Prime topological spaces

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1 Prime set of sub spaces

Blue texts indicate the words will be defined later.

This article defines new words, a prime set S of sub spaces of a topological space X, and a prime topological space. These abstract shared properties among different topological spaces.

Definition 1.1 (Prime set of sub spaces)**.** Take $\forall(X, S)$ such that: X is a topological space $\ _{and}\wedge$ S is a set of sub spaces of X. [1]Footnote.

Then S is said prime if: Let $F_{our} := \{X\} * \{S\} * S * S$. Take $\forall(f_1, f_2) :\in F_{our}$. Hence, f_i has a form as $f_i := (X, S, s_i, t_i)$. If (X, s_1, t_1) and (X, s_2, t_2) are isomorphic then (S, s_1, t_1) and (S, s_2, t_2) are also isomorphic.

That is,

$$(X, s_1, t_1) \cong (X, s_2, t_2) \quad \rightarrow \quad (S, s_1, t_2) \cong (S, s_2, t_2)$$

■

A topological space X is a pair as $(\mathrm{Space}(X), T)$ where $\mathrm{Space}(X)$ denotes the set of all points of X and T denotes a topology on $\mathrm{Space}(X)$.

Isomorphisms:

[1]"$x \ _{and}\wedge y$" is almost equivalent to "$x \wedge y$" except that it is not promised to be commutative.

For example, take $\forall(X1, X2, X3, X4)$ as topological spaces. If there exists $\exists F$ as a bijection on a set of points such that: **some subset of** F **is a homeomorphism** from $X1$ to $X3$ ${}_{and}\wedge$ **some subset of** F **is a homeomorphism** from $X2$ to $X4$, then you write as $(X1, X2) \cong (X3, X4)$.

More formally:
Take $\forall F$ as a bijection on a set of points.
Take $\forall(p1, p2)$ as points. Define *1 to be equivalent to *2.

1. **some subset of** F **is a homeomorphism** from $p1$ to $p2$.

2. $F(p1) = p2$.

Take $\forall(P1, P2)$ as sets of points. Define *3 to be equivalent to *4.

3. **some subset of** F **is a homeomorphism** from $P1$ to $P2$.

4. $P1 \subset$ domain(F) ${}_{and}\wedge$ image$(F[P1])$=$P2$.

Take $\forall x$.

x is said **point free** if: There exists no chain of set membership from x down to any point. For example: "123" is point free; $(x, 1, \{2, 3, \varnothing\})$ is point free if x is point free.

Take $\forall(q1, q2)$ such that: Both of $(q1, q2)$ are point free. Define *5 to be equivalent to *6.

5. **some subset of** F **is a homeomorphism** from $q1$ to $q2$.

6. $q1 = q2$.

x is said a **topological vertex** if: x is either a point, a set of points, a topological space or anything point free.

x is said a **topological graph** if: x is a graph $G := (V, E)$ with (C defined as a function from the vertext set V to a set of topological vertices). In detail: G can be any type of graphs if **graph isomorphisms** are defined for the type; C is called the **content map** on V.

Proposition 1. Take $\forall(v1, v2, F)$ such that: $(v1, v2)$ are topological vertices ${}_{and}\wedge$ **some subset of F is a homeomorphism** from $v1$ to $v2$. For the inverse function F^{-1}, **some subset of F^{-1} is a homeomorphism** from $v2$ to $v1$.

Proof. Take $\forall(v1, v2, F)$ as a counterexample of the proposition. Hence $(v1, v2)$ are either points as (1.), sets of points as (2.), topological spaces as (3.) or any pair of point free objects as (4.). For all cases, as F is a bijection, F^{-1} is also a bijection.

***1.** By the definition, $\neg(F^{-1}(v2) = v1)$.
Though, that $F(v1) = v2$ implies that $F^{-1}(v2) = v1$.

***2.** By the definition, $\neg($ $v2 \subset \text{domain}(F^{-1})$ ${}_{and}\wedge$ $\text{image}(F^{-1}[v2]) = v1$ $)$.
Equivalently, $\neg($ $v2 \subset \text{domain}(F^{-1})$ $)$ ${}_{or}\vee$ $\neg($ $\text{image}(F^{-1}[v2]) = v1$ $)$.
Though it contradicts to that $\text{image}(F[v1]) = v2$. [2]Footnote.

***3.** $F[\text{Space}(v1)]$ is a homeomorphism from $v1$ to $v2$ ${}_{and}\wedge$ $(F[\text{Space}(v1)])^{-1}$ is not a homeomorphism from $v2$ to $v1$.
By the definition of homeomorphisms between topological spaces, it is a contradiction.

***4.** By the definition, $v1 \neq v2$.
Hence $\neg($ some subset of F is a homeomorphism from $v1$ to $v2$ $)$.
It is a contradiction.

$\square$

Definition 1.2 (Isomorphism between topological graphs)**.** Take $\forall\{G_i\}_{i\in\{1,2\}}$ as a pair of topological graphs. Hence $G_i := (V, E, C)_i$ where C_i denotes the content map on V_i. Take $\forall f$ as a graph isomorphism from V_1 to V_2. Take $\forall F$ as a bijection on a set of points.

Then F is said an isomorphism between G_1 and G_2 if *1. And if there exists some isomorphism between G_1 and G_2 then you write as *2.

1. Take $\forall v :\in V_1$, then **some subset of F is a homeomorphism** from $C_1(v)$ to $C_2(f(v))$.

2. $G_1 \cong G_2$.

[2]"x ${}_{or}\vee$ y" is almost equivalent to "$x \vee y$" except that it is not promised to be commutative.

Proposition 2. $G_1 \cong G_2 \quad \equiv \quad G_2 \cong G_1$.

Proof. Refer to the definition of isomorphism of toplogical graphs as MainDef. Take $\forall(G_1, G_2)$ as a counterexample. By graph theory, f^{-1} is a graph isomorphism from G_2 to G_1. And F^{-1} is a bijection on a point of sets. Hence the antecedent of MainDef holds for $(G_2, G_1, f^{-1}, F^{-1})$ in place of (G_1, G_2, f, F) except *1. Hence *1 of MainDef fails for it.

1. Hence: $\exists v2 :\in V_2 \ {}_{and}\wedge\ \neg($ some subset of F^{-1} is a homeomorphism from $C_2(v)$ to $C_1(f^{-1}(v))$).

2. $G_1 \cong G_2$ implies that: Some subset of F is a homeomorphism from $C_1(f^{-1}(v))$ to $C_2(v)$.

(*1 $\ {}_{and}\wedge$ *2) contradicts to Proposition1. □

2 Prime topological space

To define prime topological spaces, two new notions as preliminaries are required.

2.1 To specify a variable

Take $\forall(x, y)$ as variables. That (x is **specified** on y) is equivalent to that: x represents exactly one case of value in the sense of that y represents exactly one case of value.

For example, let y denote R^2 as a topological space. Take $\forall M$ as a metric table to define y. Let $x := \{z \mid z$ is a circle in y with M $\}$. Then it does not specify x on y because infinitely many metric table can define y. In other words, M is not specified on y.

Meanwhile, x is specified on (y, M). [3]Footnote

2.2 Subtext

Take $\forall e$ as a logical expression. All **subtext** of e is got as follows.

[3]As I omit the proof, I need to add "probably".

Express e as a tree g of (either **binary** or **unary**) logical operations. For example, given $v = \{1, 2\}$ as e; g can be $v = \{1, 2\}$ as a trivial example. In stead take $(1 \in v \wedge (\ 2 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\})))$ as g. It is just a coincide that this g has no logical disjunction operation.

Next, select some vertex x of g. For example, x can be $v \subset \{1, 2, 3, 4, 5, 6\}$. Though, for this case, the procedure inevitably ends here; that is, you have got e itself as a subtext of e. In stead take $(\ 2 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\}))$ as x.

Next, change the operator of x so that the new x returns exactly one term. For example, the original operator "$\wedge$" can be changed to "right identity"; that is, the new x returns the right term. Namely, the new x returns $(|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\})$.

In detail, if the operator is unary then it must be changed to the identity; that is, the new x returns the only one term.

Finally the resultant expression for the modified tree is said a subtext of e. For the example, the subtext of e is $(1 \in v \wedge (|v| = 2 \wedge v \subset \{1, 2, 3, 4, 5, 6\}))$.

And the relation on the set of all logical expressions by (being pair of a subtext and the original expression) is defined to be **transitive**. That is, a subtext of a subtext of e is also a subtext of e.

2.3 Prime topological space

Definition 2.1 (Prime topological space)**.** Take $\forall X$ as a topological space. Then X is said prime if the following main propositional function holds for X in place of X.

Definition 2.2 (Main propositional function)**.** Let X be the input topological space.

Take $\forall(S1, S2, d2)$ such that $(*1 \ {}_{and}\wedge \ *2)$. Then $(*3 \ {}_{or}\vee \ (*4 \ {}_{and}\wedge \ ..... \ {}_{and}\wedge \ *6))$.

1. $S1$ is a prime set of sub spaces of X.

2. $d2$ is a definition to specify $S2$ on X so that $S2 \subset S1$.

3. $S2$ is a prime set of sub spaces of X.

4. $\exists(d3, S3)$.

5. $d3$ is a subtext of $d2 \ {}_{and}\wedge \ d3$ specifies $S3$ on X.

6. $\varnothing \neq S3 \subset S1 \ {}_{and}\wedge \ S3$ is a prime set of sub spaces of X.

3 Conjectures

3.1 Main conjecture for metric spaces

Conjecture 3.1 (Main conjecture)**.** Take $\forall(X, M)$ as a metric space where X denotes the topological space and M denotes the metric table to define X. Refer the main propositional function as MainProp. If X is prime then MainProp holds for X in place of X, with changes in the text of MainProp at (*2, *5) as teh before to (*2_2, *5_2) as the after respectively.

2. $d2$ is a definition to specify $S2$ on X so that $S2 \subset S1$.

2_2. $d2$ is a definition to specify $S2$ on (X, M) so that $S2 \subset S1$.

5. $d3$ is a subtext of $d2 \ {}_{and}\wedge \ d3$ specifies $S3$ on X.

5_2. $d3$ is a subtext of $d2 \ {}_{and}\wedge \ d3$ specifies $S3$ on (X, M).

3.2 Sub conjecture

Conjecture 3.2 (Sub conjecture)**.** Refer to the main conjecture as MainConj. The following (*1 $_{and}\wedge$ ..... $_{and}\wedge$ *3) holds true. Take $\forall M$ as a metric table to define a Euclidean space of 3-dimension. Let X denote the topological space defined by M.

1. X is a prime topological space.

Take $\forall k_u$ as a knot in X.
$K := \{k \mid (X, k_u) \cong (X, k) \ \}$.

2. (X, M, K, K_f) is an instance of $(X, M, S1, S2)$ of ManConj.

Instance: That is, the antecedent of MainConj holds for (X, M, K, K_f) in place of $(X, M, S1, S2)$.

3. For (X, M, K, K_f), the item *3 of ManConj [4]holds.

[4]Hence K_f is a prime set of sub spaces of X.

Definition of K_f:

$K_f := \{k \in K \mid f(k_u) = f(k)\ \}$.

Sub definitions with K as the domain:

- $j1(k) := \{j \mid$
  j is an orthogonal [5]projection of k onto some infinite plane $\}$.
- $j2(k) := \{j \in j1(k) \mid$
  $\neg\ (\exists p\ \ {}_{and}\wedge\ p \in \text{image}(j)\ \ {}_{and}\wedge\ \mid j^{-1}(p)\ \mid > 2)\ \}$.
- $j3(k) := \{n \mid$
  $\exists j\ \ {}_{and}\wedge\ j \in j2(k)\ \ {}_{and}\wedge\ n$ is the number of [6]double points on $j\ \}$.
- $f(k) := \{m \mid$
  m is the maximal number from $j3(k)\ \}$.

∎

3.3 Factor of a logical expression

Currently it is hard to show that the main conjecture implies the sub conjecture because to analyze subtexts of a logical expression requires computer-assisted proof software.

By the way, readers may criticize that, the item *3 of the main conjecture fails for the sub conjecture due to ($j3$ and f). As an excuse for the negative critique, let me briefly introduce a new notion named "factor".

Recall that, $j3(k)$ returns a set of natural numbers. And $f(k)$ takes the maximum number from $j3(k)$. For example $j3(k_x) = \{7, 8, 9\}$. The inverse image of $\{\{7, 8, 9\}\}$ over $j3$ is a subset of domain(f).

Let me analyze the equation which defines the inverse image; $y = \{7, 8, 9\}$. It is equivalent to $(7 \in y \wedge 8 \in y \wedge \max(y) = 9 \wedge |y| = 3)$ where each term of the logical conjunction is named as a **factor** of $y = \{7, 8, 9\}$ if the logical conjunction is not redundant. For the example above, it happens to be not redundant. And to use a factor to define an inverse image is just to use a subtext of the original equation, $y = \{7, 8, 9\}$. For example, $\max(y) = 9$.

[5]Hence, j is a function from k to an infinite plane.

[6]That is, the inverse image of a double point has exactly 2 distinct points of k; no matter the double point is a crossing or a tangent point.

As a conclusion, if the definition is not redundant, $j3$ is not a subtext of f but f is a subtext of $j3$. Though non redundant definitions tend to be of less quality in human readability. So my definitions of f and $j3$ are inevitably a little redundant.

Definition 3.1 (Redundant logical expression)**.** Take $\forall e$ as a logical expression. Then e is said **redundant** if some its subtext is equivalent to e.