Adaptive Skip-gram

Sergey Bartunov, DeepMind

(joint work with Dmitry Kondrashkin, Anton Osokin and Dmitry Vetrov)

This lecture

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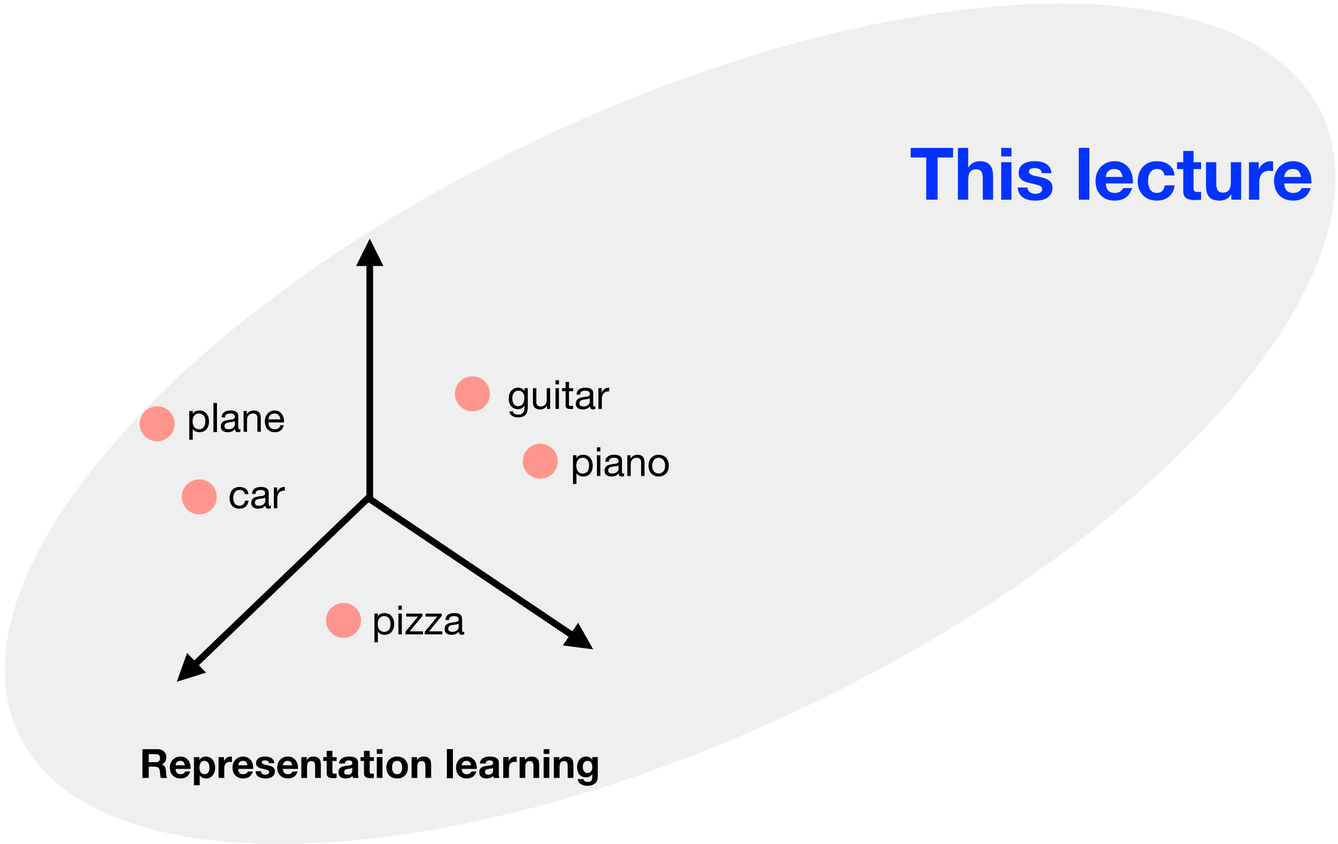

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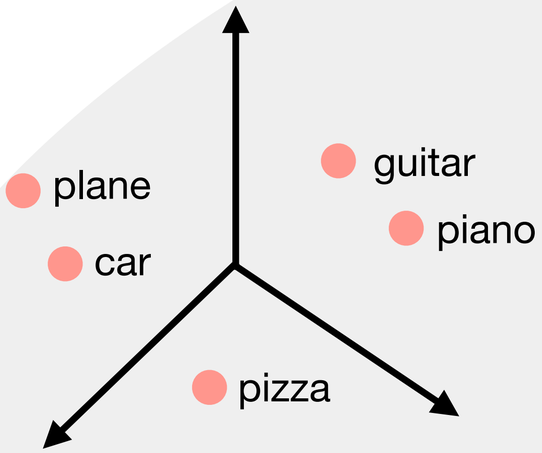
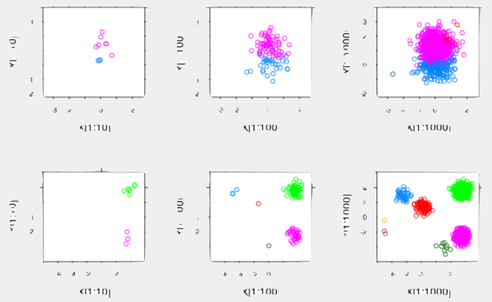
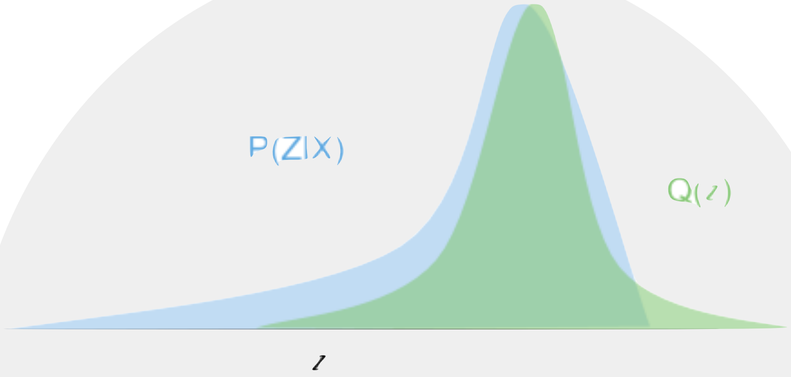


Representation learning

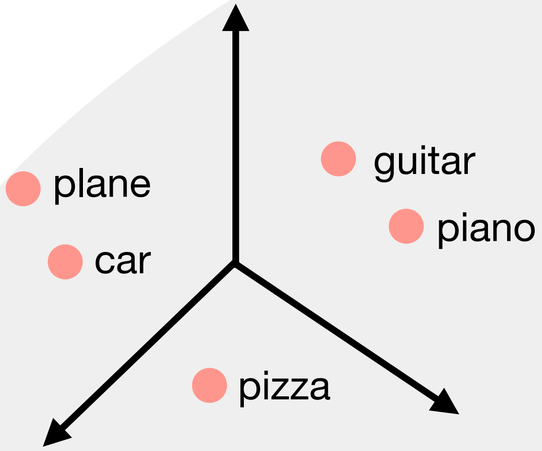


Bayesian nonparametrics



Variational inference

This lecture



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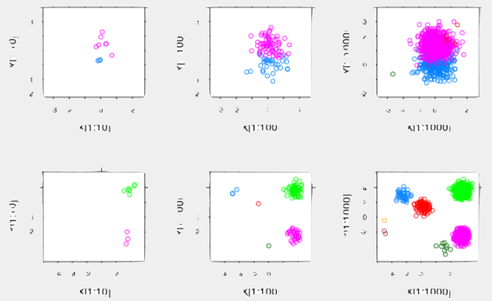

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Elephants, the largest existing land animals, are mammals of the family Elephantidae. Three species are currently recognised: the African bush elephant, the African forest elephant, and the Asian elephant.

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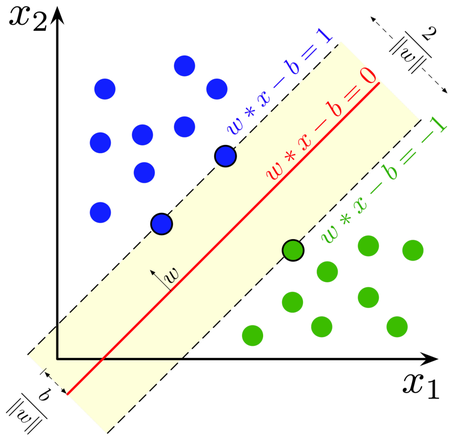

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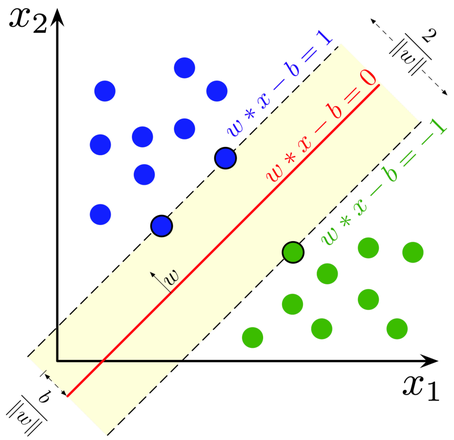

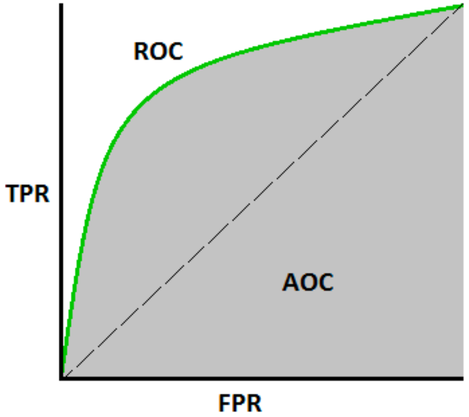

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Supervision / analysis

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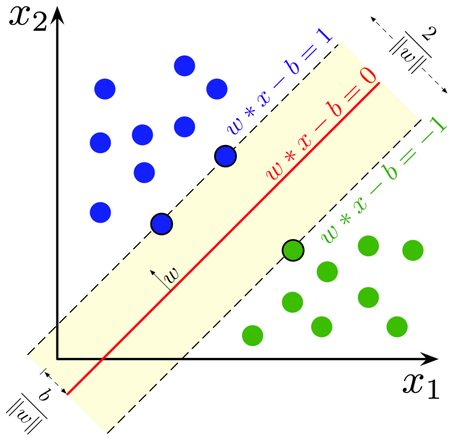
Feature extraction



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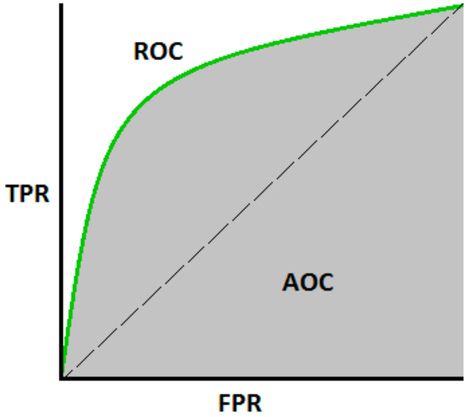

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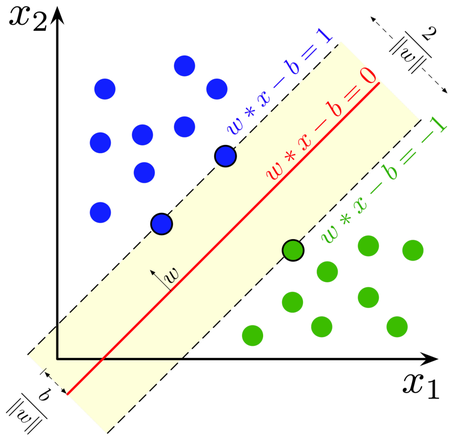

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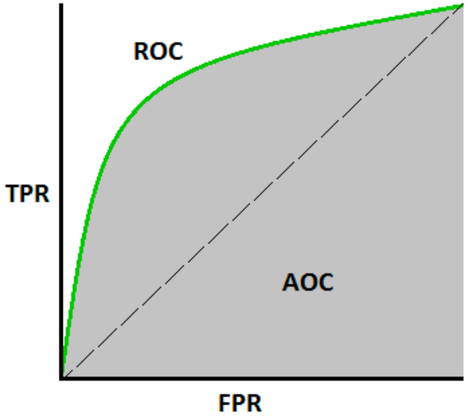

Supervision / analysis

Word embeddings

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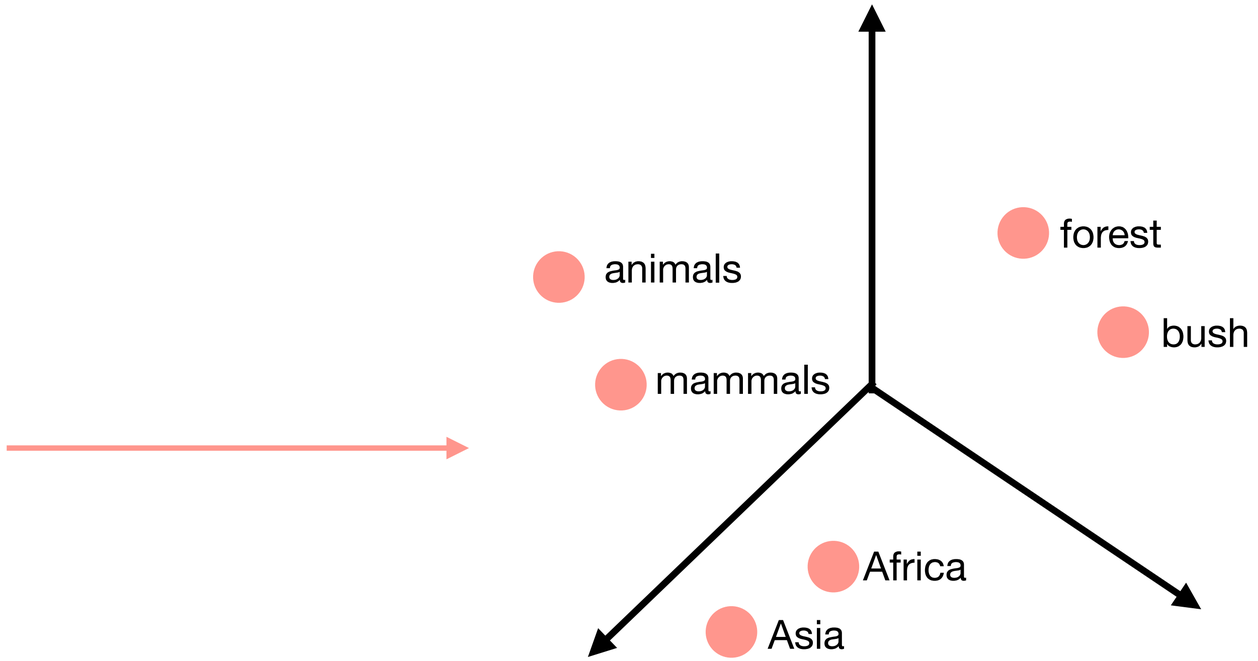
Text data



Goal: learn close embeddings for semantically similar words

Word embeddings in Skip-gram

Elephants, the largest existing land animals, are mammals of the family Elephantidae. Three species are currently recognised: the African bush elephant, the African forest elephant, and the Asian elephant.

Text data

Word	Input embedding	Output embedding
mammals	[0.5, 3.45, -1.26]	[0.5, 3.45, -1.26]
animals	[0.45, -0.5, -2.6]	[-0.93, 1.2, 0.]
land	[0.18, 0.45, 0.6]	[-1.52, -0.39, 0.89]
…		

Skip-gram model

…largest existing land animals, are **mammals** of the family Elephantidae...

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Word **x**

Skip-gram model

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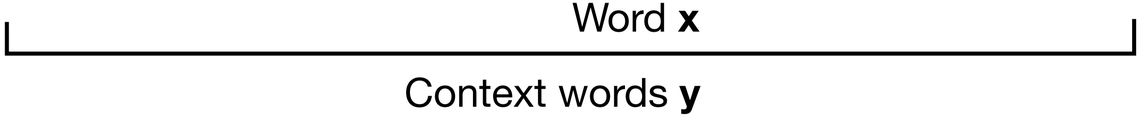

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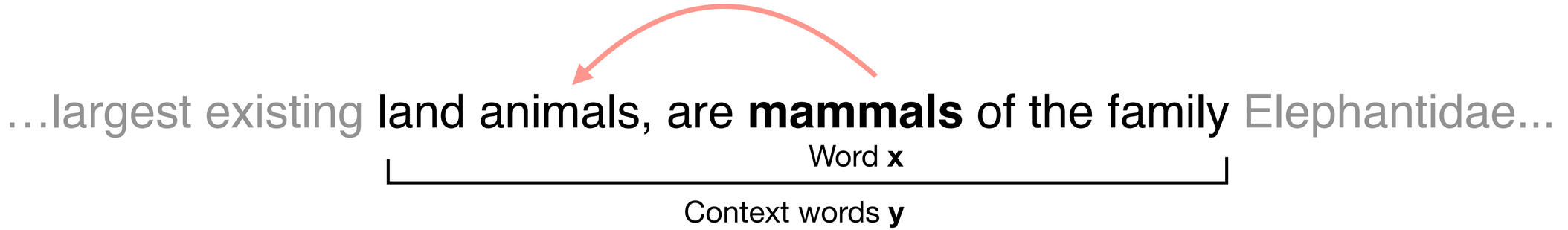
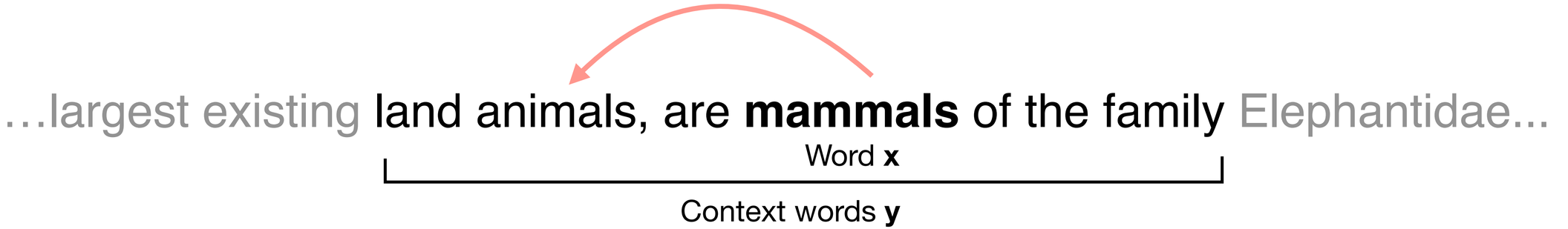


$$p(\mathbf{y}|x) = \prod_j p(y_j|x)$$

Skip-gram model



$$p(\mathbf{y}|x) = \prod_j p(y_j|x) \qquad p(v|w) = \frac{\exp(\text{in}_w^T \text{out}_v)}{\sum_{v'} \exp(\text{in}_w^T \text{out}_{v'})}$$

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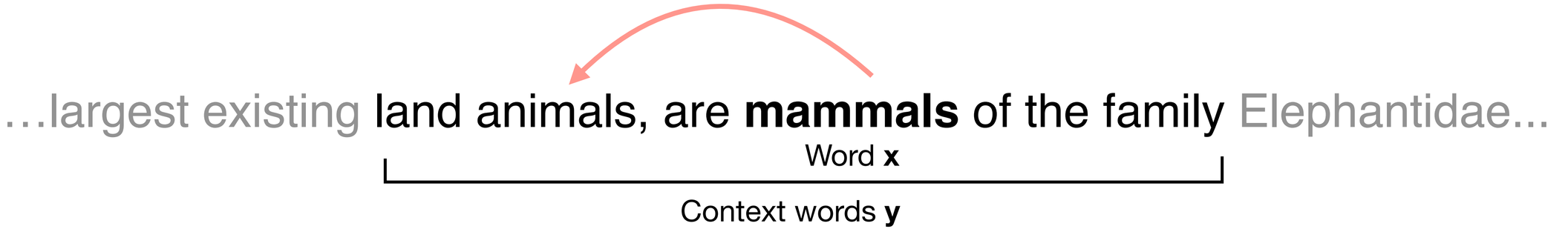

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Distributional hypothesis: similar words appear in similar contexts

Training Skip-gram

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…largest existing land **animals**, are mammals of the family Elephantidae...

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Gradient update: $\theta \leftarrow \theta + \eta \nabla \log p(\mathbf{y}_i | x_i)$

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Summary

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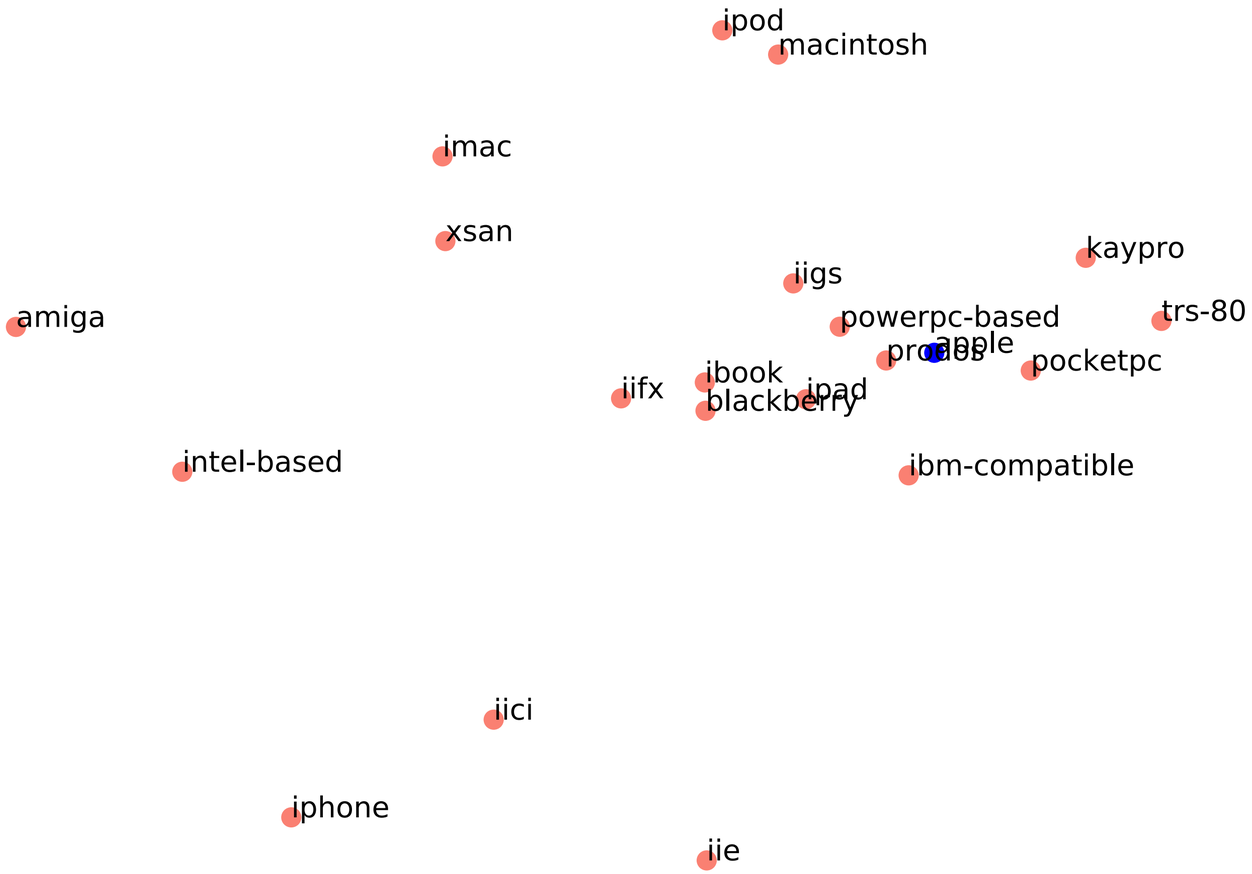
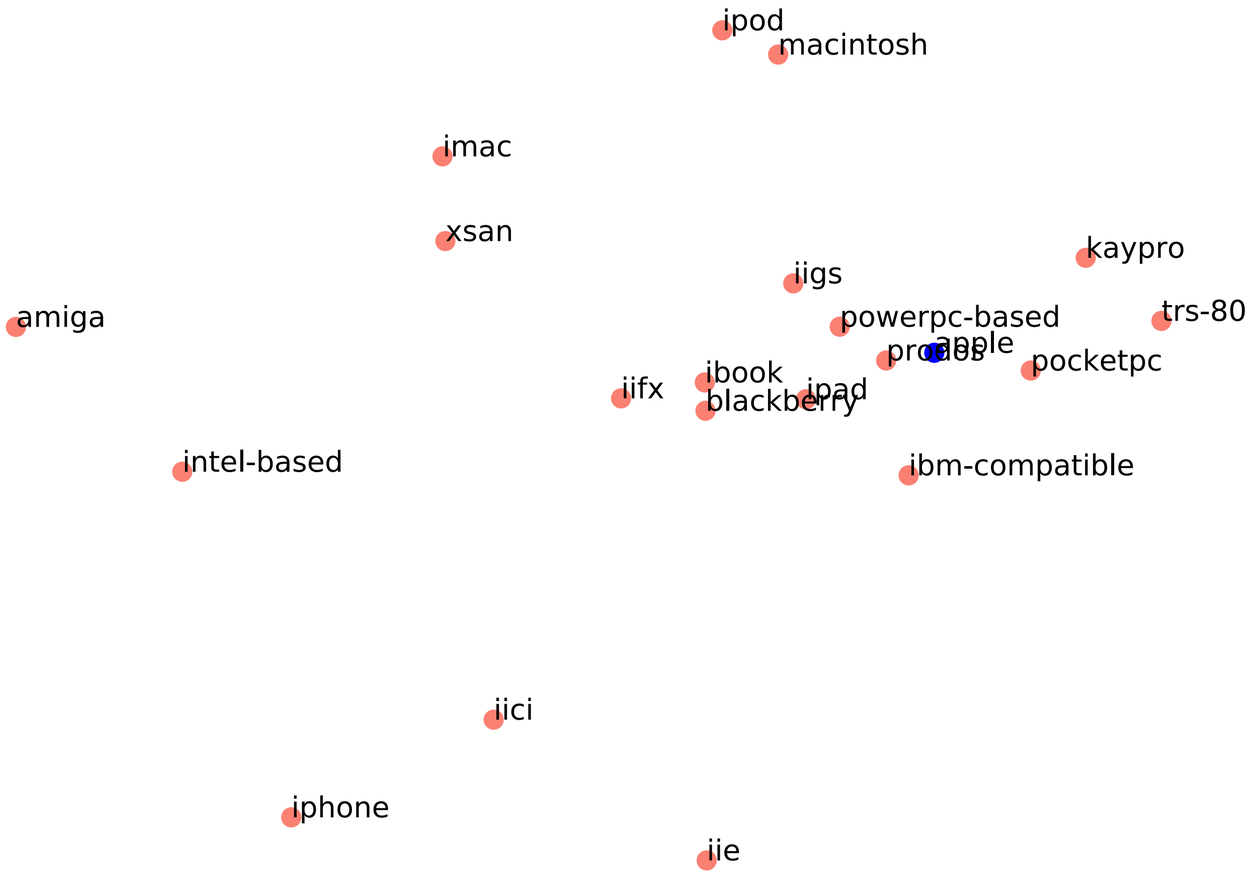
- Sparse gradients (not explaining why in this lecture)
- Very efficient parallel training
- Are we done now?

Skip-gram: embeddings



T-SNE visualisation of 20 nearest neighbours of word “apple”.
300D Skip-gram trained on wikipedia

Skip-gram: embeddings



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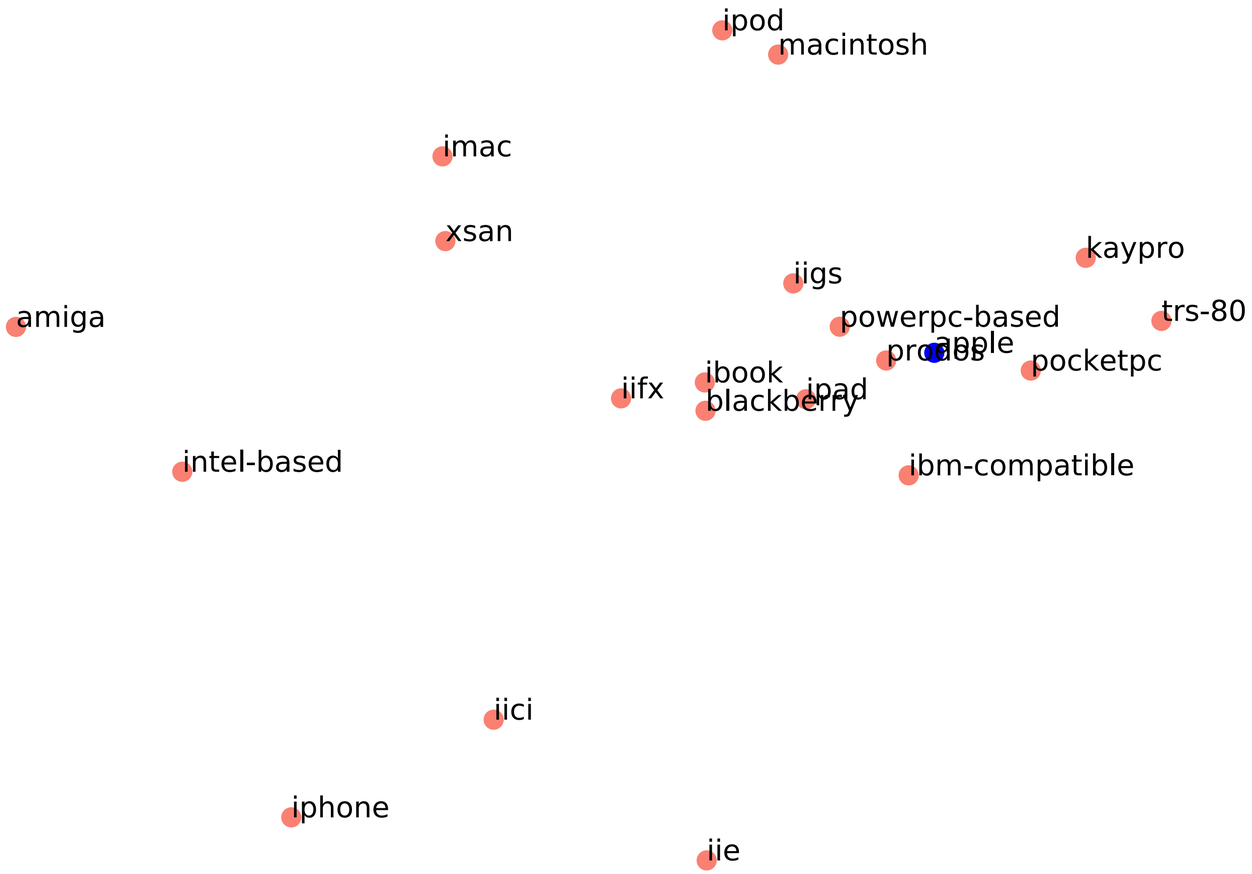

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- For some words only one *meaning* is captured
- For others meanings get uncontrollably mixed up

Word ambiguity

Mammal (disambiguation)

From Wikipedia, the free encyclopedia

A **mammal** is a member of a class of vertebrates.

Mammal, **mammals** or **mammalia** may also refer to:

- Mammal (band), an Australian band
- *Mammal* (film), a 2016 Irish film
- The Mammals, an American folk rock band
- *Mammals* (play), a play by Amelia Bullmore
- "Mammal", a song by They Might Be Giants on the album *Apollo 18*
- *Mammal* (EP), the debut EP by Mammal
- *Mammal* (album), an album by Irish black metal band Altar of Plagues
- *Mammalia*, the debut album by drum & bass band Comparative Anatomy

Look up ***bank*** in Wiktionary, the free dictionary.

A **bank** is a financial institution and a financial intermediary that accepts deposits and channels those deposits into lending activities.

Bank or **banking** may also refer to:

Geography

- Bánk, a village and municipality in the comitat of Nógrád, Hungary
- Bank, Iran, a city in Bushehr Province, Iran
- Bank junction, a major road junction in the City of London
- Bank Station (OC Transpo), a bus stop in Ottawa
- Bank Street (Ottawa), Ontario, Canada
- Bank–Monument station, a tube station in London
- Bankə, Azerbaijan
- East Bank (disambiguation)
- Left Bank (disambiguation)
- North Bank (disambiguation)
- Promysel Narimanova or Bank, Azerbaijan
- Right Bank (disambiguation)
- South Bank (disambiguation)
- West Bank (disambiguation)

People

- Bank (surname)

Arts, entertainment, and media

Films

- *DCI Banks*, a British television crime drama, aired by ITV 2010-2016
- *Overdrawn at the Memory Bank*, a 1983 telemovie
- *The Bank* (1915 film), starring Charlie Chaplin
- *The Bank* (2001 film), starring David Wenham

Music

- "Bank", a 2017 song by Brockhampton from *Saturation*
- "Bank", a 2018 song by Lil Baby from *Harder Than Ever*

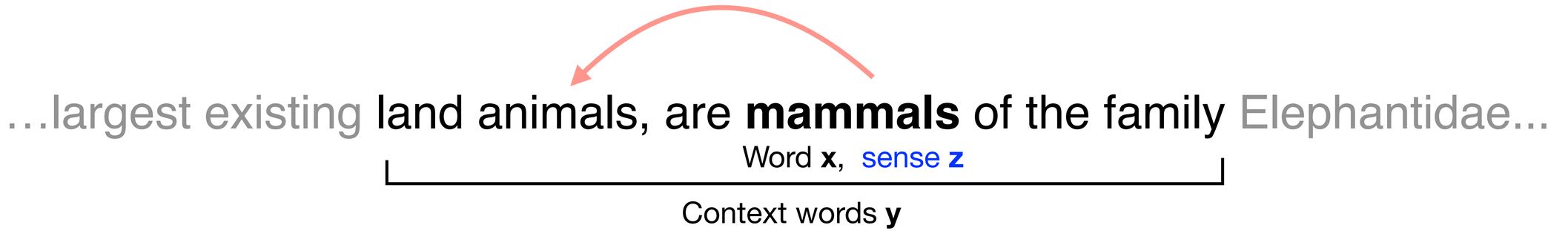
Other uses in arts, entertainment, and media

- *Memory Bank* (UK game show), a daytime game show which was shown on Five in the UK
- *Memory Banks* (comic strip), a British comic strip

Computing and technology

- Data bank, a storage area for information in telecommunications
- Memory bank, a logical unit of storage

Solution: latent-variable model



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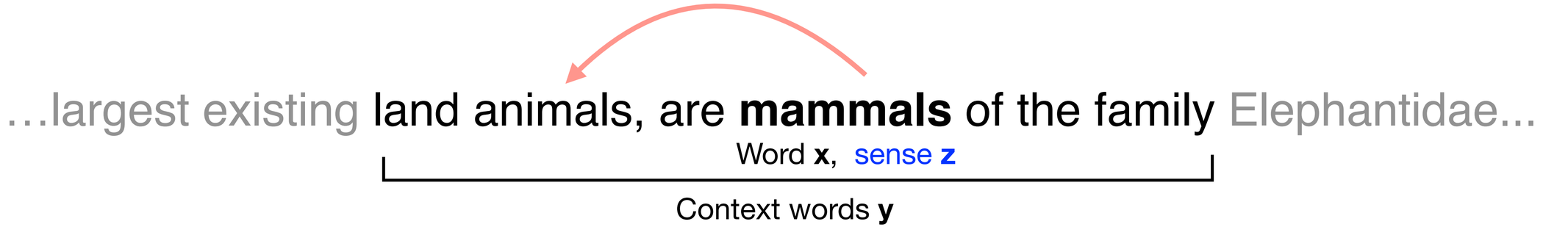

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animals	[0.76, 0.8, -1.1]	[0.3, 1.2, -1.7]	[0.52, -0.24, -0.073]	[-0.93, 1.2, 0.]
land	[0.1, -0.23, 0.58]	[2.0, 0.21, 2.0]	[0.49, -0.022, -0.71]	[-1.52, -0.39, 0.89]
	…	…	…	

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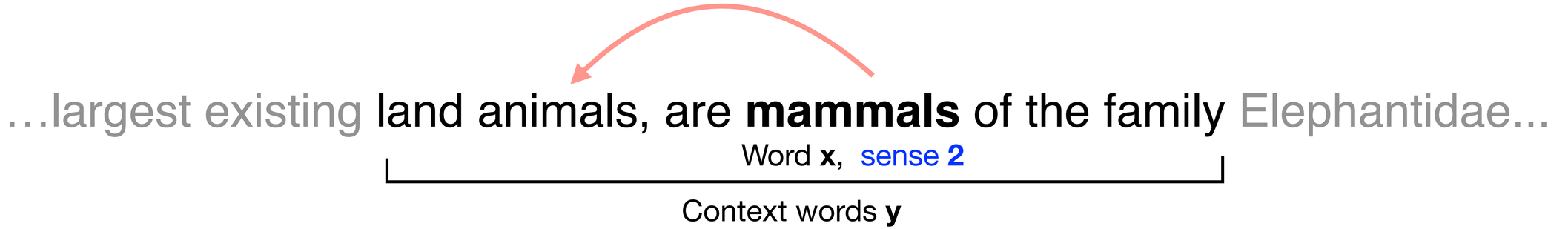

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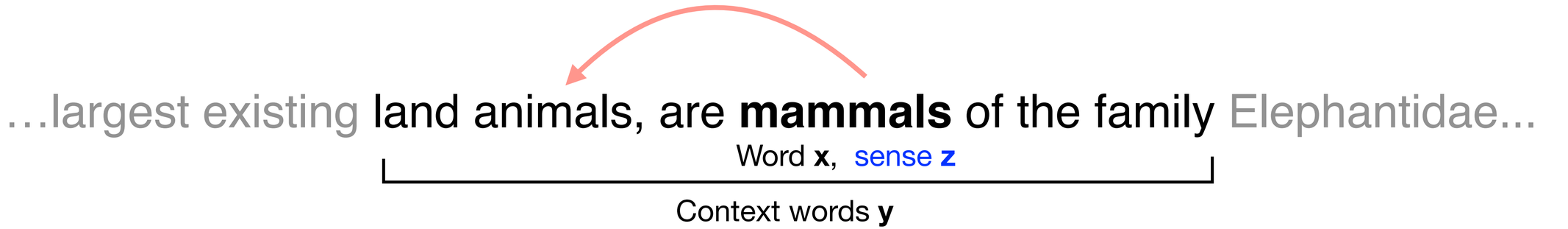

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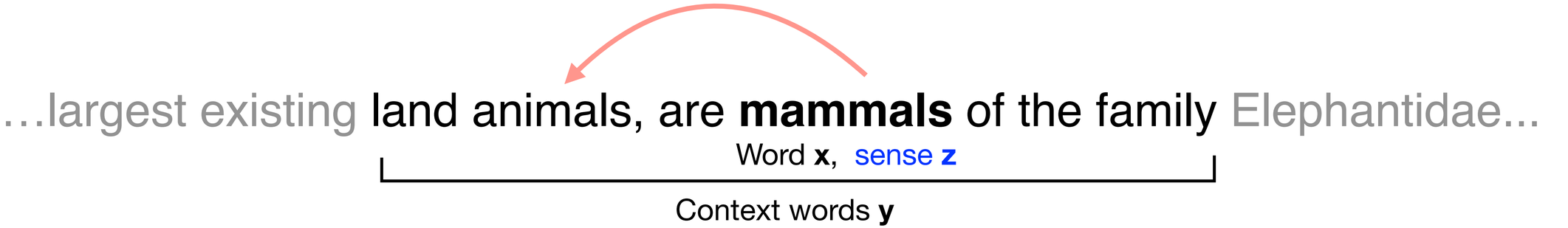

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Mixture model of word's contexts

$$\pi_w|\alpha \sim \text{Dirichlet}(\alpha,\ldots,\alpha)$$

$$z_i|\boldsymbol{\pi}, x_i \sim \text{Categorical}(\pi_{x_i})$$

$$y_{ij}|x_i,z_i \sim \text{Softmax}(\text{in}_{x_i,z_i}^T \text{out}_{y_{ij}})$$

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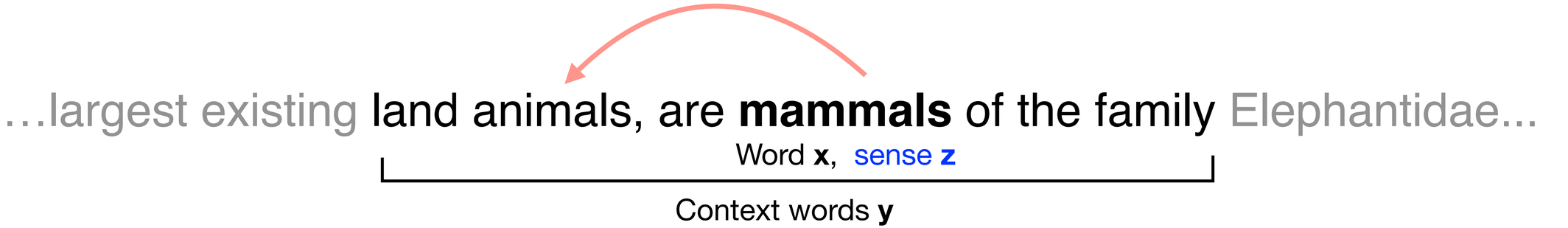

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$$\log p(\mathcal{D}|\theta) = \log \int p(\Pi|\alpha) \int p(Z|\Pi) \prod_{i=1}^N p(\mathbf{y}_i|x_i, z_i, \theta) dZ d\Pi \to \max_\theta$$

Intractable to compute or optimize directly

Training via variational EM

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- Consider a fully factored **posterior approximation**

$$q(Z, \Pi) = \prod_{w=1}^{V} q(\pi_w) \prod_{i=1}^{N} q(z_i) \approx p(Z, \Pi | \mathcal{D}, \theta)$$

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- Introduce a **variational lower bound**:

$$\begin{aligned} \mathcal{L}(q, \theta) &= \mathbb{E}_{q(Z,\Pi)} \left[\log p(\Pi, Z, Y | X, \theta) - \log q(Z, \Pi) \right] \\ &\leq \log p(\mathcal{D} | \theta) \end{aligned}$$

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$$q(Z, \Pi) = \prod_{w=1}^{V} q(\pi_w) \prod_{i=1}^{N} q(z_i) \approx p(Z, \Pi | \mathcal{D}, \theta)$$

- Introduce a **variational lower bound**:

$$\begin{aligned} \mathcal{L}(q, \theta) &= \mathbb{E}_{q(Z,\Pi)} \left[\log p(\Pi, Z, Y | X, \theta) - \log q(Z, \Pi) \right] \\ &\leq \log p(\mathcal{D} | \theta) \end{aligned}$$

- Employ **stochastic variational inference** for optimization

E-step. Getting rid of local parameters

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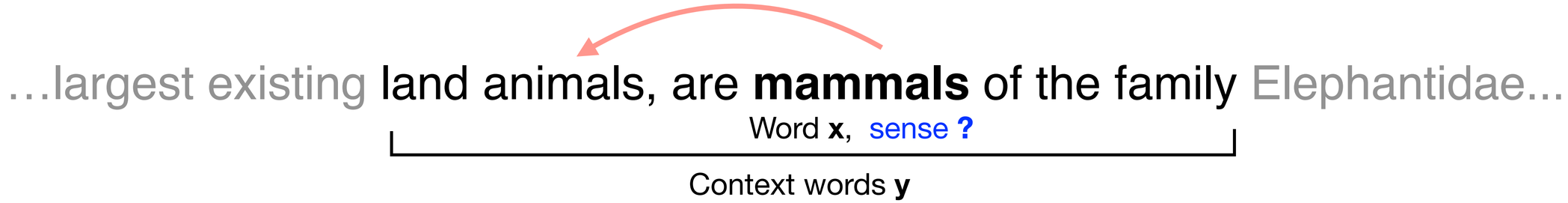

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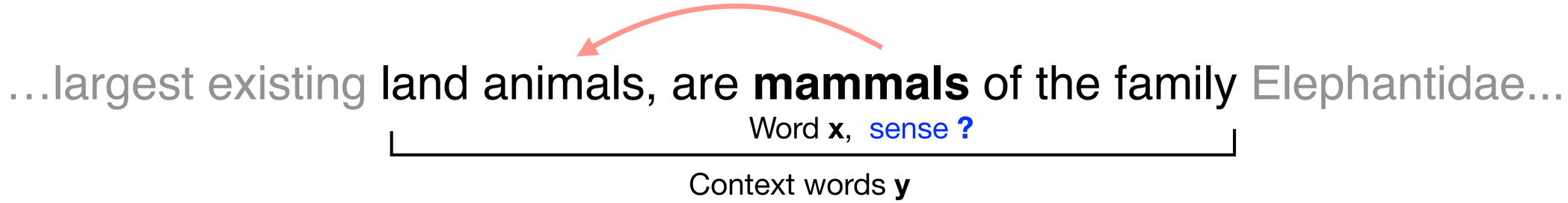

- Perform **sense disambiguation**

$$q(z_i = k)^* \propto \exp(\mathbb{E}_{q(\pi_{x_i})} \log p(z_i = k|\pi_{x_i}) + \log p(\mathbf{y}_i|x_i, z = k, \theta))$$

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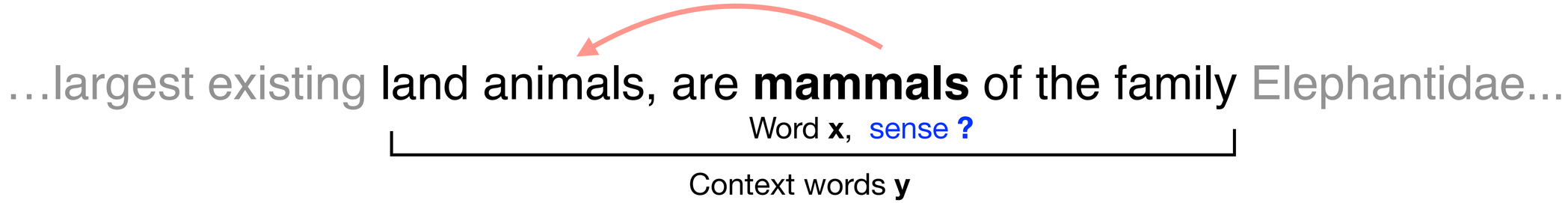

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- Consider new, **tighter** bound $\mathcal{L}^*(q(\Pi), \theta) = \mathcal{L}(q(\Pi), q^*(Z), \theta) \geq \mathcal{L}(q(\Pi), q(Z), \theta)$

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Weighted skip-gram prediction

- We can use almost the same Skip-gram training algorithm!

SVI for Multi-meaning Skip-gram

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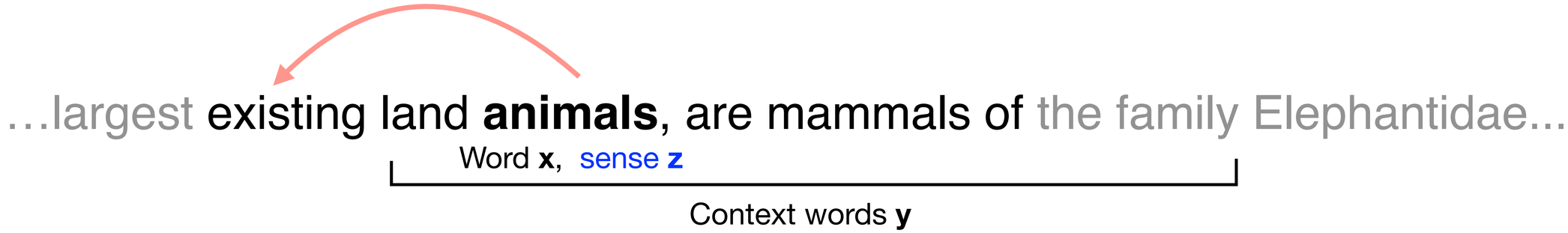

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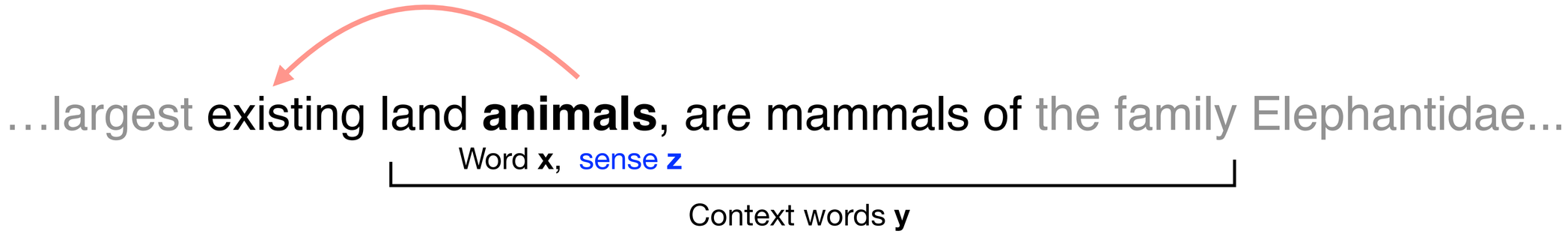

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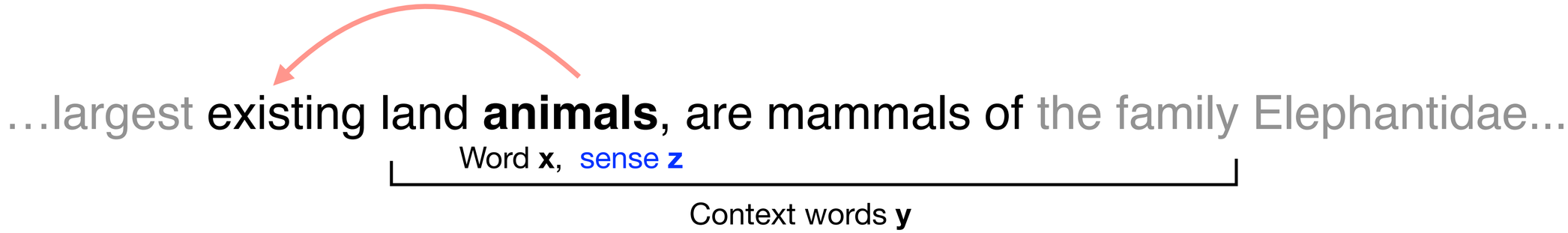

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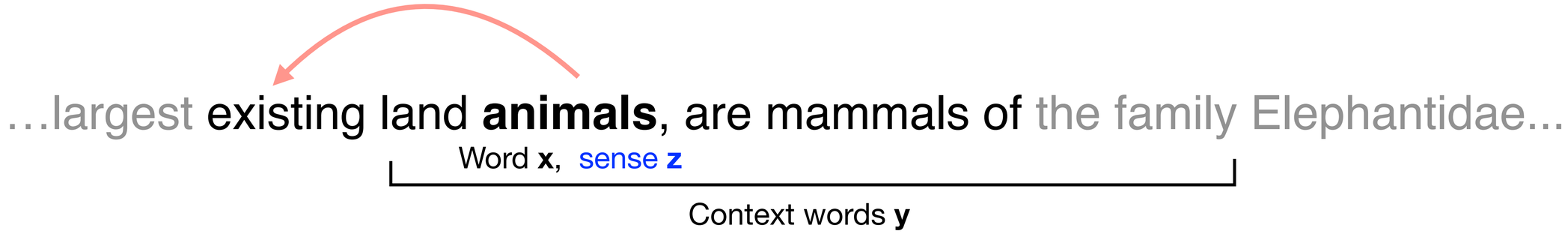

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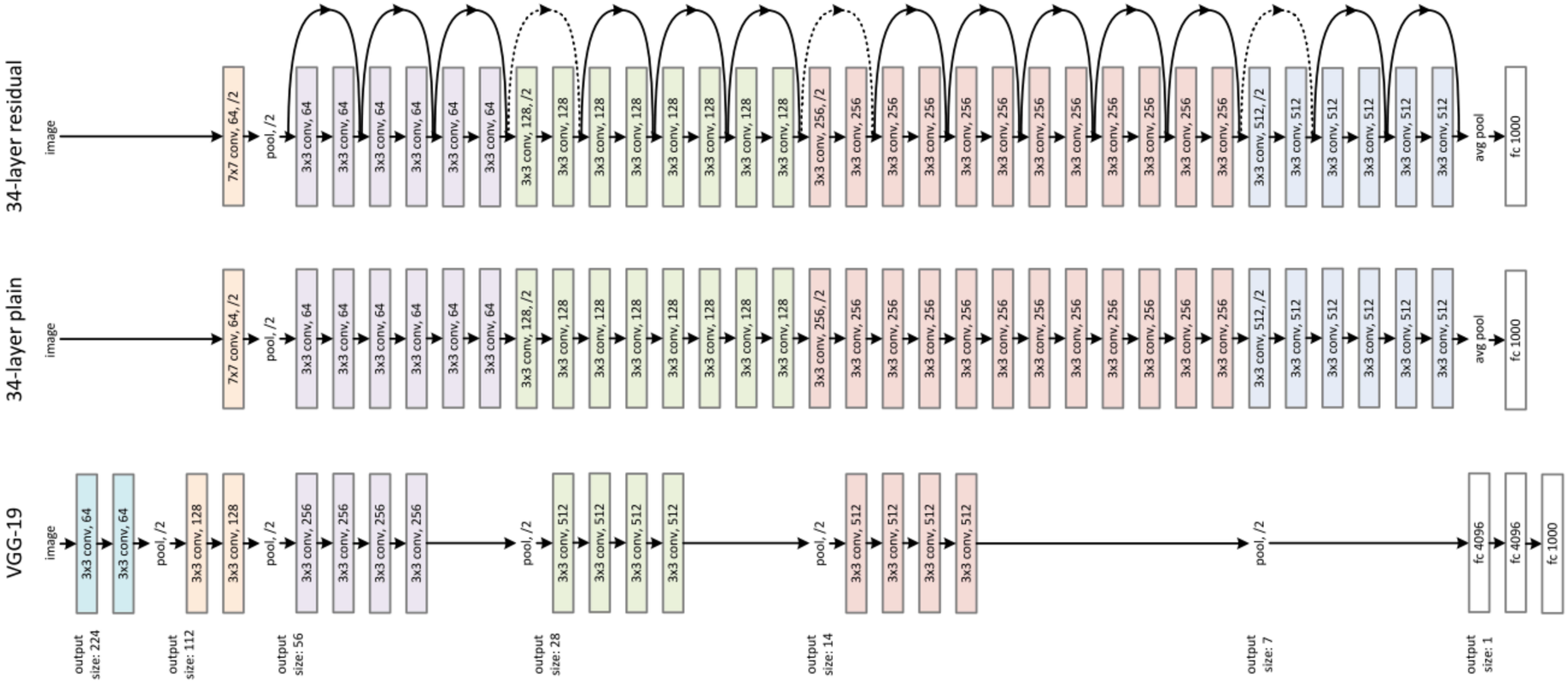
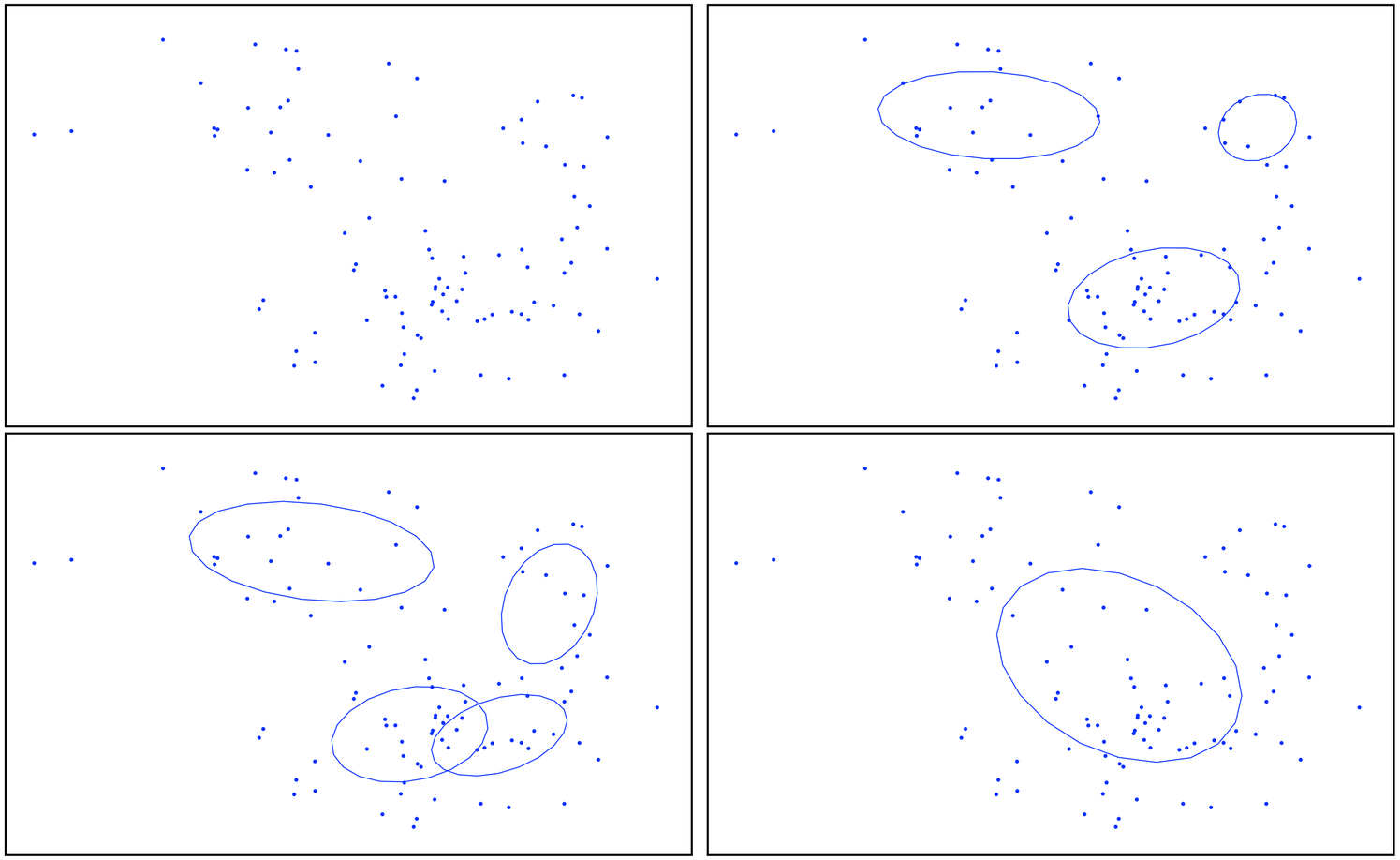
- Figure out something really smart

How many layers?



How many clusters?



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- Define a finite mixture model

$$\pi|\alpha \sim \mathrm{Dir}(\alpha_1, \ldots, \alpha_K)$$
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Model selection via Dirichlet process

- Define an infinite model density

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$$G|H, \alpha \sim \mathrm{DP}(H, \alpha)$$

- Select the model one-shot

$$p(x|X) = \int \underbrace{p(G|X)}_{\text{model}} p(x|G) dG$$

$$p(G|X) \propto \underbrace{p(G)p(X|G)}_{\text{model selection}}$$

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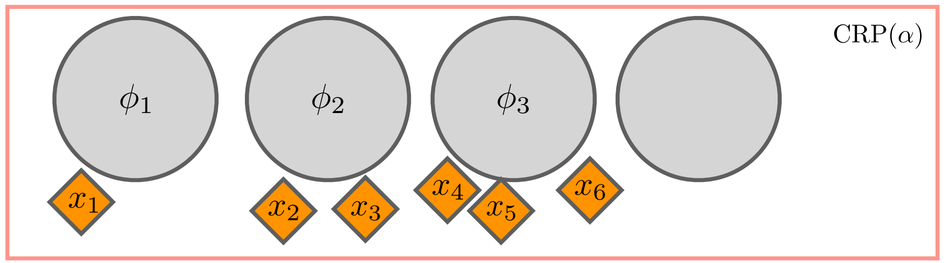

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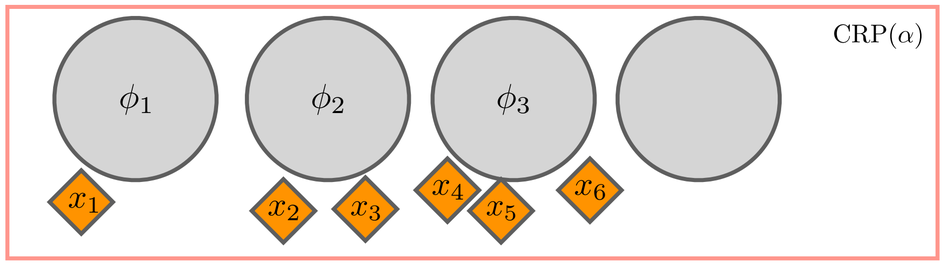

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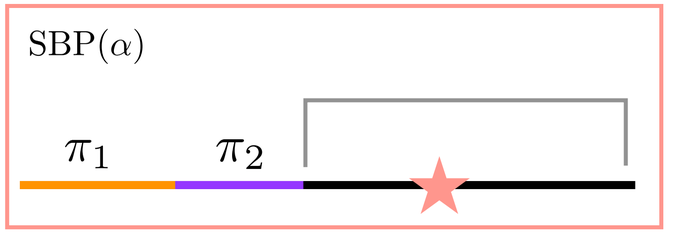

Chinese Restaurant Process

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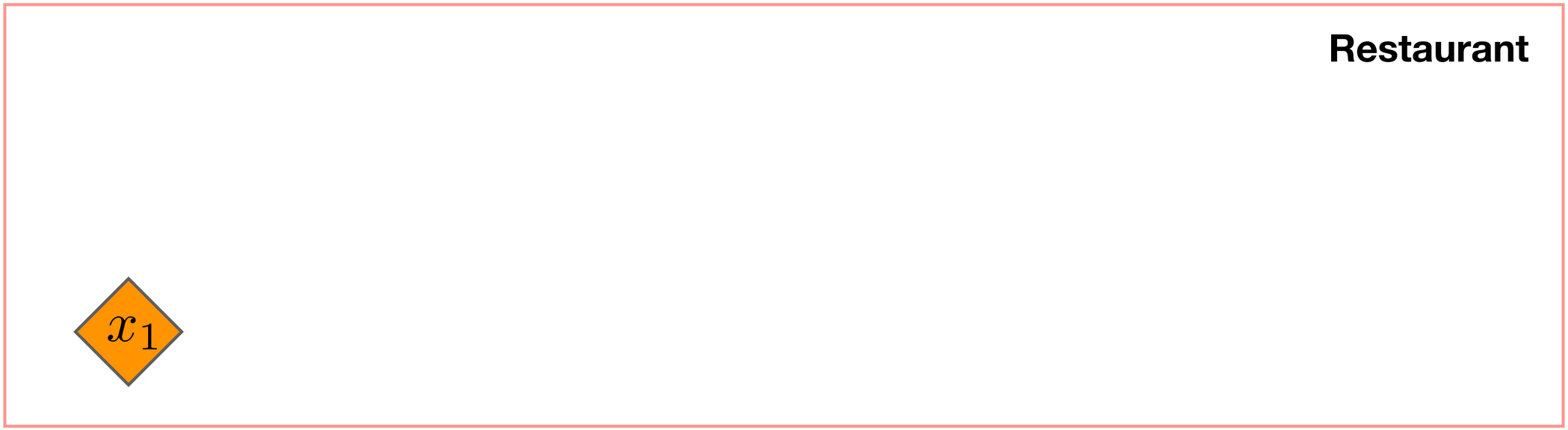

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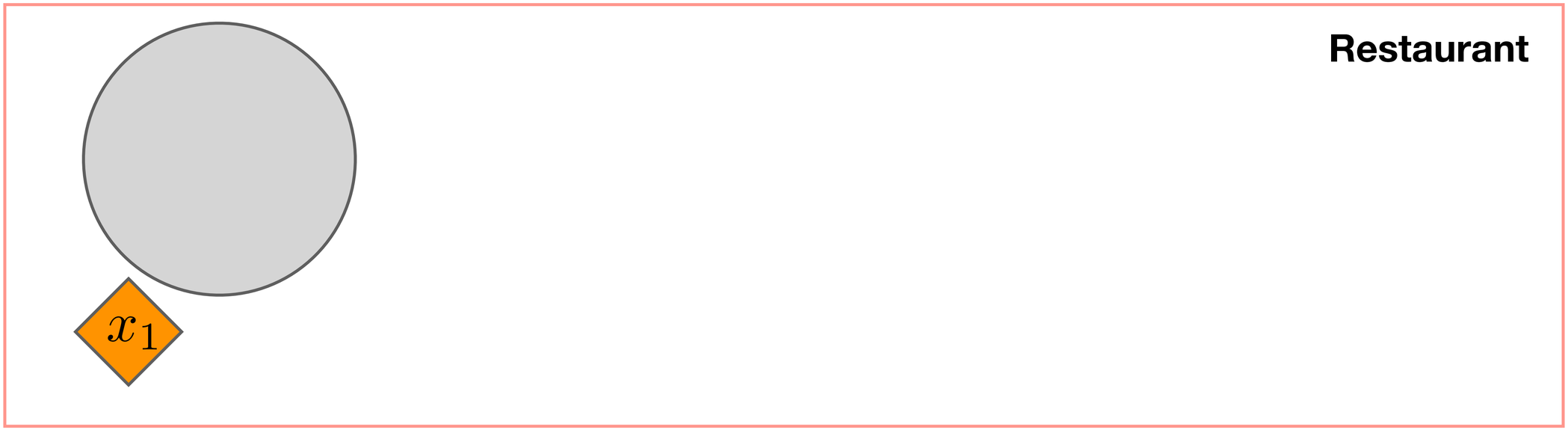

$z_1 = 1$

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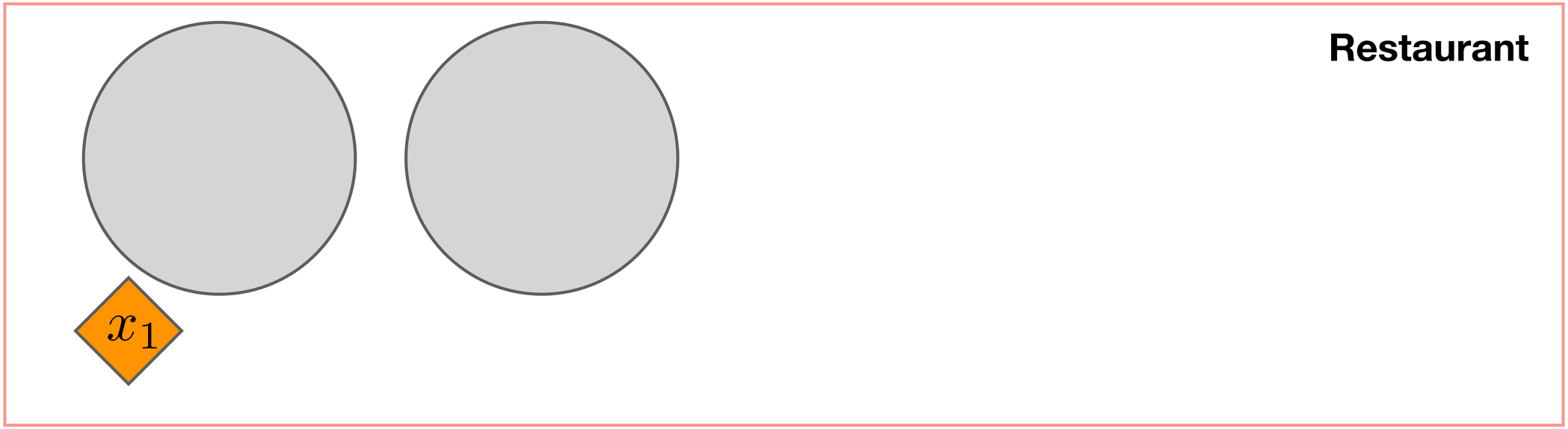

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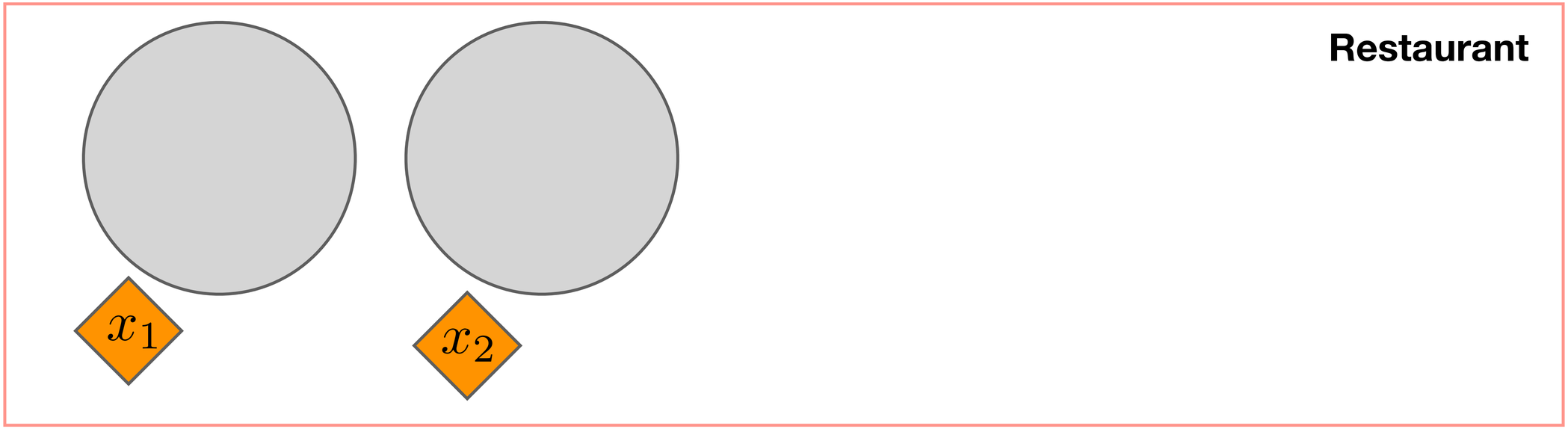

$z_1 = 1$

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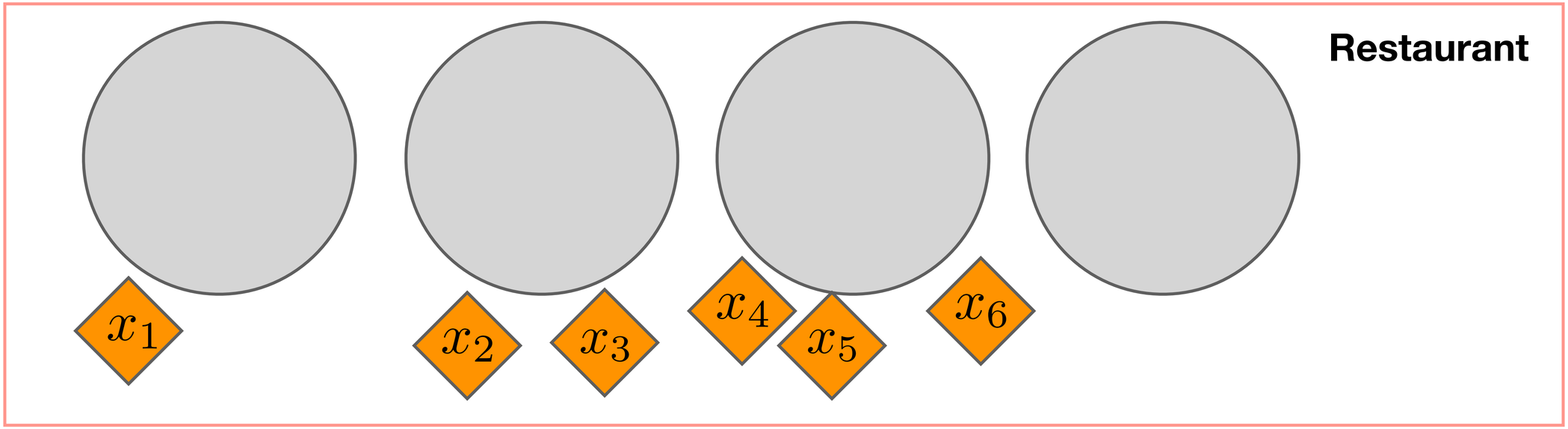
Chinese Restaurant Process



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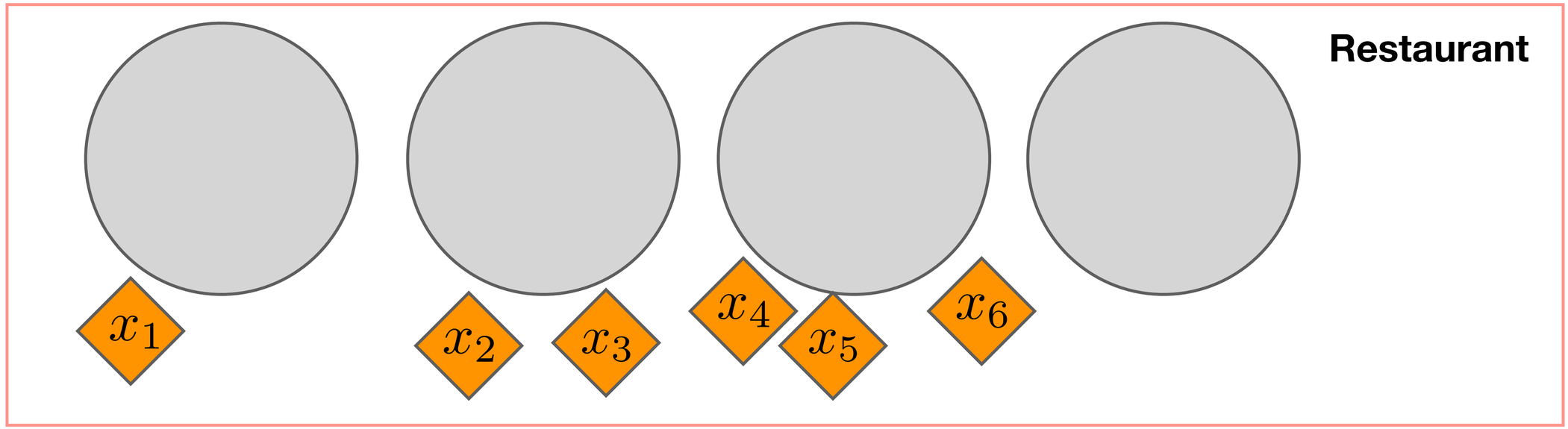

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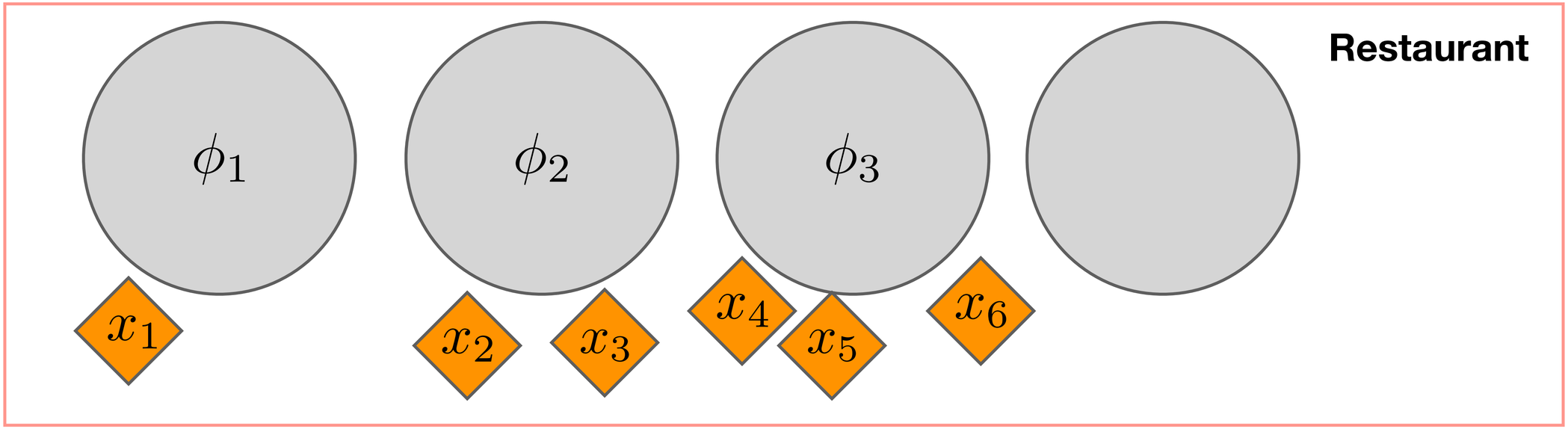

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How many meanings?

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Finite mixture of word's contexts

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Problem: data is not iid anymore, inconvenient for SVI

Stick-breaking process

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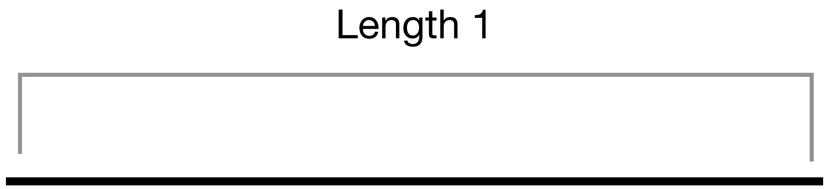
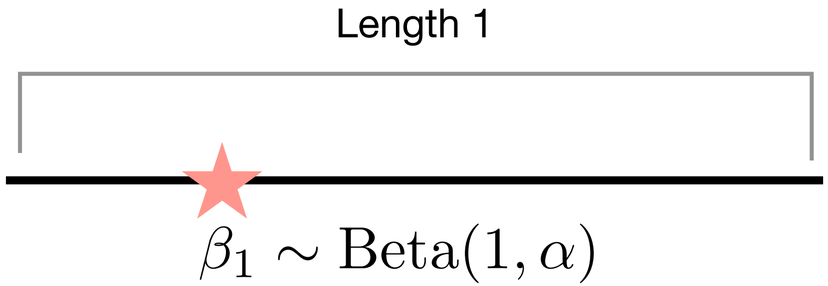
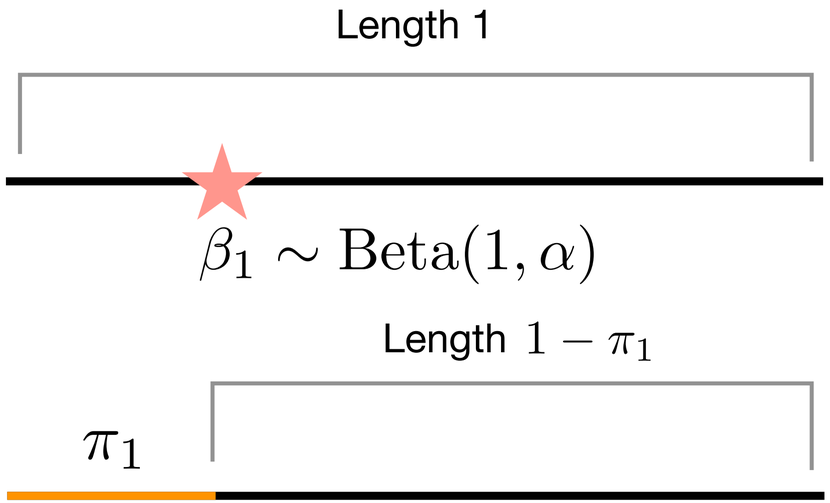


Stick-breaking process



Stick-breaking process



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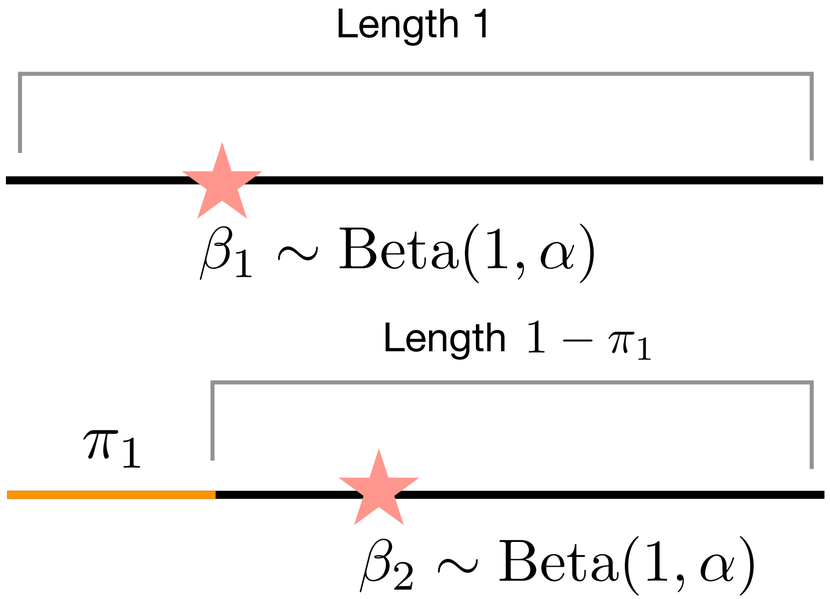

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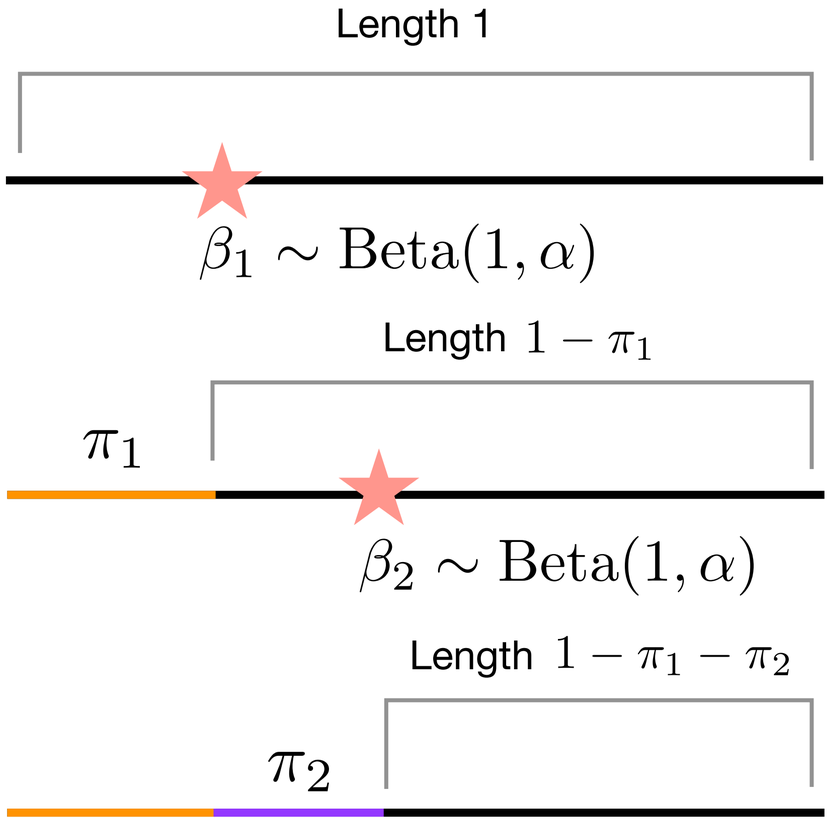
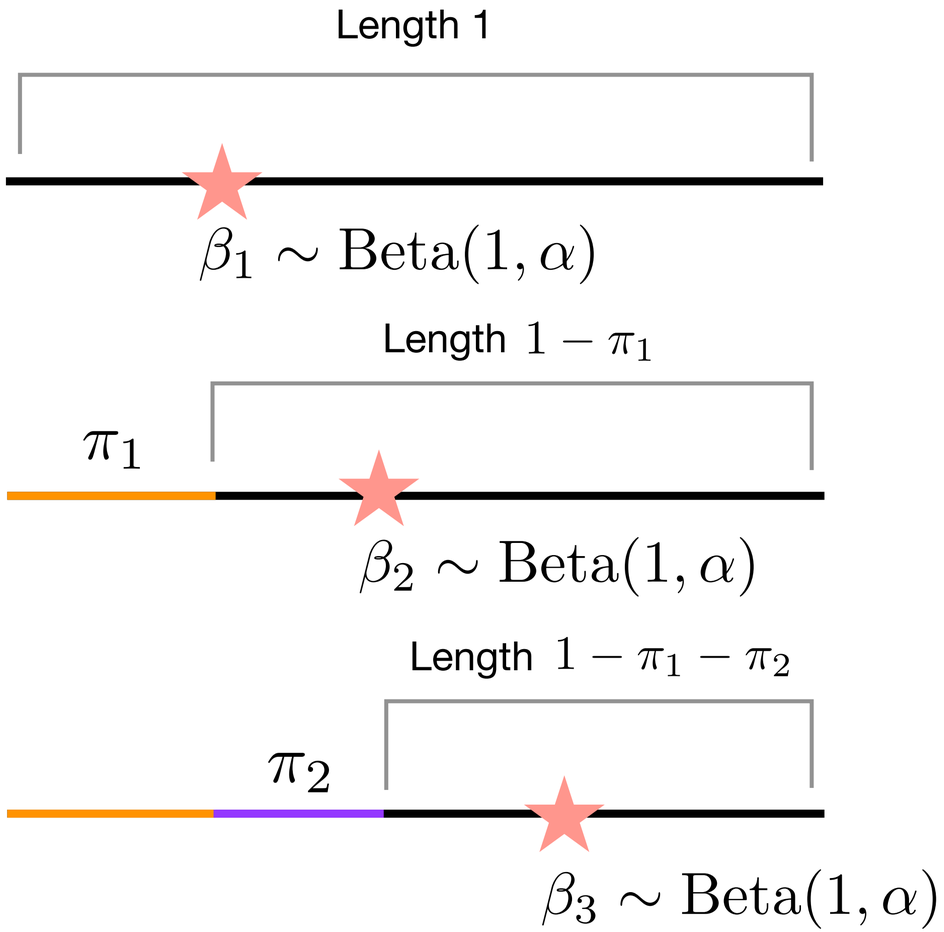


Stick-breaking process



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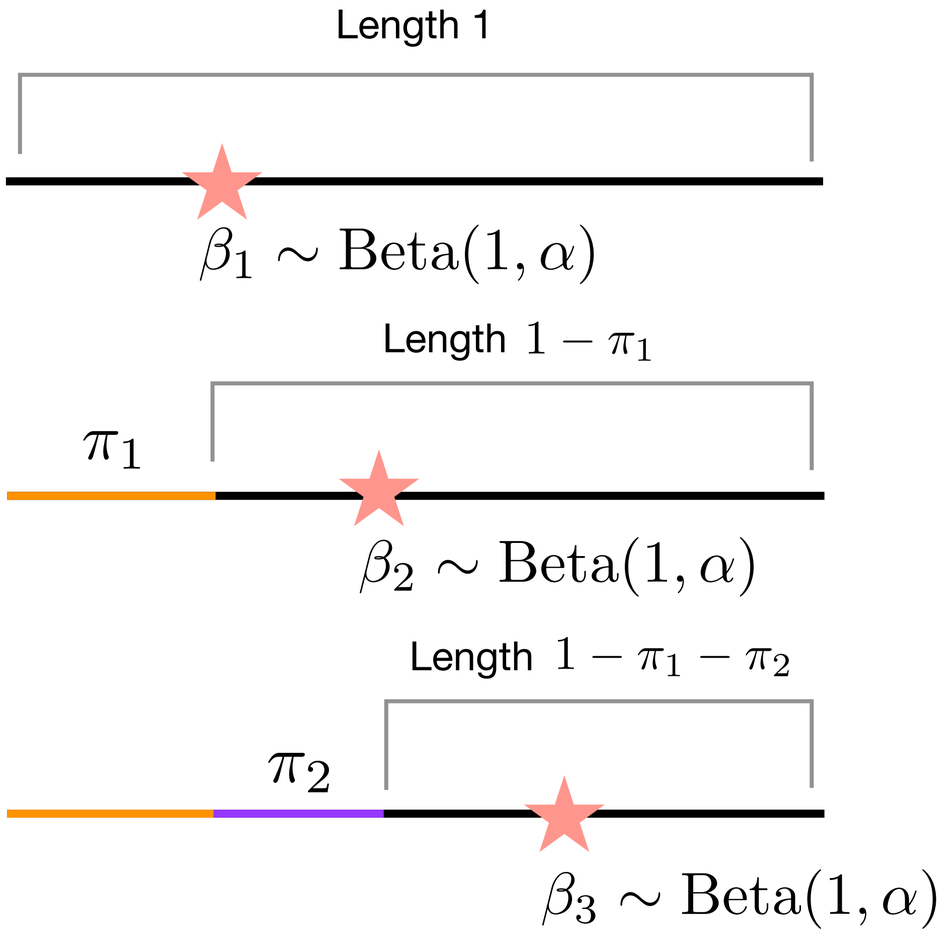
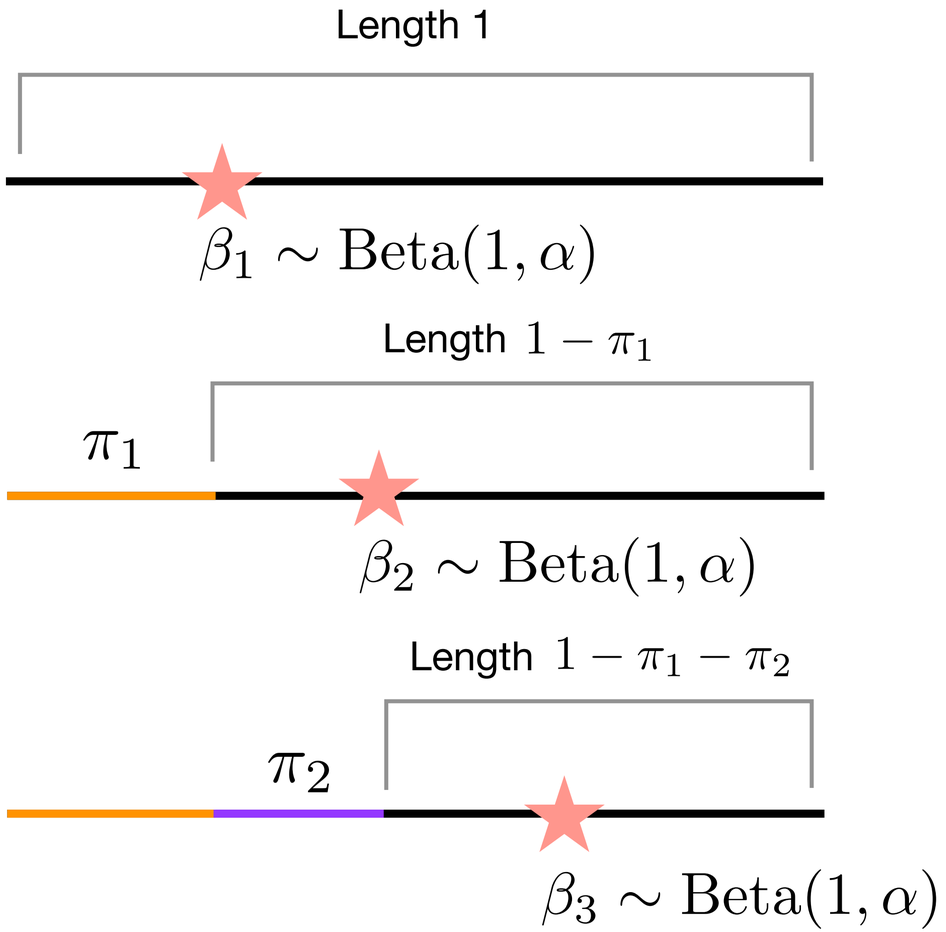


$$
\begin{aligned}
\beta_k &\sim \text{Beta}(1, \alpha) \\
\phi_k &\sim H, \quad k = 1, 2, \ldots \\
\pi_k &= \beta_k \prod_{t<k} (1 - \beta_t) = \beta_k (1 - \sum_{t<k} \pi_t) \\
z_i | \pi &\sim \text{Categorical}(\pi) \\
x_i | z_i &\sim p(x_i | \phi_k)
\end{aligned}
$$

Stick-breaking process



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$$z_i | \pi \sim \text{Categorical}(\pi)$$
$$x_i | z_i \sim p(x_i | \phi_k)$$

$$G = \sum_{k=1}^{\infty} \pi_k \delta(\phi - \phi_k) \sim \text{DP}(H, \alpha) \qquad z \sim \text{CRP}(\alpha)$$

Adaptive Skip-gram

- Prior over infinitely-many meanings
- Automatic model selection (more data - more meanings)
- Control of the meaning granularity

Infinite mixture of word's contexts

$$\beta_k|\alpha \sim \text{Beta}(1, \alpha), \quad k = 1, 2, \ldots$$

$$\pi_k = \beta_k \prod_{t<k} (1 - \beta_t)$$

$$z_i|\boldsymbol{\beta}, x_i \sim \text{Categorical}(\pi_{x_i})$$

$$y_{ij}|x_i, z_i \sim \text{Softmax}(\text{in}_{x_i,z_i}^T \text{out}_{y_{ij}})$$

Variational inference for SBP

Variational inference for SBP

- Finite-dimensional variational approximation

$$q(\boldsymbol{\beta}, \boldsymbol{\phi}, Z) = \prod_{k=1}^{\infty} q(\beta_k) q(\phi_k) \prod_{i=1}^{N} q(z_i) \approx p(\boldsymbol{\beta}, \boldsymbol{\phi}, Z | X, \alpha)$$

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- Due to prior conjugacy $q(\beta_k) = \text{Beta}(\beta_k | a_k, b_k)$

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$$\begin{aligned}
\mathcal{L}(q) &= \mathbb{E}_q \left[\log p(\mathbf{x} | \boldsymbol{\beta}, \boldsymbol{\pi}, \mathbf{z}) - \text{KL}(q(\boldsymbol{\beta}, \boldsymbol{\pi}, \mathbf{z}) || p(\boldsymbol{\beta}, \boldsymbol{\pi}, \mathbf{z}) \right] \\
&= \mathbb{E}_q [\log p(\mathbf{x} | \boldsymbol{\beta}, \boldsymbol{\pi}, \mathbf{z}) - \sum_{k=1}^{T} \text{KL}(q(\beta_k) q(\phi_k) || p(\beta_k) p(\phi_k)) \\
&\quad - \sum_{k=T+1}^{\infty} \text{KL}(p(\beta_k) p(\phi_k) || p(\beta_k) p(\phi_k)) - \text{KL}(q(\mathbf{z}) || p(\mathbf{z} | \boldsymbol{\beta}))]
\end{aligned}$$

Sense discovery

Sense discovery

ALPHA	$p(z)$	nearest neighbours
		“light”
Skip-Gram	1.00	far-red, emitting
0.075	0.28	armoured, amx-13, kilcrease
	0.72	bright, sunlight, luminous
0.1	0.09	tvärbanan, hudson-bergen
	0.17	dark, bright, green
	0.09	4th, dragoons, 2nd
	0.26	radiation, ultraviolet
	0.28	darkness, shining, shadows
	0.11	self-propelled, armored
		“core”
Skip-Gram	1.00	cores, components, i7
0.075	0.3	competencies, curriculum
	0.34	cpu, cores, i7, powerxcell
	0.36	nucleus backbone
0.1	0.21	reactor, hydrogen-rich
	0.13	intel, processors
	0.27	curricular, competencies
	0.15	downtown, cores, center
	0.24	nucleus, rag-tag, roster

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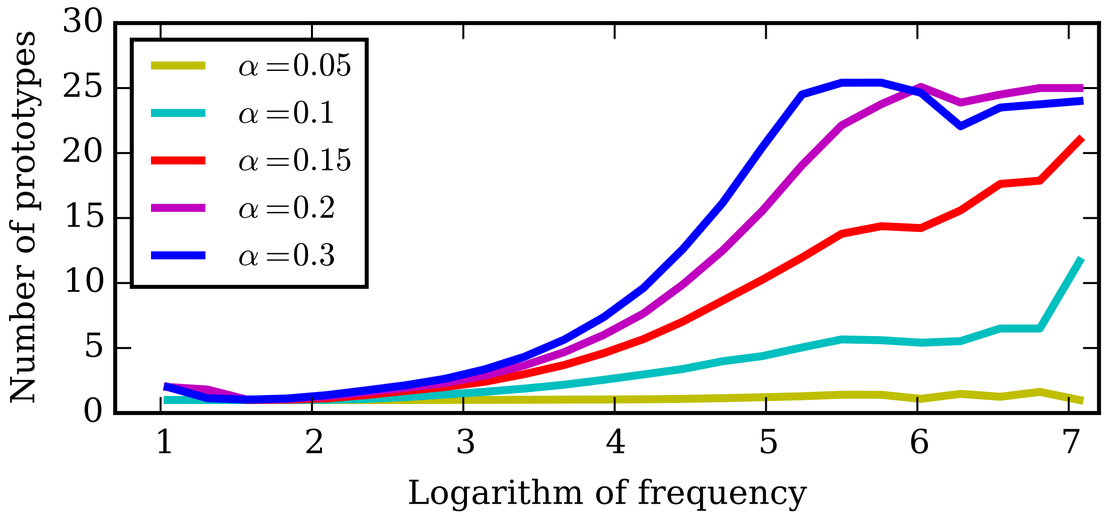

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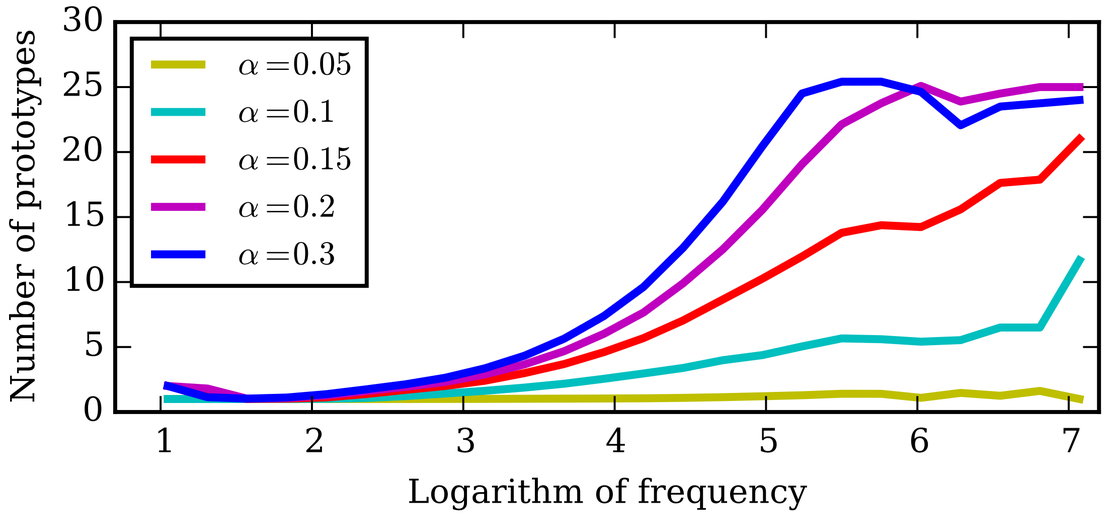
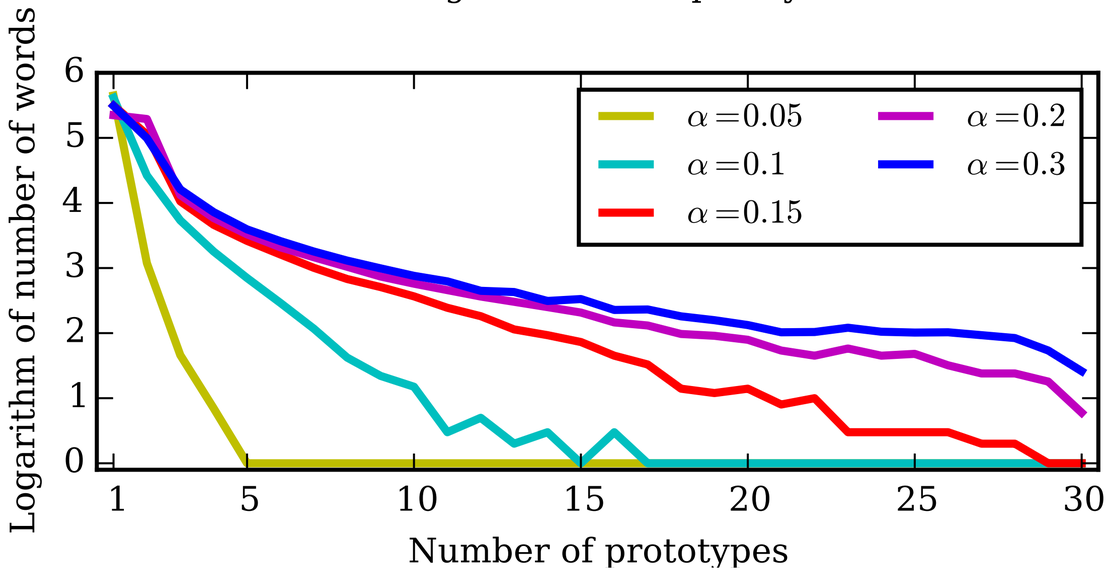




Model selection

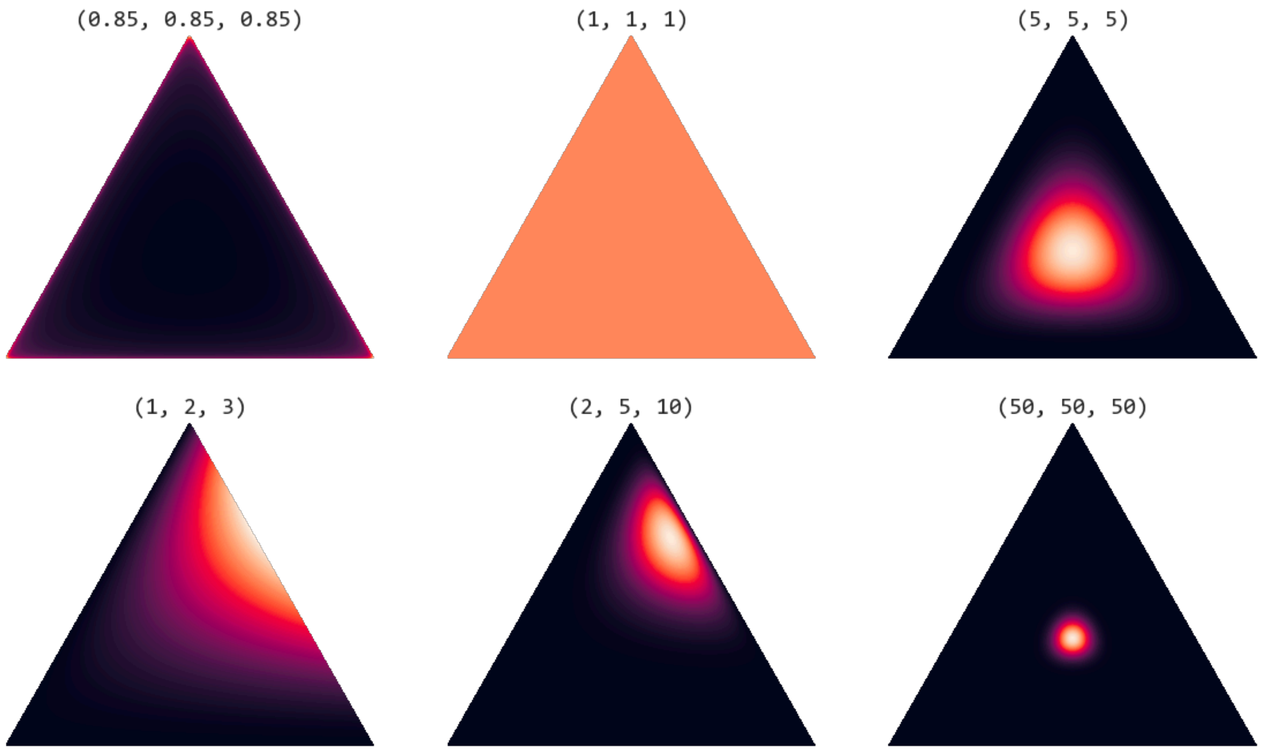
MODEL	LOG-LIKELIHOOD	ARI
Skip-Gram.300D	-7.403	-
Skip-Gram.600D	-7.387	-
AdaGram.300D $\alpha = 0.05$	-7.399	0.007
AdaGram.300D $\alpha = 0.1$	-7.385	0.226
AdaGram.300D $\alpha = 0.15$	-7.382	**0.268**
AdaGram.300D $\alpha = 0.2$	-7.378	0.254
AdaGram.300D $\alpha = 0.25$	**-7.375**	0.250
AdaGram.300D $\alpha = 0.5$	-7.387	0.230

Breaking Sticks and Ambiguities with Adaptive Skip-gram

Sergey Bartunov, Dmitry Kondrashkin, Anton Osokin and Dmitry Vetrov. In *AISTATS* 2016.

https://github.com/sbos/AdaGram.jl
https://bitbucket.org/sbos/adagram_deepbayes2019

Dirichlet distribution



Distribution over (discrete distribution)

$$\mathrm{Dir}(\pi|\alpha) = \frac{\Gamma(\sum_{k=1}^{K}\alpha_k)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\prod_{k=1}^{K}\pi_k^{\alpha_k-1}$$

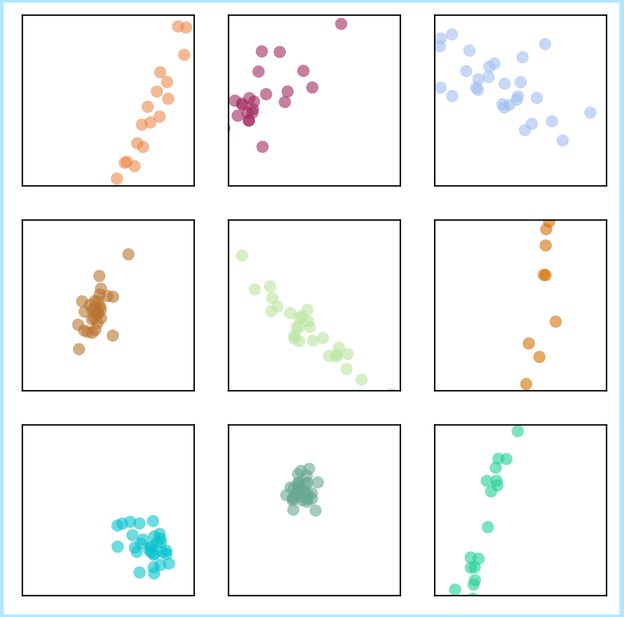
$$\pi_1,\ldots,\pi_K \sim \mathrm{Dir}(\alpha_1,\ldots,\alpha_K)$$

$$\sum_{k=1}^{K}\pi_k = 1, \quad \pi_k \geq 0$$

Dirichlet process

Base measure (prior)

$$H = \mathrm{NW}(\mu_0, \lambda, W, \nu)$$

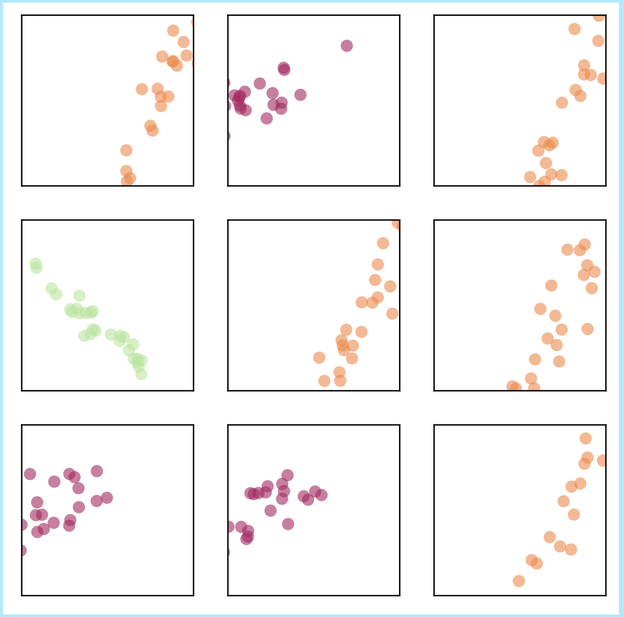


$$\mu, \Sigma \sim H$$

$$\mathrm{DP}(H, \alpha)$$

DP sample

$$G \sim \mathrm{DP}(H, \alpha)$$



$$\mu, \Sigma \sim G$$

Gaussian vs Dirichlet

Gaussian process (function estimation)

Prior

$$f \sim \mathrm{GP}(\mu, \Sigma)$$

Gaussian vs Dirichlet

Gaussian process (function estimation)

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$$f \sim \mathrm{GP}(\mu, \Sigma)$$

Posterior over functions

$$p(f|X,Y) = \frac{p(f)p(Y|f,X)}{p(X,Y)}$$

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Posterior predictive

$$p(y|x,X,Y) = \int p(f|X,Y)p(y|f,x)df$$

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Posterior over densities

$$p(G|X) = \frac{p(G)p(X|G)}{p(X)}$$

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Gaussian process (function estimation)

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Prior

$$G \sim \mathrm{DP}(H, \alpha)$$

Posterior over densities

$$p(G|X) = \frac{p(G)p(X|G)}{p(X)}$$

Posterior predictive

$$p(x|X) = \int p(G|X)p(x|G)dG$$

Finite projections

Gaussian process (function estimation)

Projection is Gaussian

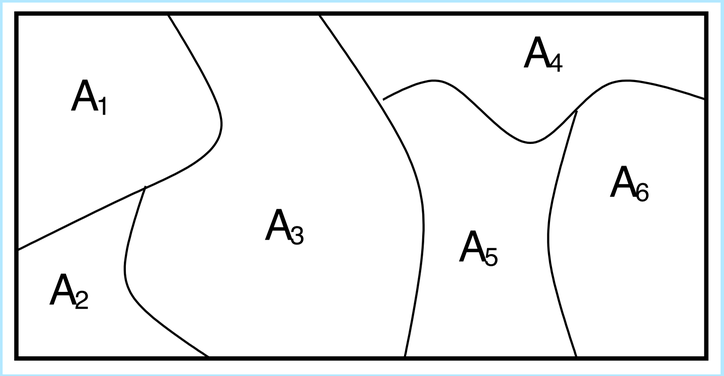
$$Y = f(X) \sim \mathcal{N}(\mu(X), \Sigma(X))$$

Dirichlet process (density estimation)

Projection is Dirichlet

$$(G(A_1), \ldots, G(A_n)) \sim \mathrm{Dir}(\alpha H(A_1), \ldots, \alpha H(A_n))$$

$$A_1 \cup \ldots \cup A_n = \mathcal{X}, \quad G \sim \mathrm{DP}(H, \alpha)$$



References

- **Skip-gram**: Mikolov, Tomas, et al. "Distributed representations of words and phrases and their compositionality." Advances in neural information processing systems. 2013.

- **Dirichlet process**

  - Ferguson, Thomas S. "A Bayesian analysis of some nonparametric problems." The annals of statistics (1973): 209-230.

  - Sethuraman, Jayaram. "A constructive definition of Dirichlet priors." Statistica sinica (1994): 639-650.

  - Teh, Yee Whye. "Dirichlet processes: Tutorial and practical course." *Gatsby Computational Neuroscience Unit, University College London* (2007).

References

- **Variational inference for Dirichlet process**

  - Blei, David M., and Michael I. Jordan. "Variational inference for Dirichlet process mixtures." Bayesian analysis 1.1 (2006): 121-143.

Image credits

- Slide 2: https://blog.evjang.com/2016/08/variational-bayes.html
- Slide 3: SHUTTERSTOCK / KYSLYNSKAHAL, https://artsyvoiceprint.com/products/tattoo-sound-wave-custom-art-black-white-digital-file, https://en.wikipedia.org/wiki/Support-vector_machine, https://towardsdatascience.com/understanding-auc-roc-curve-68b2303cc9c5
- Slide 33: https://towardsdatascience.com/dirichlet-distribution-a82ab942a879
- Slide 36: Yee Whye The, August 2007, MLSS