Model Equations for the `Orthodont` Example

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1 Growth Curve Model

$$
\begin{aligned}
y_{it} &= \beta_0 + \beta_1 \text{Age}_i + b_{0i} + b_{1i}\text{Age}_i + \epsilon_{it} \\
&\quad i = 1, \ldots, n, \quad t = 1, 2, 3, 4 \\
&\quad \mathbf{b}_i = \begin{bmatrix} b_{0i} \\ b_{1i} \end{bmatrix} \sim N(0, \Psi), \quad \Psi = \begin{bmatrix} \sigma^2_{00} & \sigma_{01} \\ \sigma_{01} & \sigma^2_{11} \end{bmatrix}, \\
&\quad \epsilon_{it} \sim N(0, \sigma^2_e),
\end{aligned}
\tag{1}
$$

where growth data y from the ith child at time t is a function of Age (centered at 11 years of age). The fixed intercept β_0 represents the model-estimated average growth at age 11 and the fixed slope β_1 represents the estimated average rowth rate per year. The random intercepts b_{0i} and the random slopes b_{1i} represent, respectively, each child's unique intercept and slope (as compared to the averages in β_0 and β_1).

2 R Syntax

```
orth_vintslope <- lmer(distance ~ I(age - 11) + (1 + I(age - 11) | Subject),
     data = orth)
```

3 SPSS Syntax

```
 MIXED distance WITH age
/FIXED INTERCEPT age
/RANDOM INTERCEPT age | SUBJECT(Subject) COVTYPE(ID)
/PRINT SOLUTION TESTCOV.
```