Axiom of Definition length

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October 11, 2020

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1 Summary

In this summary, words in blue are expected to be trivial for readers although it will be defined in the main text; words in red are meant to remain abstract.

The author once tried but failed to state that:

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- $\star$. For all Euclidean space X, M where X is the set of points and M is the metric table:
- $\star$. For all sub space , for example is a knot:
- $\star$. For the set S_K of every such $K1$ that $X, K1$ is isomorphic to X, K:
- $\star$. For all function f on S_K which naturally maps members of S_K to real numbers:
- $\star$. For all real number r, for the inverse image $f^{-1}(\{r\})$:
- $\star$. $f^{-1}(\{r\})$ is closed in terms of ambient isotopies on X.
  - $\diamond$. That is, for all $\{K1, K2\} \subset f^{-1}(\{r\})$:
- $\star$. For some ambient isotopy F on X:
  - $\diamond$. F takes $K1$ to $K2$.
  - $\diamond$. F takes $K1$ via $K3$ to $K2$ implies that $K3 \in f^{-1}(\{r\})$.

For example, $f(K) :=$ "the probabilistically expected number of crossings on a projection of K". But it was too hard to redefine the abstract term "f naturally maps ..." in strict words. For example $f(K) :=$ "the length of K" is to be said natural whereas $g(k) := (f(K)-1)^2$ is not to be said natural because it is a trivial counterexample.

Notice that, in general, $g^{-1}(\{r\})$ is a union of sets where each component set can be defined in a shorter text than $g^{-1}(\{r\})$.

As an improvement of the above failure, the author introduces Axiom of Definition length.

We give the axiom first then we give definitions for the used words, symbols and notations. Please be patient for that it may sound nothing until you have read at least a half of the page.

2 Axiom

Axiom 2.1. For all set denoted as $\psi_{(S,f,y)}$, if it is divided by the connectivity through ambient isotopies into multiple components then:

> All component C is specified in terms of (X, M) by a shorter text than $\psi_{(S,f,y)}$ is specified by.

3 Definitions

3.1 A completed chain by set member ship

Definition 3.1. Let S, p be all such that:

- $\star$. S is a set.
- $\star$. p is a finite sequence such that:
  - $\diamond$. For the first term x of p , $x = S$.
  - $\diamond$. For all pair (x, y) as a consecutive terms on p where y follows x ,
    - $\triangleright$. y is a member of x .
  - $\diamond$. For the last term x of p , x is an empty set or not a set.

Then: **28**

p is said a completed chain by set member ship with S as the root. **29**

3.2 A set to fall in a set **30**

Definition 3.2. Let $S, P, S1, S2$ be all such that: **31**

- ⋆. P is the set of all completed chains by set member ship with S as the root.
- ⋆. $S1 = \{x \mid \text{for some } p \in P \text{ , } x \text{ is the last term of } p\}$
- ⋆. $S2 = S1$ subtracted $\{\phi\}$. **32**

Then S is said: **33**

- ⋆. To fall in all super set of $S2$. **34**

Besides, we regard P as the tree to represent S in the sense of that: **35**

- ⋆. $V := \{v \mid \text{for some } p \in P \text{ , } v \text{ is a term of } p \ \}$.
- ⋆. $E := \{(v1, v2) \mid \text{for some } p \in P \text{ , } (v1, v2) \text{ is a pair of consecutive terms on } p \ \}$.
    - ⋄. In detail, $v2 \in v1$. **36**

3.3 Isomorphisms of points **37**

Definition 3.3. . **38**

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- ⋆. All set is not said a point (in the sense of this article).
- ⋆. All two points are isomorphic to each other.
- ⋆. That p is a point and p, q are isomorphic to each other implies that:
    - ⋄. q is a point. **40**

3.4 Isomorphisms of sets 41

Definition 3.4. 4 Two sets are said isomorphic to each other if: 42

Some such isomorphism f exists between their trees in terms of graph theory that:

- $\star$. For all point $p \in \text{image}(f\ )$:
  - $\diamond$. $p, f(p)$ are isomorphic to each other.

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3.5 Definition of X, M, n 44

Definition 3.5. 5 Let X, M, n be all such that: 45

- $\star$. X is a set of points.
- $\star$. M is a metric table on X to define (X, M) as an n -dimensional Euclidean space.

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3.6 Specified sets 47

As a comment, *you may need to read this definition very slowly. Remember 48
that the notions are very elemental and some how childlish*. 49

Definition 3.6. Let S be all which falls in X . 50
Then S is said specified in terms of (X, M) if: 51

- $\star$. Let d denote the definition text of S in terms of (X, M) .
  - $\diamond$. That is, d does not define (X, M) but refers to (X, M) as the predefined context.
- $\star$. For the following repetitive expression as $S := d; S := d$:
  - $\diamond$. The first S is identical to the second S .

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3.7 Examples 53

Example 3.1. Let $S := d$ as "the set of all points of X ". 54
Let us define, $S := d; S := d$. Then the two S are identical because $S = X$ 55
. Hence S is specified in terms of (X, M) . 56

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Let $S := d$ as "all subset of X such that $|S| = 2$ ".

Let us define, $S := d; S := d$. Then the two S are not identical in general. Hence S is not specified in terms of (X, M) .

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Let x be all point in X . Let $S := d$ as "the set of all points of distance 1 to x ".

Then S is specified in terms of (X, M, x) but in terms of (X, M) .

The difference is between that d refers x as the predefined context and that d defines x within it when it is repititively expressed.

3.8 Definition of $\psi_{(S,f,y)}$

Definition 3.7. Let $S, \Psi_S, f, \psi_{(S,f,y)}$ be all such that:

- ⋆. S is a set to fall in X .
- ⋆. Ψ_S is the set such that:
  - ⋄. $\forall S1$, $S1 \in \Psi_S$ is equivalent to that:
    - ▷. $(X, S1)$ is isomorphic to (X, S) .
  - ⋄. Ψ_S is specified in terms of (X, M) .
- ⋆. f is a specifed function on Ψ_S in terms of (X, M) .
- ⋆. $y \in \text{image}(f)$ and y is specified in terms of (X, M) .
- ⋆. $\psi_{(S,f,y)}$ is the subset of Ψ_S such that:
  - ⋄. $S1 \in \psi_{(S,f,y)}$ is equivalent to that:
    - ▷. $f(S1) = y$.

3.9 To divide $\psi_{(S,f,y)}$ by connectivity through ambient isotopies

Definition 3.8. Let $S1, S2$ be all such that:

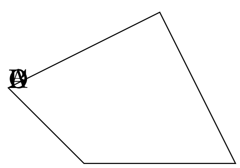
- $\star$. $\{S1, S2\} \subset \psi_{(S,f,y)}$.

$S1, S2$ are said connected through an ambient isotopy F on (X, M) if:

- $\star$. F takes $S1$ to $S2$.
- $\star$. If F takes $S1$ via $S3$ to $S2$ then $S3 \in \psi_{(S,f,y)}$.

To diveide $\psi_{(S,f,y)}$ by this relation is said:

To divide $\psi_{(S,f,y)}$ by connectivity through ambient isotopies.



References

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