Data Analysis in R

Multivariate Regression

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Syllabus: Data Analysis in R

1. Introduction
2. Causality & Basics of Statistics
3. Sampling & Measurement
4. Prediction
5. **Multivariate Regression**
6. Probability & Uncertainty
7. Hypothesis Testing
8. Assumptions & Limits of OLS
9. Interactions & Non-Linear Effects

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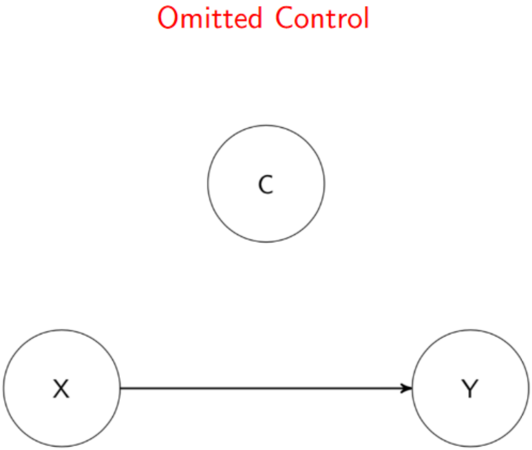
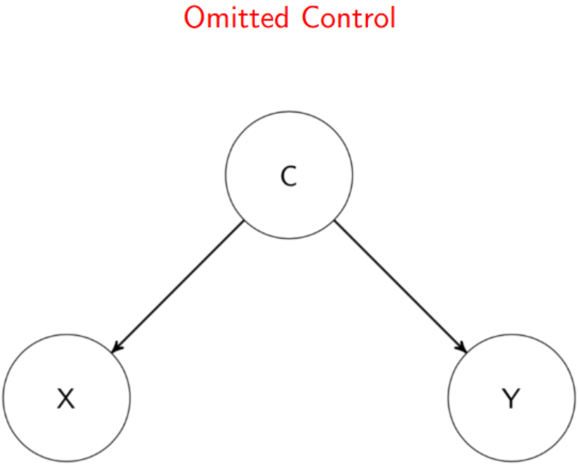
Recap: Linear Regression Analysis

- We want to make predictions minimising errors
- We could simply use the mean of a variable
- If we have another variable that we suspect may be associated with the variable we care about, we can use linear regression to help make better predictions
- A linear regression model is a *linear* approximation of the relationship between explanatory variables X and a dependent variable Y
- We do this by minimising the sum of squared errors

Omitted Variable Bias

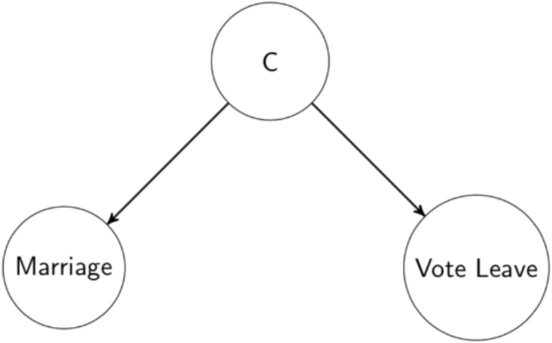


Omitted Variable Bias II



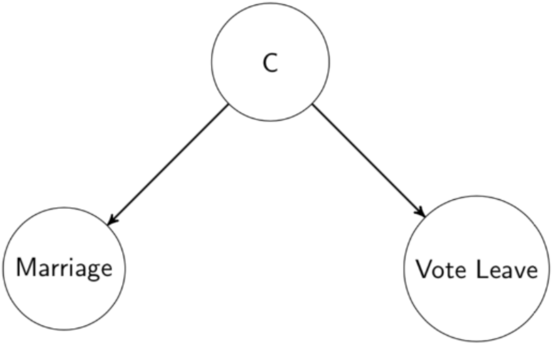
Omitted Variable Bias III

What is a potential omitted control?

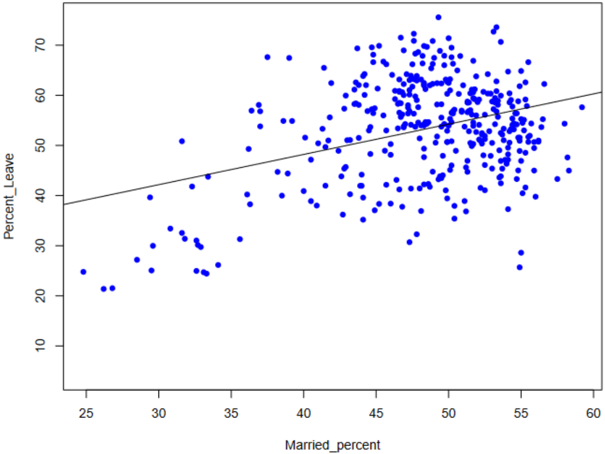


Omitted Variable Bias IV

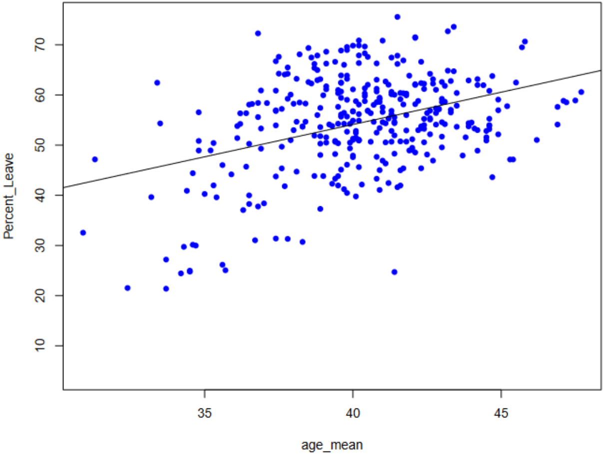
Omitted control: age



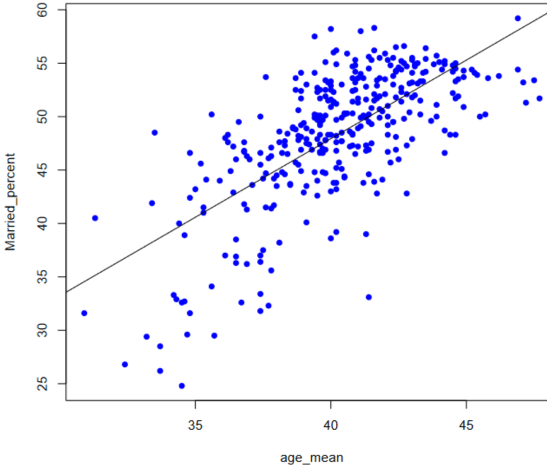
Mutual Association: Scatter-plot of % Leave and % Married



Mutual Association: Scatter-plot of % Leave and Mean Age



Mutual Association: Scatter-plot of % Married and Mean Age



Multivariate OLS: Interpretation of Coefficients

Table: Effect of % of Marriage on %Vote Leave

	(1) % Vote Leave	(2) % Vote Leave
% Married ($\hat{\beta}_1$)	0.604	0.402
Average age ($\hat{\beta}_2$)		0.697
Constant ($\hat{\alpha}$)	24.062	6.988
Observations	2,152	2,152
R Squared	0.1296	0.181

Source: British election survey 2017

- (1) *One unit* increase of % Married is associated with 0.604 increase in % Vote Leave
- (2) *One unit* increase of % Married is associated with 0.402 increase in % Vote Leave holding Average Age constant
- In (2) $\hat{\beta}_1$ estimates the partial effect of X_1 on Y

The Logic of Multivariate Regression

- In order for $\hat{\beta}$ to be unbiased we need the following condition: $E[\epsilon|X_1] = 0$ (zero conditional mean)
  - In observational studies, X_1 is likely to be determined by omitted variables in ϵ, which could be also related to Y
  - thus, $E[\hat{\beta}] \neq \beta$
  - This is know as omitted variable bias
- A common practice that aims to account for omitted variable bias is to use X_2 (the confounder) as a *'control'*:

$$Y = \underbrace{\beta_0}_{\text{intercept}} + \underbrace{\beta_1}_{\text{slope 1}} X_1 + \underbrace{\underbrace{\beta_2}_{\text{slope2}} X_2 + \underbrace{\epsilon}_{\text{error term}}}_{\text{previous error term}}$$

- Holding X_2 constant, β_1 denotes the partial **association** of X_1 with Y
- β_0 now denotes the expected value when all independent variables are 0 (whether this is useful or not)

General Extension of Multivariate Regression

- A multiple variable regression can be written as:

$$Y = \alpha + \beta_1 X_1 + \cdots + \beta_k X_k + \epsilon_i$$

- X_k are the k independent variables and β_k are the k coefficients for those X_k variables
- How many and which ones can [**should**] be added?
  - If the intention is to estimate the effect of X_1 on Y, controls *can* be useful in addressing omitted variable bias
  - Associated both with X_1 and Y
  - Number is always limited by the degrees of freedom $(N - k)$, where N is the number of observations
  - Our aim is to meet the conditional independence assumption $(E[\epsilon|X_1, ..., X_k] = 0)$

A brief example

Regression Results with Dummy Variables

- Consider the example of colonial legacy on democratisation
- $[Democracy|Colony] = \beta_0 + \beta_1 Colony$
- What is β_0 here?
  - β_0: The mean level of Democracy for non-colonies
- What about β_1
  - β_1: The difference in the level of Democracy between colonies and non-colonies.
- *Question*: How do we know the level of Democracy for colonies?

- Now imagine, we want to estimate $E[Democracy|GDP] = \beta_0 + \beta_1 GDP$. How does `colony` play into this?

Adding Categorical Covariates

- We can generalize the prediction equation:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i}$$

- This implies that we want to predict Y using the information we have about X_1 and X_2
- Therefore:

$$\widehat{Democracy} = \hat{\beta}_0 + \hat{\beta}_1 GDP + \hat{\beta}_2 Colony$$

What Does It Mean to Add Covariates?

- `Colony` is a *dummy variable*. It takes only two values:
  - 0 if the country was not a British colony
  - 1 if the country was a British colony
- Based on our regression equation, this renders two regression lines over `GDP`:
  - If $X_2 = 0$: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 * 0 = \hat{\beta}_0 + \hat{\beta}_1 X_{1i}$
  - If $X_2 = 1$: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 * 1 = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 = (\hat{\beta}_0 + \hat{\beta}_2) + \hat{\beta}_1 X_{1i}$
- We are fitting two lines with the same slope but two different intercepts
  - Think of it as adding a constant to former British colonies

Where's the Difference?

From R, we get the following estimates: $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$:

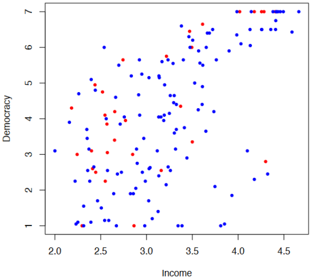
FHREVERS	Coef.
GDP90LGN	1.705888
BRITCOL	.5880665
_cons	-1.506045

Non-British Colonies:

- $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1$
- $\hat{Y} = -1.5 + 1.7 * X_{1i}$

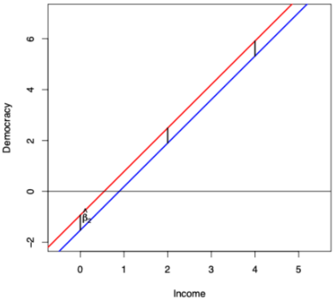
Former British Colonies:

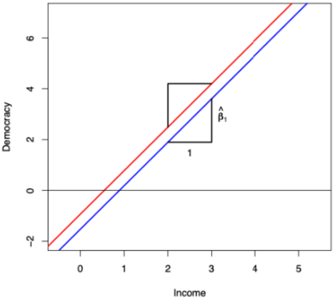
- $\hat{Y} = (\hat{\beta}_0 + \hat{\beta}_2) + \hat{\beta}_1 X_{1i}$
- $\hat{Y} = -.92 + 1.7 * X_{1i}$



Different Intercept - Same Slopes

Same slope, but two different intercepts. Two different levels, corresponding to the different values in X_2: `colony`.





Recall: Reference Categories

- Imagine we have the following set-up:
- Our dependent variable (Y) is `life satisfaction` (0 to 10 ordinal scale)
- Our independent variable (X) is `civil status`
  - $X = 1$ if married
  - $X = 2$ if divorced
  - $X = 3$ if widowed
  - $X = 4$ if single
- Can a regression coefficient be interpreted with the variable coded like this?

Recall: Reference Categories II

- For us to make sense of the results, we recode the categorical variable into a set of dummies:

$$LifeSatisfaction = \beta_0 + \beta_1 Divorced + \beta_2 Widowed + \beta_3 Single + \mu_i$$

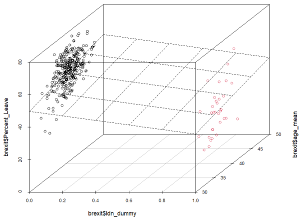
	Estimate
divorced	-0.605
widowed	-0.939
single	0.220
constant	6.640

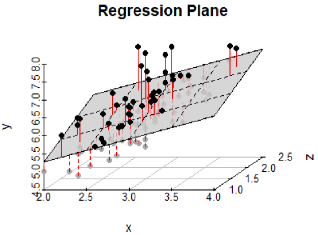
- Categorical variables (dummies or variables with more than 2 categories) are always built with a reference category
- The coefficients are all interpreted with reference to this category

Ultimately, which category you pick as reference category does not matter in statistical terms

Multivariate Regression with Continuous Variables

- We also include continuous variables as controls - the basic logic is exactly the same
- Effects are also interpreted in the same way - but this is less intuitive than with categorical variables
- Regression with two continuous variables fits a plane (*not a line*) made up of two perpendicular dimensions
- We are still trying to reduce squared errors





Worth It? Goodness of Fit Revised

- The *R-squared* remains a commonly used way of assessing goodness of fit
- The way you calculate the *R-squared* in the multivariate context is exactly the same as in the bivariate context
- If we keep adding variables, the *R-squared* will increase by design
- To keep it from doing so mechanically, we usually rely on the adjusted R-squared:

$$R^2 adjusted = \frac{(1 - R^2)(N - 1)}{N - p - 1}$$

where p = number of predictors

Some Words of Caution...

- Controls *can* be useful - but they are not a magical solution

- Their assumptions are quite (prohibitively?) strong: We'd have to think of and measure *all* confounders to estimate the unbiased effect
  - One can usually think of some additional confounders
  - Often, confounders are hard to measure (e.g., charisma in election campaigns) or unobservable entirely

- Be careful about what controls you choose - controlling for anything that can be a consequence of X reintroduces bias!
  - This is called post-treatment bias
  - All controls must be pre-treatment, i.e. realised before X
  - We also don't want to include more than one variable measure conceptually the same thing or even are perfectly collinear

Key Take Aways

- In social sciences, everything is related to virtually everything else, so there are many confounders
- Controlling them 'away' is a common approach to tackle the issues, but it comes with *[too]* strong assumptions
- Statistical interpretations of multivariate regressions are based on the *ceteris paribus* assumption
- Be aware of substantive issues when analysing dummy/categorical variables
- We got an idea of how continuous controls work
- Don't use controls that are realized after your independent variable (post-treatment)
- Remember to be skeptical of R-squared and the substantive meaning of adjusted R-squared

The Way Ahead...

- So far we have been concerned with point estimates: what is the effect of X on Y?
- Starting next week, we'll move go from how we find a single answer to understanding how precise/uncertain this answer is
- Bear in mind throughout: we still care about how "large" an effect is. But we also want to know if our estimate of that effect is precise or not. Both are crucial to interpret a result and provide substantive answers to research questions