Minors of sets

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1 Contents

The first two pages are the main part. The first page gives the main definition by examples. The second page gives the main definitions formally. The rest pages give definitions used in propositions and proofs and prove the propositions which states that the main definitions are super classes or sub classes of standard notions of mathematics.

2 Main definition by examples

Some words or some notations in this page are possibly not clear for some readers. All of them will be formally defined in the next page.

Let (X, T^2, M^2) denote the 2-dimensional Euclidean space where T^2 is the topology and M^2 is the metric table.

Let $S1 := \{L1 \mid L1$ is a subspace of X and $L1$ is a closed straight line segment of length 1 in terms of $M^2\}$.

As a remark, $L1$ represents (a subset of X) and (the restriction of T^2 at ($L1$ as a subset of X)). Meanwhile $L1$ has no information in terms of M^2.

Let $S2 := \{L2 \mid \exists L1 \in S1$ such that $L2$ is homeomorphic to $L1\}$.

Then $S1$ and $S2$ are not topologically equivalent. For example, some distinct two members of $S2$ intersect to each other exactly at two or more many countable points whereas the same fails for $S1$ in place of $S2$.

Though there are needs to state that $S1$ and $S2$ are almost topologically equivalent. For example, it is true that (A1.) $S1 \subset S2$. And it is possibly true that

(A2.) for all three members $(L1, L2, L3)$ of $S1$, if $(S2, L1, L3)$ and $(S2, L2, L3)$ are topologically equivalent, then $(S1, L1, L3)$ and $(S1, L2, L3)$ are also topologically equivalent.
If (*A1 and *A2) holds for $(S1, S2)$ then $S1$ is said a minor of $S2$.

3 Main definitions

First of all, $\forall m$ is said a **memBer** if it is a member of some set.
Take $\forall \{c\}_{i \in [1,n] \subset \mathbb{N}}$ as a chain of set [1]membership where the indexing is bijective. Then the smallest member c_1 is said a **deep member** of the maximum member c_n; and c_1 is calculated [2]**a deep number relative to** c_n as (if $n \geq 2$ then $\prod_{i \in [2,n]} |C_i|$ else 0). [3]Footnote.
And all memBer m is said a **constant-memBer** if all deep member of m is not a point. And all memBer m is said an [4]end-memBer if m is either a constant-memBer or a point.
Needless to say all topological space is a memBer and all memBer m is expressed as a deep graph. To [5]resolve "deep graph", take $\forall m$, then the **deep graph** of m is defined as the directed graph (V, E) on the set V of all deep members of m such that $E = \{(v1, v2) \in V * V \mid v1 \in v2\}$.
Ultimately, two memBers are said **isomorphic** or **isomorphic by** f if (their deep graphs are isomorphic by f as a graph isomorphism and relate-constant-memBer(f). To resolve "relate-constant-memBer",take $\forall L$ as a binary relation, then it is written as **relate-constant-memBer**(L) if (take $\forall (x, y) :\in$ Ł such that either x or y is a constant-memBer, then $x = y$).

Shifting to the notion of minors of memBers.
Take $\forall (m1, m2)$ such that $Space(m1) \subset Space(m2)$.
Then $m1$ is said a **minor** of $m2$ if *1 implies *2.

1 Take $\forall (d1, d2, d3)$ as deep members of $m1$ such that
$((m2, d1, d3), (m2, d2, d3))$ are isomorphic).

2 $((m1, d1, d3), (m1, d2, d3))$ are isomorphic.

[1]The order implies that all member is smaller than the set.
[2]This word will not be used in the rest.
[3]Be careful that, there possibly exist multiple chains of set membership between (c_1, c_n).
[4]This word will not be used in the rest.
[5]In this article, "to resolve" means to define the meaning of words after using the words.

4 Notations

Consider a proposition, e.g., a and b.
And consider a proposition, e.g., $a \wedge b$.
The two example propositions are unclear whether they are equivalent to each other.
In this article, the two are possibly different.
Speaking simply, "a and b" are not checked by the author(me) if it can be commutative.
In this sense, "a and b" is written as "$a\ {}_{and}\wedge\ b$".
And in this sense, "a or b" is written as "$a\ {}_{or}\vee\ b$".
As a remark, I don't have any actual example of "a and b" which is not commutative.

Definition 4.1 (Restriction of binary relation)**.**
Take $\forall(L, X, Y)$ as a binary relation L and sets (X, Y).
$L[X] := \{(x, y) \in L \mid x \in X\}$.
$L[, Y] := \{(x, y) \in L \mid y \in Y\}$.

5 Properties of equivalence relation

Proposition 1 (Reflexive,symmetry,transitive properties)**.**
The relation by isomorphisms of memBers has properties of reflexive, symmetry and transitive.

Proof.

- *1 has been proved in graph theory.
- It is trivial that (*2 $\ {}_{and}\wedge$ ..... $\ {}_{and}\wedge$ *5) holds.
- Hence this proposition holds.

1 The relation by graph isomorphisms has properties of reflexive, symmetry and transitive.

2 Take $\forall f_1, f_2, f_3$ as graph isomorphisms such that
$domain(f_2) = image(f_1)\ \ {}_{and}\wedge$
f_3 is the identity function on domain(f_3).

3 relate-constant-memBer(f_3) $\ {}_{and}\wedge$

4 relate-constant-memBer(f_1) $\equiv$ relate-constant-memBer(f_1^{-1}) $_{and}\wedge$

5 (relate-constant-memBer(f_1) $_{and}\wedge$ relate-constant-memBer(f_2)) $\equiv$
relate-constant-memBer($f_2 \circ f_1$)

□

6 Homeomorphic topological spaces as isomorphic memBers

Definition 6.1.
Take $\forall(m1, m2, c)$ such that (
c is a chain of set membership $_{and}\wedge$
$m1$ is the [6]minimum member of c
$m2$ is the [7]maximum member of c.
).
Then define (*1 $_{and}\wedge$ ..... $_{and}\wedge$ *5).

1 $m1$ is said a deep member of $m2$.

Hence all memBer is a deep member of itself.

2 $|c| - 1$ is said a power of $(m1, m2)$.

3 It is written as $m1 \in^{|c|-1} m2$.

4 Let p be the maximum power of $(m1, m2)$.
Then $depth(m1, m2) := p$.

5 Let $S := \{d \mid \text{there exists } \exists m \text{ such that } d = depth(m, m2)\}$.
Then $depth(m2)$:="the maixmum member of S".

■

Definition 6.2 (Space of memBer)**.**
Take $\forall m$.
Then define that
$Deep(m) := \{d \mid d \text{ is a deep member of } m\ \}$.
$Space(m) := \{p \in Deep(m) \mid p \text{ is a point }\}$. ■

[6]No member of c is a member of $m1$.
[7]No member of c has $m2$ as a member.

Proposition 2 (Isomorphism of vertices)**.**
Take $\forall(m1, m2, f, v1)$ such that ($(m1, m2)$ are isomorphic by $f \ \ _{and}\wedge\ v1 \in \mathit{Deep}(m1)$). Then $v1, f(v1)$ are isomorphic by $f[\mathit{Deep}(v1)]$.

Proof.

- Let $v2 := f(v1)$.
- As **C1**, claim that $\mathit{Deep}(v2) \subset \mathit{image}(f[\mathit{Deep}(v1)])$.
- Assume that the claim fails.
- There exists $\exists w2 :\in \mathit{Deep}(v2)$ as a minimum counterexample to *C1 compared by $\mathit{depth}(w2, v2)$.
- It is trivial that $w2 \neq v2$.
- There exists $\exists x2 :\in \mathit{Deep}(v2)$ such that $w2 \in x2$.
- Hence x2 is not a counterexample to *C1 because $\mathit{depth}(w2, v2) < \mathit{depth}(x2, v2)$.
- Hence There exists $\exists x1 :\in \mathit{Deep}(v1)$ such that $f(x1) = x2$.
- Hence There exists $\exists w1 :\in x1$ such that $(f(w1) = w2 \ \ _{and}\wedge\ w1 \in \mathit{Deep}(v1))$. A contradiction.
- Hence The assumption on $(\neg$ *C1) is false.
- As **C2**, claim that ($\mathit{Deep}(v1) \subset \mathit{image}(\ f^{-1}[\mathit{Deep}(v2)]\ ))$.
- Though it is trivial that the same logic for the proof of *C1 proves *C2.
- Hence $\mathit{Deep}(v2) = \mathit{image}(f[\mathit{Deep}(v1)])$.
- Hence $f[\mathit{Deep}(v1)]$ is a graph isomorphism from*to $\mathit{Deep}(v1) * \mathit{Deep}(v2)$.
- And it is trivial that relate-constant-memBer(f) $\Rightarrow$ relate-constant-memBer($f[\mathit{Deep}(v1)]$).

□

Proposition 3 (Isomorphism of Spaces)**.**
Take $\forall(m1, m2, f)$ such that $(m1, m2)$ are isomorphic by f.
Then $f[Space(m1)]$ is a bijection from*to $Space(m1) * Space(m2)$.

Proof.

- Assume it is false.
- $image(f[Space(m1)]) \neq Space(m2)$.
- $image(f[Space(m1)]) \not\subset Space(m2)$ $_{or}\vee$ $image(f[Space(m1)]) \not\supset Space(m2)$.
- Then there exists $\exists(m1, m2, f, p1, p2)$ as a counterexample such that (*A0 $_{and}\wedge$ (*A1 $_{or}\vee$ *A2)) holds.
  A0 $(p1, p2) :\in Space(m1) * Space(m2))$.
  A1 $f(p1) \notin Space(m2)$.
  A2 $p2 \notin image(f[Space(m1)])$.
- Assume *A1 holds.
- Then $f(p1)$ is either a constant-memBer (or a non-constant-memBer as a set).
- Though $f(p1)$ can not be a constant-memBer by that relate-constant-memBer(f).
- Hence $f(p1)$ is a non-constant-memBer as a set.
- Though it contradicts to that f is a graph isomorphism because $f(p1)$ has edge to some its member.
- Hence the assumption of *A1 is false $_{and}\wedge$ *A2 holds.
- There exists $\exists c1 :\notin Space(m1)$ such that $f(c1) = p2$.
- Hence $f^{-1}(p2) = c1$
- Though this condition has been denied in the disproof of *A1.
- Hence the assumption of *A2 is false $_{and}\wedge$ the main assumption is false.

$\square$

Proposition 4 (Pair of member's isomorphisms)**.**
Take $\forall(I := \{1,2,3,4\},\ \ \{m_i\}_{i \in I},\ \ f_{1,2},\ \ f_{3,4})$
such that $(*1\ \ {}_{and}\wedge\ .....\ \ {}_{and}\wedge\ *4\ )$ holds.
Then $(*5\ \ {}_{and}\wedge\ *6)$ holds.

1 $(m1, m2)$ are isomorphic by $f_{1,2}$.

2 $(m3, m4)$ are isomorphic by $f_{3,4}$.

3 Let $f := f_{1,2} \cup f_{3,4}\ \ {}_{and}\wedge$ let $f_s := f[Space(f)]$.

4 Then f_s is a bijection.

5 f is a function.

6 f is a bijection.

7 relate-constant-memBer(f).

$\blacksquare$

*Proof of *5.*

- Let $(V, E)_{i :\in \{1,2,3,4\}}$ be the deep graph of m_i.

- Assume it is false.

- Then there exists $\exists((m1, m3), (m2, m4))$ as a minimum counterexample by $depth((m1, m3))$ such that f is not a function.

- Let us make sure that f is a union of a set of bijections.

- There exists $\exists v :\in V_1 \cap V_3$ such that $|f[\{v\}]| \geq 1\ \ {}_{and}\wedge\ v \notin \{m1, m3\}$.

- By the way, this proposition accepts the following $args_v$
  in place of $(m1, m2, m3, m4, f_{1,2}, f_{3,4})$.

- $args_v := ($
  $v,$
  $f_{1,2}(v),$
  $v,$
  $f_{3,4}(v),$
  $f_{1,2}[Deep(v)],$
  $f_{3,4}[Deep(v)]$
  $).$

- In the rest, this $args_v$ is proved to be a counterexample smaller than a minimal counterexample.
- As the first step, the such-that clause of this proposition holds for $args_v$ as follows.
- Equivalently (*1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *4) holds for $args_v$ as follows.
- Assume *1 fails for $args_v$.
- Hence $(v, f_{1,2}(v))$ is not isomorphic by $f_{1,2}[Deep(v)]$.
- Though it contradicts to the proposition titled as "Isomorphism of vertices".
- Hence the last assumption is false.
- Hence *1 holds for $args_v$.
- Hence *2 holds for $args_v$ because (for $args_v$, *1 and *2 are logically equivalent).
- Assume *4 fails for $args_v$.
- Let $f_v := f_{1,2}[Deep(v)] \cup f_{3,4}[Deep(v)]$ ${}_{and}\wedge$
  let $f_{v,s} := f_{1,2}[Deep(v)][Space(f_v)] \cup f_{3,4}[Deep(v)][Space(f_v)]$.
- Then $f_{v,s}$ is not a bijection.
- Though it is false because $f_{v,s} \subset f_s$. Hence *4 holds for $args_v$.
- Hence (*1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *4) holds for $args_v$.
- Moreover *5 fails for $args_v$ as follows.
- Assume *5 holds for $args_v$.
- Then f_v is a function.
- Though
  $v \in Deep(v)$ ${}_{and}\wedge$
  $f_v[\{v\}] = f[\{v\}]$ ${}_{and}\wedge$
  $|f_v[\{v\}]| = |f[\{v\}]| \geq 1$.
- Hence *5 fails for $args_v$.
- $args_v$ is a counterexample.

- And the size as a counterexample of $args_v$ equals to $depth((v, v))$.
- Though $depth((v, v)) < depth((m1, m3))$ [8]because $depth((v, v)) = depth(v) + 2 < depth(m1) + 2 \leq depth(m1, m3)$.
- Hence arg_v is a counterexample smaller than a minimum counterexample.
- Hence the main assumption is false.

$\square$

*Proof of *6.*

- Consider the proposition $*\mathrm{P}_S$ titled as "Reflexive,symmetry,transitive properties".
- Consider the proposition $*\mathrm{P}_I$ titled as "Isomorphism of spaces".
- Then $((*\mathrm{P}_S \ {}_{and}\wedge \ *\mathrm{P}_I) \ {}_{and}\wedge \ (*1 \ {}_{and}\wedge \ \ldots\ldots \ {}_{and}\wedge \ *4\ ))$ implies $(*\mathrm{S}1 \ {}_{and}\wedge \ \ldots\ldots \ {}_{and}\wedge \ *\mathrm{S}4\ )$.

S1 $(m2, m1)$ are isomorphic by $f_{1,2}^{-1}$ as an isomorphism.

S2 $(m4, m3)$ are isomorphic by $f_{3,4}^{-1}$ as an isomorphism.

S3 Let $f_{-1} := f_{1,2}^{-1} \cup f_{3,4}^{-1} \ {}_{and}\wedge$ let $f_{s,-1} := f_{-1}[Space(f_{-1})]$.

S4 Then $f_{s,-1}$ is a bijection.

- For *S4, it holds because (it is trivial that ($f_{-1} = f^{-1} \ {}_{and}\wedge \ f_{s,-1} = f_s^{-1}$).
- Moreover *5 implies that f_{-1} is a function.
- Hence f^{-1} is a function.
- Hence *5 implies that f is an injection.
- By the way,f is surjective because f is not defined the codomain.
- Hence f is a bijection.

$\square$

*Proof of *7.*

[8] $(x, y) := \{\{x\}, \{x, y\}\}$

- Assume it is false.
- There exists $\exists(x, y) :\in f$ such that (either x or y is a constant-memBer) $_{and}\wedge$ $(x \neq y)$.
- Though $f = f_{1,2} \cup f_{3,4}$.
- Hence $(x, y) \in f_{1,2}$ $_{or}\vee$ $(x, y) \in f_{3,4}$.
- There exists $\exists g :\in \{f_{1,2}, f_{3,4}\}$ such that $\neg$(relate-constant-memBer(g)).
- It contradicts to (*1 $_{and}\wedge$ *2).
- The assumption is false.

$\square$

Definition 6.3 (Constant space)**.**
A constant space D is most likely a function to be used to state conditions on variables.
For example, let D be a function and let $x, y, z :\in Z * Z * Z$ such that $x = D(z)$ and $y = D(z)$.
Then $x = y$.
In this case, D is used to make sure that variables hold equal values.
Be careful that all constant space is just a usual variable but a global constant.

Proposition 5 (Isomorphism by member's isomorphisms)**.**
Let *P$_P$ denote the proposition titled as "Pair of member's isomorphisms".
Take $\forall(S1, S2, f, F)$ as sets $(S1, S2)$ such that (*A1 $_{and}\wedge$ ..... $_{and}\wedge$ *A7).
Then (*10 $_{and}\wedge$ ..... $_{and}\wedge$ 12) holds.

A1 $\mid$ $Deep(\{S1, S2\})$ $\mid \leq$ continuum.

A2 f is a bijection from*to $S1 * S2$.

A3 There exists $\exists D$ as a function and as a constant space.

A4 Take $\forall((m1, m2), (m3, m4)) :\in f^2$.

A5 There exists $\exists f_{1,2}, f_{3,4}$
such that $f_{1,2} = D((m1, m2))$ $_{and}\wedge$ $f_{3,4} = D((m3, m4))$.

A5 Let $args :=$ (
$m1, m2, m3, m4,$
$f_{1,2},$
$f_{3,4}$
).
Then $*\mathrm{P}_P$ accepts $args$
in place of $(m1, m2, m3, m4, f_{1,2}, f_{3,4})$.

A6 $*\mathrm{P}_P.(*1 \ {}_{and}\wedge \ ..... \ {}_{and}\wedge \ *4)$ holds for $args$.

A7 Let $D_{1,2} := \{D((m1, m2)) \mid (m1, m2) \in f\}$.
Then $F =$ union $D_{1,2}$.

C10 $F[Space(F)]$ is bijective.

C11 F is a function.

C12 F is bijective.

C13 relate-constant-memBer(F).

C14 $(S1, S2)$ are isomorphic by $F \cup \{S1, S2\}$.

■

*Proof of *C10.*

- First of all, it is trivial that
  $domain(F[Space(F)]) = Space(S1) \ {}_{and}\wedge$
  $image(F[Space(F)]) = Space(S2)$.
- Assume it is false.
- There exists $\exists(p1, p2) :\in Space(S1) * Space(S2)$ such that
  $|F(p1)| \geq 1 \ {}_{or}\vee \ |F^{-1}(p2)| \geq 1$.
- Though it implies that the antecedent of this proposition have failed.
- Namely, there exists $\exists((m1, m2), (m3, m4))$
  which has been taken as $\forall((m1, m2), (m3, m4))$ in *A4
  such that, of *A6, $*\mathrm{P}_P.(*4)$ have failed for $((m1, m2), (m3, m4))$.
- Hence the assumption is false.

□

*Proof of (*C11 ${}_{and}\wedge$ *C12 ${}_{and}\wedge$ *C13).*

- First of all, consider the proposition titled as "Pair of member's isomorphisms".
- The proposition implies that the antecedent of this proposition implies that *A6 can be modified as the following *A6 typed in red.
- That is, the original "*4" has been replaced with "*7".
- **A6** *P$_P$.(*1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *7) holds for $args$.
- Call this modified antecedent as the modified antecedent.
- By the way, assume (*C11 ${}_{and}\wedge$ *C12 ${}_{and}\wedge$ *C13) is false.
- (*B1 ${}_{or}\vee$ *B2) holds.
- **B1** There exists $\exists(x1, x2) :\in S1 * S2$ such that
  $|F(x1)| \geq 1$ ${}_{or}\vee$ $|F^{-1}(x2)| \geq 1$.
- **B2** There exists $\exists f_{1,2} :\in D_{1,2}$ such that
  $\neg$ relate-constant-memBer($f_{1,2}$).
- Though it implies that the modified antecedent have failed.
- Namely, there exists $\exists((m1, m2), (m3, m4))$
  which has been taken as $\forall((m1, m2), (m3, m4))$ in *A4
  such that, of *A6,
  P$_P$.(*5 ${}_{and}\wedge$ *6 ${}_{and}\wedge$ *7) have failed for $((m1, m2), (m3, m4))$.
- Hence the assumption is false.

$\square$

*Proof of *C14.*

- Assume it is false.
- Let $F_+ := F \cup \{S1, S2\}$, Then (*B1 ${}_{or}\vee$ *B2) holds.
- As **B1**, $(S1, S2)$ are not graph-isomorphic by F_+.
- As **B2**, $\neg$ relate-constant-memBer(F_+).
- Assume *B2 holds.
- Hence $\neg$ relate-constant-memBer($\{S1, S2\}$).

- Hence there exists$\exists (T1, T2) :\in \{(S1, S2), (S2, S1)\}$ such that $T1$ is a constant-memBer $\ _{and}\wedge$ $T2$ is not a constant-memBer.

- There exists $\exists (c_1, p_2) :\in F$ such that (c_1 is a constant-memBer $\ _{and}\wedge$ p_2) is not a point. By this contradiction, the assumption on *B2 is false.

- Hence *B1 holds.

- There exists $\exists (v1, v2) :\in S1 * S2$ such that $F(v1) \notin S2 \ _{or}\vee F^{-1}(v2) \notin S1$.

- Though there exists $\exists f_{1,2} :\in D_{1,2}$ such that ($(v1, F(v1)) \in f_{1,2} \ _{and}\wedge$ $f_{1,2}$ is a bijection from*to Deep($v1$)*Deep($F(v1)$)).

- Moreover $F \supset f_{1,2}$.

- Hence the assumption on *B1 is false.

- The main assumption is false.

□

Definition 6.4 (Variations of Indexed set)**.**
As you know, for example, $\{x_i\}_{i\in\{1,2\}} := \{x_1, x_2\}$, in mathematics.
In this article,
analogously, $(x_i)_{i\in\{1,2\}} := (x_1, x_2)$.
As an alternative simplified form, $(x)_{i\in\{1,2\}} := (x_1, x_2)$.
As one of many variations, $(\{x\})_{i\in\{1,2\}} := (\{x_1\}, \{x_2\})$.
As a comment, the order on the composed sequence should respect the most natural order on the index set. ■

Proposition 6 (Isomorphisms by spaces)**.**
Take $\forall (S)_{i:\in\{1,2\}}, \forall (f, g)$ such that (
$(S)_{i:\in\{1,2\}}$ are isomorphic by f and also by $g \ _{and}\wedge$
f[Space(f)]=g[Space(g)]
).
Then $f = g$.

Proof.

- Assume it is false.

- There exists $\exists v_1 :\in Deep(S1)$ as a minimum counterexample compared by depth(v_1) such that $f(v_1) \neq g(v_1)$.

- It is trivial that depth(v_1)> 0.

- Hence v_1 is a set.

- $f[v_1] = g[v_1]$ because (
  take $\forall w_1 :\in v_1$,
  then (depth(w_1) <depth(v_1) ${}_{and}\wedge$ w_1 is not a counterexample)
).

- Hence $f(v_1) = image(f[v_1]) = image(g[v_1]) = g(v_1)$.

- The assumption is false.

$\square$

Definition 6.5 (Isomorphism by spaces)**.**
Take $\forall(S)_{i:\in\{1,2\}}, \forall(f, F)$ such that
$(S)_{i:\in\{1,2\}}$ are isomorphic by F ${}_{and}\wedge$ $Space(F) \subset f \subset F$.
Then $(S)_{i:\in\{1,2\}}$ are also said isomorphic by f.

Proposition 7 (Homeomorphism as isomorphism)**.**
As you know, the set theory defines that
$(x, y) := \{\{x\}, \{x, y\}\}$.
Take $\forall((X, T))_{i:\in\{1,2\}}, \forall H$ such that (
$((X, T))_{i:\in\{1,2\}}$ is a pair of topological spaces ${}_{and}\wedge$
H is a bijection from*to $X_1 * X_2$ ${}_{and}\wedge$
$((X, T))_{i:\in\{1,2\}}$ are homeomorphic by H
).
Then (*1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *5) holds.

1. $(X)_{i:\in\{1,2\}}$ are isomorphic by H.

2. Take $\forall(t_1, t_2) :\in T1 * T2$ such that $t_2 = image(H[t_1])$.
Then $(t)_{i:\in\{1,2\}}$ are isomorphic by $H[t_1]$.

3. $(T)_{i:\in\{1,2\}}$ are isomorphic by H.

4. $(\{X\})_{i:\in\{1,2\}}$ are isomorphic by H.

5. $(\{X, T\})_{i:\in\{1,2\}}$ are isomorphic by H.

6. $(\{\{X\},\{X,T\}\})_{i:\in\{1,2\}}$ are isomorphic by H.

$\blacksquare$

*Proof of *1.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".
- $(X)_{i:\in\{1,2\}}$ are isomorphic by $H \cup \{(X1, X2)\}$.

$\square$

*Proof of *2.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".
- $(t)_{i:\in\{1,2\}}$ are isomorphic by $H[t_1] \cup \{(t1, t2)\}$.

$\square$

*Proof of *3.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".
- Consider *2.
- Let $t_{1,2} := \{(t_1, t_2) \in T1 * T2 \mid t_2 = image(H[t_1])\}$.
- $(T)_{i:\in\{1,2\}}$ are isomorphic by $H \cup t_{1,2} \cup \{(T1, T2)\}$.

$\square$

*Proof of *4.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".
- Consider *1.
- $(\{X\})_{i:\in\{1,2\}}$ are isomorphic by $H \cup \{(X1, X2), (\{X1\}, \{X2\})\}$.

$\square$

*Proof of *5.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".

- Consider *1 and *3.

- $(\{X,T\})_{i:\in\{1,2\}}$ are isomorphic by $H \cup \{(X1,X2),(T1,T2),(\{X1,T1\},\{X2,T2\})\}$.

$\square$

*Proof of *6.*

- Consider the proposition titled as "Isomorphism by member's isomorphisms".

- Consider *4 and *5.

- $(\{\{X\},\{X,T\}\})_{i:\in\{1,2\}}$ are isomorphic

- by $H \cup \{$
  $(X1,X2)$,
  $(T1,T2)$,
  $(\{X1,T1\},\{X2,T2\})$,
  $(\{\{X1\},\{X1,T1\}\},\{\{X2\},\{X2,T2\}\})$
  $\}$.

$\square$