Question 1.

A bag contains red, blue, and green marbles. The number of each color marble is unknown, but the total number of marbles is known to be 30. You are to design a game involving drawing a marble from the bag without looking, where players win a prize if they draw a red marble. Create a set of rules for the game that includes the probability of winning a prize, and explain how changing the number of red marbles affects the probability of winning. Assume you have the freedom to decide how many marbles of each color are in the bag, but there must be at least one marble of each color. Illustrate your answer with at least two different scenarios and provide the calculations for the probability of winning in each case.

Answer:

The solution will vary depending on the rules and scenarios created by the student. Here is an example of a possible solution:

Rules of the Game:

- The game is played by drawing a single marble from the bag.

- If the player draws a red marble, they win a prize.

Scenario 1:

- Suppose there are 10 red marbles, 10 blue marbles, and 10 green marbles.

- The probability of drawing a red marble is $\frac{10}{30} = \frac{1}{3}$

Scenario 2:

- Suppose there are 15 red marbles, 10 blue marbles, and 5 green marbles.

- The probability of drawing a red marble is $\frac{15}{30} = \frac{1}{2}$

By increasing the number of red marbles from 10 to 15, the probability of winning the game increases from $\frac{1}{3}$ to $\frac{1}{2}$

Grading Rubric:

- Explanation of game rules: 2 points

- Calculation of probability in Scenario 1: 3 points

- Calculation of probability in Scenario 2: 3 points

- Explanation of how changing the number of red marbles affects the probability: 2 points

Total: 10 points

'Topic': 'Probability', 'Subtopic': 'Experimental Probability', 'Difficulty Level': '5', 'Blooms Class': 'Creating'

Question 2.

Consider the function $f(x) = ax^2 + bx + c$, where a , b, and c are constants. The graph of f(x) passes through the points $(1, 6)$, $(2, 11)$, and $(3, 18)$. Analyze the given points and the form of the function to determine the values of a, b, c. Show all your steps and justify your reasoning.

Answer

To find the values of a, b and c, we can set up a system of equations using the given points by substituting them into the function f(x).

For (6,1): $6 = a * 1^2 + b * 1 + c$

For (2,11): $11 = a * 2^2 + b * 2 + c$

For (3,18): $18 = a * 3^2 + b * 2 + c$

Solving this system of equations will give us the values of a, b and c.

Grading rubric:

- Setting up the system of equations correctly (2 points)

- Correctly using algebraic methods to solve for a, b and c. (3 points)

- Justifying each step of the solution (2 points)

- Arriving at the correct solution for a, b and c. (3 points)

'Topic': 'Algebra', 'Subtopic': 'Quadratic Equations', 'Difficulty Level': '5', 'Blooms Class': 'Analyzing'

Question 3.

Simplify the expression: 2(x + 3) + 4(x - 2). Show each step of your simplification process.

Answer

Step 1: Distribute the 2 into the first parenthesis: $2 \times x + 2 \times 3$ which gives $2x + 6$.

Step 2: Distribute the 4 into the second parenthesis: $4 \times x + 4 \times (-2)$ which gives $4x - 8$.

Step 3: Combine like terms: $2x + 6 + 4x - 8$ which simplifies to $6x - 2$.

Grading Rubric:

For correctly distributing into the first parenthesis (1 point)

For correctly distributing into the second parenthesis (1 point)

For combining like terms correctly (1 point)

For the final simplified expression (1 point)

'Topic': 'Algebra', 'Subtopic': 'Simplifying Expressions', 'Difficulty Level': '1', 'Blooms Class': 'Remembering'

Question 4.

A recipe for a cake requires 250 grams of flour for every 75 grams of sugar. If a baker has 1 kilogram of sugar, how much flour is needed to keep the proportions the same?

Answer

First, we need to find the ratio of flour to sugar in the recipe.

We have 250 grams of flour for every 75 grams of sugar, which simplifies to a ratio of $\frac{250}{75}$.

Simplifying this ratio gives us $\frac{10}{3}$.

This means that for every 3 parts of sugar, we need 10 parts of flour.

Since the baker has 1 kilogram (or 1000 grams) of sugar, we can set up a proportion to find out how much flour is needed.

Let's use x to represent the amount of flour needed. We have: $\frac{10}{3} = \frac{x}{1000}$

Cross-multiplying gives us: $3x = 10000$

Dividing both sides by 3 gives us: $x = \frac{10000}{3}$

So, x = 3333.33 grams of flour are needed. Therefore, the baker needs 3333.33 grams (or 3.333 kilograms) of flour to keep the proportions the same.

Grading Rubric:

Step 1: Identifying the ratio': '1 point

Step 2: Simplifying the ratio': '1 point

Step 3: Setting up the proportion': '2 points

Step 4: Cross-multiplying to solve for x': '2 points

Step 5: Dividing to find the value of x': '1 point

Step 6: Stating the final answer with correct units': '1 point

Question 5.

Consider the function $f(x) = ax^2 + bx + c$. Given that the graph of f(x) passes through the point (1, 6) and has a vertex at the point (3, 2), analyze the information to determine the values of a, b, and c. Show all steps in your analysis.

Answers:

To find a, b, and c, we use the vertex form of a quadratic function, $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola.

1. Since the vertex is (3, 2), we have h = 3 and k = 2. Thus, the function can be written as $f(x) = a(x - 3)^2 + 2$

2. Using the point (1, 6), we substitute x = 1 and f(x) = 6 into the equation to find a:

$6 = a(1 - 3)^2 + 2$

6 = 4a + 2

4 = 4a

a = 1

3. Now that we have a, we can expand the vertex form to find b and c:

$f(x) = (x - 3)^2 + 2$

$f(x) = x^2 - 6x + 9 + 2$

$f(x) = x^2 - 6x + 11$

 Therefore, a = 1, b = -6, and c = 11.

Grading Rubric:

- Identifying the vertex form and substituting the vertex (2 points)

- Substituting the point (1, 6) and solving for a (3 points)

- Expanding the vertex form to find b and c (3 points)

- Writing the final equation in standard form (2 points)

'Topic': 'Algebra', 'Subtopic': 'Quadratic Functions', 'Difficulty Level': '4', 'Blooms Class': 'Analyzing'