# Isomorphism between general objects

with Fundamental applications

Shigeo Hattori

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bayship.org@gmail.com

https://github.com/bayship-org/mathematics

https://orcid.org/0000-0002-2297-2172

## 1 Preface

Embeddings into the space

Depending entirely on $(X, T, M)$

Roughly speaking, this paper gives **Claim1** of which **antecedent** says that: e.g., you take two *objects $\{k\}_{i\in\{1,2\}}$ from a same ambient isotopy class in a Euclidean space $(X, T, M)$ where $T$ is the topology and $M$ is the metric space and $X$ must be continuum; you compare the 2 objects by their ***inherent** values as $(f(k_0), f(k_1))$ where $f(k_i) \in ((+R^1) \cup \{0, \infty\})^n$, i.e., an $n$-length Cartesian product of the set of non-negative real numbers.

And the **consequent** of Claim1 says that: you can transform $k_0$ into $k_1$ through some ambient isotopy $F : X * [0, 1] \to X$ in terms of $(X, T)$ so that: for the image $k_t$ of $k_0$, $f(k_t)$ simply approaches to $f(k_1)$, i.e., n terms simply do so respectively.

As a supplement, the term "inherent" indicates that the differences between outputs from $f$ entirely depend on $(X, T, M)$ but on differences of logic.

As a supplement, $f$ must be specified by $(X, T, M)$. That is, $(X, T, M)$ decides $f$ uniquely. For example, take $\forall x :\in X$ then $x$ is not specified by $(X, T, M)$ whereas the set of all straight lines of length 1 in $(X, T, M)$ is specified by $(X, T, M)$.

For example, you probably agree that length$(k)$ is not a counterexample of the claim for almost all classes for $k$, e.g., $k$ is a finite curved line. Meanwhile $f(x) := (\text{length}(x) - 1)^2$ is apparently a trivial counterexample of the claim if $f(x)$ is said an inherent value of $k$. Though if you carefully study the definition of $f$, it is clear that the differences between outputs of $f$ depend not entirely on $(X, T, M)$ but also on differences of logic.

Less roughly, this paper claims, say **Claim2**, that the Claim1 does not depend on the topological class of $k$, if the definition of the function is general over the topological classes to apply Claim1. Because, if so then inherency of $f$ does not change over classes.

Readers probably agree that the following definition of $f(k)$ is general between all two topological classes $(K_1, K_2)$ if: $|\text{image}(f[K_1])| = |\text{image}(f[K_2])|$.

Let the dimension of the Euclidean space be 3; project $k$ onto each infinitely far plane $P$; for each $P$, let $D_P$ is the set to collect all multi$_2$ points with a definition that all point $p$ on the plane $P$ is said a **multi**$_2$ point if exactly two points of $k$ are projected onto $p$; let f(x):= ( the maximum cardinality of $S_P$ over the set of all infinitely far planes ).

For the $f(k)$, it is trivial that Claim1 holds for some topological class $K$, e.g., members of $K$ are sets of countably infinite points. Hence Claim2 implies Claim1 also, e.g., for the topological class of finite curved lines and for all knot classes.

All claims in this paper has given neither proofs nor disproofs. If we regard these claims as science then "no proofs" is just one side of them. Meanwhile the claims are extremely fundamental among theorems and conjectures of mathematics so that they are widely exposed to be disproved to all sub fields of mathematics if there existed some disproof.

## 2 Introduction

To write down the main conjectures, some definitions need to be given. As you know, two objects are regarded as equivalent if they are isomorphic. In other words, the mathematics on each of the two are equivalent. First we define when two given general objects, say $(x, y)$, are said [1]isomorphic, written $x \cong y$.

Before we go ahead, let me give some trivial examples.

For example, if $\{p_i\}_{i \in \{1,2,3\}}$ is a set of 3 objects pairwise isomorphic then $(p_1, p_2) \cong (p_1, p_3)$; $(p_1, p_2) \ncong (p_1, p_1)$.

Two homeomorphic topological spaces $((X_1, T_1), (X_2, T_2))$ are not isomorphic in general because their points are not promised to be pairwise isomorphic,

[1] In other words, generally isomorphic.

e.g., the homeomorphism $f$ relates all points $p$ as $f(p) = \{p\}$.

$$(X_1, T_1) \cong_h (X_2, T_2)$$
$$(X_1, T_1) \ncong (X_2, T_2)$$

Take $\forall(x, y)$ as numbers, then it will be defined that: $x \cong y \equiv x = y$. Contrary there exists a class of points where all points are pairwise isomorphic. For example points of some elementary geometry belong to such a class.

A topological space $X$ is a set of points defined the topology $T$. $(X, T)$ also may be said a topological space.

In the rest, even if $X$ is meant to be a set of points defined a topology, that will be ignored **inside expressions of isomorphism,** $\cong$, i.e., $X$ will be regarded as just a set of points, no topology will be implicitly accompanied;

$\blacksquare$

# 3 Isomorphism

**Definition 3.1** (Deep member)**.** Take $\forall(c, n, x, y)$ such that: $c$ is a chain of set membership; $|c| = n$; $x$ is the maximum member of $c$. $y$ is a minimum member of $c$. Then $c$ is said a deep chain of $x$; $y$ is said a deep member of $x$; and you write $y \in^{deep} x$; $y \in^{n-1} x$; $(x, y)$ are also written as ( $\max(c, 0)$, $\min(c, 0)$ ) respectively. $\blacksquare$

For example:

$$y \in .... \in x$$

For example:

$$\{y1, y2\} \in^0 \{y1, y2\}$$
$$y \in^2 \{1, \{2, y\}\}$$

**Axiom 3.1** (Identity)**.** Conceptually, the identity of an entity is a non literal unique name. Take $\forall(x, y)$, then $\exists z$ written $z$=ID$(x)$ such that (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *4).

**1.** ID$(x)$ =ID$(y) \equiv x = y$;

**2.** ID$(x)$ has no deep member other than itself;

**3.** ID$(x) \neq \varnothing$;

**4.** the mathematics on $\mathrm{ID}(x)$ and the mathematics on $\mathrm{ID}(y)$ are equivalent;

**Definition 3.2** (Box)**.** This definition is just for making the texts shorter but for fundamental mathematics.

All box $b$ is a tuple of either 1 or 2 entities. We write $b$ as $\mathrm{box}(i, v)$ or $\mathrm{box}(i, )$ depending on the length; $i$ is said the **index** and $v$ is said the **value**; the index $i$ must be an identity.

**Definition 3.3** (Deep graph and tree)**.** Take $\forall(x, V, E)$ such that (*1 ... $\overset{\text{and}}{\wedge}$ *3). Define (*4 *5).

**1.** let $V_c := \{g(c) \mid c \text{ is a deep chain of } x\ \}$;

**2.**

$V_2 = \{c \mid c \in V_c \ \overset{\text{and}}{\wedge}\ c \text{ is maximal on } V_1\ \}$;

$V_1 = V_c - V_2$;

$V = \{\mathrm{box}(\mathrm{ID}(c), \min(c)) \mid c \in V_2\ \} \cup \{\mathrm{box}(\mathrm{ID}(c),) \mid c \in V_1\ \}$;

**3.** $E$ is the set of directed edges on $V$ such that:

$E := \{(b_1 > b_2) \in V^2 \mid (@b_1 \supset @b_2) \ \overset{\text{and}}{\wedge}\ (|@b_1| - |@b_2| = 1)\}$;

$@b := \mathrm{ID}^{-1}(\text{the index of } b)$;

**4.** $(V, E)$ is said the deep tree of $x$;

**5.** all vertex $v$ of a deep tree is said an end vertex if $v \in V_2$;

**Definition 3.4** (Isomorphism)**.** Take $\forall(x, y, F, f)$ such that (*0 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *5). Define $x \cong^{F,f} y \ \overset{\text{and}}{\wedge}\ x \cong^{F} y \ \overset{\text{and}}{\wedge}\ x \cong y$.

**0.** let $G_i$ be the deep tree of $\forall i \in \{x, y\}$;

**1.** let $G_i$ decomposed as $(V, E)_i := G_i$;

**2.** $F$ is a bijection on some set of indentities;

**3.** $f$ is a graph isomorphism from*to $G_x * G_y$;

**4.** take $\forall v$ as an end vertex of $G_x$,

$@v$ :=(the value of $v$);

**5.** (*5a $\overset{\text{or}}{\vee}$ *5b);

**5a.** $F(@v) = F(@f(v))$;

**5b.** $@v \notin^2 F \overset{\text{and}}{\wedge} @v = @f(v)$;

■

**Definition 3.5.** Take $\forall(x, G_1, G_2, F, f)$ such that: $G_1$ is a deep tree of $x$ $\overset{\text{and}}{\wedge}$ $G_1 \cong^{F,f} G_2$ $\overset{\text{and}}{\wedge}$ $F$ is an identity function. Then $G_2$ too is said a deep tree of $x$. ■

# 4 Point abstraction

Take $\forall((X_1, T_1), (X_2, T_2))$ as homeomorphic topological spaces where $T_{\forall i}$ is a topology. In the rest we prefer that $((X_1, T_1), (X_2, T_2))$ are also isomorphic. In other words we prefer all points in $X_1 \cup X_2$ to be identities.

In the rest, if the condition is not satisfied then we transform the topological space into its point abstraction.

**Definition 4.1** (Point abstraction)**.** Take $\forall(x, G, P)$ such that (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *5). Do (*6 $\overset{\text{and}}{\wedge}$ *9) to get the point abstraction $y$ of $x$.

**1.** $G$ is a deep tree of $x$;

**2.** let $G$ be decomposed as $(V, E) := G$;

**3.** $P \subset V$;

**4.** $P$ is the set of all such vertices $p$ that you regard $p$ as a point of $x$;

Recall that $p$ is either a box$(c, m)$ or box$(c)$ where $c$ is a deep chain of $x$.

**5.** members of $P$ are pairwise $(p_1, p_2)$ in a condition that $@p_1 \not\supset @p_2$;

$@p$ :=ID$^{-1}$(the index of $p$);

**6.** update G by removing all such vertices $v$ from $V$ that $\exists p :\in P \overset{\text{and}}{\wedge} @v \supset @p$;

**7.** for each $p :\in P$, update $G$ at $p$ as *7a;

**7a** let $c$:=ID$^{-1}$(the index of $p$), then replace $p$ with box( ID($c$), ID(min($c$)) );

**8.** let $(V, E)_2$ be the output of *7;

**9.** take $\forall y$ such that $(V, E)_2$ is a deep tree of $y$;

The proof of the uniqueness of $y$ is omitted.

■

# 5 Applications in geometrical topology

## 5.1 Natural automorphism

**Definition 5.1.** Take $\forall(F, X, T)$ such that $(X, T)$ is a topological space and $F$ is an ambient isotopy on $X$.

$$F : X * [0, 1] \to X$$

Take $\forall(t, f)$ such that $f$ is the function as $f : X \to X$, $f(x) := F(x, t)$. Then $f$ is said a **natural automorphism** on $(X, T)$; alternatively $F$ or $(F, t)$ are used to describe $f$.

Take $\forall(x, y)$ such that $(X, T, x) \cong f\ (X, T, y)$, then $(x, y)$ are said $(X, T)$-natural-automorphic.

## 5.2 Ideal set of sub spaces

**Definition 5.2** (Ideal set of sub spaces)**.**
Take $\forall(X, T, S)$ such that: $(X, T)$ is a topological space. $S$ is a set of sub spaces of $X$.

For example, $(X, T, M)$ is a Euclidean space of dimension 1 where $M$ is the metric table, and $S$ is the set of all open intervals of length 1 in terms of $M$.

Be careful that, neither $(X, T)$ nor $S$ is defined the notion of lengths; instead $M$ defines lengths.

$S$ is said **ideal** if: (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *5).

**1.** $\exists B$ as an open basis to generate $(X, T)$.

Hence $B$ is a subset of the power set of $X$.

**2.** Let $S_B := \{@S_b \mid \exists b \in B \overset{\text{and}}{\wedge} S_b = \{s \mid s \in S \overset{\text{and}}{\wedge} P(s) \subset b\ \}\ \}$.

That is, $P(s)$ denotes the set of all points of $s$;

$\forall A$, define $@A$:= $\{$ID$(a) \mid a \in A\ \}$;

**3.** let $S_d := @S$;

**4.** $\exists T_d$ such that $S_B$ is an open basis to generate $(S_d, T_d)$.

**5.** Members of $S_d$ are pairwise $(X, T)$-natural-automorphic.

**Conjecture 5.1** (Ideal set of sub spaces and ambient isotopies)**.** Take $\forall(X, T, S, F, A)$ such that: $(X, T)$ is a topological space where $T$ is the topology. $S$ is an ideal set of sub spaces of $(X, T)$. $F$ is the set to collect: $\forall f$: $X$*[0,1] $\rightarrow X$ such that $f$ is an ambient isotopy. $A$ is the set to collect $\forall(g, S_1, S_2)$ such that: $g$ is a natural automorphism on $(X, T)$ $\overset{\text{and}}{\wedge}$ $(S1, S2)$ are subsets of $S$ $\overset{\text{and}}{\wedge}$ $(S_1, T) \cong^g (S_2, T)$.

Then (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *4) holds.

**1.** take $\forall(g, S_1, S_2) :\in A$;

**2.** $\exists f :\in F$;

**3.** take $\forall t :\in [0, 1]$ $\overset{\text{and}}{\wedge}$ define $f_t$ as the natural automorphism in terms of $(X, T, f, t)$;

**4.** $(f_t, S_1, S_2) \in A$ $\overset{\text{and}}{\wedge}$ if $t = 1$ then $f_t = g$;

**Definition 5.3** (Prime topological space)**.** Take $\forall(X, T)$ as a topological space. Then $(X, T)$ is said prime if *1.

**1.** $\exists S$ as a set of sub spaces of $(X, T)$ $\overset{\text{and}}{\wedge}$ $S$ is ideal $\overset{\text{and}}{\wedge}$ $@S$ is an open basis to generate $X$.

$@S := \{s \mid (s, t) \in S\ \}$ where $t$ is the topology;

**Conjecture 5.2** (Ideal set of sub spaces)**.** Take $\forall(X, T, S)$ such that: $(X, T)$ is a prime topological space. $S$ is a set of sub spaces of $(X, T)$. Then $S$ is ideal if *1.

**1.** let $S_{Xp} := \{(S, X_s, p) \mid \exists s \in S$ $\overset{\text{and}}{\wedge}$ $(X_s, T_s) := s$ $\overset{\text{and}}{\wedge}$ $p \in X_s\ \}$;

Members of $S_{Xp}$ are pairwise $(X, T)$-natural-automorphic.

# 6 Abstract conjectures

## 6.1 Main abstract conjecture

**Definition 6.1** ( $\overset{\text{ID}}{Deep}$)**.** Take $\forall X$.

$\overset{\text{ID}}{Deep}(X) := \{p \mid p \in^{deep} X \overset{\text{and}}{\wedge} p \text{ is an identity } \}$.

**Conjecture 6.1** (Abstract conjecture of ideal set and metric)**.**
Take $\forall(M, X, T, S_1, f)$ such that *A.

Consider (*B $\rightarrow$ *C). It is independent from the topological class of members of $S_1$ if $f$ is **enough general** over ( different solutions of $S_1$), in terms of topological classes of members.

The claim converges to true if generality approaches to the perfect by removing inequalities. And you can achieve it in finite steps proportional to the length of the original definition.

**A.** *1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *3.

  **1.** $M$ is a metric table to define $(X, T)$ as a topological space $\overset{\text{and}}{\wedge}$ $(X, T)$ is prime.

  **2.** $S_1$ is an ideal set of sub spaces of $X$.

  **3.** $f$ is a function on $S_1$.

**B.** *1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *3.

  **1.** Take $\forall k_1 :\in S_1$

  **2.** Let $S_2 := \{k_2 \in S_1 \mid f(k_2) = f(k_1) \}$.

  **3.** $S_2$ is unique for $(M, X)$.

  Unique?: For example, take $\forall x :\in \overset{\text{ID}}{Deep}(X)$. It is trivial that $x$ is not unique for $(M, X)$ **in general**. Hence, if $S_2$ is the set to collect $\forall k :\in S_1$ such that $x \in \overset{\text{ID}}{Deep}(k)$ then $S_2$ is not unique for $(M, X)$ **in general**. Instead $S_2$ is unique for $(M, X, x)$.

**C.** $S_2$ is ideal.

### 6.2 Application on knots

Let Conj be an alias for Conjecture 6.1. Let Def be an alias for the following Definition 6.2. The antecedent of Conj apparently holds for $(M, X, T, K, K_f, f)$ of Def in place of $(M, X, T, S_1, S_2, f)$. And $f$ is apparently enough general as required in Conj.

**Definition 6.2** (A set of knots)**.** Take $\forall(M, X, T, K, K_f)$ such that:
$M$ is a metric table to define $(X, T)$ as a Euclidean space of 3-dimension.
Take $\forall k_0$ as a knot and a subspace of $(X, T)$.
$K$ is the set to collect $\forall k$ such that: $(k, k_0)$ are $(X, T)$-natural-automorphic.
$K_f := \{k \in K \mid f(k) = f(k_0)\ \}$.

Definition of $f$:

- $j_1(\forall k :\in K) := \{j \mid$
  $j$ is an orthogonal [2]projection of $k$ onto some infinite plane $\}$.
- $j_2(\forall k :\in K) := \{j \in j_1(k) \mid$
  $\neg\ (\exists p \overset{\text{and}}{\wedge} p \in \text{image}(j) \overset{\text{and}}{\wedge} \mid j^{-1}(p) \mid > 2)\ \}$.
- $j_3(\forall k :\in K) := \{n \mid$
  $\exists j \overset{\text{and}}{\wedge} j \in j_2(k) \overset{\text{and}}{\wedge} n$ is the number of [3]double points in image$(j)$ $\}$.
- $f(\forall k :\in K) := \{m \mid$
  $m$ is the maximal member from $j_3(k)$ $\}$.

■

# 7 Notation

- take $\forall x \equiv$ for $\forall x \equiv \forall x$.

  In other words, "take" means nothing.
- $\forall x$ as a set $\equiv \forall x$ such that $x$ is a set.
- assume that $y$ has been introduced as dependent on $z$; if $(x_1, x_2)$ are introduced as solutions of $y$; then $(x_1, x_2)$ are dependent on a same $z$.

[2]Hence, $j$ is a function from $k$ to an infinite plane.
[3]Double point?: That is, the inverse image of a double point has exactly 2 distinct points of $k$; no matter the double point represents a crossing or a tangent point.

- $\{x \mid p(x)\ \} \equiv$ the set to collect $\forall x$ such that $p(x)$.

- All tuple of length 1 is written with parentheses and a comma, e.g., $(x,)$.

In definitions, I rarely write "if and only if". In stead I write "if" even if I know that "if and only if" can replace the "if".

## References


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[2] Reinhard Diestel, Graph Theory, Springer-Verlag, ISBN 0-387-98976-5