Isomorphism between general objects

with Fundamental applications

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1 Introduction

To write down the main conjectures, some definitions need to be given. As you know, two objects are regarded as equivalent if they are isomorphic. In other words, the mathematics on each of the two are equivalent. First we define when two given general objects, say (x, y), are said [1]isomorphic, written $x \cong y$.

Before we go ahead, let me give some trivial examples.

For example, if $\{p_i\}_{i \in \{1,2,3\}}$ is a set of 3 objects pairwise isomorphic then $(p_1, p_2) \cong (p_1, p_3)$; $(p_1, p_2) \ncong (p_1, p_1)$.

Two homeomorphic topological spaces $((X_1, T_1), (X_2, T_2))$ are not isomorphic in general because their points are not promised to be pairwise isomorphic, e.g., the homeomorphism f relates all points p as $f(p) = \{p\}$.

$$(X_1, T_1) \cong_h (X_2, T_2)$$

$$(X_1, T_1) \ncong (X_2, T_2)$$

Take $\forall(x, y)$ as numbers, then it will be defined that: $x \cong y \equiv x = y$. Contrary there exists a class of points where all points are pairwise isomorphic. For example points of some elementary geometry belong to such a class.

A topological space X is a set of points defined the topology T. (X, T) also may be said a topological space.

[1]In other words, generally isomorphic.

In the rest, even if X is meant to be a set of points defined a topology, that will be ignored **inside expressions of isomorphism,** $\cong$, i.e., X will be regarded as just a set of points, no topology will be implicitly accompanied;

2 Isomorphism

Definition 2.1 (Deep member)**.** Take $\forall(c, n, x, y)$ such that: c is a chain of set membership; $|c| = n$; x is the maximum member of c. y is a minimum member of c. Then c is said a deep chain of x; y is said a deep member of x; and you write $y \in^{deep} x$; $y \in^{n-1} x$; (x, y) are also written as ($\max(c, 0)$, $\min(c, 0)$) respectively.

For example:

$$y \in .... \in x$$

For example:

$$\{y1, y2\} \in^0 \{y1, y2\}$$
$$y \in^2 \{1, \{2, y\}\}$$

Axiom 2.1 (Identity)**.** Conceptually, the identity of an entity is a non literal unique name. Take $\forall(x, y)$, then $\exists z$ written z=ID(x) such that (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *4).

1. ID(x) =ID$(y) \equiv x = y$;

2. ID(x) has no deep member other than itself;

3. ID$(x) \neq \varnothing$;

4. the mathematics on ID(x) and the mathematics on ID(y) are equivalent;

Definition 2.2 (Box)**.** This definition is just for making the texts shorter but for fundamental mathematics.

All box b is a tuple of either 1 or 2 entities. We write b as box(i, v) or box$(i,)$ depending on the length; i is said the **index** and v is said the **value**; the index i must be an identity.

Definition 2.3 (Deep graph and tree)**.** Take $\forall(x, V, E)$ such that (*1 ... $\overset{\text{and}}{\wedge}$ *3). Define (*4 *5).

1. let $V_c := \{g(c) \mid c \text{ is a deep chain of } x\ \}$;

2.

$V_2 = \{c \mid c \in V_c \ \overset{\text{and}}{\wedge}\ c \text{ is maximal on } V_1\ \}$;

$V_1 = V_c - V_2$;

$V = \{\text{box}(\text{ID}(c), \min(c)) \mid c \in V_2\ \} \cup \{\text{box}(\text{ID}(c),) \mid c \in V_1\ \}$;

3. E is the set of directed edges on V such that:

$E := \{(b_1 > b_2) \in V^2 \mid (@b_1 \supset @b_2) \ \overset{\text{and}}{\wedge}\ (|@b_1| - |@b_2| = 1)\}$;

$@b := \text{ID}^{-1}(\text{the index of } b)$;

4. (V, E) is said the deep tree of x;

5. all vertex v of a deep tree is said an end vertex if $v \in V_2$;

■

Definition 2.4 (Isomorphism)**.** Take $\forall(x, y, F, f)$ such that (*0 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *5). Define $x \cong^{F,f} y \ \overset{\text{and}}{\wedge}\ x \cong^{F} y \ \overset{\text{and}}{\wedge}\ x \cong y$.

0. let G_i be the deep tree of $\forall i \in \{x, y\}$;

1. let G_i decomposed as $(V, E)_i := G_i$;

2. F is a bijection on some set of indentities;

3. f is a graph isomorphism from*to $G_x * G_y$;

4. take $\forall v$ as an end vertex of G_x,

$@v :=$(the value of v);

5. (*5a $\overset{\text{or}}{\vee}$ *5b);

5a. $F(@v) = F(@f(v))$;

5b. $@v \notin^2 F \ \overset{\text{and}}{\wedge}\ @v = @f(v)$;

■

Definition 2.5. Take $\forall(x, G_1, G_2, F, f)$ such that: G_1 is a deep tree of x $\overset{\text{and}}{\wedge}$ $G_1 \cong^{F,f} G_2$ $\overset{\text{and}}{\wedge}$ F is an identity function. Then G_2 too is said a deep tree of x. ■

3 Point abstraction

Take $\forall((X_1, T_1), (X_2, T_2))$ as homeomorphic topological spaces where $T_{\forall i}$ is a topology. In the rest we prefer that $((X_1, T_1), (X_2, T_2))$ are also isomorphic. In other words we prefer all points in $X_1 \cup X_2$ to be identities.

In the rest, if the condition is not satisfied then we transform the topological space into its point abstraction.

Definition 3.1 (Point abstraction)**.** Take $\forall(x, G, P)$ such that $(*1 \overset{\text{and}}{\wedge} ... \overset{\text{and}}{\wedge} *5)$. Do $(*6 \overset{\text{and}}{\wedge} *9)$ to get the point abstraction y of x.

1. G is a deep tree of x;

2. let G be decomposed as $(V, E) := G$;

3. $P \subset V$;

4. P is the set of all such vertices p that you regard p as a point of x;

Recall that p is either a box(c, m) or box(c) where c is a deep chain of x.

5. members of P are pairwise (p_1, p_2) in a condition that $@p_1 \not\supset @p_2$;

$@p :=$ID^{-1}(the index of p);

6. update G by removing all such vertices v from V that $\exists p :\in P \overset{\text{and}}{\wedge} @v \supset @p$;

7. for each $p :\in P$, update G at p as *7a;

7a let c:=ID^{-1}(the index of p), then replace p with box(ID(c), ID(min(c)));

8. let $(V, E)_2$ be the output of *7;

9. take $\forall y$ such that $(V, E)_2$ is a deep tree of y;

The proof of the uniqueness of y is omitted.

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4 Applications in geometrical topology

4.1 Natural automorphism

Definition 4.1. Take $\forall(F, X, T)$ such that (X, T) is a topological space and F is an ambient isotopy on X.

$$F : X * [0, 1] \to X$$

Take $\forall(t, f)$ such that f is the function as $f : X \to X,\ f(x) := F(x, t)$. Then f is said a **natural automorphism** on (X, T); alternatively F or (F, t) are used to describe f.

Take $\forall(x, y)$ such that $(X, T, x) \cong f\ (X, T, y)$, then (x, y) are said (X, T)-natural-automorphic.

4.2 Ideal set of sub spaces

Definition 4.2 (Ideal set of sub spaces)**.**
Take $\forall(X, T, S)$ such that: (X, T) is a topological space. S is a set of sub spaces of X.

For example, (X, T, M) is a Euclidean space of dimension 1 where M is the metric table, and S is the set of all open intervals of length 1 in terms of M.

Be careful that, neither (X, T) nor S is defined the notion of lengths; instead M defines lengths.

S is said **ideal** if: (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *5).

1. $\exists B$ as an open basis to generate (X, T).

Hence B is a subset of the power set of X.

2. Let $S_B := \{@S_b \mid \exists b \in B \overset{\text{and}}{\wedge} S_b = \{s \mid s \in S \overset{\text{and}}{\wedge} P(s) \subset b\ \}\ \}$.

That is, $P(s)$ denotes the set of all points of s;

$\forall A$, define $@A := \{\mathrm{ID}(a) \mid a \in A\ \}$;

3. let $S_d := @S$;

4. $\exists T_d$ such that S_B is an open basis to generate (S_d, T_d).

5. Members of S_d are pairwise (X, T)-natural-automorphic.

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Conjecture 4.1 (Ideal set of sub spaces and ambient isotopies)**.**
Take $\forall(X, T, S, F, A)$ such that: (X, T) is a topological space where T is the topology. S is an ideal set of sub spaces of (X, T). F is the set to collect: $\forall f$: X*[0,1] $\to X$ such that f is an ambient isotopy. A is the set to collect $\forall(g, S_1, S_2)$ such that: g is a natural automorphism on (X, T) $\overset{\text{and}}{\wedge}$ $(S1, S2)$ are subsets of S $\overset{\text{and}}{\wedge}$ $(S_1, T) \cong^g (S_2, T)$.

Then (*1 $\overset{\text{and}}{\wedge}$ ... $\overset{\text{and}}{\wedge}$ *4) holds.

1. take $\forall(g, S_1, S_2) :\in A$;

2. $\exists f :\in F$;

3. take $\forall t \ :\in \ [0,1] \ \overset{\text{and}}{\wedge}$ define f_t as the natural automorphism in terms of (X, T, f, t);

4. $(f_t, S_1, S_2) \in A \ \overset{\text{and}}{\wedge}$ if $t = 1$ then $f_t = g$;

$\blacksquare$

Definition 4.3 (Prime topological space)**.** Take $\forall(X, T)$ as a topological space. Then (X, T) is said prime if *1.

1. $\exists S$ as a set of sub spaces of $(X, T) \ \overset{\text{and}}{\wedge} \ S$ is ideal $\overset{\text{and}}{\wedge}$ @S is an open basis to generate X.

   @$S := \{ s \mid (s, t) \in S \}$ where t is the topology;

$\blacksquare$

Conjecture 4.2 (Ideal set of sub spaces)**.** Take $\forall(X, T, S)$ such that: (X, T) is a prime topological space. S is a set of sub spaces of (X, T). Then S is ideal if *1.

1. let $S_{Xp} := \{(S, X_s, p) \mid \exists s \in S \ \overset{\text{and}}{\wedge} \ (X_s, T_s) := s \ \overset{\text{and}}{\wedge} \ p \in X_s \ \}$;

   Members of S_{Xp} are pairwise (X, T)-natural-automorphic.

$\blacksquare$

5 Abstract conjectures

5.1 Main abstract conjecture

Definition 5.1 ($\overset{\text{ID}}{Deep}$)**.** Take $\forall X$.

$$\overset{\text{ID}}{Deep}(X) := \{p \mid p \in^{deep} X \ \overset{\text{and}}{\wedge} \ p \text{ is an identity } \}.$$

$\blacksquare$

Conjecture 5.1 (Abstract conjecture of ideal set and metric)**.**
Take $\forall(M, X, T, S_1, f)$ such that *A.

Consider (*B $\to$ *C). It is independent from the topological class of members of S_1 if f is **enough general** over (different solutions of S_1), in terms of topological classes of members.

The claim converges to true if generality approaches to the perfect by removing inequalities. And you can achieve it in finite steps proportional to the length of the original definition.

A. $*1 \overset{\text{and}}{\wedge} \ldots \overset{\text{and}}{\wedge} *3$.

1. M is a metric table to define (X, T) as a topological space $\overset{\text{and}}{\wedge}$ (X, T) is prime.

2. S_1 is an ideal set of sub spaces of X.

3. f is a function on S_1.

B. $*1 \overset{\text{and}}{\wedge} \ldots \overset{\text{and}}{\wedge} *3$.

1. Take $\forall k_1 :\in S_1$

2. Let $S_2 := \{k_2 \in S_1 \mid f(k_2) = f(k_1)\ \}$.

3. S_2 is unique for (M, X).

Unique?: For example, take $\forall x :\in \overset{\text{ID}}{Deep}(X)$. It is trivial that x is not unique for (M, X) **in general**. Hence, if S_2 is the set to collect $\forall k :\in S_1$ such that $x \in \overset{\text{ID}}{Deep}(k)$ then S_2 is not unique for (M, X) **in general**. Instead S_2 is unique for (M, X, x).

C. S_2 is ideal.

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5.2 Application on knots

Let Conj be an alias for Conjecture 5.1. Let Def be an alias for the following Definition 5.2. The antecedent of Conj apparently holds for (M, X, T, K, K_f, f) of Def in place of (M, X, T, S_1, S_2, f). And f is apparently enough general as required in Conj.

Definition 5.2 (A set of knots)**.** Take $\forall (M, X, T, K, K_f)$ such that:
M is a metric table to define (X, T) as a Euclidean space of 3-dimension.
Take $\forall k_0$ as a knot and a subspace of (X, T).
K is the set to collect $\forall k$ such that: (k, k_0) are (X, T)-natural-automorphic.
$K_f := \{k \in K \mid f(k) = f(k_0)\ \}$.

Definition of f:

- $j_1(\forall k :\in K) := \{j \mid$ j is an orthogonal [2]projection of k onto some infinite plane $\}$.

[2]Hence, j is a function from k to an infinite plane.

- $j_2(\forall k :\in K) := \{j \in j_1(k) \mid$
  $\neg\ (\exists p \ \overset{\text{and}}{\wedge}\ p \in \text{image}(j) \ \overset{\text{and}}{\wedge}\ \mid j^{-1}(p) \mid > 2)\ \}$.

- $j_3(\forall k :\in K) := \{n \mid$
  $\exists j \ \overset{\text{and}}{\wedge}\ j \in j_2(k) \ \overset{\text{and}}{\wedge}\ n$ is the number of [3]double points in image(j) $\}$.

- $f(\forall k :\in K) := \{m \mid$
  m is the maximal member from $j_3(k)$ $\}$.

■

6 Notation

- take $\forall x \equiv$ for $\forall x \equiv \forall x$.

  In other words, "take" means nothing.

- $\forall x$ as a set $\equiv \forall x$ such that x is a set.

- assume that y has been introduced as dependent on z; if (x_1, x_2) are introduced as solutions of y; then (x_1, x_2) are dependent on a same z.

- $\{x \mid p(x)\ \} \equiv$ the set to collect $\forall x$ such that $p(x)$.

- All tuple of length 1 is written with parentheses and a comma, e.g., $(x,)$.

■

In definitions, I rarely write "if and only if". In stead I write "if" even if I know that "if and only if" can replace the "if".

[3]Double point?: That is, the inverse image of a double point has exactly 2 distinct points of k; no matter the double point represents a crossing or a tangent point.

References

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[2] Reinhard Diestel, Graph Theory, Springer-Verlag, ISBN 0-387-98976-5