Data Analysis in R

Prediction

Ken Stiller

12th January 2024

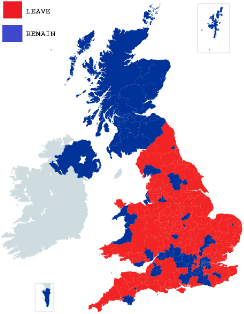
Syllabus: Data Analysis in R

1. Introduction
2. Causality & Basics of Statistics
3. Sampling & Measurement
4. **Prediction**
5. Multivariate Regression
6. Probability & Uncertainty
7. Hypothesis Testing
8. Assumptions & Limits of OLS
9. Interactions & Non-Linear Effects

Table of Contents

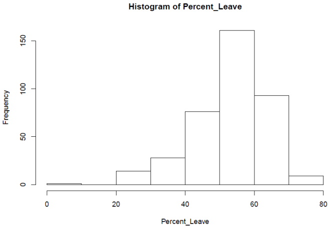
Predicting Brexit I

- Context:
  - On the 23rd of June 2016 the UK voted to leave the EU
  - 51.89% of voters who turned out (72.21%), voted to leave the EU



Predicting Brexit II

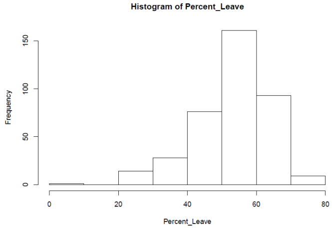
- We want to predict the % `Vote Leave` in a given constituency.
- We want to minimise errors (in stats generally) and be parsimonious (e.g. don't add variables unless we learn from them)
- All we know is the actual Leave Vote in each constituency.
  - We want to take one at random, predict its % `Vote leave`. How?



- Now let's say that we know one other variable that we can use as a predictor: *mean age in constituency.*
  - Does this help us? What would you do with it?

Predicting Brexit II

- We want to predict the % `Vote Leave` in a given constituency.
- We want to minimise errors (in stats generally) and be parsimonious (e.g. don't add variables unless we learn from them)
- All we know is the actual Leave Vote in each constituency.
  - We want to take one at random, predict its % `Vote leave`. How?

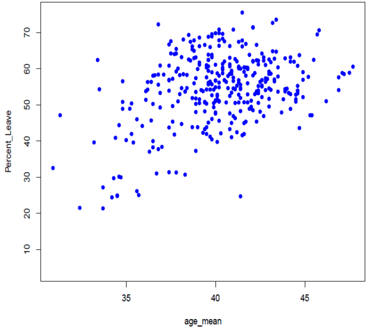


- Now let's say that we know one other variable that we can use as a predictor: *mean age in constituency.*
  - Does this help us? What would you do with it?

The Big Picture

- Our goal always is to make statements (predictions) about a *population*, minimising errors
- We can add increasingly more information about the data to help predict an outcome
- You might think of tools to do this as a continuum:
  1. Descriptive statistics: we draw upon one variable only
  2. **Bivariate regression**: we draw upon two variables
  3. Multivariate regression: we draw upon more than two variables

Linear Regression: Motivation



- How would you summarize the relationship between X and Y?
- It seems that counties with older population also have a higher leave vote
- **What we are interested in now: By how much?** (Note that correlation can't tell!)

Linear Regression: Intuition

What we often want to summarise is conditional expectation of a variable (Y) dependent on another variable (X)

- This is written as $E[Y|X]$, where E stands for *expectation*
- $E[Y]$ is the *population mean* and known as expectation only
- $E[Y|X=x]$ is the expectation of Y given a value of x

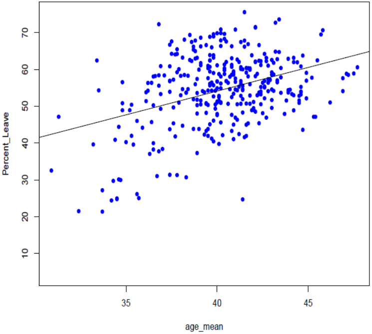
Regression allows us to provide overall estimates about how Y changes with X - without having to rely on specific values of X.

- Linear regression assumes Y varies in X in the same way through the range of values of X
- This allows us to predict the value of Y for each value of X
- It's a simple linear form of the conditional expectation function:

$$E[Y|X] = \beta_0 + \beta_1 X$$

Linear Regression: Equation

$$E[Y|X] = \beta_0 + \beta_1 X$$



- What does β_0 stand for?

$$E[Y|X = 0]$$

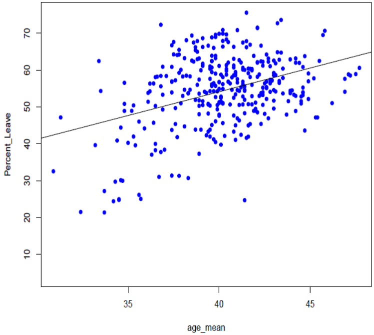
- And β_1?

$$E[Y|X = x] - E[Y|X = x - 1]$$

- Does the value of X matter?
  No - there is a single, uniform slope.

Linear Regression: Equation

$$E[Y|X] = \beta_0 + \beta_1 X$$



- What does β_0 stand for?

$$E[Y|X=0]$$

- And β_1?

$$E[Y|X=x] - E[Y|X=x-1]$$

- Does the value of X matter? **No** - there is a single, uniform slope.

Linear Regression: Notation

So, a linear regression model is a *linear* approximation of the relationship between explanatory variables X and a dependent variable Y

$$E[Y|X] = \underbrace{\beta_0}_{\text{Intercept}} + \underbrace{\beta_1}_{\text{Slope}} X$$

- Y: Dependent variable (outcome)
- X: Explanatory/independent variable
- β_0: Intercept (or constant)
- β_1: Slope coefficient (**association** between X and Y)

Quick interpretation:

- $\beta_0 + \beta_1 X$: Conditional mean of Y given a value of X
- β_0: Value of Y when $X = 0$
- β_1: Change in Y associated with a one unit increase in X across

Linear Regression: Notation II

$$E[Y|X] = \underbrace{\beta_0}_{\text{Intercept}} + \underbrace{\beta_1}_{\text{Slope}} X$$

- Sign: denotes the direction of the relationship:
  - $\beta_1 > 0$: Increase in X is associated with an increase in values of Y
  - $\beta_1 < 0$: Increase in X is associated with a decrease in values of Y
- Magnitude: β_1 tells us the extent to which Y changes with a one unit increase in X

Linear Regression: Prediction v Causality

Prediction

Regression allows us to *predict* the value of Y for any value of X, even if the specific x is not included in the sample

$$E\left[Y \mid X = x_{any}\right] = \hat{\beta}_0 + \hat{\beta}_1 \times x_{\text{any}}$$

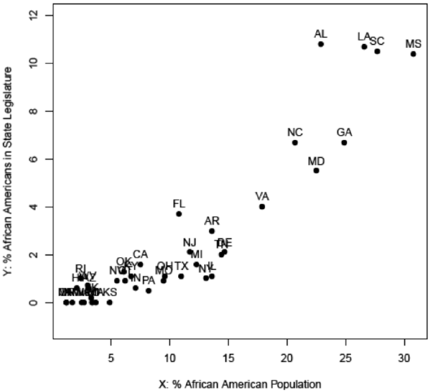
In a predictive model, $\hat{\beta}_1$ is interpreted as the expected difference in Y when there is a one unit increase in X

Causality

Prediction $\neq$ causality! Predicting Y on the basis of X does not imply that it is the change in X that causes Y!

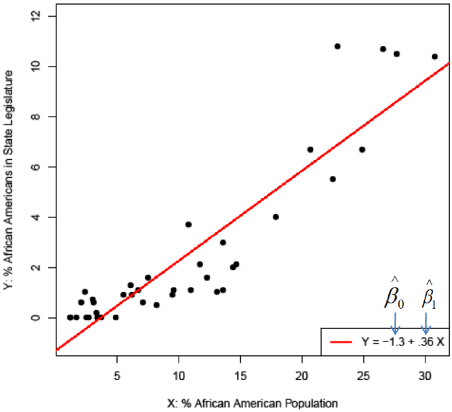
- Causality implies that we make sure other factors (confounders) that can cause a change in both X and Y are being accounted for
- Regression is helpful only because it provides a framework to account for other observed confounders
- It's a means to end - not more, not less

A Simple Bivariate Relationship



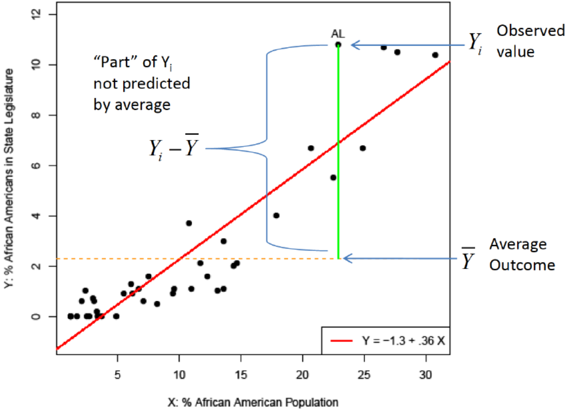
- How do we choose which line to fit to the data?
- In principle, infinite number of lines available
- Recall our aims as social scientists

Fit the Line



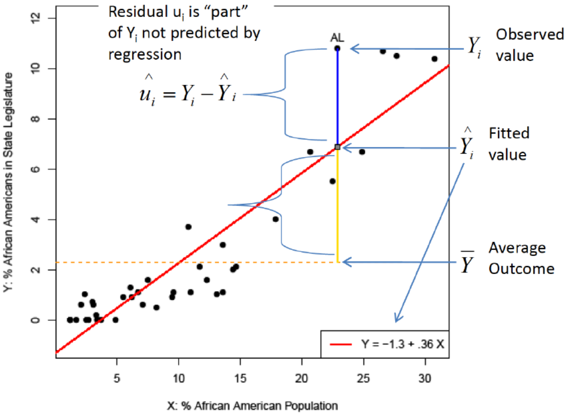
Why choose this line though?

Compare with Benchmark



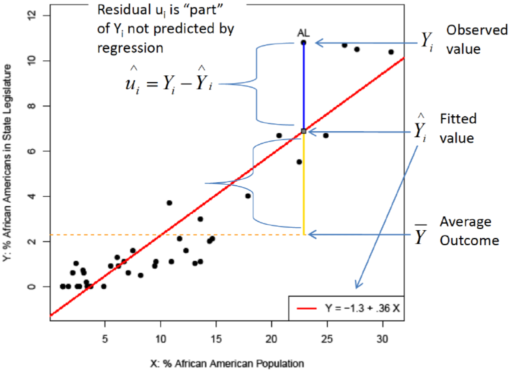
Our benchmark is the veil of ignorance: predicting Y without knowledge of x.

Compare with Benchmark II



Is this helpful? Let's decompose the distance between Y_i and $\bar{Y}$ to find out.

Decompose Distance between Y_i and $\bar{Y}$



From $\bar{Y}$ to $\hat{Y}_i$: Improvement in prediction from veil of ignorance: `Predicted Y`

From $\hat{Y}_i$ to Y_i: Remaining mistakes we make in prediction: `Residual`

Linear Regression Model

- Model:

$$Y = \underbrace{\alpha}_{\text{Intercept}} + \underbrace{\beta}_{\text{Slope}} X + \underbrace{\epsilon}_{\text{Error Term}}$$

  - (α, β): coefficients (parameters of the model)
  - ϵ : unobserved error/disturbance term (mean zero)

- Fitted model:
  $\hat{Y} = \hat{\alpha} + \hat{\beta}x$: predicted/fitted values
  $\hat{\mu} = Y - \hat{Y}$: residuals

  - $(\hat{\alpha}, \hat{\beta})$: estimated coefficients

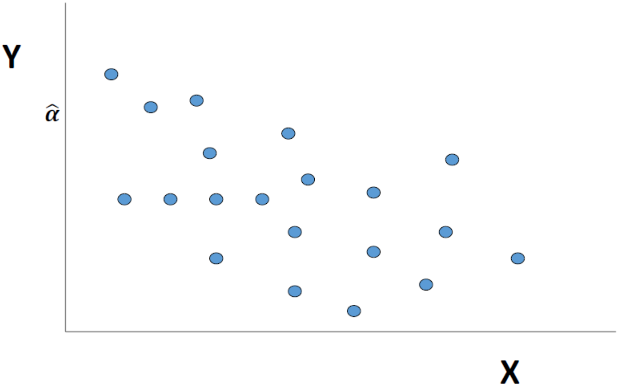
Ordinary Least Squares: OLS

- Estimating the model parameters from the data, we usually obtain these estimates via the *least squares method*
- Minimise the sum of squared residuals (SSR):

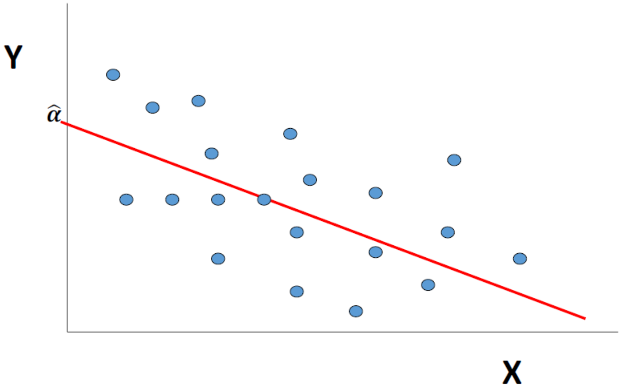
$$\text{SSR} = \sum_{i=1}^{n} \left(Y_i - \widehat{Y}\right)^2 = \sum_{i=1}^{n} (\hat{\mu}_i)^2 = \sum_{i=1}^{n} \left(Y_i - \hat{\alpha} - \hat{\beta} X_i\right)^2$$

- There is only one line that satisfies this criteria: Ordinary Least Sqaures (OLS) regression. OLS estimates β_0 and β_1 in so that `SSR` are being minimised
- Simply speaking, OLS estimates a β_1 that on average minimises the (squared) errors between the dots and the line.

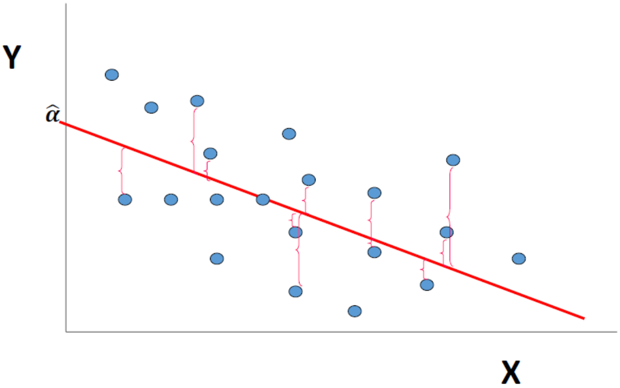
OLS II



OLS II



OLS II



OLS III

- In OLS, the mean of residuals is always zero (only one possible line satisfies the condition):

$$\text{mean of } \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{\alpha} - \hat{\beta}X_i\right) = \bar{Y} - \hat{\alpha} - \hat{\beta}\bar{X} = 0$$

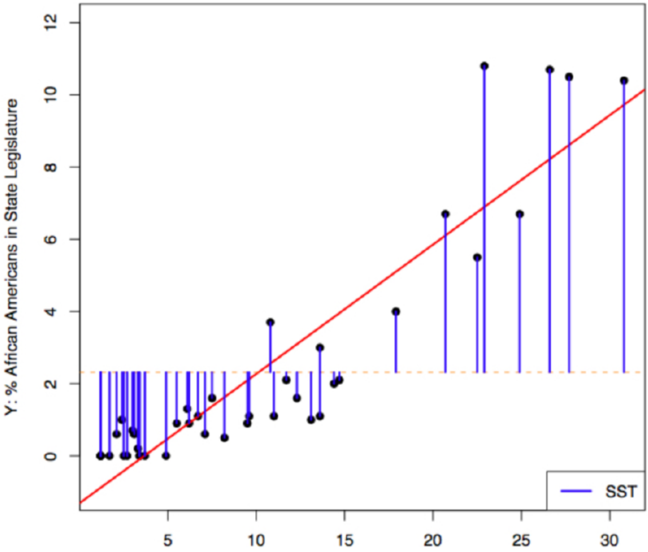
- How do we compute the OLS estimators? The slope....

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)}{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2} = \frac{\text{COV}(X,Y)}{\text{VAR}(X)}$$

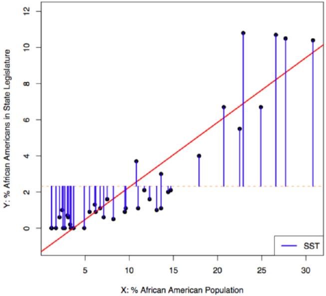
  - Notice that, unlike with correlation, order matters
- ...and the intercept:

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1\bar{X}$$

Goodness of Fit



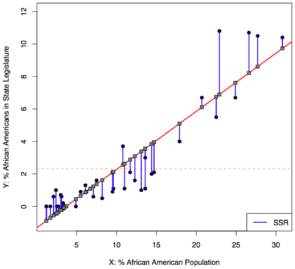
Goodness of Fit



$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = \text{TSS}$$

- This represents the sum of squares in the *null modell* - i.e., when we don't know anything but values of the dependent variable
- You can think about it as a measure of how "off" your prediction is from the real data, if all you rely on is the average

Goodness of Fit II



$$\sum_{i=1}^{n} (y_i - \widehat{y})^2 = RSS$$

- This represents the sum of squares in the *model* i.e., when we do know something beyond values of the dependent variable
- You can think about it as a measure of how "off" your prediction is from the real data, if you rely on an *independent variable* to explain variation in Y

Model Fit - Goodness of Fit

- How well does our model perform? Do we learn anything from adding the independent variable (vis-a-vis the null model)?
- The R-squared gives us a measure of this

$$\text{TSS} = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$$

$$R^2 = \frac{\text{TSS} - \text{RSS}}{\text{TSS}} = 1 - \frac{\text{RSS}}{\text{TSS}} = 1 - \frac{\sum_{i=1}^{n} \hat{\epsilon}_i^2}{\sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2}$$

- R^2 represents the proportion of total variation in the outcome variable explained by the predictor(s) included in the model
- R^2 is bounded between 0 and 1
- Do we care?

Guidelines

- R^2 denotes the goodness of fit, but not relevance of the variable in explaining the outcome
- We are interested in β_1, statistical significance, slope and its magnitude

$$\%\text{VoteLeave}|\text{MeanAge} = \underbrace{2.61}_{\text{intercept}} + \underbrace{1.28}_{\text{slope}}\ \text{MeanAge}$$

- Magnitude: Can you interpret what the slope means here?

Take Away

Always interpret the magnitude of the findings. A finding may be significant but too small for us to care; or vice-versa.