Aircraft landing fees: a game theory approach

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Discussion of airport pricing policy has largely centered on the problem of congestion, but at most airports congestion is negligible. In the present paper we explore the implications of an alternative pricing philosophy where the common costs of runway construction are shared among the different aircraft types according to a club principle. Linear programming and game theory techniques are used to clarify the notions of optimal runway size and fair and efficient landing fees, and to suggest new rules of thumb for allocating common costs based on the Shapley value and the nucleolus. The model is applied to Birmingham Airport to assess investment and pricing policy in 1968–1969. To the authors' knowledge this paper is the first explicit application of the "club principle" and the largest numerical application of game theory to date.

1. Introduction

■ In recent years, airport pricing and investment policy has received some attention from economists, mainly with a view to determining appropriate peak-load prices to reflect congestion costs (see, for example, Carlin and Park, 1970; Levine, 1969; Little and MacLeod, 1972; Walters, 1984; and Warford, 1971). However, for all but the largest airports, congestion is in practice quite negligible. What policy is therefore appropriate? The costs of an airport movement area[1] have a simple but interesting structure: the cost of building a runway depends essentially upon the "largest" aircraft for which the runway is designed, while the cost of subsequently using the runway is proportional to the number of movements of each type of aircraft. Economists generally agree that user costs should be incorporated in charges, but there is dispute as to whether, or how, common capacity

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[1] A movement is a take-off or landing. The movement area (with which the present paper is chiefly concerned) includes the runways, taxiways, and apron areas, as distinguished from the terminal area.

costs should be recovered. One frequently floated suggestion is that common costs should be shared among users according to a "club principle." (See Coase, 1946, and for a critical appraisal, Wiseman 1957.) Recently, the techniques of game theory have been used to spell out the implications of this principle for efficient and equitable pricing and investment policies (cf. Littlechild, 1975, and Telser, 1972).

The purpose of the present paper is to show how these ideas may be applied to airport pricing, and to illustrate the application by calculations based on the municipal airport at Birmingham, England, in 1968–1969. More precisely, we shall attempt to ascertain whether Birmingham airport runway was built to the "correct" size, whether the movement charges were both "fair" and "efficient," and how these charges compare to rules of thumb suggested by various game theory criteria. To our knowledge, this paper represents the first explicit application of the club principle in public utility pricing. Moreover, since there were over thirteen thousand aircraft movements at Birmingham in the year under consideration, each of which represents a player in the game, the paper also incorporates by far the largest actual numerical application of game theory to date.

We begin in Section 2 with a discussion of alternative pricing policies and explore the case for a pricing policy based on the club principle. The next section presents the mathematical model and certain qualitative results. Section 4 contains the relevant empirical data and Section 5 presents the results. A concluding section contains a discussion of extensions, qualifications, and reservations.

2. Alternative pricing and investment policies

■ The general problem is to design a framework within which decisions can be made about the volume of traffic at each airport at each period of the day and year, about the growth in traffic over time, about the kinds of aircraft utilized and the future development of new aircraft, about the location, nature, and timing of extensions to and removals of existing airport capacity, and about the provision of entirely new airports. These decisions are not made by a single organization, since passengers, airlines, airport managers, aircraft manufacturers, and the government all contribute certain decisions.

□ **Present institutional arrangements.** Current government policy in the United Kingdom, as represented by the 1967 White Paper,[2] requires nationalized industries to set prices equal to long run marginal cost, subject to covering overall accounting costs (and subject to various other qualifications). The White Paper observes that "two-part and differential pricing systems are also used to improve financial results without distorting the allocation of resources when there are important elements in costs which cannot easily be allocated to specific services or products (e.g., costs incurred jointly by several services or costs which do not vary proportionately with output)" (p. 9). The British Airports Authority, which is responsible, *inter alia,* for Heathrow and Gatwick Airports, was required to earn a return of 14 percent on average net assets in 1969.

[2] *Nationalised Industries—A Review of Economic and Financial Objectives,* HMSO Cmnd. 3437, November 1967.

Birmingham Airport, which is owned and operated by the City, does not fall directly within these guidelines. Along with many other municipal airports, it adopted in 1968–1969 the schedule of landing fees recommended by the Board of Trade (Doganis and Thompson, 1974, Appendix B). The rate of charge was related to aircraft weight, and in 1968–1969 (converted to new pence) was £0.30 per 1000 lbs. for the first 26,000 lbs., £0.39 per 1000 lbs. for the next 174,000 lbs., and £0.43 per 1000 lbs. thereafter. In addition, a navigation services charge of £0.15 per 1000 lbs. was levied to cover the provision of Air Traffic Control facilities by the Board of Trade and a variety of rebates and surcharges applied.[3] Charges were also made for use of the terminal area, with which we are not presently concerned. Generally, municipal airport revenues cover operating costs, with capital costs financed by government grants, local rates, or borrowing. At the time of writing, the Department of Trade had just issued the second part of its consultative document on Airport Strategy for Great Britain (1976). To allocate investment resources efficiently, a small number of major regional airports is envisaged—probably including Birmingham—and surcharges at London airports are proposed to divert traffic to the regions.

□ **Implications of economic theory.** We now examine briefly some alternative theoretical prescriptions.

Congestion cost pricing. The cost of an aircraft movement is taken as the value of the time and expense imposed upon other users who are forced to wait. There have been several excellent calculations of these costs with discussions of their implications for time of day pricing and for the expansion of runway capacity (see Carlin and Park, 1970; Levine, 1969; Little and MacLeod, 1972; Walters, 1974; and Warford, 1971). However, in Britain only Heathrow and Gatwick Airports experience any significant traffic congestion (defined by the International Civil Aviation Organization as an average delay of over four minutes per movement). At Birmingham, which is Britain's second largest city and which had in 1968–1969 the sixth busiest airport in the country, there was at that time an average of less than three movements per working hour. Whatever its merits at the few busy airports, congestion pricing is superfluous at the majority of airports.

Short-run marginal cost pricing. Price per movement of any aircraft is set equal to the expense of wear and tear on the runway imposed by that type of aircraft. The Board of Trade schedules of weight-related fees to some extent, but not entirely, reflect this principle.[4] While all economists would presumably agree that maintenance costs should be included, maintenance costs at Birmingham account for less than half of the expenditures associated with the movement area. An SRMC

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[3] Notably intercontinental surcharge 66⅔ percent; international short-haul (within 115 miles) rebate 50 percent; intra-UK short-haul rebates 80 percent (within 45 miles), 70 percent (45–80 m.), and 55 percent (80–115 m.); new service rebates of varying amounts and durations. In fact, hardly any of these applied to Birmingham in 1968–1969.

[4] For example, runway wear and tear largely depend upon "footprint pressure," rather than weight. Levine (1969) points out that a Boeing 727-2000 has a higher footprint pressure than some models of the Boeing 747 which weigh three times as much.

pricing policy would therefore have three significant drawbacks. First, when constructing new facilities or extending existing ones, inadequate guidance would be available concerning the value of different runway sizes. Second, aircraft manufacturers and airlines would have no incentive to consider the implications of their aircraft design for runway construction (as distinct from wear on the runway once constructed). Third, there would be an arbitrary transfer of income from whoever financed the runway construction (taxpayers or users of the terminal area) to passengers or owners of aircraft requiring the largest runways.

Optimal departure from SRMC pricing. (See Baumol and Bradford, 1970, and references therein.) To recover total costs, fees would be increased above maintenance costs for those aircraft movements where demand is least elastic. This presumably happens at present insofar as there are rebates for flying new or short-distance routes. However, without further guidelines, the resulting fee structure need not reflect construction costs. Consequently, it could give misleading incentives to aircraft designers, it could generate arbitrary transfers of income between passengers or owners of different aircraft types, and it need not provide the most useful information to airports about values of different runway sizes.

Two-part tariff. (Cf. Coase, 1946.) Movement fees would reflect maintenance costs, while construction costs would be recovered via lump-sum charges independent of usage. This theoretically attractive policy (which is used by other public enterprises such as telephones, electricity, etc.) has a serious drawback in the present instance. On whom would the fixed charges be levied? Suppose the fixed charge were imposed (say, annually) on each airline using the airport. Since an airline typically operates many different types of aircraft, it would have no incentive to choose those aircraft with minimal runway construction requirements. On the other hand, if a fixed charge were imposed on each aircraft type for use of the airport, regardless of owners, then this charge would presumably be paid by the aircraft manufacturer, but it throws upon the manufacturer the burden of an operating decision (i.e., which airports to buy licenses for) which he is not well equipped to make.

Long-run marginal cost pricing. The movement fee for any aircraft would equal maintenance cost plus a contribution to capacity costs—but how is that contribution calculated when the latter are joint or common (specifically, capacity is here determined simply by the "largest" aircraft to be accommodated)? The club principle set out in the next section is addressed to that very problem. The second difficulty with long-run marginal cost pricing is that a price above maintenance cost may deter some traffic which is just willing to cover its user cost.

☐ **Discussion.** Which of these five theoretical principles is most appropriate? Current opinion among economists seems to favor short-run marginal cost pricing, plus congestion costs where appropriate and with increases based on demand elasticity where necessary to raise revenue. The justification of this policy is the relative infrequency of airport construction decisions and the desire to utilize fully existing capacity.

However, for two reasons the case for explicitly incorporating capacity costs in movement charges deserves further consideration. First, it turns out that movement fees are such a small proportion of the cost to the final consumer (or even of the cost to the airline) that the derived demand for landings is likely to be rather inelastic,[5] at least up to the level where it becomes profitable for the airline to switch to the nearest alternative airport. Providing movement fees do not exceed this level, pricing based on capacity costs seems likely to involve negligible misallocation of resources.

Second, in Britain today the airport network is not fixed, but remarkably fluid. With the general growth in international traffic, and the capacity limitations experienced at the London airports, the major regional airports are likely to expand, while the minor ones are likely to contract (cf. F. P. Thompson, 1974, and Department of Trade, 1976). The design capacities of these airports are now to be considered as variables. In these circumstances, greater importance should be attached to the feedback of information concerning the value of different sizes of runway, in order to improve the investment decisions, where large sums of money are at stake. (See Coase, 1946 and 1970, on the need to look at the problem as a whole.) Moreover, a very wide range of aircraft types is now available or in process of development (including vertical and short take-off, large bodied, supersonic, etc.), so it is important to induce manufacturers and airlines to take runway costs into account.

The above considerations suggest that it would be worthwhile to explore the implications of an airport pricing policy which reflects capacity costs. The next subsection specifies what exactly is meant by this with reference to the "club principle."

□ **The club principle.** The club principle, as a way of sharing common costs, seems to have been developed in connection with the two-part tariff, but is equally applicable to a single-part tariff where, as here, demand is assumed to be inelastic within a certain range. Let us first define the *optimal* airport runway size and pattern of aircraft movements as those which maximize the net present value of total benefits. We shall say that a pricing policy is *efficient* if it satisfies the following three conditions:

(1) It should give the airport authority the incentive to build the optimal size of runway and accept the optimal pattern of movements; moreover, the authority should just break even.

(2) There should be no cross subsidization whereby any group of aircraft pays more than the cost of accommodating that group alone. Hence, there should be no incentive for the airport authority (or for any entrepreneur) to build another airport to cater for any group of aircraft.

(3) The pricing scheme should give all aircraft which can cover runway user cost the incentive to use the airport by setting movement charges in such cases within "ability to pay."

[5] In 1968–1969 aircraft landing charges at Birmingham averaged about £15 per flight, which amounts to only a few pence per person. Landing fees amount to at most 7 percent of airline direct costs (Doganis and Thompson, 1973). Assuming elasticity of demand for air travel to be less than −2, the elasticity of derived demand for landings is likely to be at most −0.2 (G. F. Thompson, 1974).

These efficiency conditions might equally be called *competitive* conditions, since they presumably characterize the charges that would come about if there were adequate knowledge and free bargaining between potential runway users and runway builders under a system of private property rights.

If one set of movement charges is efficient (and it is not yet established that such an efficient pricing policy exists), then it is possible that many efficient sets of charges will exist. How should one choose among them? A first step might be to introduce some basic notions of equity. Let us say that a pricing policy is *fair* if (a) smaller aircraft pay no more than larger ones and (b) the amount by which the charge to a larger aircraft exceeds that for a smaller one does not exceed the difference in costs of providing for the two types.[6]

Even with this further restriction, it will probably be necessary to choose some specific rule of thumb for allocating costs. We shall therefore look to game theory for some suggestions as to rules to follow; two obvious possibilities are the Shapley value and the nucleolus.

In the context of the present numerical application we wish to answer the following four specific questions:

(1) Was Birmingham airport runway the optimal size in 1968–1969?

(2) Were the 1968–1969 movement charges efficient?

(3) Were those charges fair?

(4) How do those charges compare with various rules of thumb derived from game theory?

3. The model

■ We shall make the following simplifying assumptions.[7] We deal with a single airport, not with a system of airports, and take a single period of time (one year) rather than a multiperiod model. The analysis is limited to the movement area, and excludes the terminal area. Aircraft movements are chosen as the basic units of analysis, because they are the natural basis for pricing, although in practice the bargaining entities would more likely be the airlines, the airport, the passengers, the government, the city, etc. Passengers are assumed not to switch between different aircraft movements. There is no uncertainty about demand or cost. No account is taken of noise or other externalities such as commercial benefits to the surrounding towns. Government policy, affected by instruments such as subsidies and planning permission, is not explicitly treated. The airport authority is assumed to maximize net social benefit as defined below. No collusion among airlines to influence charges is allowed, although the game

[6] Fairness condition (b) is distinct from efficiency condition (2), above. For example, charging entire runway cost to the largest aircraft type would avoid cross subsidization, but it would not be fair because the second largest aircraft type would pay nothing. On the other hand, charging the same fee to each aircraft would be fair but would involve cross subsidization.

[7] The model described in this section is developed with greater rigor and generality by Littlechild (1975). The technique of dual linear programs to characterize the core is due to Shapley (1967) and Charnes and Kortanek (1967); it is used also by Telser (1972). Faulhaber (1975) has developed a game theory model for analyzing cross subsidization in which demand curves are incorporated explicitly.

theory solutions may be interpreted as reflecting conditional bargaining power. In the final section we examine the possibilities of relaxing these various, rather severe, assumptions.

Let $N = \{1, 2, \ldots, n\}$ denote a set of potential aircraft movements over the (one year) planning period, with the typical member denoted by subscript j, and the typical subset denoted by S. Let

b_j = gross benefit (to airline and passengers) if movement j is affected,

c_j = runway user cost of movement j, and

$G(S)$ = capital (construction) cost of a runway designed to accommodate subset S of movements.[8]

The net social benefit of constructing and operating a runway for subset S of movements, denoted $\pi(S)$, is defined by

$$\pi(S) = \sum_{j \in S} (b_j - c_j) - G(S),\ S \subseteq N. \tag{1}$$

Suppose there are m different aircraft types, denoted by subscript i, with n_i movements of type i (where $\sum_{i=1}^{m} n_i = n$) and N_i the set of movements type i (where $\bigcup_{i=1}^{m} N_i = N$. Let g_i denote the capital cost of constructing a runway for aircraft type i. We shall argue later that the aircraft types can be ranked by "size" and that the cost of accommodating any subset of aircraft is equal to the cost of accommodating the largest aircraft type in that subset. Hence

$$G(S) = \max\{g_i : S \cap N_i \neq \phi\}. \tag{2}$$

Let x denote a 2^n-dimensional decision vector whose typical component x_S takes the value unity if a runway is built to accommodate precisely the subset of movements S, and takes the value of zero otherwise. To answer the questions posed in the previous section, consider the following pair of dual linear programs:

(I)	(II)
$\max \sum_{S \subseteq N} \pi(S) x_S$	$\min \sum_{j=1}^{n} u_j$
$\sum_{S \ni j} x_S \leq 1, j = 1, \ldots, n$	$\sum_{j \in S} u_j \geq \pi(S),\ S \subset N$
$x_S \geq 0,\ S \subseteq N$	$u_j \geq 0, j = 1, \ldots, n.$

□ **Optimal size of runway.** The primal problem (I) chooses that pattern of runway construction which maximizes total net social benefit. The constraint ensures that each potential aircraft movement is allotted to at most one runway. In principle, it might be optimal to build more than one runway, or even fractional runways, but because the

[8] The construction cost of a runway for any set of movements S is here supposed to be independent of what other runways are constructed. In practice, since suitable land is limited, this would not be the case. As it happens, this is not a problem because it will be optimal to build only one runway.

runway cost function $G(S)$ exhibits economies of scale,[9] it can be shown that it will be optimal to build precisely one runway.

In the empirical application of the next section, we shall for simplicity take gross benefits and runway costs for each movement to be equal to the *average* for a movement of that type of aircraft; it will not cause any confusion to use the notation b_i and c_i for $i = 1, \ldots, m$. It follows the optimal policy will be to design for some critical aircraft type k^*, where $0 \leq k^* \leq m$, and to accommodate all movements of all aircraft types $\{1, 2, \ldots, k^*\}$ which do not exceed that size. The primal problem (I) with 2^n variables thus reduces to a comparison of the $m + 1$ values of $\pi(\bigcup_{i=0}^{k} N_i) = \sum_{i=1}^{m} n_i(b_i - c_i) - g_k$ over $k = 0, 1, \ldots, m$.

☐ **Efficient movement charges.** Let p_j denote the fee for movement j, and assume it consists of two components: one equal to the runway user cost c_j and the other a contribution to capital costs y_j. Interpret the dual variable u_j as the "net payoff" to movement j; that is, the gross benefit b_j less the movement fee.

Then

$$u_j = b_j - p_j = b_j - (c_j + y_j), j = 1, 2, \ldots, n. \tag{3}$$

Substituting from (1) and (3) into the dual problem (II), cancelling, and rearranging yield the equivalent problem

$$\text{(II}'\text{)}$$

$$\max \sum_{j=1}^{n} y_j$$

$$\sum_{j \epsilon S} y_j \leq G(S), \quad S \subset N$$

$$c_j + y_j \leq b_j, \quad j = 1, 2, \ldots, n.$$

The equivalent dual problem (II′) determines that schedule of capital components of the movements fees which maximizes total contribution. The constraints require that no subset of movements contribute more than the cost of building a runway for that subset alone and no movement is charged more than its associated net benefit (i.e., gross benefit less user cost). But these constraints correspond precisely to the "no cross subsidization" and "within ability-to-pay" conditions (2) and (3) of an efficient pricing policy. Moreover, by complementary slackness, we know that total contribution will just equal total capital cost for whichever airport runway is actually built, which is condition (1). Consequently, the set of optimal solutions to problem (II′) is precisely the set of efficient capital charges y_j; the efficient movement fees p_j are obtained simply by adding user costs c_j. Since problem (I) is feasible and bounded, we are assured that an efficient set of movement fees exists, but it will generally not be unique.

Since we shall work with average benefits b_i and costs c_i for each type of aircraft, we shall set identical capital components y_i and fees p_i for all movements of each aircraft type i. Recalling that k^* denotes the optimal size of runway, the efficient pricing conditions of (II′) reduce to the set of capital components y_i satisfying

[9] More precisely, $G(S \cup T) \leq G(S) + G(T) - G(S \cap T)$ for all $S,T \subseteq N$.

$$\sum_{i=1}^{k} y_i \leq g_k,\ k = 1, 2, \ldots, k^* - 1;$$

$$y_i \leq b_i - c_i,\ i = 1, 2, \ldots, k^* \quad (4)$$

$$\sum_{i=1}^{k^*} y_i = g_{k^*}; \qquad y_i = b_i - c_i,\ i = k^* + 1, \ldots, m.$$

☐ **Fair movement charges.** The two fairness criteria (a) and (b) laid down in the previous section may be formally represented by the conditions

$$y_{i-1} \leq y_i \leq y_{i-1} + \frac{g_i - g_{i-1}}{n_i}, \quad i = 1, 2, \ldots, m. \quad \text{(5a,b)}$$

Note that these conditions are independent of benefits.

☐ **Game theory pricing rules.** Let $\pi^0(S)$ be the maximum net benefit obtainable from the subset S of movements *or any smaller set,* i.e., $\pi^0(S) = \max_{T \subseteq S} \pi(T)$. Formally, π^0 is the monotonic cover of π. Consider the set of aircraft movements N as a set of *players* in a *game* and π^0 as a *characteristic function*. It may be shown (see Littlechild, 1975) that the set of optimal solutions to the dual problem (II) comprises precisely the *core* of the game (N, π^0). Thus, the set of payoffs which cannot be improved upon by any coalition of aircraft movements is precisely the set of payoffs which corresponds to efficient movement charges.

This gives us access to many other game theory solution concepts. Two are of particular interest here. The *nucleolus* (cf. Schmeidler, 1969) is that unique vector of payoffs which makes the coalition which is worst off (relative to its characteristic function value) as well off as possible, then makes the second worst off coalition as well off as possible, and so on lexicographically. Essentially, the nucleolus maximizes the minimum benefit, and in that sense minimizes the maximum complaint; it corresponds in many ways to Rawls' (1972) maximin fairness criterion.[10] It will also be of interest to explore that solution which lexicographically maximizes the minimum complaint, which we may call the *antinucleolus*. In general, neither the nucleolus nor antinucleolus can be written down in explicit form, but may be calculated by a sequence of linear programs which, in the present example, take a particularly simple form.

The *Shapley value* (1953)[11] is a unique payoff vector which satisfies certain general and reasonable criteria; it gives to each player a weighted average of his contribution to each coalition, namely

$$u_j = \sum_{\substack{S \subseteq N \\ S \ni j}} \frac{(n - s)!\,(s - 1)!}{n!}$$

$$[\pi^0(S) - \pi^0(S - \{j\})],\ j = 1, 2, \ldots, n, \quad (6)$$

[10] The nucleolus is more general in that it incorporates coalitions as well as individuals, and the benchmark for complaints its the characteristic function whose values are not generally zero.

[11] Shubik (1962) first proposed the Shapley value as a device for allocating overheads in the private firm; the idea was later applied to public utilities by Littlechild (1970) and Loehman and Whinston (1971).

where $s = |S|$. For convex games such as the present one, both these solution concepts lie in the core, hence the corresponding movement charges are efficient. We examine more detailed implications of these two rules in the next section.

It may also be of interest to determine pricing rules which depend upon the relatively objective and ascertainable elements of cost and numbers of movements, but not upon benefits. For this reason, we shall also consider the "airport cost game" $(N, -G)$, in which the characteristic function is the negative of the runway cost function, as a basis for determining game theory pricing rules.

4. An application to Birmingham Airport

■ In the financial year ending October 31, 1969, Birmingham Airport incurred accounting costs of some £511,000 and earned revenues of some £393,000.[12] There was thus a deficit to be written off amounting to nearly one quarter of expenditure, even after taking into account a government grant towards capital expenditure. In the present paper, we are primarily interested in the costs and revenues of the movement area. Landing fees and other movement area charges totalled £208,087, about half of total revenues earned. Maintenance costs of the movement area amounted to £90,411. The airport accounts do not identify capital expenditures by functional activity areas. Discussions with airport executives enabled us to identify runway area investments. Calculations based on a series of assumptions about depreciation policy, financial policy, interest rates, government grants, etc., suggested that an annual charge for the movement area would be of the same order of magnitude as the 1968–1969 movement area revenues less maintenance expenses, *viz.* £208,087 – £90,441 = £117,676. However, the notion of calculating an annual charge in this way is debatable, and we are concerned not so much with the total cost to be covered in any year as with the way in which the contribution is shared among the different aircraft movements. We therefore assume that if the same size runway is built, then the contribution to total capital cost which is to be recovered is the amount of £117,676, and that if a smaller runway is built, then a proportionally smaller contribution is required.

In the year 1968–1969 eleven different types of aircraft made a total of 13,572 movements (i.e., 6786 take-offs and 6786 landings) at Birmingham Airport. Table 1 sets out the number of movements n_i of each aircraft type i, and the average movement charges p_i actually paid.[13] We now discuss briefly the estimation of the benefit and cost parameters b_i, c_i and g_i.

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[12] The empirical data in this section are taken mainly from G. F. Thompson (1971) and Doganis and Thompson (1973 and 1974), supplemented by the present authors' own calculations using timetables, fee schedules, etc.

[13] The movement charges p_i^o are half the landing charges applicable in 1968–1969 to aircraft of the specified weights. The Board of Trade navigational surcharge is not included, nor have other surcharges and rebates been added or subtracted. It appears that only the London flights were eligible for a rebate (of 55 percent). The average movement charges actually paid by Vickers Viscounts, BAC 1-11's, and Vanguards would have been lower than shown in Table 1 by £0.63, £1.19, and £2.30, respectively.

TABLE 1

BIRMINGHAM AIRPORT DATA: MOVEMENTS, BENEFITS AND COSTS 1968/69

AIRCRAFT TYPE	SUBSCRIPT i	ACTUAL 1968/69 NUMBER OF MOVEMENTS n_i	ACTUAL 1968/69 CHARGES PER MOVEMENT p_i^o	ESTIMATED DIVERSION COSTS PER MOVEMENT d_i	BENEFIT PER MOVEMENT b_i	RUNWAY USER INDEX I_i	MAINTENANCE COST PER MOVEMENT c_i	CAPITAL COST (ASSUMED CONTRIBUTION) g_i
FOKKER FRIENDSHIP 27	1	42	£ 5.80	£ 85.38	£ 91.18	0.560	£ 5.23	£ 65,899
VICKERS VISCOUNT 800	2	9,555	11.40	89.37	100.77	0.652	6.09	76,725
HAWKER–SIDDELEY TRIDENT	3	288	21.70	89.25	110.95	0.809	7.55	95,200
BRITANNIA 100	4	303	29.80	142.31	172.11	0.826	7.71	97,200
CARAVELLE CI R	5	151	20.30	118.85	139.15	0.828	7.73	97,436
BAC 1–11(500)	6	1,315	16.70	143.91	160.61	0.834	7.79	98,142
VANGUARD 953	7	505	26.40	288.48	314.88	0.871	8.13	102,496
COMET 4 B	8	1,128	29.40	76.04	105.44	0.891	8.32	104,849
BRITANNIA 300	9	151	34.70	170.77	205.47	0.963	8.99	113,322
CONVAIR CORONADO	10	112	48.30	156.54	204.84	0.981	9.16	115,440
BOEING 707	11	22	66.70	185.00	251.70	1.000	9.34	117,676
	TOTAL	13,572						

☐ **Benefits from aircraft movements.** We have argued that, because airport landing charges are so low compared with other expenses, the elasticity of demand for aircraft movements with respect to the landing charge is likely to be negligible in the region of the existing charges. However, if charges at one airport are increased too much, flights are likely to be rearranged from alternative airports whenever the difference in charges exceeds the additional transport costs. The gross benefit of landing at one particular airport is the fee actually paid plus the consumer surplus, where the latter is calculated as the increase in transport costs from using the best alternative airport, net of any difference in landing fees.

The nearest airport to Birmingham is in Manchester, where the same schedule of landing fees was in operation in 1968–1969. We therefore defined, for each aircraft type, the average benefit of a movement at Birmingham (b_i) as the average movement fee actually paid (p_i^o), plus the average estimated cost of diversion to Manchester (d_i). Diversion cost was calculated as average cost per passenger (including both financial and time costs of additional ground and air journeys) multiplied by the appropriate load factors. We did not attempt to estimate benefits associated with potential flights which were not actually made in 1968–1969.

☐ **Movement area costs.** The cost of building and operating a runway depends upon a large number of factors, and designers typically do not have in mind comprehensive functions relating cost to detailed patterns of usage. Discussion and correspondence with the chief engineer of the British Airports Authority suggested that the three most important elements were take-off distance, runway pressure, and maneuverability. For each of these three elements, an index was developed in terms of which the different aircraft types were ranked.

The three indices were then standardized and weighted in the proportions 5 : 2 : 1. The resulting overall Runway Use Index was thought by the present authors to give a reasonable estimate of the proportions in which both capital and maintenance costs would be generated by different aircraft types.[14]

These index numbers (I_i), standarized to unity for the "largest" aircraft type (Boeing 707), are given in Table 1. The aircraft types have been labelled in increasing order of "size." Maintenance and capital costs for different types are related by the equations

$$c_i = I_i c_{11} \text{ and } g_i = I_i g_{11}. \tag{7}$$

Maintenance costs total £90,411 and capacity cost contribution for the largest runway is £117,676, hence

$$\sum_{i=1}^{11} n_i c_i = 90{,}411 \text{ and } g_{11} = 117{,}676. \tag{8}$$

Equations (7) and (8) may be solved for the parameters c_i and g_i given in Table 1.[15]

5. Optimal investment and pricing policy

■ We shall consider in order the four questions posed at the end of Section 2.

□ **Optimal runway size and usage.** With our present data we are unable to estimate whether a runway larger than the one existing in Birmingham should have been built. However, the calculations show that it was optimal to extend the runway to accommodate all the aircraft types which actually did land there, i.e., critical size $k^* = 11$. To illustrate, the extra cost of accommodating the largest aircraft type (Boeing 707) over the next largest type (Convair Coronado) is £(117,676 − 115,441) = £2,235, whereas the net benefit generated by the Boeing movements is 22 × £(251.70 − 9.34) = £5,332, thereby justifying the final runway extension (provided that the smaller runway size was already justified.)

□ **Efficient movement fees.** For this section we may restrict our discussion of movement fees to the capital components thereof. The capital components implied by the actual 1968–1969 charges (i.e., actual fees p_i^o less maintenance costs c_i) are set out in the second column of Table 2.

The 1968–1969 landing charges almost but not quite meet the efficiency conditions (4). Specifically, after covering runway mainte-

[14] This relationship is thus based on hypothetical investments primarily under British conditions; for an alternative approach based on actual costs of extensions at different runways in the United States, see Baker (1965). We emphasize again that the present index is an oversimplification; conceivably a more adequate weighting could be obtained from a linear regression study of either hypothetical or actual investments.

[15] These runway construction cost figures g_i are an order of magnitude less than annualized current construction costs. There are several possible reasons: the "base" for Birmingham runway was purchased from the Ministry of Defense after WWII (probably at a low price); the cost of subsequent additions has been reduced by government and municipal subsidies; prices and interest rates have risen sharply in the last few years. It follows that the movement charges calculated in this paper will be much lower than charges based upon current construction costs.

TABLE 2

CAPITAL COMPONENTS OF MOVEMENT FEES FOR ALTERNATIVE GAME THEORY SOLUTION CONCEPTS+

AIRCRAFT TYPE	SUBSCRIPT i	IMPLIED CAPITAL COMPONENT IN ACTUAL FEES* 1968/69	NUCLEOLUS (MINIMUM CONVEXITY FAIRNESS BOUND)**	ANTINUCLEOLUS (MAXIMUM CONVEXITY FAIRNESS BOUND)	SHAPLEY VALUE (LIMITING APPROXIMATION)	SHAPLEY VALUE (BASED ON COST ONLY)
FOKKER FRIENDSHIP 27	1	£ 0.57	£ 7.89	£ 0	£ 6.12	£ 4.86
VICKERS VISCOUNT 800	2	5.31	7.89	0	6.74	5.66
HAWKER SIDDELEY TRIDENT	3	14.15	7.89	11.59	7.36	10.30
BRITANNIA 100	4	22.09	7.89	18.19	11.71	10.85
CARAVELLE CI R	5	12.57	7.89	19.75	9.36	10.92
BAC 1–11(500)	6	8.91	7.89	20.29	10.88	11.13
VANGUARD 953	7	18.27	7.89	28.91	21.84	13.40
COMET 4 B	8	21.08	7.89	30.99	6.91	15.07
BRITANNIA 300	9	25.71	40.16	87.10	40.34	44.80
CONVAIR CORONADO	10	39.14	40.16	106.01	40.17	60.61
BOEING 707	11	57.36	103.46	207.64	101.64	162.24

+ THE TOTAL FEE PER MOVEMENT, CONSISTING OF THE CAPITAL COMPONENT LISTED HERE PLUS THE MAINTENANCE COST c_i, IS SET OUT IN TABLE 3.

* I.E., ACTUAL FEES p_i^o LESS MAINTENANCE COSTS c_i.

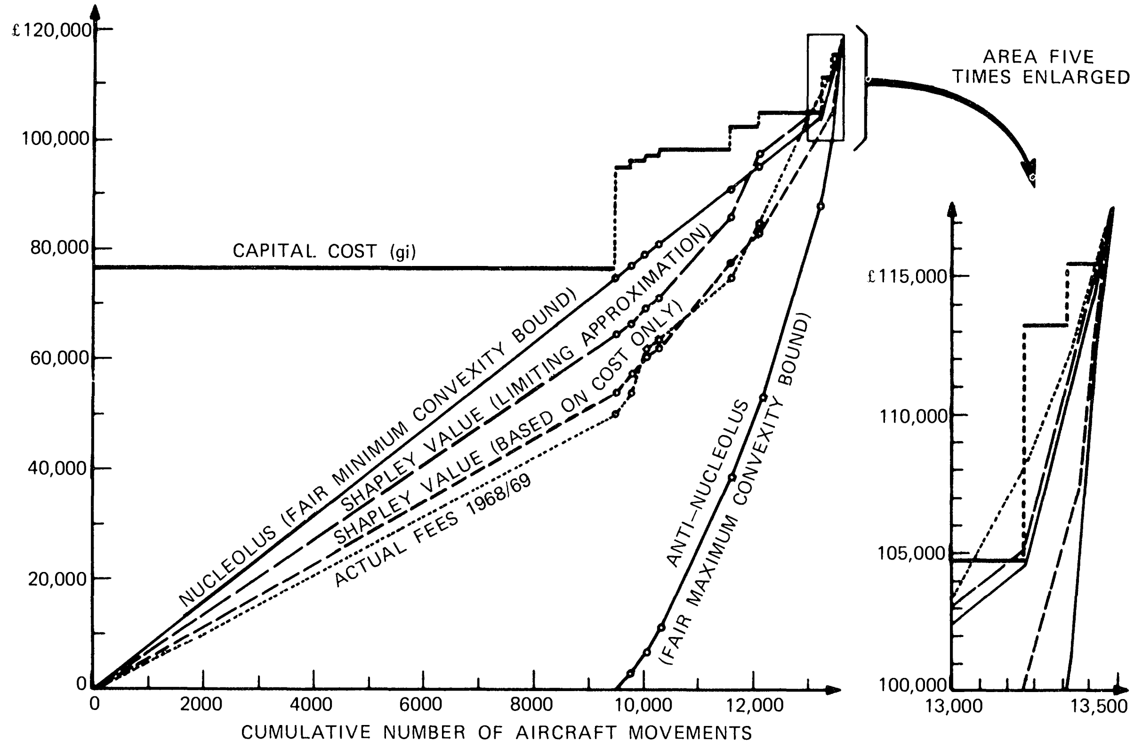
** STRICTLY SPEAKING, THESE ARE VALUES OF THE NUCLEOLUS ONLY. THE MINIMUM CONVEXITY FAIRNESS BOUND, WHICH IS EQUAL TO THE CONVEX ENVELOPE OF THE COST FUNCTION, TAKES THE VALUES £7.89 FOR $i = 1, \ldots, 8$, £40.26 FOR $i = 9, 10$ AND £101.63 FOR $i = 11$.

nance costs, the eight smallest aircraft types are being charged £108,148 for runway facilities which could have been provided for £104,849, so they are collectively overcharged by £3,299; similarly, the ten smallest types are being overcharged by £973. Put another way, the Britannia 300's, the Convairs, and the Boeing 707's are being charged only £9,528 for facilities which cost an extra £12,827 to provide, and similarly, the Boeing 707's are being charged only £1,262 for facilities which cost an extra £2,236 to provide. Thus, the largest aircraft types are being slightly cross subsidized by the smaller ones.

The special structure of the runway cost function enables us to represent graphically in Figure 1 the capital components of the movement charges. Along the horizontal axis, order the aircraft movements by increasing size of aircraft type. The piecewise-constant curve from the origin to the point (13,572, £117,676) represents the annual capital cost function with height g_i over all movements of the ith type of aircraft. Any set of movement fees can be represented in Figure 1 by plotting the cumulative curve of capital contributions. The slope of such a curve at any point (i.e., at any movement j) represents the capital component y_j charged to that movement. A nondecreasing line implies nonnegative capital components and a linear segment implies equal capital components over that set of movements. Any line from the origin to the point (13,572, £117,676) corresponds to a set of movement charges which just recoups total costs. A line wholly below the cost curve implies no cross subsidization (since if the line crossed the cost curve at any point (movement) it would imply capital charges to those movements which exceeded the cost of providing runway facilities for them).

FIGURE 1

COMPARISON OF PRICING POLICIES



The set of efficient capital components thus consists of the set of nondecreasing lines from the origin to the point (13,572, £117,676) which lie wholly below the cost curve and whose slope over the segment corresponding to aircraft type i does not exceed ability-to-pay $b_i - c_i$. The line corresponding to the actual 1968–1969 landing fees is drawn in Figure 1; evidently it lies above the cost curve for the larger aircraft types, as just described. Unfortunately, ability-to-pay cannot easily be represented visually, but, of course, the 1968–1969 charges by definition met this criterion.

□ **Fair movement fees.** To represent graphically the twin fairness conditions (5a,b) observe that they represent lower and upper bounds, respectively, on the rate at which charges increase with aircraft size. The first says that the cumulative charge curve must be convex, the second that it must not be "too" convex. The minimum degree of convexity consistent with the efficiency conditions is, in fact, the convex envelope (from below) of the runway cost function. The cumulative charge curve with the maximum degree of convexity is also easily calculated. Both are plotted in Figure 1. For a set of movement charges to be fair in the twin sense defined as well as efficient, it is necessary (but not sufficient) that the corresponding cumulative charge curve lie wholly within these two bounds.

The actual 1968–1969 curve lies, for the most part, rather centrally between the two bounds except for the larger aircraft which violate the efficiency conditions. However, the central part of the 1968–1969 curve is evidently not convex, in that Caravelles and BAC 1-11's are each charged less than the smaller Hawker Siddeleys and Britannia

100's. Moreover, although it is not immediately evident from the graph, four aircraft types (Vickers Viscounts, Britannia 100's, Vanguards, and Comets) are each being charged more than the next smallest aircraft types by amounts which exceed the difference in runway costs. Thus, the 1968–1969 charges did not fully satisfy either of the fairness conditions.

☐ **Game theory pricing rules.** Table 2 sets out the capital components of the movement charges corresponding to various game theory solution concepts; they are shown graphically in Figure 1. Table 3 adds

TABLE 3

TOTAL MOVEMENT FEES FOR ALTERNATIVE GAME THEORY CONCEPTS

AIRCRAFT TYPE	SUBSCRIPT i	ACTUAL FEES 1968/69	NUCLEOLUS (MINIMUM CONVEXITY FAIRNESS BOUND)	ANTINUCLEOLUS (MAXIMUM CONVEXITY FAIRNESS BOUND)	SHAPLEY VALUE (LIMITING APPROXIMATION)	SHAPLEY VALUE (BASED ON COST ONLY)
FOKKER FRIENDSHIP 27	1	£ 5.80	£ 13.12	£ 5.23	£ 11.35	£ 10.09
VICKERS VISCOUNT 800	2	11.40	13.98	6.09	12.83	11.75
HAWKER SIDDELEY TRIDENT	3	21.70	15.44	19.14	14.91	17.85
BRITANNIA 100	4	29.80	15.60	25.90	19.42	18.56
CARAVELLE CI R	5	20.30	15.62	27.48	17.09	18.92
BAC 1–11(500)	6	16.70	15.68	28.08	18.67	18.92
VANGUARD 953	7	26.40	16.02	37.04	29.97	21.53
COMET 4 B	8	29.40	16.21	39.31	15.23	23.39
BRITANNIA 300	9	34.70	49.15	96.09	49.33	53.79
CONVAIR CORONADO	10	48.30	49.32	115.17	49.33	69.77
BOEING 707	11	66.70	112.80	216.98	110.98	171.58

the maintenance components to form the movement charges themselves. Details of the computations have been presented elsewhere as footnoted.

It turns out that, for a given optimal size of runway, the *nucleolus* is independent of movement benefits, and that its cumulative capital charge curve is very closely approximated by the convex envelope of the cost function, which is, in turn, precisely the minimum convexity fairness bound just described (see Littlechild, 1974, and Littlechild and Owen, 1973). Thus, the nucleolus vector of movement fees may be obtained by taking the aircraft types in increasing order of size and raising the capital component as high as possible, consistent with (a) not charging larger aircraft types less than smaller types, (b) not charging any group of aircraft types more than the cost of providing for them, and (c) just recovering total cost. Equivalently (though it sounds more attractive), one might take the aircraft types in *decreasing* order of size and make the capital component as *low* as possible subject to (a), (c), and (b) not charging any group of aircraft types less than the additional cost of providing for them. Either method of calculation will generate the cumulative charge curve which "hugs" the cost curve (i.e., the convex envelope).

The cumulative charge curve corresponding to the *antinucleolus* turns out to be precisely the maximum convexity fairness bound. The antinucleolus vector of movement fees may thus be obtained by taking the aircraft types in increasing order of size and making the capital contributions as low as possible consistent with (a) nonnegativity, (b) not charging larger aircraft types more than smaller types by an amount which exceeds the difference in runway costs, and (c) just covering total cost. Equivalently (but this time less attractively) one might take the aircraft types in decreasing order of size and raise the capital contributions as high as possible, consistent with the same conditions.

Neither the nucleolus nor the antinucleolus is a good approximation of the actual 1968–1969 fee structure, which is located about midway between these two extreme bounds.[16]

We have not been able to calculate exactly the *Shapley value* for this game, but we have calculated an approximate value which is correct "in the limit" for games with an infinite number of players, and which is conjectured to be a close approximation in the present case.[17] This limiting Shapley value yields a set of charges which is slightly closer than the nucleolus to the actual 1968–1969 charges, especially for the smaller aircraft types. In contrast to the nucleolus, ability-to-pay has a noticeable effect on charges, but this effect is excessive as a predictor of actual fees. For example, the remarkably low diversion cost of Comets[18] leads to a capital charge to those aircraft of only one third of the actual level. Like the actual charges, the limiting Shapley value charges often violate the first fairness condition, but only fractionally violate the second one.

As remarked at the end of Section 3, it is possible to define a game based only upon the airport cost function, ignoring benefits. The nucleolus for such a game is the same as before. As demonstrated by Littlechild and Owen (1977), there is a remarkable simplification in the formula (6) for the Shapley value to

$$y_i = y_{i-1} + \frac{g_i - g_{i-1}}{\sum_{k=i}^{m} n_k}, \quad i = 1, 2, \ldots, m. \tag{9}$$

Intuitively, with the Shapley value of the "airport cost game," the cost of the runway required by the smallest aircraft type is divided equally among the movements of all aircraft types, the cost of the next increment of runway is divided among all movements except those of the smallest aircraft type, and so on until the cost of the last increment of runway capacity is divided only among the movements of the largest aircraft type. What is particularly interesting is that precisely this rule for allocating runway construction costs has been earlier proposed by airport economists (Baker (1965) and G. F. Thompson (1971)). The present discussion shows that this rule yields

[16] Define any coalition's "propensity to disrupt" as the ratio of what all other players stand to lose from abandoning a proposed solution to what the coalition itself stands to lose; Littlechild and Vaidya (1977) show that the "disruption nucleolus" equals the "disruption antinucleolus" and, in the present airport example, this solution is very close to the nucleolus.

[17] "The Limiting Shapley Value of a Game," unpublished manuscript by Littlechild and L. S. Shapley, October 1974.

[18] Over half the Comet flights are to Glasgow, so that a diversion from Birmingham to Manchester would generate savings in aircraft time and fuel to offset increased passenger costs.

charges which are fair (in both senses) and also efficient (providing they are within ability-to-pay).

Moreover, Figure 1 suggests that the charges thus computed are not a bad approximation to the actual charges obtaining at Birmingham in 1968–1969, at least in the general trend geometrically if not in detail.

6. Conclusions

■ This paper has been concerned with the problem of setting aircraft landing fees in the absence of congestion. In order to provide better guidance and incentives to both aircraft and aircraft designers, we have argued the case for reflecting runway construction costs in movement charges. The fee per movement would consist of runway user cost plus a contribution to common capital costs based on the "club principle." Linear programming and game theory techniques were used to clarify the notions of optimality, efficiency, and fairness in these circumstances, and to derive particular rules of thumb for allocating costs. The model was applied to Birmingham airport in 1968–1969. It was shown that, in terms of the definitions and computations used (1) the existing runway size was optimal, (2) the existing fees were nearly, but not quite, efficient (since they involved a slight cross subsidization of the largest aircraft types), (3) the fees did not meet the twin criteria of fairness proposed, but (4) they were, nonetheless, not too dissimilar from a fair and efficient set of fees calculated by using a rule of thumb based on the Shapley value.

The calculations of benefits and costs used in the present paper were exceedingly rough and ready; they suffice to illustrate the principles involved, but in a more serious application a great deal more care would be required. There seems to be no difficulty in principle in generalizing the model to incorporate a more complex runway cost function, multiple time periods, and a whole network of airports. This would generate an optimal system-wide pattern of runway investment and extensions over time, together with an associated efficient and fair schedule of landing fees over time. Of course, numerical computation would becaome more difficult.

It is conceivable that the model could be modified to allow passengers to switch between flights and to incorporate externalities, but there would be difficulties: in the first case the units of decision, and hence pricing, would probably need to be the passengers, rather than the aircraft movements, and in the second case an additional set of taxes and subsidies would probably need to be introduced.

A more fundamental difficulty concerns the relationship between time and cost. In the present single-period model, where a runway is envisioned but not yet built, the runway construction and user costs are well defined. But in the real world, as time progresses and investments take place, the opportunity costs change: the day after it is constructed, the cost of providing a runway falls to zero. At what point in time, then, should cost calculations be based, and when and how often should they be revised?

Finally, since it is possible to make different forecasts about future demand, benefits, and costs, it is evident that different managers may propose quite different pricing and investment policies while still following the general principles laid down herein. If there is no "right" solution, how then is one to assess the manager's efficiency?

If there is no adequate direct check on whether the policy is being carried out, it is not likely that the manager will respond to the variety of other pressures which undoubtedly come from the airlines, the passengers, the city authorities, the government, etc., by choosing the line of least resistance, whatever that may seen to him to involve?

These difficulties are not peculiar to airports, nor to the "club principle," nor even to marginal cost pricing. They are inherent in any approach which ignores the actual competing pressures in any situation and which specifies subjective criteria, the satisfaction of which is not subject to direct and objective verification.[19] It may be that the increasing disillusion with such pricing and investment criteria will stimulate an exploration of alternative regulatory systems for airports as well as for other public enterprises. Such issues, however, are beyond the scope of the present paper.

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[19] See Wiseman (1957) for a thorough critique of public utility pricing along these lines.

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