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Research Project Overview

1 Concept overview

Given

$$\sigma(z,t) = H_A \int_0^t G(t-s)\frac{d\epsilon}{ds}(z,s)ds = H_A G * \frac{d\epsilon}{dt} \tag{1}$$

where

$$G(t) = 1 + c\int_{\tau_1}^{\tau_2} g(t,\tau)dF(\tau) = G_\infty + G_d\mathbb{E}_F[g] \tag{2}$$

In other words, $G_\infty \equiv 1, G_d \equiv c$ with

$$g(t,\tau) = \frac{e^{-t/\tau}}{\tau}, \quad \tau = \text{ ”relaxation time”} \tag{3}$$

Note that

$$\hat{g} = \frac{1}{1+i\omega\tau} \tag{4}$$

Then, $\sigma(z,t)$ can be expressed as

$$\sigma(z,t) = H_A G_\infty \left.\epsilon\right|_0^t + H_A P \tag{5}$$

where $P := G_d\mathbb{E}_F[g] * d\epsilon/dt$, giving

$$\hat{P} = G_d\mathbb{E}_F[\hat{g}] \cdot \frac{d\hat{\epsilon}}{dt} = \underbrace{\int_{\tau_1}^{\tau_2} \frac{G_d}{1+i\omega\tau}dF(\tau)}_{\text{“}\hat{G}_r(\omega)\text{”}} \cdot \frac{d\hat{\epsilon}}{dt} \tag{6}$$

“Can show that” $P = \mathbb{E}_F[\mathcal{P}]$ where $\mathcal{P}$ satisfies an (auxiliary) ODE:

$$\tau\dot{\mathcal{P}} + \mathcal{P} = G_d\frac{d\epsilon}{dt}, \text{ with } \tau \sim F(\tau). \tag{7}$$

2 Project overview

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Weeks	Planned Activities
1, 2	Show this problem is analogous to Debye Polarization in Electromagnetics (auxiliary ODE same, although PDEs differ)
3, 4	Implement polynomial chaos to solve random ODE for $\mathcal{P}$
5...	Rigorously prove approach is valid recover Gauss-Legendre as special case