Prime topological spaces

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1 Prime set of sub spaces

Blue texts indicate the words will be defined later.

This article defines new words, a prime set S of sub spaces of a topological space X, and a prime topological space. The main purpose of the new notions is to research that same conditions hold across different topological spaces.

A topological space X is a pair as $(\text{Space}(X), T)$.

Definition 1.1 (Deep member)**.** Take $\forall(c, x, y)$ such that c is a chain of set [1]membership of which the maximum member is x and the minimum member is y. Then y is said a deep member of x and you write as $y \in^{deep} x$. ■

For example, $\{y1, y2\} \in^{deep} \{y1, y2\} \overset{\text{and}}{\wedge} y \in^{deep} \{1, \{2, y\}\}$.
[2]Footnote.

Definition 1.2 (Space)**.** $\text{Space}(X) := \{p \mid p \in^{deep} X \overset{\text{and}}{\wedge} p \text{ is a point }\}$. ■

A metric space (X, M) is a topological space X with the metric table M to define X. M is said a non topological property of X.

[1] That is, the set is larger than the member.

[2] "$a \overset{\text{and}}{\wedge} b$" is almost equivalent to "$a \wedge b$" except that "$a \overset{\text{and}}{\wedge} b$" is not promised to be commutative. The same holds for "$a \overset{\text{or}}{\vee} b$" and "$a \vee b$".

Let (X, M, x_0) denote R^1, Euclidean space of 1-dimension where x_0 denote the origin point. Take $\forall I$ as a closed interval on X. Take $\forall I_h$ as a homeomorphism from the interval $[0, 1]$ on (X, M, x_0) to I so that you can use I_h an index system on I.

Take $\forall(S, X)$ such that: X is a topological space $\overset{\text{and}}{\wedge}$ $S \subset 2^{Space(X)}$. Let X_{*I} denote the topological space of the Cartesian product $X * I$. Hence you can use $X * I$ as the index set for Space(X_{*I}) although $X * I$ is not any part of X_{*I}.

Take $\forall F$ as a bijection from Space(X)$*$Space(I) to Space(X_{*I}). Hence F takes a pair of points as the input and F outputs a point represented by a pair of points. Let F_0 be the instance of F such that: $F(x, i) = (x, i)$.

Take $\forall F_i$ as an instance of F such that: (*1 $\overset{\text{and}}{\wedge}$ ..... $\overset{\text{and}}{\wedge}$ *5).

1. $(F_0) \cong (F_i)$.

2. $F_0[X * 0] = F_i[X * 0]$.

3. image($F_0[\ X * t\ ]$) = image($F_i[\ X * t\ ]$).

4. Take $\forall s1 :\in S, \forall t :\in I$.

5. $\exists s2 :\in S$ $\overset{\text{and}}{\wedge}$ image($F_0[\ s1 * t\ ]$) = image($F_i[\ s2 * t\ ]$).

Then the set A of all instances of F_i is said the **ambient system** of (X, S). Especially, if (*4 $\overset{\text{and}}{\wedge}$ *5) is exempted from the required condition for F_i, then A is said the **ambient system** of X.

Instance of x: That is, $\forall y$ such that you can substitute y into x.

Definition 1.3 (Prime set of sub spaces)**.** Take $\forall(S, X)$ such that: X is a topological space $\overset{\text{and}}{\wedge}$ $S \subset 2^{Space(X)}$. If (*1 $\overset{\text{and}}{\wedge}$ ((*2 $\overset{\text{and}}{\wedge}$ ..... $\overset{\text{and}}{\wedge}$ *4) $\rightarrow$ ($*5$ $\overset{\text{and}}{\wedge}$ ..... $\overset{\text{and}}{\wedge}$ *7))) holds then S is said a **prime set** of sub spaces of X.

1. Take $\forall f$ as a bijection between subsets of S.

2. $\exists F$ as a member of the ambient system of X.

3. Take $\forall s :\in$ domain(f).

4. image($F[\ s * 1\ ]$) = $f(s) * 1$.

5. $\exists H$ as a member of the ambient system of (X, S).

6. Take $\forall s :\in \mathrm{domain}(f)$.

7. $\mathrm{image}(H[\ s * 1\ ]) = f(s) * 1$.

1.1 Isomorphism

[3]Informally speaking:
Take $\forall (X_i)_{i \in I}$, $\forall (Y_i)_{i \in I}$ as a pair of sequences of topological spaces.

If there exists $\exists F$ as a bijection between sets of points such that: Take $\forall i :\in I$. Then **some subset of F is a homeomorphism** from X_i to Y_i.

Then you write as $(X_i)_{i \in I} \cong (Y_i)_{i \in I}$.

Formally:
Take $\forall F$ as a bijection between sets of points .

Take $\forall (p1, p2)$. Define *1 to be equivalent to $(*2 \overset{\text{or}}{\vee} *3)$.

1. $p1 \cong_F p2$.

2. $F(p1) = p2$.

3. $\mathrm{Space}(\{p1, p2\}) \cap \mathrm{Space}(F) = \varnothing \overset{\text{and}}{\wedge} p1 = p2$.

Proposition 1. $p1 \cong_F p2 \quad \equiv \quad p2 \cong_{F^{-1}} p1$.

Proof. Take $\forall (p1, p2, F^{-1})$ as a counterexample. Hence $(*2 \overset{\text{or}}{\vee} *3)$ holds for $(p1, p2, F)$ in place of $(p1, p2, F)$

Assume *2 holds. Then *2 holds for $(p2, p1, F^{-1})$ in place of $(p1, p2, F)$. A contradiction.

Assume *3 holds. Then *3 holds for $(p2, p1, F^{-1})$ in place of $(p1, p2, F)$. A contradiction. □

[3]Speaking with no proof.

Take $\forall G := (V, E, C)$ such that: (V, E) is a graph $\overset{\text{and}}{\wedge}$ C is a function from the vertex set V to a set. In detail: (V, E) can be any type of graphs if **graph isomorphisms** are defined for the type; C is called the **content map** on V.

Then G is said a **topological graph**.

Definition 1.4 (Isomorphism between topological graphs)**.** Take $\forall \{G_i\}_{i \in \{1,2\}}$ as a pair of topological graphs. Hence $G_i := (V, E, C)_i$ where C_i denotes the content map on V_i. Take $\forall f$ as a graph isomorphism from V_1 to V_2. Take $\forall F$ as a bijection between sets of points.

Then F is said an isomorphism from G_1 to G_2 if *1; you write the fact as *2. And it is defined that: *2 $\to *3$.

1. Take $\forall v :\in V_1$, then $C_1(v) \cong_F C_2(f(v))$.
2. $G_1 \cong^F G_2$.
3. $G_1 \cong G_2$.

Proposition 2. $G_1 \cong^F G_2 \quad \equiv \quad G_2 \cong^{F^{-1}} G_1$.

Proof. Refer to the definition of isomorphism between toplogical graphs as MainDef. Take $\forall (G_1, G_2, F)$ as a counterexample. By graph theory, f^{-1} is a graph isomorphism from G_2 to G_1. And F^{-1} is a bijection between sets of points. Hence the antecedent of MainDef holds for $(G_2, G_1, f^{-1}, F^{-1})$ in place of (G_1, G_2, f, F) except *1. Hence *1 of MainDef fails for it.

1. Hence: $\exists v :\in V_2 \ \overset{\text{and}}{\wedge} \ \neg(\ C_2(v) \cong_{F^{-1}} C_1(f^{-1}(v))\ )$.

2. Though: $(G_1 \cong^F G_2) \to (C_1(f^{-1}(v)) \cong_F C_2(v))$.

(*1 $\overset{\text{and}}{\wedge}$ *2) contradicts to Proposition1. □

2 Prime topological space

To define prime topological spaces, two new notions as preliminaries are required.

2.1 To specify a variable

Take $\forall(x, y)$ as variables. That (x is **specified** on y) is equivalent to that: x has exactly one solution in the sense of that y has exactly one solution.

For example, let y denote R^2 **as a topological space**. Take $\forall M$ as a metric table to define y. Let $x := \{z \mid z \text{ is a circle in } y \text{ with } M\ \}$. Then it does not specify x on y because infinitely many metric table can define y. In other words, M is not specified on y.

Meanwhile, x is specified on (y, M). [4]Footnote

2.2 Subtext

Take $\forall e$ as a mathematical expression; for example $v = \{1, 2\}$. All **subtext** of e is got as follows.

Interpret e into a tree g of (either **binary** or **unary**) logical operations. As a trival example, g can be $v = \{1, 2\}$. For more interesting example, g can be $(1 \in v \wedge (\ 2 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\})))$. It is just a coincide that this g has no logical disjunction operation.

Next, select some vertex x of g. For example, x can be $(|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\})$. Or x can be $(\ 2 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\}))$ as x.

Next, change the operator of x so that the new x returns exactly one term. For example, the original operator "$\wedge$" can be changed to "right identity"; that is, the new x returns the right term. For the example, the new x returns $(|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\})$.

In detail, if the operator is unary then it must be changed to the identity; that is, the new x returns the only one term.

Finally the resultant expression for the modified tree is said a subtext of e. For the example, the subtext of e is $(1 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\}))$.

Moreover the relation on the set of all mathematical expressions by (being pair of a subtext and the original expression) is defined to be **transitive**. That is, a subtext of a subtext of e is also a subtext of e.

2.3 Prime topological space

Definition 2.1 (Prime topological space)**.** Take $\forall X$ as a topological space. Then X is said prime if the following main propositional function holds for X in place of X.

[4] As I omit the proof, I need to add "probably".

Definition 2.2 (Main propositional function)**.** Let X be the input topological space.

Take $\forall(S1, S2, d2)$ such that $(*1 \overset{\text{and}}{\wedge} *2)$. Then $(*3 \overset{\text{or}}{\vee} (*4 \overset{\text{and}}{\wedge} \ldots\ldots \overset{\text{and}}{\wedge} *6))$.

1. $S1$ is a prime set of sub spaces of X.

2. $d2$ is a definition to specify $S2$ on X so that $S2 \subset S1$.

3. $S2$ is a prime set of sub spaces of X.

4. $\exists(d3, S3)$.

5. $d3$ is a subtext of $d2 \overset{\text{and}}{\wedge} d3$ specifies $S3$ on X.

6. $\varnothing \neq S3 \subset S1 \overset{\text{and}}{\wedge} S3$ is a prime set of sub spaces of X.

■

3 Conjectures

3.1 Main conjecture for metric spaces

Conjecture 3.1 (Main conjecture)**.** Take $\forall(X, M)$ as a metric space where X denotes the topological space and M denotes the metric table to define X. Refer the main propositional function as MainProp. If X is prime then MainProp holds for X in place of X, with changes in the text of MainProp at (*2, *5) as the before to (*2_2, *5_2) as the after respectively.

2. $d2$ is a definition to specify $S2$ on X so that $S2 \subset S1$.

2_2. $d2$ is a definition to specify $S2$ on (X, M) so that $S2 \subset S1$.

5. $d3$ is a subtext of $d2 \overset{\text{and}}{\wedge} d3$ specifies $S3$ on X.

5_2. $d3$ is a subtext of $d2 \overset{\text{and}}{\wedge} d3$ specifies $S3$ on (X, M).

■

3.2 Sub conjecture

Conjecture 3.2 (Sub conjecture)**.** Refer to the main conjecture as MainConj. The following ($*1 \overset{\text{and}}{\wedge} \ldots\ldots \overset{\text{and}}{\wedge} *3$) holds true. Take $\forall M$ as a metric table to define a Euclidean space of 3-dimension. Let X denote the topological space defined by M.

1. X is a prime topological space.

Take $\forall k_0$ as a knot in X.
$K := \{k \mid (X, k_0) \cong (X, k)\ \}$.

2. (X, M, K, K_f) is an instance of $(X, M, S1, S2)$ of ManConj.

3. For (X, M, K, K_f), the item *3 of ManConj [5]holds.

Definition of K_f:

$K_f := \{k \in K \mid f(k_0) = f(k)\ \}$.

Sub definitions with K as the domain:

- $j1(k) := \{j \mid$
  j is an orthogonal [6]projection of k onto some infinite plane $\}$.
- $j2(k) := \{j \in j1(k) \mid$
  $\neg\ (\exists p \overset{\text{and}}{\wedge} p \in \text{image}(j) \overset{\text{and}}{\wedge} \mid j^{-1}(p) \mid > 2)\ \}$.
- $j3(k) := \{n \mid$
  $\exists j \overset{\text{and}}{\wedge} j \in j2(k) \overset{\text{and}}{\wedge} n$ is the number of [7]double points on j $\}$.
- $f(k) := \{m \mid$
  m is the maximal number from $j3(k)$ $\}$.

■

[5]Hence K_f is a prime set of sub spaces of X.
[6]Hence, j is a function from k to an infinite plane.
[7]That is, the inverse image of a double point has exactly 2 distinct points of k; no matter the double point is a crossing or a tangent point.

3.3 Factor of a logical expression

Currently it is hard to show that the main conjecture implies the sub conjecture because to analyze subtexts of a logical expression requires some computer-assisted proof software available.

By the way, readers may criticize that, the item *3 of the main conjecture fails for the sub conjecture due to ($j3$ and f). As an excuse for the possible negative critique, let me briefly introduce a new notion named "factor".

Recall that, $j3(k)$ returns a set of natural numbers. And $f(k)$ takes the maximum number from $j3(k)$. For example, assume $j3(k_x) = \{7, 8, 9\}$. The inverse image of $\{\{7, 8, 9\}\}$ over $j3$ is a subset of domain(f).

Let me analyze the equation which defines the inverse image; $y = \{7, 8, 9\}$. It is equivalent to $(7 \in y \wedge 8 \in y \wedge \max(y) = 9 \wedge |y| = 3)$ where each term of the logical conjunction is named as a **factor** of $y = \{7, 8, 9\}$ if the logical conjunction is not redundant. For the example above, it happens to be not redundant. And to use a factor of $y = \{7, 8, 9\}$ to define an inverse image is just to use a subtext of $y = \{7, 8, 9\}$. For example, $\max(y) = 9$.

As a conclusion, if the definition is not redundant: $j3$ is not a subtext of f; in stead f is a subtext of $j3$. Though non redundant definitions tend to have less quality in human readability. So my definitions of f and $j3$ are inevitably a little redundant.

Definition 3.1 (Redundant logical expression)**.** Take $\forall e$ as a logical expression. Then e is said **redundant** if some its subtext is equivalent to e. ■