Bayesian Analysis to Constrain Symmetry Energy with HI collisions

Genie Jhang
Research Associate
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Symmetry Energy

- An equation that describes the relations among the pressure, energy, temperature, density and isospin asymmetry of nuclear systems.

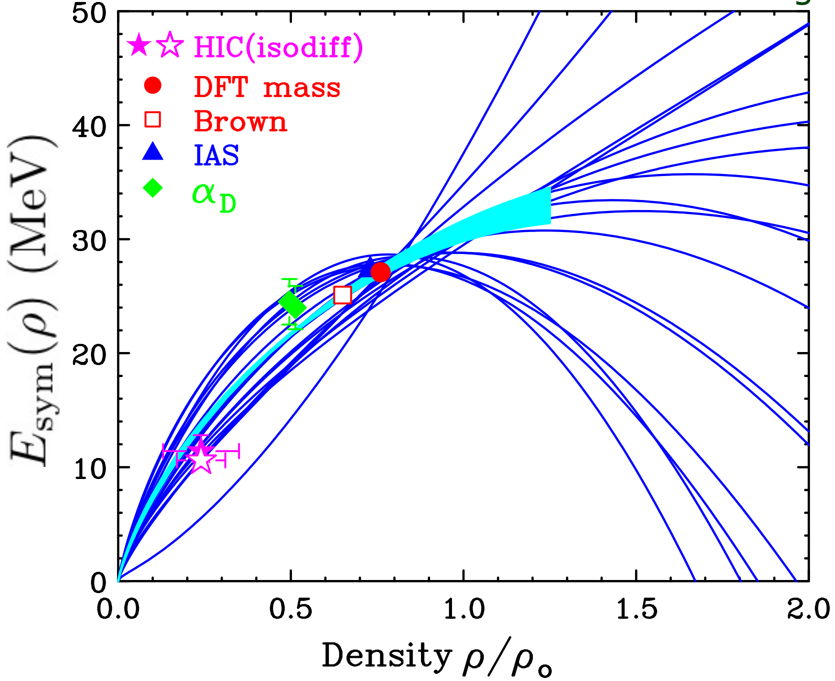
$$E(\rho,\delta) = E(\rho,\delta=0) + \underline{E_{\mathrm{sym}}(\rho)}\delta^2 + O(\delta^4), \quad \delta = \frac{\rho_{\mathrm{n}} - \rho_{\mathrm{p}}}{\rho}$$

Symmetry energy

$$E_{\mathrm{sym}}(\rho) = S_0 + \frac{L}{3}\frac{\rho-\rho_0}{\rho_0} + \frac{K_{\mathrm{sym}}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2 + \cdots$$

- Affects the relationship between neutron star radius and mass.
- Very few laboratory constraints
- Experimental constraints are emerging in sub-saturation density.

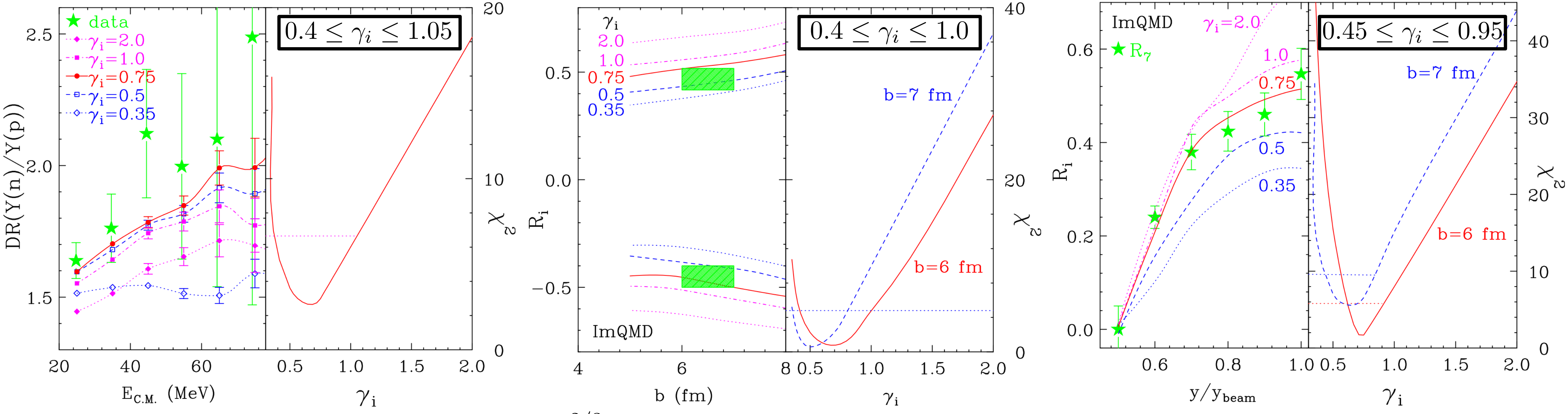
Private communication with M. B. Tsang



M. B. Tsang *et al*., Phys. Rev. C **86** (2012) 015803
C. J. Horowitz *et al*., J. Phys. G: Nucl. Part. Phys. **41** (2014) 093001

Before Bayesian Analysis

M. B. Tsang *et al.*, Phys. Rev. Lett. **102** (2009) 122701

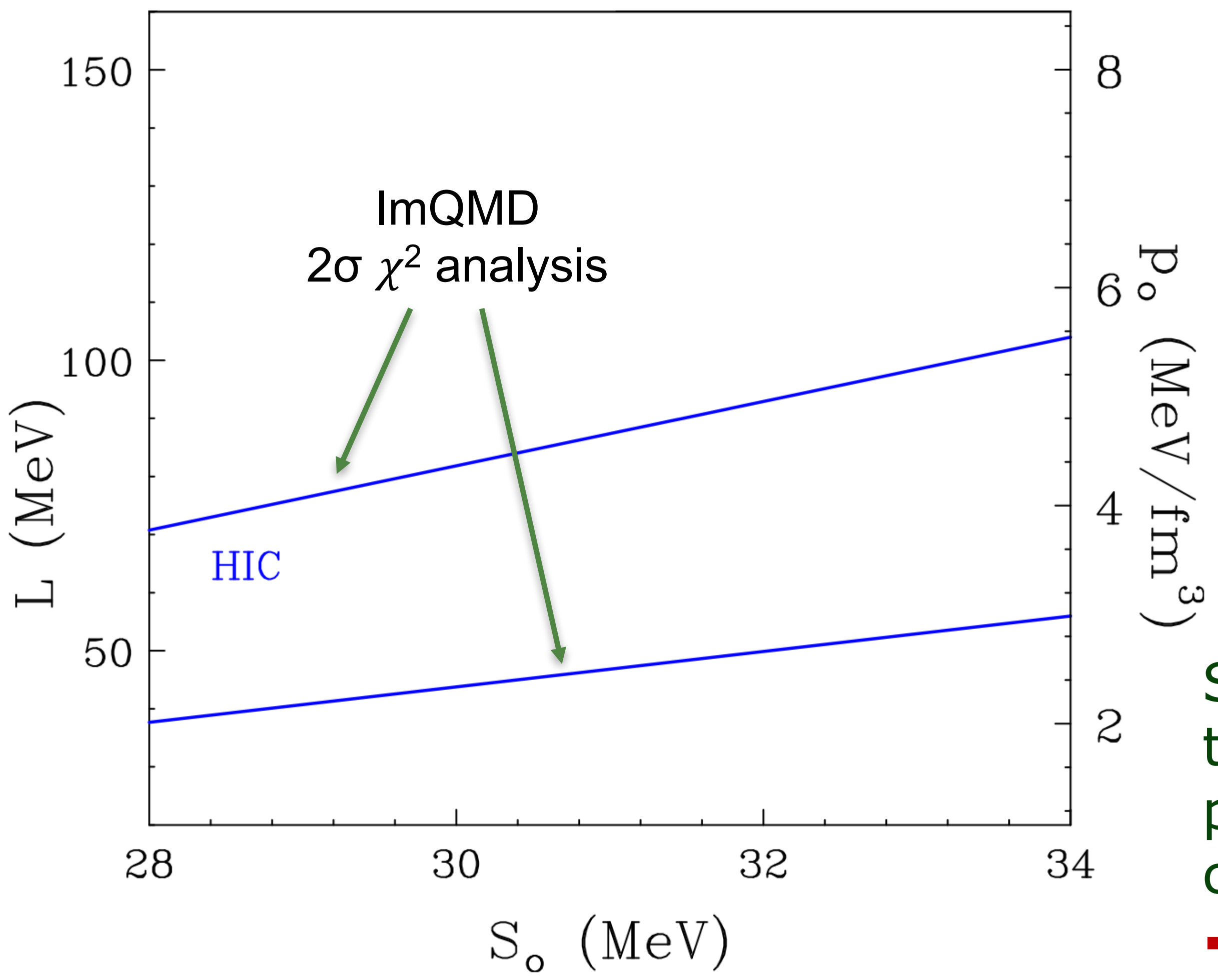


$$S(\rho) = 12.5\left(\frac{\rho}{\rho_0}\right)^{2/3} + C_{s,p}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i} \qquad C_{s,p} = 17.6\ \text{MeV},\ 0.4 \le \gamma_i \le 1.0$$

- For the first time, transport model can describe neutron to proton ratios and two isospin diffusion measurements.
- χ^2 analysis, with fixing the other parameters and changing γ_i and $C_{s,p}$, constrained two parameters with two different observables in consistent range.

Before Bayesian Analysis

M. B. Tsang *et al.*, Phys. Rev. Lett. **102** (2009) 122701



$$S(\rho) = 12.5\left(\frac{\rho}{\rho_0}\right)^{2/3} + C_{s,p}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

$$L = 3\,\rho_0 \frac{\partial S}{\partial \rho}\bigg|_{\rho=\rho_0} = \frac{3}{\rho_0} P_{\mathrm{sym}}$$

- From the obtained parameter constraints, model parameters with 2σ range are extracted.
- Constraints on the symmetry pressure is also obtained by the relation.

Since we now change the effective masses in the model with new data, the number of parameters are increasing and they need to be changed simultaneously.

➔ Bayesian analysis

Nucleon Effective Mass

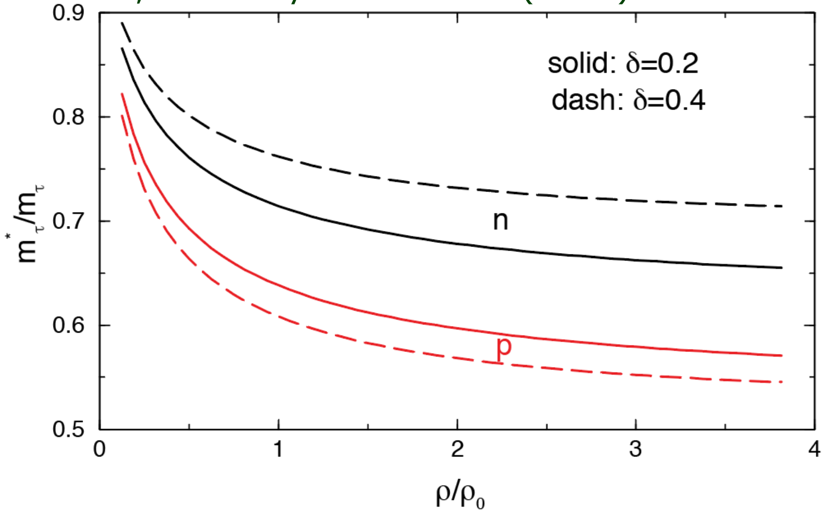
- Fundamental quantity characterizes the propagation of a nucleon in nuclear medium.
  - Describes momentum-dependent properties of neutron-rich matter such as neutron star, radioactive heavy-ion collisions
- Effective mass splitting in a neutron-rich matter
  - Gives different acceleration to neutrons and protons
  - Changes the ratios of emitted neutron to proton spectra

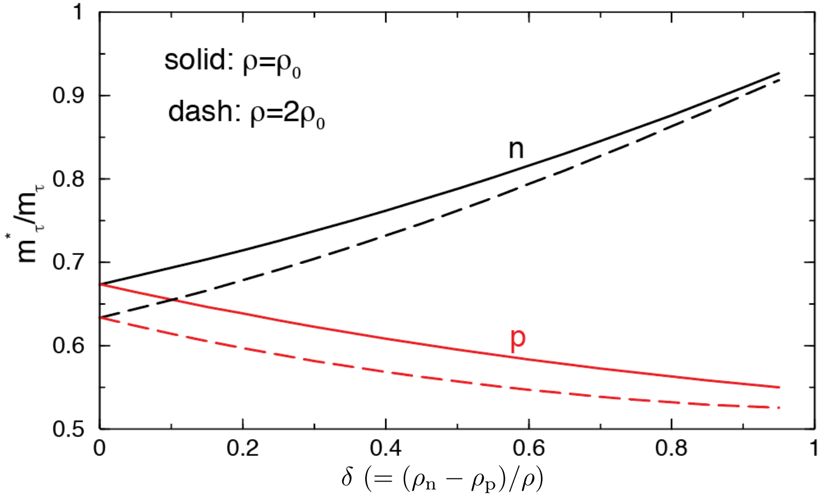
$$m^*_{\text{n-p}} = \frac{m^*_{\text{n}}}{m} - \frac{m^*_{\text{p}}}{m}$$

- ImQMD uses the following instead of the above.

$$f_I = \frac{m}{m^*_s} - \frac{m}{m^*_v} = \frac{1}{2\delta}\left(\frac{m}{m^*_{\text{n}}} - \frac{m}{m^*_{\text{p}}}\right)$$

B.-A Li *et al.*, Mod. Phys. Lett. A **30** (2015) 1530010-1





Experiment and Observables

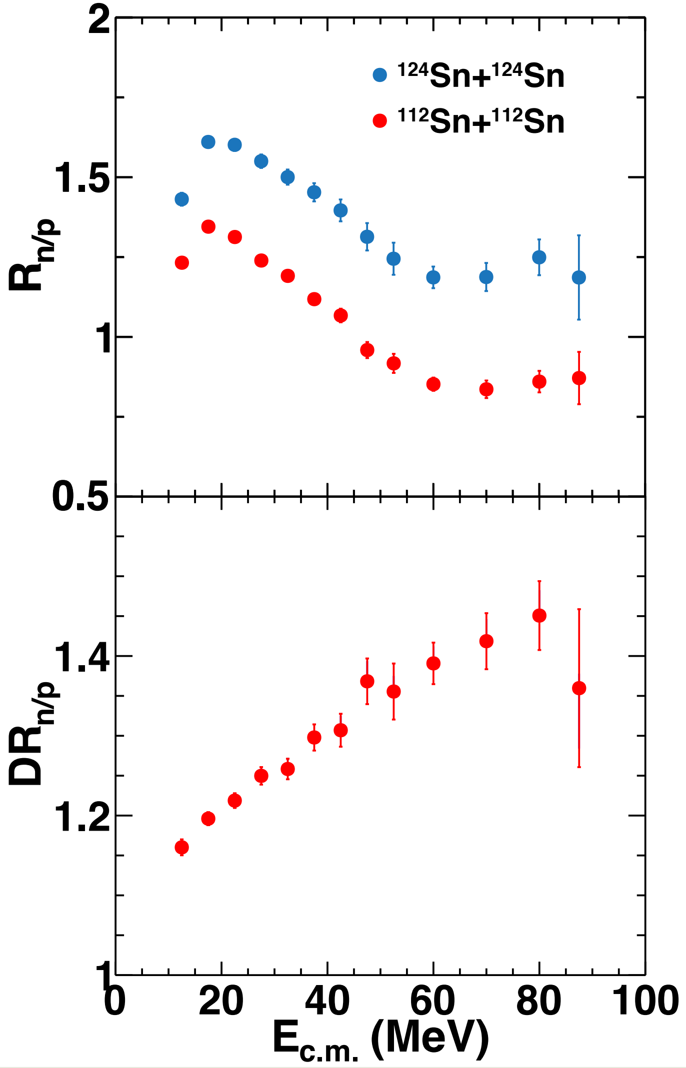
- Neutron and proton yield spectra are measured as a function of CM energy with the central collisions of neutron-rich and neutron-deficient systems.
  - $^{124}Sn+^{124}Sn$ (p=100, n=148) @ 120 AMeV
  - $^{112}Sn+^{112}Sn$ (p=100, n=124) @ 120 AMeV

- Neutron to proton ratios are constructed from the spectra.

$$R(\mathrm{n/p}) = \frac{dY_\mathrm{n}/dE_\mathrm{CM}}{dY_\mathrm{p}/dE_\mathrm{CM}}$$

- To eliminate the sensitivity to detection efficiencies, systematic uncertainties and coulomb effect, double ratio is employed.

$$DR(\mathrm{n/p}) = \frac{R_{124+124}(\mathrm{n/p})}{R_{112+112}(\mathrm{n/p})}$$



ImQMD Model

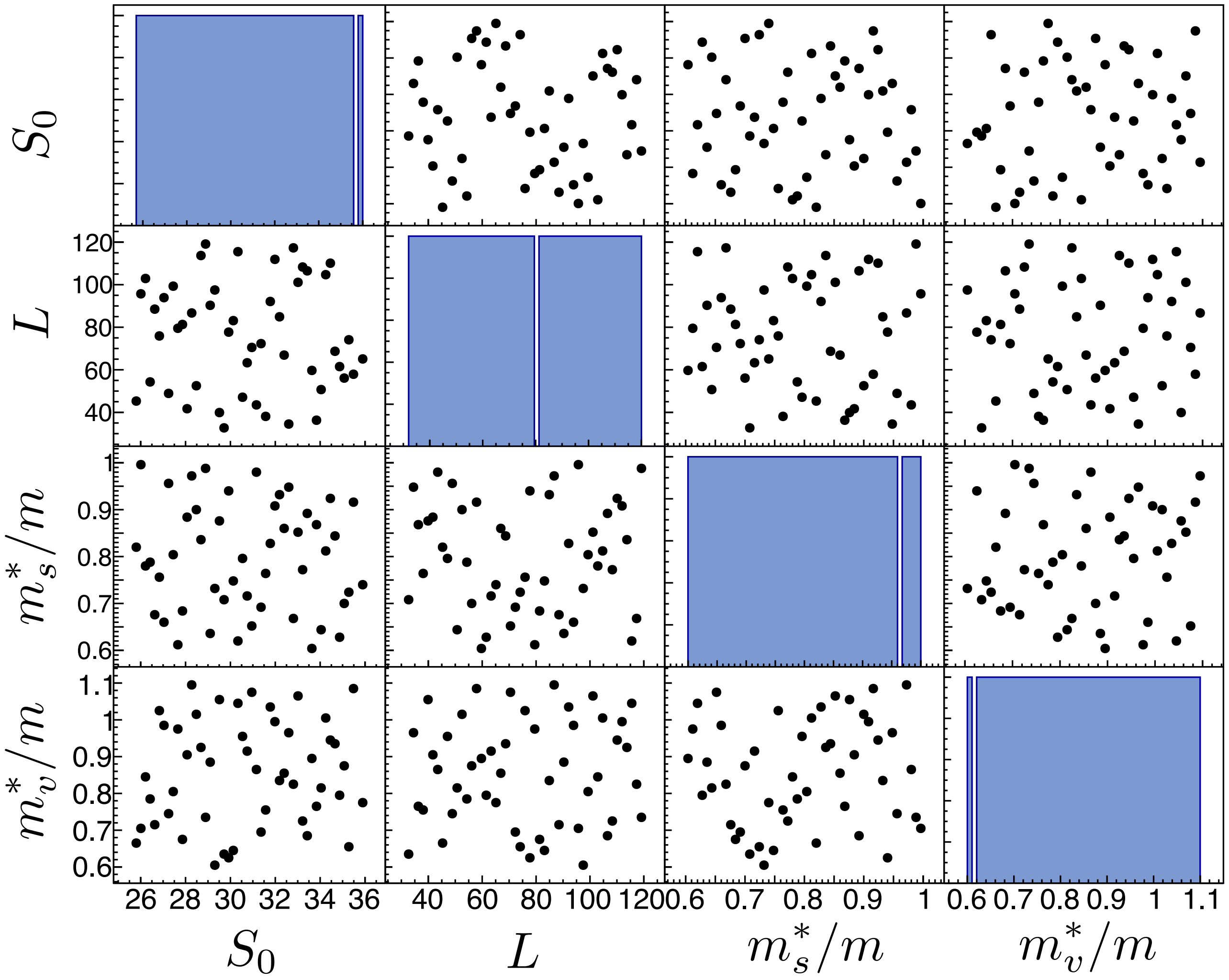
N. Wang *et al.*, Phys. Rev. C **65** (2002) 064608
N. Wang *et al.*, Phys, Rev. C **69** (2004) 034608

- **Im**proved **Q**uantum **M**olecular **D**ynamics model
  - Tuned to study low energy HI reactions.
  - Fermionic nature of nucleons is included. (Pauli blocking)
  - Mean field potential is obtained from Skyrme interaction parameters.

Y. Zhang *et al.*, Phys. Rev. C **74** (2006) 014602
L. W. Chen *et al.*, Phys. Rev. C **80** (2009) 014322

$$\rho_0,\ E_0(\rho_0),\ K_0,\ \boxed{m_s^*/m,\ m_v^*/m,\ S_0,\ L,}\ g_{\mathrm{sur}},\ g_{\mathrm{sur,asy}}$$

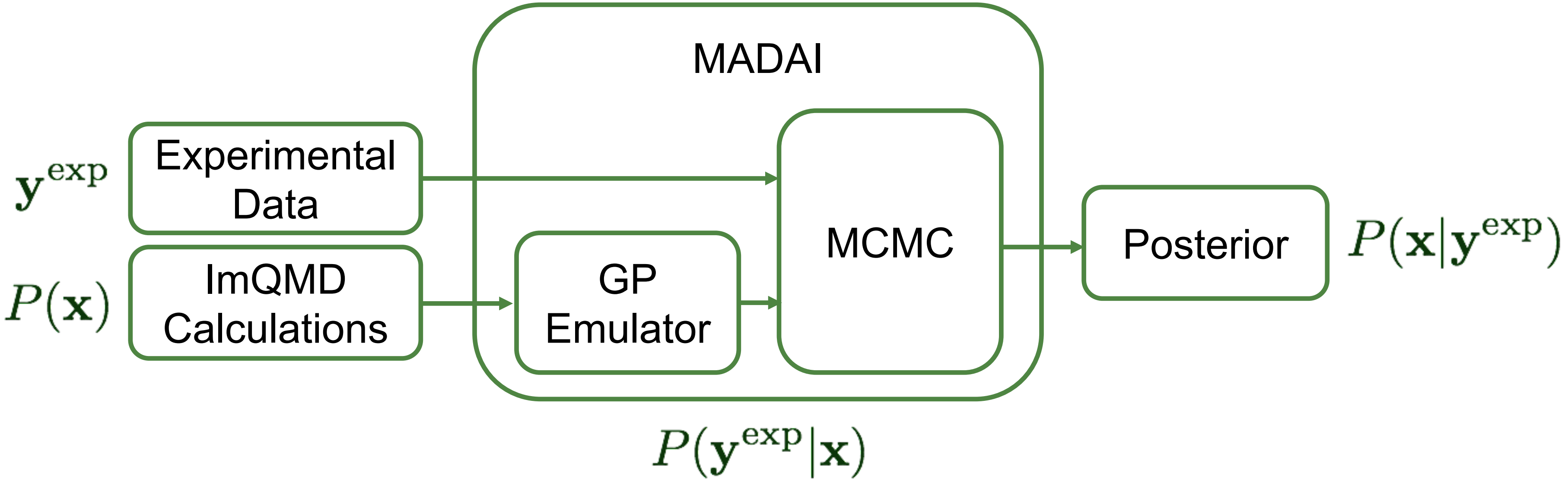
  - Momentum-dependent effective masses are parameterized.
- Parameters are sampled using Latin-hypercube sampling method.



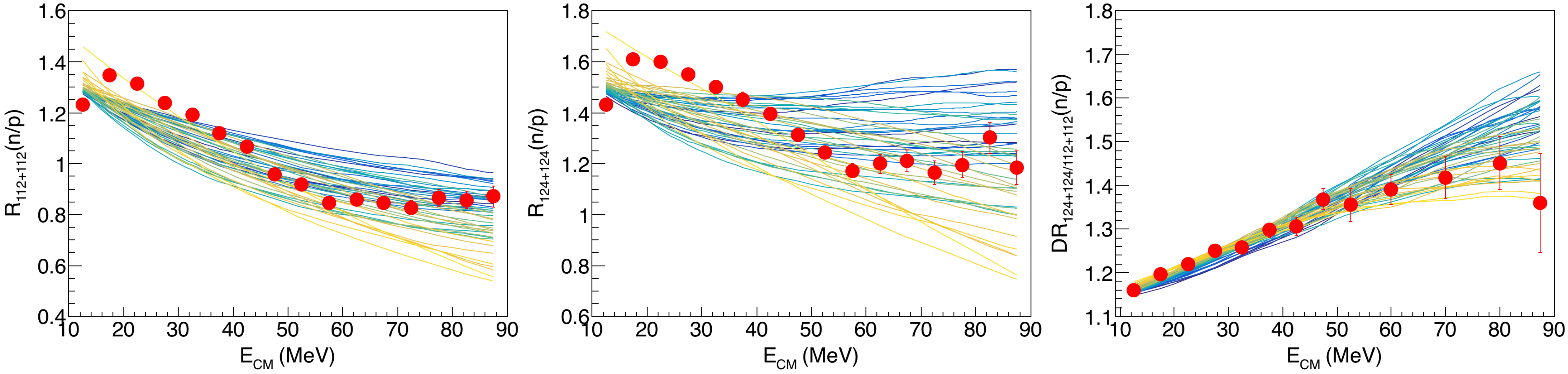
MADAI Package

http://madai.us

- **M**odel **A**nd **D**ata **A**nalysis **I**nitiative
- Distribution Sampling Tool
  - MCMC sampling with Metropolis-Hastings algorithm
  - Gaussian-process based emulator for time-expensive model calculation
- Flow chart



Model Check I

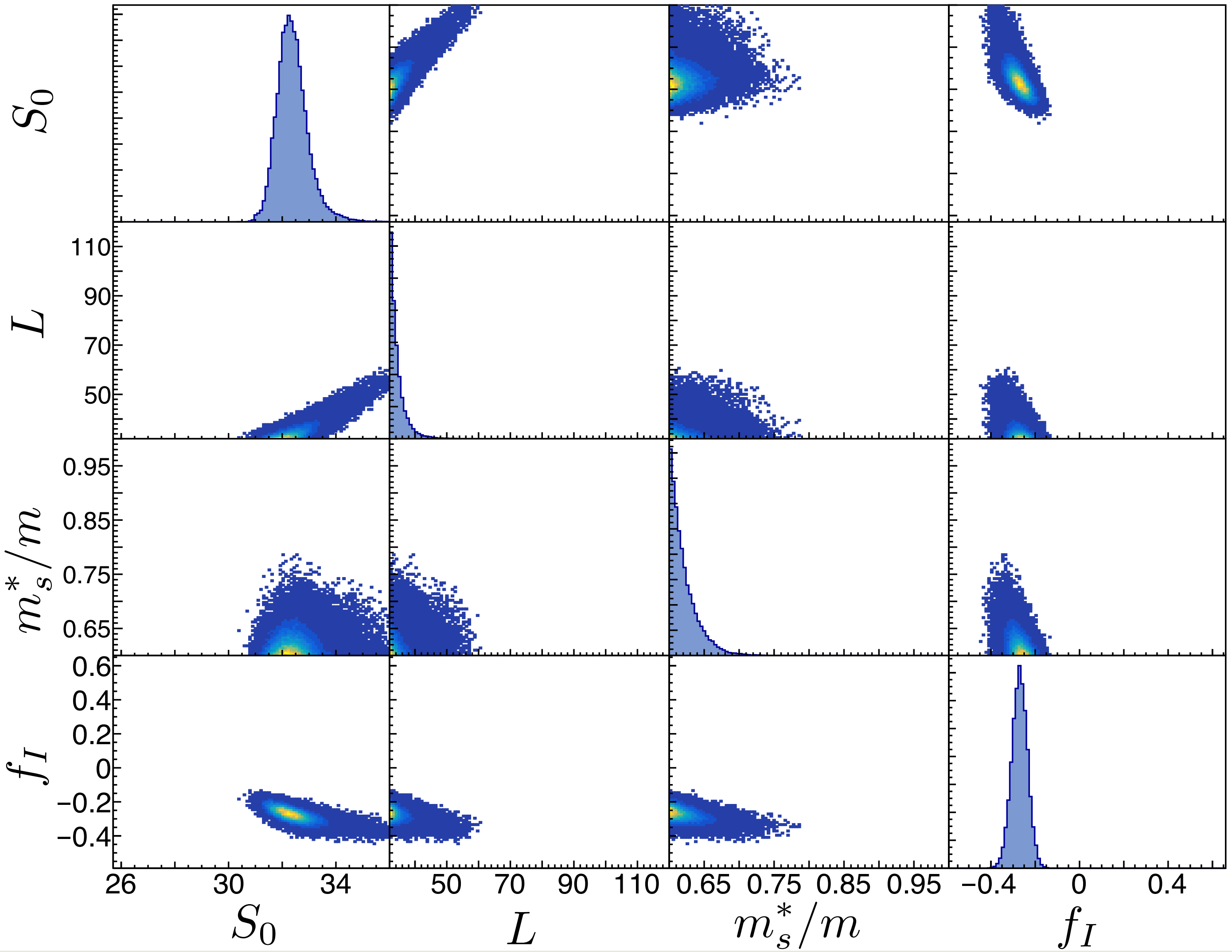


- Neutron to proton data points below 40 MeV in both systems are not well described by ImQMD model due to the missing physics in the model.
- Taking double ratio canceled the effect of missing physics making the model calculation and the data matches within the spanned parameter space.

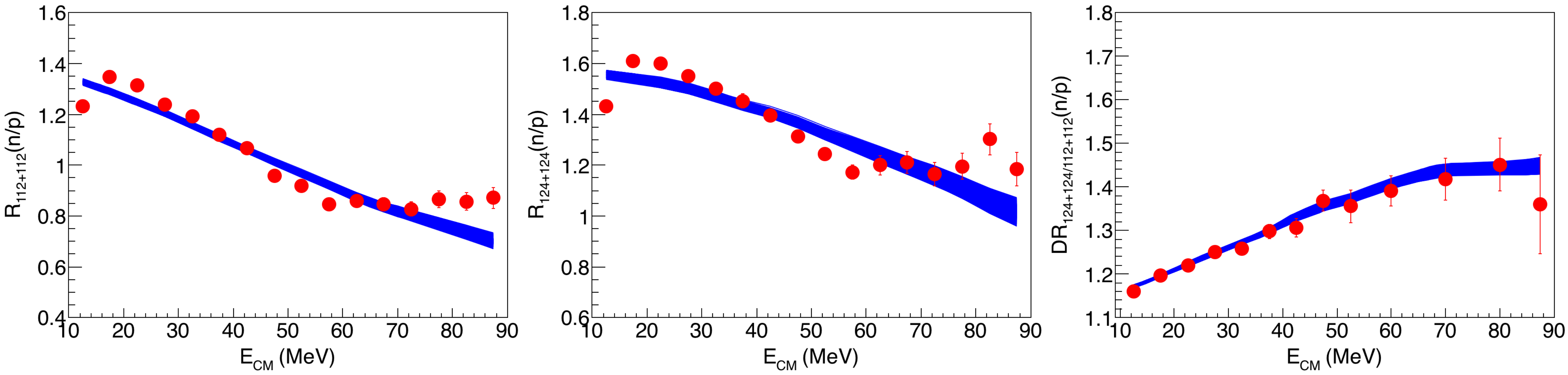
Posterior Distribution I

Parameter	Range
S_0	[25.7, 36]
L	[32, 120]
m_s^*/m	[0.6, 1.0]
f_I	[-0.59, 0.66]

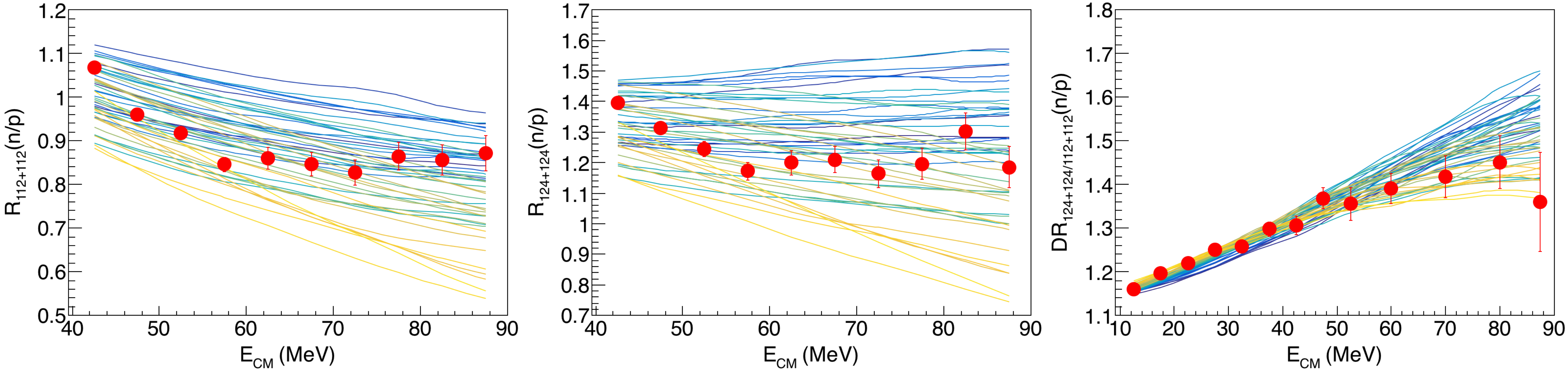
- Because of the model deficiency below E_{CM}<40 MeV in single ratios, m_s^*/m and L are not converged within the spanned parameter space.



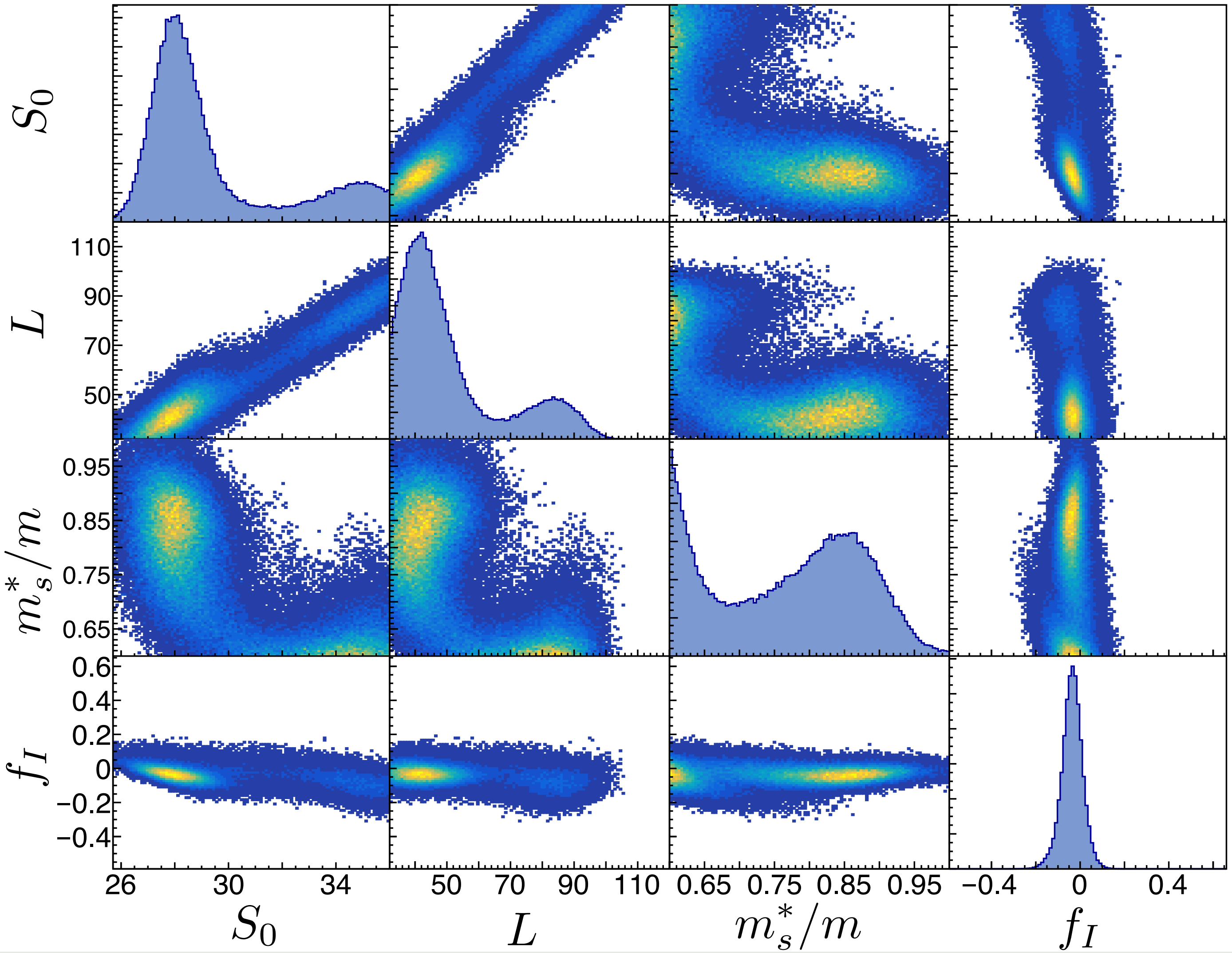
Posterior samples I



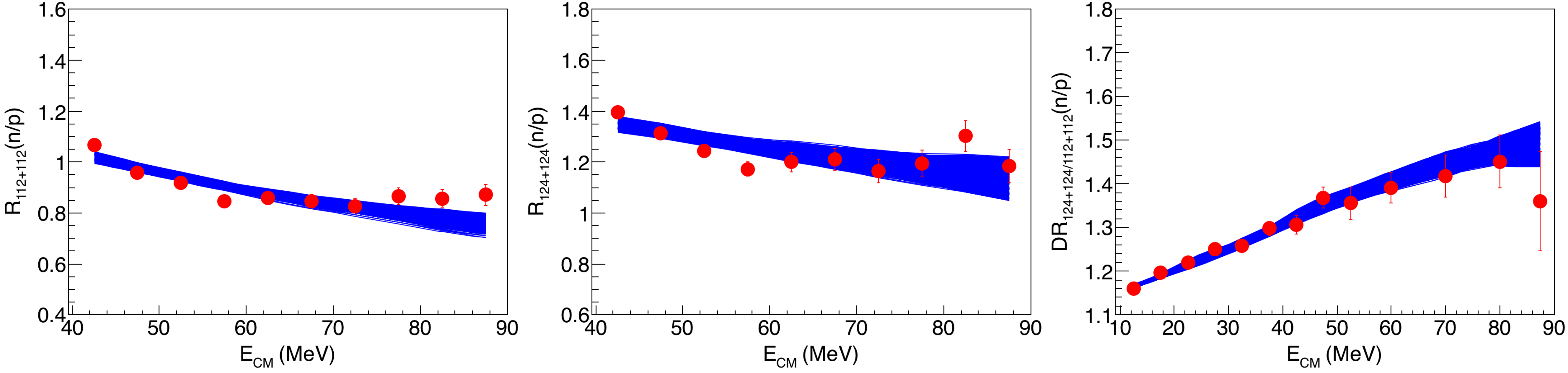
Model check II (cutting below 40 MeV)



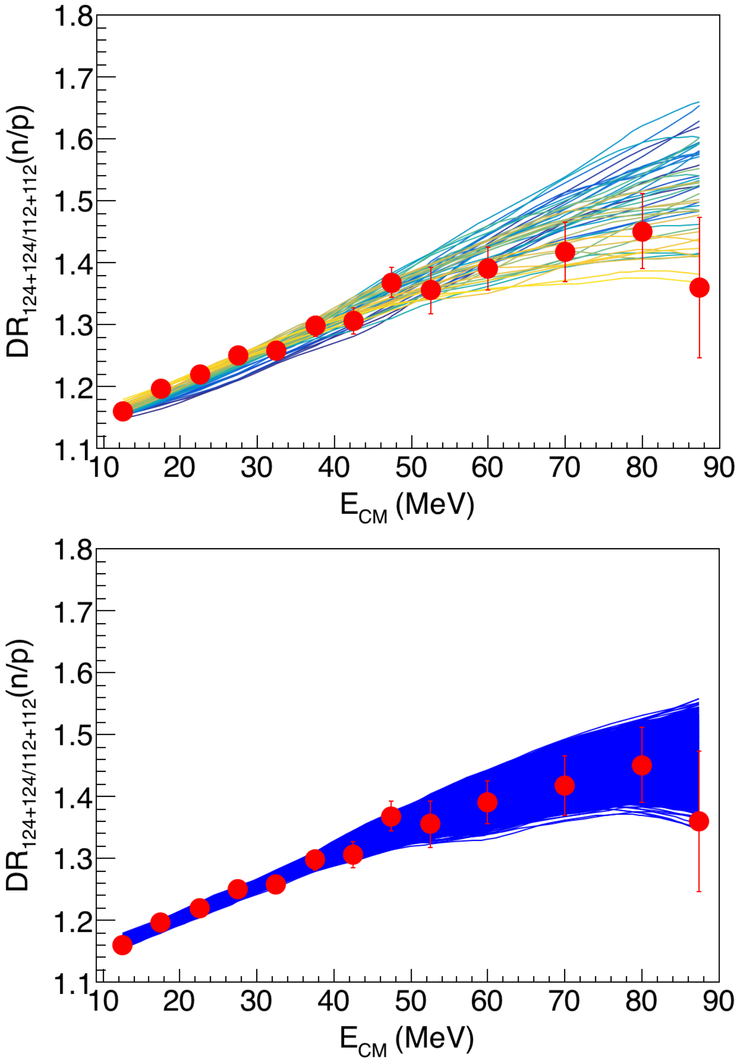
Posterior Distribution II

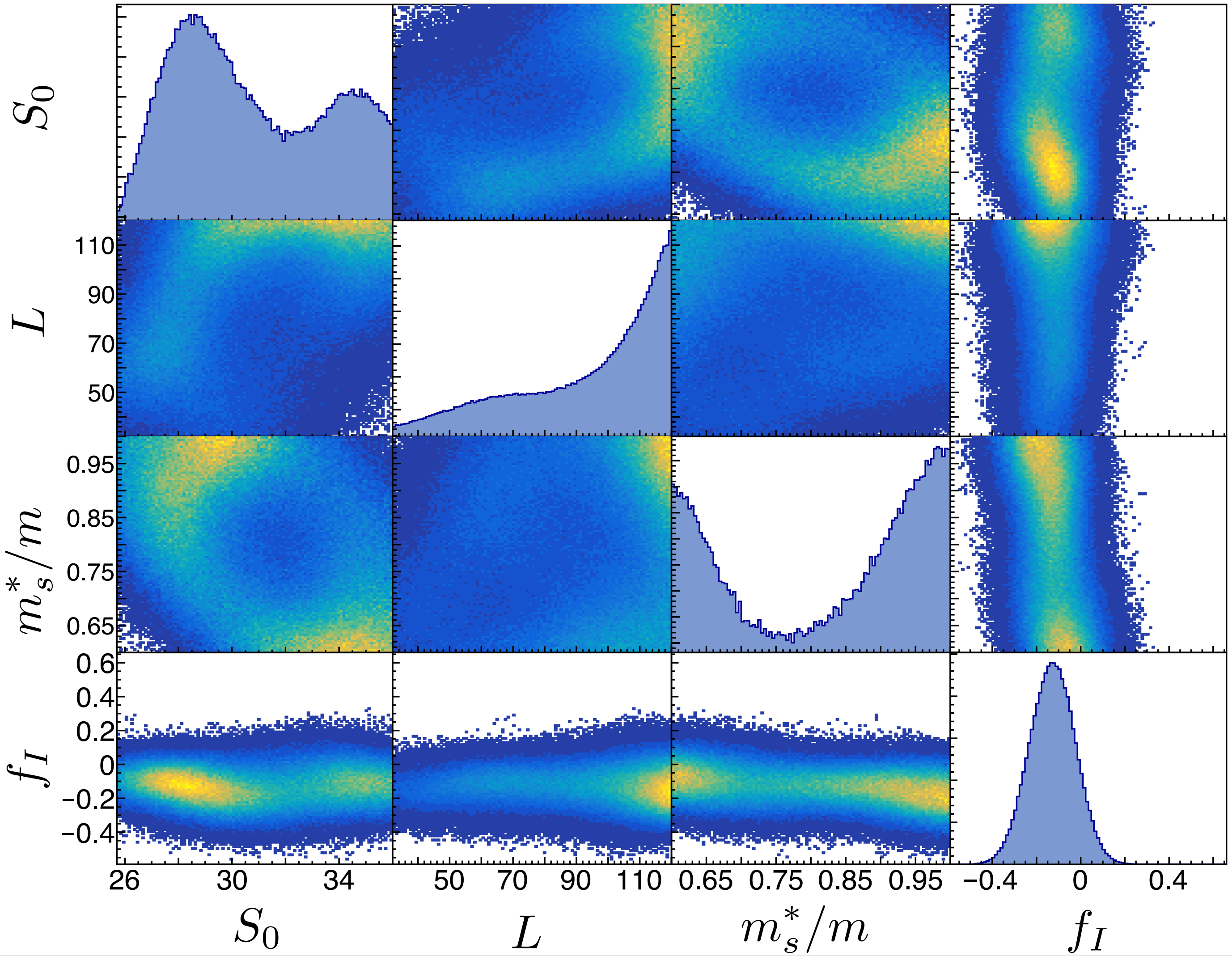


Posterior samples II



Only with DR





Summary and Outlook

- We have applied Bayesian analysis method to our data.
- Investigation is on going on the low energy part with theories and the high energy part with experimentalists.

- Recently we measured high density reaction data and they will be used to make constraints on high density region.

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