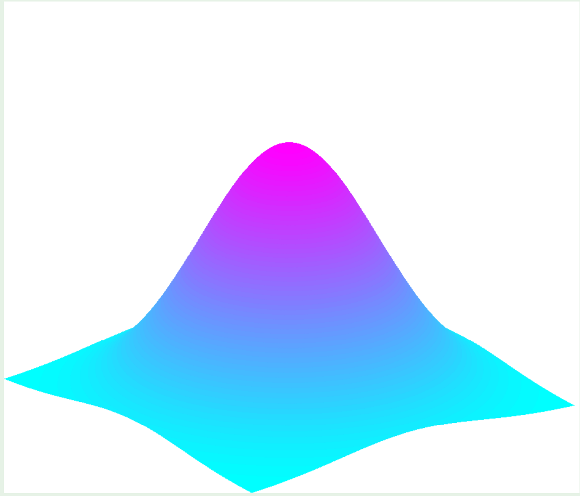
Review of Lecture 16

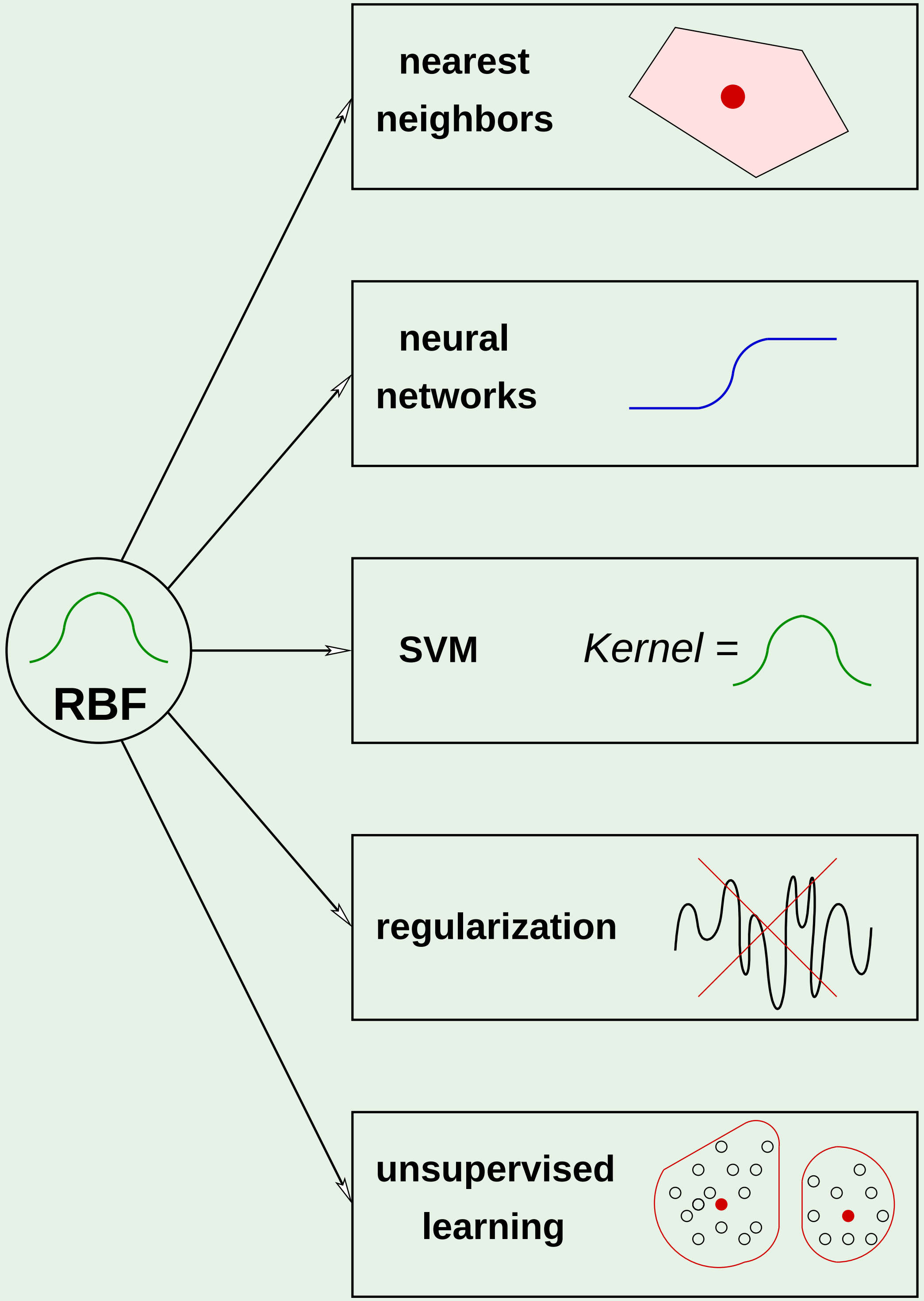
- Radial Basis Functions

$$h(\mathbf{x}) = \sum_{k=1}^{K} w_k \exp\left(-\gamma \left\|\mathbf{x} - \boldsymbol{\mu}_k\right\|^2\right)$$



Choose $\boldsymbol{\mu}_k$'s: Lloyd's algorithm

Choose w_k's: Pseudo-inverse



Learning From Data

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Lecture 17: Three Learning Principles

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Outline

- Occam's Razor

- Sampling Bias

- Data Snooping

Recurring theme - simple hypotheses

A "quote" by Einstein:

> An explanation of the data should be made *as simple as possible, but no simpler*

The razor: symbolic of a principle set by William of Occam

Occam's Razor

$$\boxed{\text{The simplest model that fits the data is also the most plausible.}}$$

Two questions:

1. What does it mean for a model to be simple?

2. How do we know that simpler is better?

First question: 'simple' means?

Measures of complexity - two types: complexity of h and complexity of $\mathcal{H}$

Complexity of h: MDL, order of a polynomial

Complexity of $\mathcal{H}$: Entropy, VC dimension

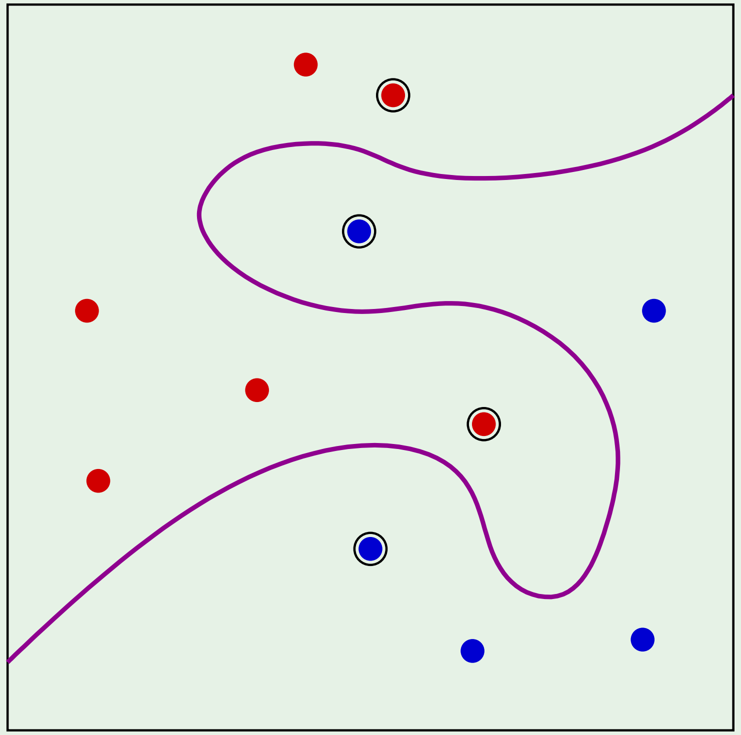
- When we think of simple, it's in terms of h
- Proofs use simple in terms of $\mathcal{H}$

and the link is ...

counting: ℓ bits specify h $\implies$ h is one of 2^ℓ elements of a set $\mathcal{H}$

Real-valued parameters? **Example:** 17th order polynomial - complex and one of "many"

Exceptions? Looks complex but is one of few - **SVM**



Puzzle 1: Football oracle

00000000000000001111111111111111	0
00000000111111110000000011111111	1
00001111000011110000111100001111	0
00110011001100110011001100110011	1
01010101010101010101010101010101	1
$\uparrow$	

- Letter predicting game outcome
- Good call!
- More letters - for 5 weeks
- Perfect record!
- Want more? \$50 charge ☺
- **Should you pay?**

Second question: Why is simpler better?

Better doesn't mean more elegant! It means better out-of-sample performance

The basic argument: (formal proof under different idealized conditions)

Fewer simple hypotheses than complex ones

$$m_{\mathcal{H}}(N)$$

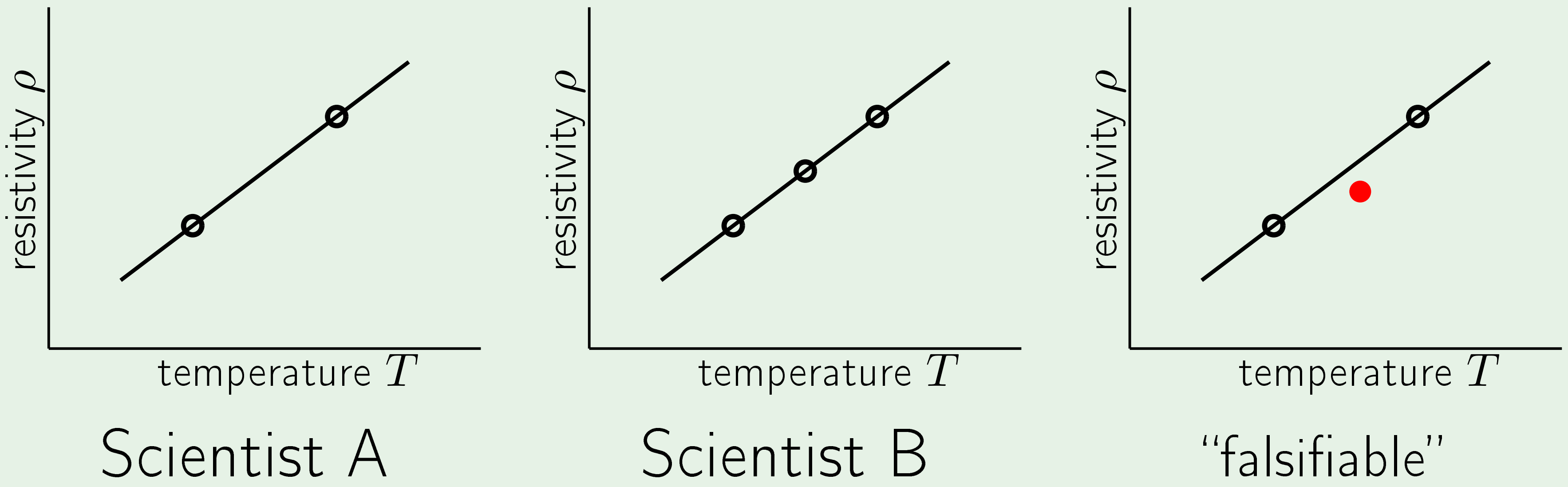
$\Rightarrow$ less likely to fit a given data set

$$m_{\mathcal{H}}(N)/2^N$$

$\Rightarrow$ more significant when it happens

The postal scam: $m_{\mathcal{H}}(N) = 1$ versus 2^N

A fit that means nothing



Conductivity linear in temperature?

Two scientists conduct experiments

What evidence do A and B provide?

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Puzzle 2: Presidential election

In 1948, **Truman** ran against **Dewey** in close elections

A newspaper ran a phone poll of how people <u>voted</u>

Dewey won the poll decisively - newspaper declared:

On to the victory rally ...

... of Truman ☺

It's not δ's fault:

$$\mathbb{P}\left[\,|E_{\text{in}} - E_{\text{out}}| > \epsilon\,\right] \;\leq\; \delta$$

The bias

In 1948, phones were expensive.

If the data is sampled in a biased way, learning will produce a similarly biased outcome.

Example: normal period in the market

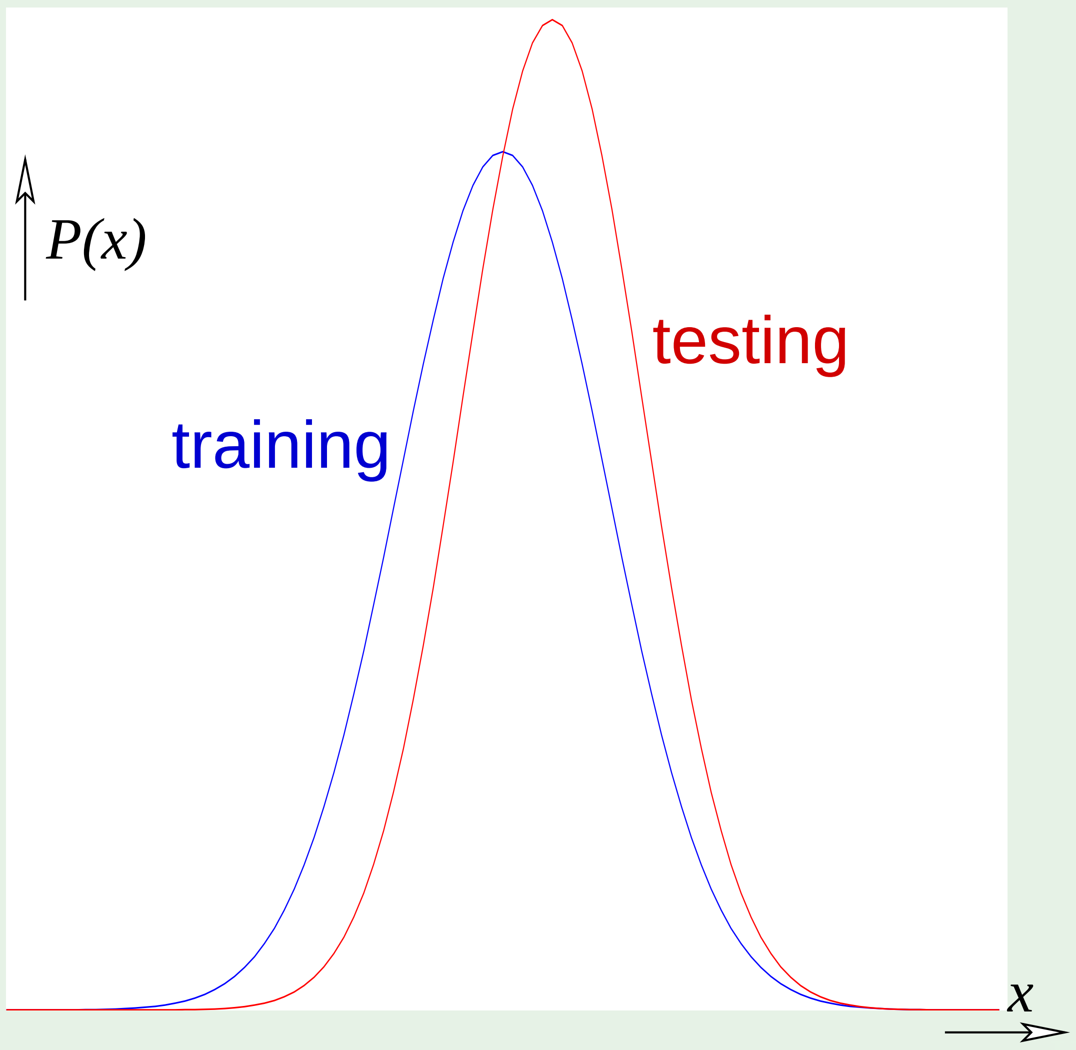
Testing: live trading in real market

Matching the distributions

Methods to match training and testing distributions

Doesn't work if:

Region has $P = 0$ in training, but $P > 0$ in testing



Puzzle 3: Credit approval

Historical records of customers

Input: information on credit application:

Target: profitable for the bank

age	23 years
gender	male
annual salary	$30,000
years in residence	1 year
years in job	1 year
current debt	$15,000
...	...

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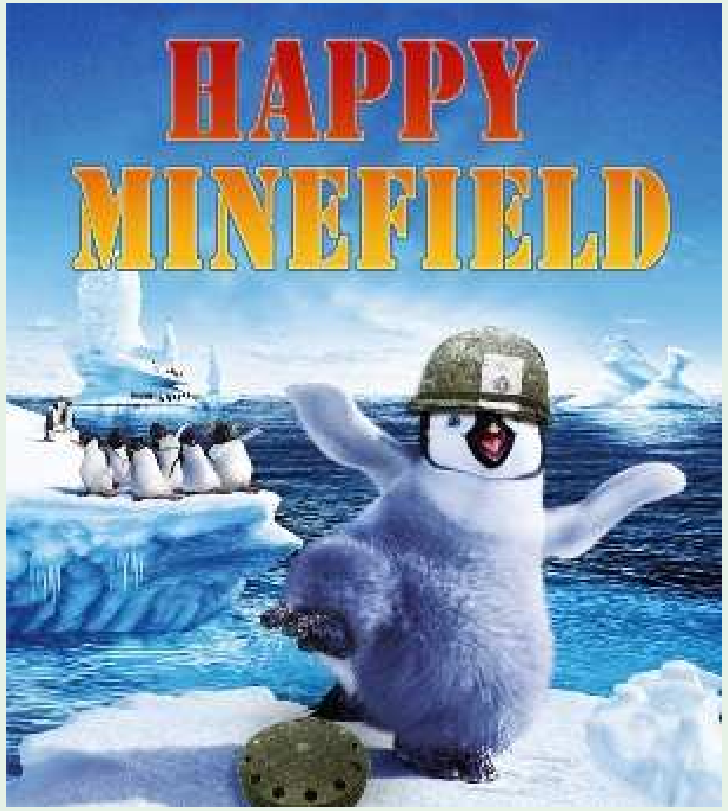
The principle

$$\boxed{\text{If a data set has affected any step in the learning process, its ability to assess the outcome has been compromised.}}$$

Most common trap for practitioners - many ways to slip ☹



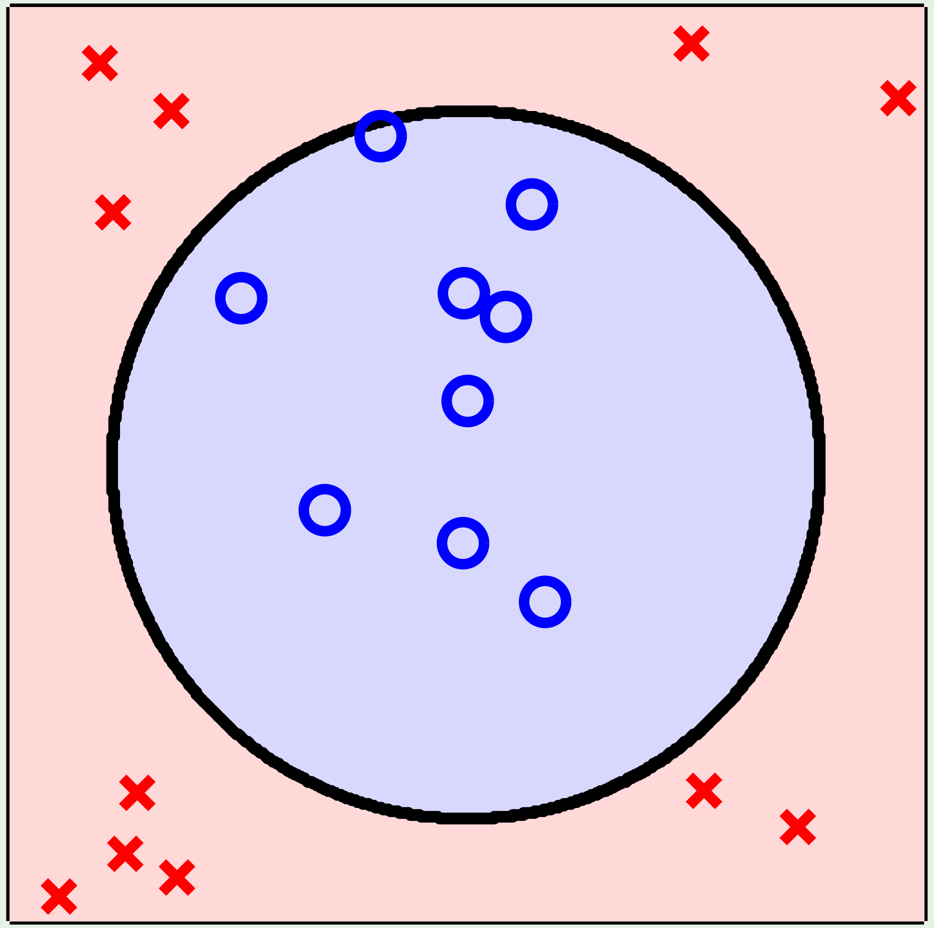
Looking at the data

Remember nonlinear transforms?

$\mathbf{z} = (1, x_1, x_2, x_1 x_2, x_1^2, x_2^2)$

or $\mathbf{z} = (1, x_1^2, x_2^2)$ or $\mathbf{z} = (1, x_1^2 + x_2^2)$

Snooping involves $\mathcal{D}$, not other information

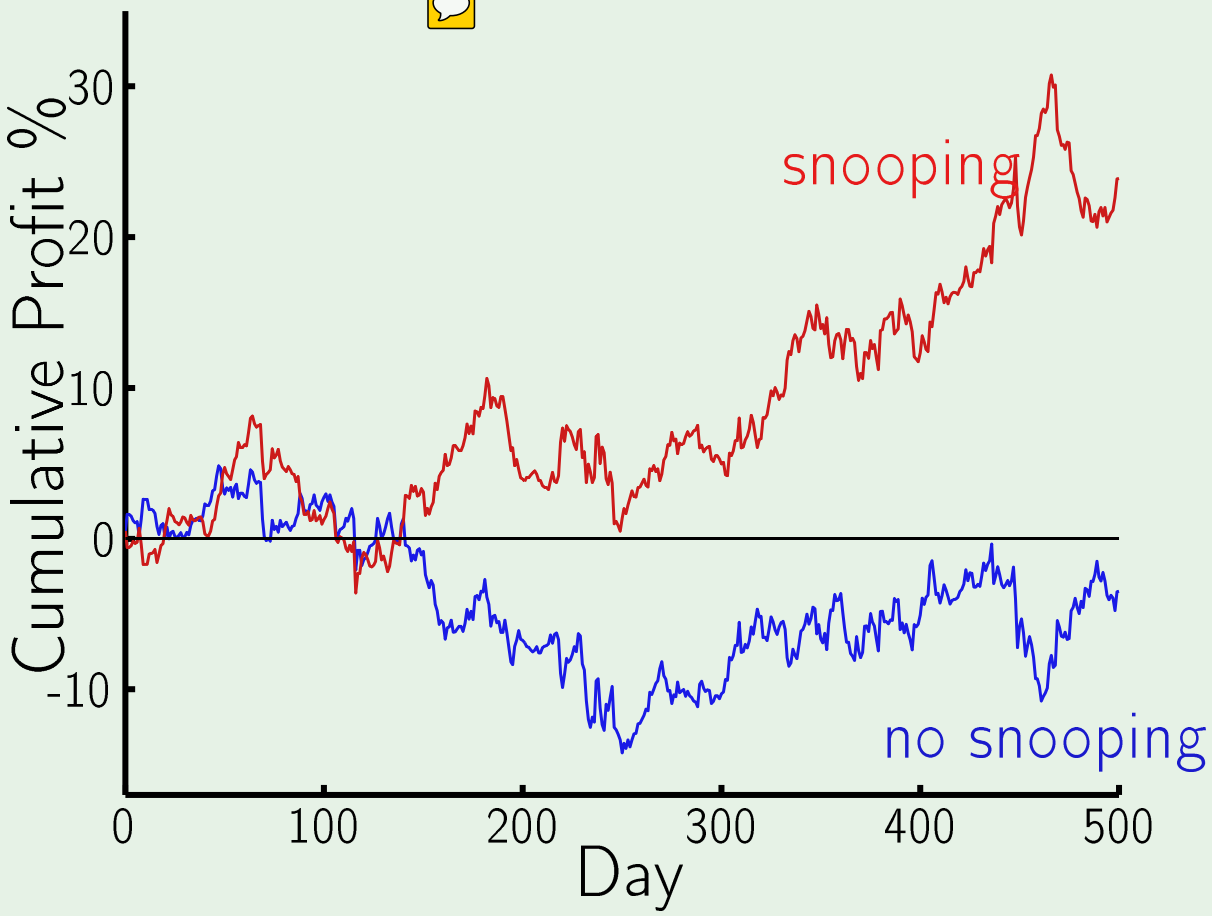


Puzzle 4: Financial forecasting

Predict US Dollar versus British Pound

Normalize data, split randomly: $\mathcal{D}_{\text{train}}$, $\mathcal{D}_{\text{test}}$

Train on $\mathcal{D}_{\text{train}}$ only, test g on $\mathcal{D}_{\text{test}}$



$$\Delta r_{-20}, \Delta r_{-19}, \cdots, \Delta r_{-1} \rightarrow \Delta r_0$$

Reuse of a data set

Trying one model after the other **on the same data set**, you will eventually 'succeed'

If you torture the data long enough, it will confess

VC dimension of the **total** learning model

May include what **others** tried!

Key problem: matching a *particular* data set

Two remedies

1. **Avoid** data snooping

   strict discipline

2. **Account for** data snooping

   how much data contamination

Puzzle 5: Bias via snooping

Testing long-term performance of "buy and hold" in stocks. Use **50 years** worth of data

- All currently traded companies in S&P500
- Assume you strictly followed buy and hold
- Would have made great profit!

Sampling bias caused by 'snooping'