Order consistent logic

Shigeo Hattori

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bayship.org@gmail.com

https://github.com/bayship-org/mathematics

1 Prerequisite definitions and notations

https://github.com/bayship-org/mathematics/blob/master/Minor_of_memBer.pdf

In the article above, in its first two pages, all prerequisite definitions for this article are given. Especially, the second page of the above article defines that two memBers (x, y) are said (x is a **minor** of y).

Definition 1.1.
"And" is also written as " $_{and}\wedge$ ".
"Or" is also written as " $_{or}\vee$ ".

2 Introduction

This article defines new words, **"a logical expression is order consistent"**, and gives a conjecture for the new words and the notion of minors of memBers. In the rest of this introduction, the main conjecture is roughly introduced in a style of an example.

Let (X, T^2) be the Euclidean space of 2-dimension.
Let $x1$ be some closed line segment in terms of some coordinate system.
Let $C := \{x \mid ((T^2, x), (T^2, x1))$ are isomorphic $\}$.
Let f be a function on C as $f(x) =$length(x).

Take $y :\in \text{image}(f)$,
Let $k := \{x \mid f(x) = y\}$.
Then the conjecture claim that
if the definition of k is order consistent
then k is a minor of (X, T^2).

3 Definitions

Definition 3.1 (Order consistent)**.**
Take $\forall L$ as a logical expression such that *1 holds.
Then L is said order consistent.

1. Take $\forall(f, p, q, r, d1, d2)$
 such that $(*2 \ {}_{and}\!\wedge \ \ldots\ldots \ {}_{and}\!\wedge \ *9)$ holds
 then $(*10 \ {}_{and}\!\wedge \ *11)$ holds.

2. L defines f.

3. f is a function.

4. $\{p, q, r\} \subset \text{domain}(f)$.

5. $(d1, d2)$ are total orders on $\{p, q, r\}$.

6. Take $\forall d :\in \{d1, d2\}$.

7. Take $\forall(s, t) :\in d$.

8. There exists $\exists x :\in \text{domain}(f)$.

9. Evaluation of $f(x)$ refers to (s, t).

10. $d1 = d2$.

11. If $p < q < r$
 then $f(p) \leq f(q) \leq f(r) \vee f(r) \leq f(q) \leq f(p)$.

4 Main conjecture

Conjecture 4.1 (Main conjecture)**.**
Let (n, X, T^n) be the Euclidean space of n-dimension.
Take $\forall(L, f, k)$ such that $(*1 \ {}_{and}\!\wedge \ \ldots\ldots \ {}_{and}\!\wedge \ *3)$.
Then k is a minor of (X, T^n).

1. L is an order consistent logical expression.

2. L takes (n, X, T^n) as the antecedent.

3. L as a consequent specifies a function f relatively to the antecedent,(n, X, T^n).

4 There exists $\exists y :\in \text{image}(f)$. Then $k = \{x \in \text{domain}(f) \mid f(x) = y\}$.

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5 Examples

This section just gives examples of substituting actual values into variables of the antecedent of the main conjecture.

Definition 5.1 (Unknot)**.**
For the main conjecture,this example substitutes values
into L as (*1 $_{and}\wedge$ ..... $_{and}\wedge$ *10).

1. n:=3.

2. (n, X, T^n) is the Euclidean space (X, T^n) of n-dimension.

3. Let $x1$ be an unknot.

4. Let $C := \{x \mid ((T^n, x), (T^n, x1))$ are isomorphic $\}$.

5. Take $\forall x :\in C$.

6. $f1(x) := \{d \mid d$ is a proper knot diagram of $x\}$.

7. $f2(x) := \{r \mid$
$\exists d :\in f1(x)$ $_{and}\wedge$
r is the number of crossings on d
$\}$.

8. $f(x)$ returns the maximum number of $f2(x)$.

19. Take $\forall y :\in \text{image}(f)$.

10. [1] Let $k := \{x \in \text{domain}(f) \mid f(x) = y\}$

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[1] For example, you can specify y as $y := 10$.

Proposition 1 (Non order consistent)**.**
Refer to f of the previous definition.
Let $g(x) := (f(x) - 2)^2$.
Then the definition of g is not order consistent.

Proof.

- Refer to the previous definition for $f2$.
- The definition of g is dependent on $f2$ and the standard order on image$(f2)$.
- By the way, let $(p, q, r) := (1, 2, 3)$.
- Then $p<q<r \quad \wedge \quad g(q)<g(p) = g(r)$.

$\square$

6 Appendix-Specification

In mathematics, to specify an entity is always relative to the context. For example, let the antecedent take $\forall(n, X, T, M)$ as a Euclidean space (X, Y, M) of n dimension then as the consequent you can not specify any single point of the space. Contrary, if the antecedent has taken a coordinate system C, the consequent can specify any point of the space relatively to the antecedent.
Although the Euclidean space (X, T, M) is not specified the specific dimension, (X, T, M) is said specified for the consequent. But (X, T, M) is not said specified for the antecedent.

Definition 6.1. For all variable x of the consequent, if exactly one entity of the antecedent can be substituted into x then x is said having been specified relatively to the antecedent.

References