated images and pretrained models
- Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, Yoshua Bengio. Generative Adversarial Networks