

Isomorphism of memBers 1

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1 Introduction 5

Definition 1.1 (memBer). Take $\forall x$ such that (there exists $\exists S$ such that $x \in S$). 6

Then x is said a memBer. 7

■ 8

This article:(1) defines a binary relation that (x,y) as memBers are isomor- 9
phic,(2) proves that the binary relation is an equivalence relation, (3) proves 10
that all homeomorphic topological spaces are isomorphic as memBers,(4) de- 11
fines that a memBer $S1$ is a minor of a memBer $S2$. 12

I expect that readers will realize that the newly defined isomorphisms are 13
somehow more fundamental than ,e.g., homeomorphisms of topological spaces. 14
Because "homeomorphisms" logically resolve to "isomorphisms of memBers" 15
whereas the inverse of it does not hold. 16

2 Notation 17

Definition 2.1. Consider "A and B". It is almost equivalent to " $B \wedge A$ ". But 18
some times they are different. Because the meaning of "B" may depend on "A". 19

20

"A _{then} \wedge B" \equiv "A holds then B holds" \equiv "A holds and B holds". 21

"A _{else} \vee B" \equiv "if A fails then B holds". 22

" $\forall x : \in S$ " \equiv "for all x such that $x \in S$ ". 23

" $\forall x$ as an integer" \equiv "for all x such that x is an integer". ■ 24

3 Deep member 25

Definition 3.1 (Deep member of memBer). This definition uses a style of recursion. 26
27

Take $\forall(x, y)$ such that *1 holds. Then define *2 *then* \wedge *3. 28

1 $x = y$ else (there exists $\exists z$ such that $x \in z \in^{\geq 0} y$). 29

2 x is a deep member of y . 30

3 $x \in^{\geq 0} y$ 31

■ 32

Definition 3.2 (Space of memBer). Take $\forall(x, y)$ such that *1 holds. Then define *2. 33
34

1 $y = \{d \mid d \in^{\geq 0} x \text{ then } d \text{ is a point}\}$. 35

2 y is the space of x . 36

■ 37

4 Notations 38

Definition 4.1 (Restriction of binary relation). Take $\forall(L, X, Y, X1, Y1)$ such that *1 holds. Then define (*2 *then* \wedge *3 *then* \wedge *4). 39
40

1 L is a binary relation on $X * Y$ *then* \wedge $X1 \subset X$ *then* \wedge $Y1 \subset Y$. 41

2 $L[X1] := \{(x, y) \in L \mid x \in X1\}$. 42

3 $L[, Y1] := \{(x, y) \in L \mid y \in Y1\}$. 43

4 $L[X1, Y1] := \{(x, y) \in L \mid x \in X1 \text{ then } \wedge y \in Y1\}$. 44

■

5 Isomorphic memBers 45

Definition 5.1 (Isomorphic memBers). Take all $\forall x$. Then (x, x) are said isomorphic. 46
47

Definition 5.2 (Isomorphic memBers by binary relation). This definition uses a style of recursion. 48
49

Take $\forall(x, y, F)$ such that $*A$ holds. Then define $(*B1 \text{ then } \wedge *B2)$. 50
51

A $(F \text{ is a binary relation then } \wedge *0)$ holds. 52

0 If there exists $\exists v : \in \{x, y\}$ such that $space(v) = \emptyset$ Then $x = y$ Else $*1$. 53
54

1 If there exists $\exists v : \in \{x, y\}$ such that v is a point Then $((x, y) \text{ are points then } \wedge (x, y) \in F)$ Else $(*2 \text{ then } \wedge *3)$. 55
56

2 $F[space(x), space(y)]$ is a ¹bijection from $*to \text{space}(x) * space(y)$. 57

3 There exists $\exists f$ such that $(*4 \text{ then } \wedge *5 \text{ then } \wedge *6)$. 58

4 f is a bijection from $*to x * y$. 59

5 Take $\forall(m1, m2) \in f$. 60

6 $*A$ holds for $(m1, m2, F)$ in place of (x, y, F) . 61

B1 (x, y) are said isomorphic by F as an isomorphism. 62

B2 Take $\forall(x, y, F)$ such that (x, y) are isomorphic by F . Then (x, y) are said isomorphic. 63
64

¹To weaken the definition, replace "bijection" with "function" or with "binary relation".

6 Minors of memBers 65

Definition 6.1 (Minors). Take $\forall(x, y)$ such that $*A$ holds. Then it is said as 66
 $*B$. 67

A $*1$ *then* \wedge $*2$. 68

1 Take $\forall d$. Then $d \in^{\geq 0} x \Rightarrow d \in^{\geq 0} y$. 69

2 Take $\forall(d1, d2, d3)$ such that (take $\forall d \in \{d1, d2, d3\}$, then $d \in^{\geq 0} x$). 70
Then $(*3 \leftarrow *4)$. 71

3 $((x, d1, d3), (x, d2, d3))$ are isomorphic. 72

4 $((y, d1, d3), (y, d2, d3))$ are isomorphic. 73

B $*5$ *then* \wedge $*6$. 74

5 x is a minor of y . 75

6 $x \leq^{minor} y$. 76

■ 77

7 Notations 78

Definition 7.1 (Family). Take $\forall(x, I, X)$ as a family X , the index set I and 79
the function x , then x is surjective. 80

In other words, $X = \{x_i \mid i \in I\}$. 81

And x is said a family's function. 82

83

Definition 7.2 (Chain). Take $\forall C$ as a chain. Then C is regarded as a family 84
and ²defined (I, C) such that $(*1$ *then* \wedge $*2$ *then* \wedge $*3)$. 85

1 I is the index set $I := [min := 1, max := |C|] \subset N$. ³Footnote. 86

2 C as a family's function is a bijection from I to C . 87

3 Take $\forall(i, j) : i \in I * I$. Then $i < j \equiv C_i < C_j$. 88

²The same name as the chain C .

³ N denotes the set of all natural numbers.

8 Depth of memBer 89

Definition 8.1 (Powers of set membership). Take $\forall(C, x, y)$ such that *1. Then 90
define *2 $\text{then} \wedge$ *3. 91

1 C is a chain between $C_{min} = x$ and $C_{max} = y$ by set ⁴membership. 92

2 $\text{power}(C) := |C| - 1$. 93

3 $x \in^{\text{power}(C)} y$. 94

For example: let $y := \{1, \{1\}\}$. 95

Then $1 \in^1 y \text{ then} \wedge 1 \in^2 y$. 96

Definition 8.2 (Depth of deep membership). Take $\forall(C, x, y)$ such that *1. 98
Then define *2. 99

1 C is a longest chain between $C_{min} = x$ and $C_{max} = y$ by set ⁵membership. 100

2 $\text{depth}(x, y) := \text{power}(C)$. 101

For example: let $y := \{1, \{1\}\}$. 102

Then $\text{depth}(1, y) = 2$. 103

Definition 8.3 (Sum of depths of deep membership). Take $\forall C$ such that *1. 104
Then define *2. 105

1 C is a chain by deep ⁶membership. 106

2 $\text{depth}(C) := \sum_{i=1}^{|C|-1} \text{depth}(C_i, C_{i+1})$. 107

■ 108

Proposition 1 (Depth of deep member). Take $\forall(x, y, z)$ such that $z \in y \in^{\geq 0} x$. 109
Then $\text{depth}(z, x) > \text{depth}(y, x)$. 110

■ 111

Proof. 112

• Assume it is false. 113

⁴For example, $x \in C_2$.

⁵For example, $x \in C_2$.

⁶For example, $C_1 \in^n C_2$

- There exists $\exists(x, y, z)$ such that it is a counterexample. 115
- Hence $depth(z, x) \leq depth(y, x)$. 116
- Hence $depth(z, x) \geq depth(z, y, x) > depth(y, x) \geq depth(z, x)$. 117
- The assumption is false. 118

□ 119

Proposition 2 (Depth of memBer). Take $\forall(x, y)$ such that $y \in x$. Then $depth(y) < depth(x)$. 120

■ 121

Proof. 122

- Assume it is false. 123
- There exists $\exists(x, y)$ such that it is a counterexample. 124
- Hence $depth(y)(x)$. 125
- There exists $\exists v : \in y$ such that (126
 $depth(y) = depth(v, y) \geq depth(v, y, x) \leq depth(x)$ 127
 $)$. 128
- Though $depth(v, y) + 1 = depth(v, y, x)$. 129
- The assumption is false. 130

□ 131

9 Isomorphic memBers as equivalence relation 132

Definition 9.1. In this section, *Def refers to the definition titled as "Isomor- 133
 phic memBers by binary relation". 134

And *1 \equiv *2, without any explicit proof because it is trivial by *Def. 135

And *3 holds, without any explicit proof because it is trivial by *Def. 136

1 (x_i, y_i) are isomorphic by F_i . 137

2 *Def.A holds for (x_i, y_i, F_i) in place of (x, y, F) . 138

3 Take $\forall(x, y, F)$ such that (*4 $_{then} \wedge$ (*5 $_{else} \vee$ *6)). Then *7 holds. 139

4 F is a binary relation. 140

5	$(space(x) = \emptyset \text{ then } \wedge x = y).$	141
6	$((x, y) \text{ are points } \text{ then } \wedge (x, y) \in F).$	142
7	$(x, y) \text{ are isomorphic by } F.$	143
Proposition 3	(Restriction). Take $\forall(x, y, F1, F2)$ such that $(*A1 \text{ then } \wedge *A2 \text{ then } \wedge *A3)$ holds. Then $*B$ holds.	144 145
A1	$(F1, F2)$ are binary relations.	146
A2	$F1[space(x)] = F2[space(x)].$	147
A3	Def.A holds for $(x, y, F1).$	148
B	Def.A holds for $(x, y, F2).$	149
		150
<i>Proof.</i>		151
•	Assume it is false.	152
•	There exists $\exists(x, y, F1, F2)$ such that it is a minimum counterexample by $depth(x).$	153 154
•	Let us follow $*Def.A$ for $(x, y, F1).$	155
•	Assume the antecedent of $*0$ holds.	156
•	Hence $space(x) = \emptyset \text{ then } \wedge x = y.$	157
•	Then $*0$ holds for $(x, y, F2).$	158
•	The last assumption is false.	159
•	Assume the antecedent of $*1$ holds.	160
•	Hence (x, y) are points $\text{ then } \wedge (x, y) \in F1.$	161
•	Then $*1$ holds for $(x, y, F2).$	162
•	The last assumption is false.	163
•	Then $(*2 \text{ then } \wedge *3)$ holds.	164
•	Hence $*2$ holds for $(x, y, F2).$	165
•	Hence $*3$ fails for $(x, y, F2).$	166

- Hence there exists $\exists(m1, m2) \in f$ such that 167
- $*\text{Def.A}$ holds for $(m1, m2, F1)$ $\text{then} \wedge$ $*\text{Def.A}$ fails for $(m1, m2, F2)$. 168
- Hence $(m1, m2, F1, F2)$ is a counterexample smaller than $(x, y, F1, F2)$. 169
- The first assumption is false. 170

□ 171

Proposition 4 (Members' isomorphisms as consequent). Take $\forall(x, y, F)$ such 172
that $(*A1 \text{ then} \wedge *A2)$. Then $(*B1 \text{ then} \wedge *B2)$ holds. 173

A1 $*\text{Def.A}$ holds for (x, y, F) in place of (x, y, F) . 174

A2 F is an injection. 175

B1 Take $\forall m1 : \in^{\geq 0} x$. Then there exists $\exists m2 : \in^{\geq 0} y$ such that $*\text{Def.A}$ holds 176
for $(m1, m2, F)$ in place of (x, y, F) . 177

B2 Take $\forall m2 : \in^{\geq 0} y$. Then there exists $\exists m1 : \in^{\geq 0} x$ such that $*\text{Def.A}$ holds 178
for $(m1, m2, F)$ in place of (x, y, F) . ■ 179

*Proof of *B1.* 180

- Assume it is false. 181
- Then there exists $\exists(x, y, F, m1)$ such that it is a minimum counterexample 182
by $\text{depth}(m1, x)$. 183
- It is trivial that $(x \neq m1)$. 184
- Consider the proposition titled as "Depth of deep member". 185
- There exists $\exists x1$ such that $(m1 \in x1 \text{ then} \wedge (x, y, F, x1)$ is not a coun- 186
terexample). 187
- Hence $*B1$ holds for $x1$ in place of $m1$. 188
- Hence there exists $2 : \in^{\geq 0} y$ such that $*\text{Def.A}$ holds for $(x1, y2, F)$. 189
- Let us follow $*\text{Def.A}$ for $(x1, y2, F)$. 190
- Assume the antecedent of $*0$ holds. 191
- Then $\text{space}(x1) = \emptyset \text{ then} \wedge x1 = y2$. 192
- Hence $\text{space}(m1) = \emptyset \text{ then} \wedge m1 = m1 \text{ then} \wedge m1 \in^{\geq 0} y$. 193

- Hence *B1 holds for $m1$ in place of $m1$. 194
- Hence $(x, y, F, m1)$ is not a counterexample. 195
- Hence the last assumption is false. 196
- Assume the antecedent of *1 holds. 197
- Hence $(x1 \text{ is a point}) \text{ then } \wedge (m1 \in x1)$. 198
- Hence the last assumption is false. 199
- Hence $(*2 \text{ then } \wedge *3)$ must hold. 200
- Hence, $(*4 \text{ then } \wedge *5 \text{ then } \wedge *6)$ holds. 201
- Hence *B1 holds for $(x, y, F, m1)$ in place of $(x, y, F, m1)$. 202
- Hence $(x, y, F, m1)$ is not a counterexample. 203
- The first assumption is false. 204

□ 205

*Proof of *B2.* 206

- Assume it is false. 207
- Then there exists $\exists(x, y, F, m2)$ such that $(x, y, F, m2)$ is a minimum coun- 208
terexample by $depth(m2, y)$. 209
- It is trivial that $(y \neq m2)$. 210
- There exists $\exists y2$ such that $(m2 \in y2 \text{ then } \wedge (x, y, F, y2) \text{ is not a coun-}$ 211
terexample). 212
- Hence *B2 should hold for $y2$ in place of $m2$. 213
- Hence there exists $1 : \in^{\geq 0} x$ such that *Def.A holds for $(x1, y2, F)$. 214
- Let us follow *Def.A for $(x1, y2, F)$. 215
- Assume the antecedent of *0 holds. 216
- Then $space(x1) = \emptyset \text{ then } \wedge x1 = y2$. 217
- Hence $space(m2) = \emptyset \text{ then } \wedge m2 = m2 \text{ then } \wedge m2 \in^{\geq 0} x$. 218
- Hence *B2 holds for $m2$ in place of $m2$. 219

- Hence $(x, y, F, m2)$ is not a counterexample. 220
- Hence the last assumption is false. 221
- Assume the antecedent of $*1$ holds. 222
- Hence $(y2 \text{ is a point}) \text{ then } \wedge (m2 \in y2)$. 223
- Hence the last assumption is false. 224
- Hence $(*2 \text{ then } \wedge *3)$ must hold. 225
- Hence, $(*4 \text{ then } \wedge *5 \text{ then } \wedge *6)$ holds. 226
- Hence $*B2$ holds for $(x, y, F, m2)$ in place of $(x, y, F, m2)$. 227
- Hence $(x, y, F, m2)$ is not a counterexample. 228
- The first assumption is false. 229

□ 230

Proposition 5 (Symmetric property). Take $\forall B$ such that B is a binary relation. 231
 Then let B^{-1} denote $\{(b2, b1) \mid (b1, b2) \in B\}$. 232
 Take $\forall(x, y, F)$. Then $*A1$ implies $*A2$. 233

A1 Def.A holds for (x, y, F) . 234

A2 Def.A holds for (y, x, F^{-1}) . 235

■ 236

Proof. 237

- Assume it is false. 238
- There exists $\exists(x, y, F)$ such that it is a minimum counterexample by $depth(x)$. 239
240
- Let us follow $*Def.A$ for (x, y, F) in terms of $*A1$. 241
- Assume the antecedent of $*0$ holds for (x, y, F) in terms of $*A1$. 242
- Hence $space(x) = \emptyset \text{ then } \wedge x = y$. 243
- Hence $*0$ holds for (x, y, F) in terms of $*A2$. 244
- Hence the last assumption is false. 245

- Assume the antecedent of *1 holds for (x, y, F) in terms of *A1. 246
- Hence (x, y) are points $\text{then} \wedge (x, y) \in F$. 247
- Hence (y, x) are points $\text{then} \wedge (y, x) \in F^{-1}$. 248
- Hence *1 holds for (x, y, F) in terms of *A2. 249
- Hence the last assumption is false. 250
- Hence $(*2 \text{ then} \wedge *3)$ must hold for (x, y, F) in terms of *A1. 251
- Hence $F[\text{space}(x), \text{space}(y)]$ is a bijection from *to $\text{space}(x) * \text{space}(y)$. 252
- Hence $F^{-1}[\text{space}(y), \text{space}(x)]$ is a bijection from *to $\text{space}(y) * \text{space}(x)$. 253
- Hence *2 holds for (x, y, F) in terms of *A2. 254
- Hence *3 must fail for (x, y, F) in terms of *A2. 255
- At same time, *3 hold for (x, y, F) in terms of *A1. 256
- Hence there exists $\exists(m1, m2) \in f$ such that (257
 - Def.A holds for $(m1, m2, F) \text{ then} \wedge$ 258
 - Def.A fails for $(m2, m1, F^{-1})$. 259
 - item). 260
- Hence $(m1, m2, F)$ is a counterexample. 261
- Consider the proposition titled as "Depth of memBer". 262
- Moreover $\text{depth}(m1) < \text{depth}(x)$. 263
- It contradicts to the title of (x, y, F) as a minimum counterexample. 264
- Hence the first assumption is false. 265

□ 266

Proposition 6 (Reflexive property). Take $\forall(x, F)$ such that *A holds. Then 267
 *B holds. 268

A F is the identity function on $\text{space}(x)$. 269

B Def.A holds for (x, x, F) . 270

■ 271

<i>Proof.</i>	272
• Assume it is false.	273
• There exists $\exists(x, F)$ such that it is a minimum counterexample by $depth(x)$.	274
• Let us follow *Def.A for (x, x, F) .	275
• Assume the antecedent of *0 holds.	276
• Then *0 holds.	277
• The last assumption is false.	278
• Assume the antecedent of *1 holds.	279
• Then *1 holds.	280
• The last assumption is false.	281
• It is trivial that *2 holds. Hence *3 must fail.	282
• Let $f1$ be the identity function on x .	283
• Then *3 must fail for $f1$ in place of f .	284
• Though *4 holds.	285
• Hence $(*5 \text{ then } \wedge *6)$ must fail.	286
• Hence there exists $\exists(m1, m1) : \in f1$ such that *Def.A fails for $(m1, m1, F)$.	287
• Meanwhile *Def.A holds for $(m1, m1, F[space(m1)])$ because otherwise $(m1, F[space(m1)])$ would be a counterexample smaller than a minimum counterexample, (x, F) .	288 289 290
• Though consider the proposition titled as "Restriction".	291
• Then *Def.A holds for $(m1, m1, F)$.	292
• The first assumption is false.	293
□	294

Proposition 7 (Transitive property). Take $\forall(B1, B2)$ such that $(B1, B2)$ are binary relations. Then let $B2 \circ B1$ denote $\{(b1, b3) \mid \exists b2 \text{ such that } (b1, b2) \in B1 \text{ then } \wedge (b2, b3) \in B2\}$. Take $\forall(x, y, z, F1, F2)$ such that $(*A1 \text{ then } \wedge *A2)$ holds. Then *B holds.

A1 Def.A holds for $(x, y, F1)$.	299
A2 Def.A holds for $(y, z, F2)$.	300
B Def.A holds for $(x, z, F2 \circ F1)$.	301
	■ 302
<i>Proof.</i>	303
• Assume it is false.	304
• There exists $\exists(x, y, z, F1, F2)$ such that it is a minimum counterexample by $depth(x)$.	305 306
• Let us follow *Def.A for $(x, y, F1)$ and for $(y, z, F2)$.	307
• Assume the antecedent of *0 holds for $(x, y, F1)$.	308
• Hence $space(x) = \emptyset \quad then \wedge x = y$.	309
• Hence the antecedent of *0 holds for $(y, z, F2)$.	310
• Hence $x = y = z$.	311
• Hence *0 holds for $(x, z, F2 \circ F1)$.	312
• The last assumption is false.	313
• Assume the antecedent of *0 holds for $(y, z, F2)$.	314
• Hence $space(y) = \emptyset \quad then \wedge y = z$.	315
• Hence the antecedent of *0 holds for $(x, y, F1)$.	316
• The last assumption is false.	317
• Assume the antecedent of *1 holds $(x, y, F1)$.	318
• Hence (x, y) are points $then \wedge (x, y) \in F1$.	319
• Hence *1 also hold for $(y, z, F2)$ because otherwise *G.A cannot hold for $(y, z, F2)$.	320 321
• Hence (y, z) are points $then \wedge (y, z) \in F2$.	322
• Hence (x, z) are points $then \wedge (x, z) \in F2 \circ F1$.	323
• Hence *1 holds for $(x, z, F2 \circ F1)$.	324

- The last assumption is false. 325
- Assume the antecedent of *1 holds $(y, z, F2)$. 326
- Hence (y, z) are points $\text{then} \wedge (y, z) \in F2$. 327
- Hence the antecedent of *1 also hold for $(x, y, F1)$ because otherwise *G.A 328
cannot hold for $(x, y, F1)$. 329
- The last assumption is false. 330
- Hence $(*2 \text{ then} \wedge *3)$ holds for $(x, y, F1)$ and for $(y, z, F2)$. 331
- Hence $F1[\text{space}(x), \text{space}(Y)]$ is a bijection from *to $\text{space}(x) * \text{space}(y)$. 332
- And $F2[\text{space}(y), \text{space}(z)]$ is a bijection from *to $\text{space}(y) * \text{space}(z)$. 333
- Hence $(F2 \circ F1)[\text{space}(x), \text{space}(z)]$ is a bijection from *to $\text{space}(x) * \text{space}(z)$ 334
- Hence *2 holds for $(x, z, F2 \circ F1)$. 335
- Hence *3 fails for $(x, z, F2 \circ F1)$. 336
- By the way, there exists $(f1, f2)$ such that (337
3 holds for $(x, y, F1, f1)$ in place of (x, y, F, f) $\text{then} \wedge$ 338
3 holds for $(y, z, F2, f2)$ in place of (x, y, F, f) 339
). 340
- Then *3 fails for $(x, z, F2 \circ F1, f2 \circ f1)$ in place of (x, y, F, f) . 341
- Hence, there exists $\exists(m1, m2, m3)$ such that (342
 $(m1, m2) \in f1 \text{ then} \wedge$ 343
 $(m2, m3) \in f2 \text{ then} \wedge$ 344
(the antecedent of this proposition accepts 345
 $(m1, m2, m3, F1, F2)$ as $(x, y, z, F1, F2)$ 346
) $\text{then} \wedge$ 347
 $(m1, m2, m3, F1, F2)$ is a counterexample 348
). 349
- Though $(m1, m2, m3, F1, F2)$ is smaller than a minimum counterexample. 350
- The first assumption is false. 351

□ 352

10 Homeomorphism as Isomorphism 353

Proposition 8 (Members' isomorphisms as antecedent). Take $\forall(x, y, F, f)$ such 354
that $(*A1 \text{ then} \wedge *A2 \text{ then} \wedge *A3)$. Then $*B$ holds. 355

A1 F is an injection. 356

A2 f is a bijection from $*$ to $x*y$. 357

A3 Take $\forall(m1, m2) : \in f$. Then $*Def.A$ holds for $(m1, m2, F)$. 358

B $*Def.A$ holds for (x, y, F) in place of (x, y, F) . ■ 359

Proof. 360

- Assume B fails. 361
- Hence there exists $\exists(x, y, F)$ such that $*Def.A$ fails for (x, y, F) . 362
- Let us follow $*Def.A$ for (x, y, F) . 363
- (the antecedent of $*0$ fails $\text{ then} \wedge$ the antecedent of $*1$ fails $\text{ then} \wedge$ ($*2$ fails 364
 $\text{ else} \vee *3$ fails)). 365
- Hence $(space(x) \neq \emptyset \neq space(y)) \text{ then} \wedge$ both of (x, y) are not points. 366
- Assume $*2$ fails. 367
- Hence $F[space(x), space(y)]$ is not a bijection from $*$ to $space(x) * space(y)$. 368
- Consider $*A1$ which says F is an injection. 369
- Hence there exists $\exists(p_x, p_y) : \in space(x) * space(y)$ such that 370
 $p_x \notin domain(F) \text{ else} \vee p_y \notin image(F)$. 371
- Consider $*A2, *A3$ and the proposition titled as "Members' isomorphisms 372
as the consequent". 373
- There exists $\exists y2 \in^{\geq 0} y$ such that $*Def.A$ holds for $(p_x, y2, F)$. 374
- There exists $\exists x1 \in^{\geq 0} x$ such that $*Def.A$ holds for $(x1, p_y, F)$. 375
- Meanwhile, for each of the 2 lines just above, 376
($*Def.A$ holds only by the if-then condition of $*1$) because 377
(each of (p_x, p_y) is a point). 378
- Hence $p_x \in domain(F) \text{ then} \wedge p_y \in image(F)$. 379

• Hence the last assumption is false.	380
• Hence *3 must fail.	381
• Hence *3 fails for f in place of f .	382
• Though by $(*A2 \text{ then } \wedge *A3), (*4 \text{ then } \wedge *5 \text{ then } \wedge *6)$ holds.	383
• Hence the first assumption is false.	384
	□ 385
Definition 10.1 (Pair). Take $\forall\{x, y\}$. ⁷ Then $(x, y) := \{\{x\}, \{x, y\}\}$.	386
	■ 387
Proposition 9 (Topological space). Take $\forall((X1, T1), (X2, T2))$ such that *A	388
holds. Then *B1 \Rightarrow *B2.	389
A Take $i \in \{1, 2\}$. Then (X_i, T_i) is a topological space.	390
B1 $((X1, T1), (X2, T2))$ are homeomorphic.	391
B2 There exists $\exists F$ such that $((X1, T1), (X2, T2))$ are isomorphic by F .	392
	■ 393
<i>Proof.</i>	394
B1 implies *C.	395
	396
C There exists $\exists(G, g)$ such that $(*C1 \text{ then } \wedge \dots \text{ then } \wedge *C4)$.	397
	398
C1 G is a bijection from $X1$ to $X2$.	399
C2 G is a homeomorphism for *B1.	400
C3 g is a bijection from $T1$ to $T2$.	401
C4 Take $\forall(t1, t2) : \in g$. Then $(G \text{ takes } t1 \text{ to } t2)$.	402
	403
Consider the previous proposition titled as Members' isomorphisms as antecedent	404
and refer it as *P.	405
Then *P accepts arguments as $(*D1 \text{ then } \wedge \dots \text{ then } \wedge *D6)$.	406

⁷By Kazimierz Kuratowski.

	407
D1 *P accepts $(X1, X2, G, G)$ in place of (x, y, F, f) .	408
D2 *P accepts $(\{X1\}, \{X2\}, G, \{(X1, X2)\})$ in place of (x, y, F, f) .	409
D3 Take $\forall(t1, t2) : \in g$. Then *P accepts $(t1, t2, G, G)$ in place of (x, y, F, f) .	410
D4 *P accepts $(T1, T2, G, g)$ in place of (x, y, F, f) .	411
D5 *P accepts (412
$\{X1, T1\}$,	413
$\{X2, T2\}$,	414
G ,	415
$\{(X1, X2), (T1, T2)\}$	416
) in place of (x, y, F, f) .	417
D6 *P accepts (418
$\{\{X1\}, \{X1, T1\}\}$,	419
$\{\{X2\}, \{X2, T2\}\}$,	420
G ,	421
$\{(\{X1\}, \{X2\}), (\{X1, T1\}, \{X2, T2\})\}$	422
) in place of (x, y, F, f) .	423
	424
Hence *P implies (*E1 to *E6) combined by <i>then</i> \wedge .	425
Finally, *E6 implies this proposition.	426
	427
E1 $(X1, X2)$ are isomorphic by G .	428
E2 $(\{X1\}, \{X2\})$ are isomorphic by G .	429
E3 Take $\forall(t1, t2) : \in g$. Then $(t1, t2)$ are isomorphic by G .	430
E4 $(T1, T2)$ are isomorphic by G .	431
E5 $\{X1, T1\}, \{X2, T2\}$ are isomorphic by G .	432
E6 $(\{\{X1\}, \{X1, T1\}\}, \{\{X2\}, \{X2, T2\}\})$ are isomorphic by G .	433
	434
□	435

11 Restriction of memBer by space 436

Definition 11.1. This definition uses a style of recursion. 437

Take $\forall(S, X, N_{null})$ such that $(*A1 \text{ then } \wedge *A2 \text{ then } \wedge *A3)$ ⁸ holds. Then define 438
 $*B$. 439

A1 X is a ⁹space. 440

A2 N_{null} is not a set. 441

A3 $N_{null} \notin^{\geq 0} (S, X)$. 442

B 443

1 If $space(S) \subset X$ Then $S[X] := S$ Else $*2$. 444

2 If S is not a set Then $S[X] := N_{null}$ Else $*3$. 445

3 $S[X] := \{s[X] \mid s \in S \text{ then } \wedge s[X] \neq N_{null}\}$. 446

■ 447

12 Deep space 448

Definition 12.1. Take $\forall(S1, S2)$ such that $(*1 \text{ then } \wedge *2 \text{ then } \wedge *3)$. Then 449
define $*4$. 450

1 $S2 \subset \{m \mid m \in^{\geq 0} S1\}$. 451

2 Take $\forall(p, C)$ such that 452

$(p : \in space(S1) \text{ then } \wedge C \text{ is a chain from } S1 \text{ down to } p \text{ by set member ship})$. 453
454

3 Then $C \cap S2 \neq \emptyset$ 455

4 $S2$ is a deep space of $S1$. 456

⁸*A2 says that $\neg(N_{null} \in^{\geq 0} S)$.

⁹That is, x is a set of points.

References

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