We return to #10, except undiscretized:

$$(10)\quad \underbrace{\left(r[\Lambda_{ij}]_{p\times p} + m[F_{ij}]_{p\times p}\right)}_{*}\left([\dot{\alpha}_{ij}]_{m\times p}\right)^T + t_c\left([\alpha_{ij}]_{m\times p}\right)^T = Gd\,\hat{e}_1\left(\vec{v}_{m\times 1}\right)^T$$

where $\vec{v}$ is a column vector (m rows (excluding boundaries) and 1 column).
Note that for any two matrices, to have a defined matrix product $A \cdot B$, A must have the same number of columns as B has rows, eg. $A_{m\times n} \cdot B_{j\times k}$ must have $n = j$. Ensuring (10) obeys this property:

$$([p\times p] + [p\times p])\,[m\times p]^T + [m\times p]^T = [?\times 1][m\times 1]^T$$

$$[p\times p][p\times m] + [p\times m] = [?\times 1][1\times m]$$

$$\begin{bmatrix} 1 & \cdots & p \\ \vdots & \ddots & \\ p & & p \end{bmatrix}\begin{array}{c} \begin{bmatrix} 1 & - & m \\ p & & \end{bmatrix} \\ \begin{bmatrix} 1 & - & m \\ \vdots & & \\ p & & \end{bmatrix}\end{array} \qquad \begin{bmatrix} 1 \\ \vdots \\ ? \end{bmatrix}\begin{array}{c}\begin{bmatrix} 1 & -- & m \end{bmatrix} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\end{array}$$

$$\downarrow$$

$$[p\times m] + [p\times m] = [?\times m]$$

We then see that $\hat{e}_1$ in this context has dimensions:

$$\boxed{\hat{e}_{p\times 1}}$$

I propose use of "T" to denote $*$ as capital tau.
Rewriting,

$$(10a)\quad \left([T_{ij}]_{p\times p}\right)\left([\dot{\alpha}_{ij}]_{m\times p}\right)^T + t_c\left([\alpha_{ij}]_{m\times p}\right)^T = Gd\,\hat{e}_{p\times 1}\left(\vec{v}_{m\times 1}\right)^T$$

Solving for $\dot{\alpha}$

$$(10b)\quad \left(\left([\dot{\alpha}_{ij}]_{m\times p}\right)^T\right)^T = \left(\left([T_{ij}]_{p\times p}\right)^{-1}\left[Gd\,\hat{e}_{p\times 1}\left(\vec{v}_{m\times 1}\right)^T - t_c\left([\alpha_{ij}]_{m\times p}\right)^T\right]\right)^T$$

Based on the definition of α as:

$$\alpha_j(t_n, z_i) = [\alpha_{ij}]_{m\times p} = \begin{bmatrix} \alpha_0(z_0) & \cdots & \alpha_p(z_0) \\ \vdots & \ddots & \\ \alpha_0(z_m) & & (\alpha_p(z_m)) \end{bmatrix}$$

therefore $P = \alpha_0$ implies that the first column of α should be extracted. In order input (10a) into "Model B", we can replace $\frac{\partial \bar{P}}{\partial \bar{t}}$ with the extracted first column of $\mathring{\alpha}$ done by the $\hat{e}_1$ operator, eg.

$$\begin{bmatrix} \alpha_0 & \cdots & \alpha_p \\ \vdots & \ddots & \\ \alpha_m & & \alpha_p \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{\partial \bar{P}}{\partial \bar{t}} = [\mathring{\alpha}_{ij}]_{m\times p}\, \hat{e}_{p\times 1}$$

Recall model B

$$\frac{\partial \bar{v}}{\partial \bar{t}} = G_{\rho 0} \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + \frac{\partial^2}{\partial \bar{z}^2}\left[\frac{\partial \bar{P}}{\partial \bar{t}}\right]$$

Substituting for $\partial\bar{P}/\partial\bar{t}$ in matrix notation, dropping bars:

$$\text{(Model B)} \quad \frac{\partial}{\partial t}\vec{v}_{m\times 1} = G_{\rho 0}\frac{\partial^2}{\partial z^2}\vec{v}_{m\times 1} + \frac{\partial^2}{\partial z^2}[\mathring{\alpha}_{ij}]_{m\times p}\,\hat{e}_{p\times 1}$$

Note that our substitution is also $m\times 1$.

Returning to (10b), we can distribute the transpose operator before substitution, giving

$$\text{(10c)} \quad [\mathring{\alpha}_{ij}]_{m\times p} = \left[G_1 d\,\vec{v}_{m\times 1}\hat{e}^T_{p\times 1} - t_c\,[\alpha_{ij}]_{m\times p}\right]\left([T_{ij}]_{p\times p}\right)^{-T}$$

Plugging (10c) into (Model B)

$$\frac{\partial}{\partial t}\vec{V}_{m\times 1} - G_{d0}\frac{\partial^2}{\partial z^2}\vec{V}_{m\times 1} = \frac{\partial^2}{\partial z^2}\left[G_d \vec{V}_{m\times 1}\hat{e}_{p\times 1}^T - t_c\,[\alpha_{ij}]_{m\times p}\right]\left([T_{ij}]_{p\times p}\right)^{-T}$$

Distributing out terms

$$-\frac{\partial}{\partial t}\vec{V}_{m\times 1} + G_{d0}\frac{\partial^2}{\partial z^2}\vec{V}_{m\times 1} + G_d\frac{\partial^2}{\partial z^2}\vec{V}_{m\times 1}\hat{e}_{p\times 1}^T[T_{ij}]_{p\times p}^{-T} = t_c\frac{\partial^2}{\partial z^2}[\alpha_{ij}]_{m\times p}[T_{ij}]_{p\times p}^{-T}$$

Note the $G_d \sim [T_{ij}]_{p\times p}^{-T}$ term, similar to G_d/τ.

Now, taking a stab at discretization

$$\frac{V_{m\times 1}^{n} - V_{m\times 1}^{n+1}}{\Delta t} + G_{d0} A\,\vec{V}_{m\times 1}^{n+\theta} + G_d A\,\vec{V}_{m\times 1}^{n+\theta}\hat{e}_{p\times 1}^T[T_{ij}]_{p\times p}^{-T}$$

$$= t_c\,A[\alpha_{ij}]_{m\times p}[T_{ij}]_{p\times p}^{-T}$$

Not sure what next step is, α undiscretized? Not sure how to simplify ✱.