

# Minors of sets

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## 1 Contents

The first two pages are the main part. The first page gives the main definition by examples. The second page gives the main definitions formally. The rest pages give definitions used in propositions and proofs and prove the propositions which states that the main definitions are super classes or sub classes of standard notions of mathematics.

## 2 Main definition by examples

Some words or some notations in this page are possibly not clear for some readers. All of them will be formally defined in the next page. 1 2  
Let  $(X, T^2, M^2)$  denote the 2-dimensional Euclidean space where  $T^2$  is the topology and  $M^2$  is the metric table. 3 4  
Let  $S1 := \{L1 \mid L1 \text{ is a subspace of } X \text{ and } L1 \text{ is a closed straight line segment of length 1 in terms of } M^2\}$ . 5 6  
As a remark,  $L1$  represents (a subset of  $X$ ) and (the restriction of  $T^2$  at ( $L1$  as a subset of  $X$ )). Meanwhile  $L1$  has no information in terms of  $M^2$ . 7 8  
Let  $S2 := \{L2 \mid \exists L1 \in S1 \text{ such that } L2 \text{ is homeomorphic to } L1\}$ . 9  
Then  $S1$  and  $S2$  are not topologically equivalent. For example, some distinct two members of  $S2$  intersect to each other exactly at two or more many countable points whereas the same fails for  $S1$  in place of  $S2$ . 10 11 12  
Though there are needs to state that  $S1$  and  $S2$  are almost topologically equivalent. For example, it is true that (A1.)  $S1 \subset S2$ . And it is possibly true that 13 14

(A2.) for all three members  $(L1, L2, L3)$  of  $S1$ , if  $(S2, L1, L3)$  and  $(S2, L2, L3)$  are topologically equivalent, then  $(S1, L1, L3)$  and  $(S1, L2, L3)$  are also topologically equivalent.  
 If (\*A1 and \*A2) holds for  $(S1, S2)$  then  $S1$  is said a minor of  $S2$ .

### 3 Main definitions

First of all,  $\forall m$  is said a **memBer** if it is a member of some set.  
 Take  $\forall \{c\}_{i \in [1, n] \subset \mathbb{N}}$  as a chain of set <sup>1</sup>membership where the indexing is bijective.  
 Then the smallest member  $c_1$  is said a **deep member** of the maximum member  $c_n$ ; and  $c_1$  is calculated <sup>2</sup>**the deep number relative to**  $c_n$  as ( if  $n \geq 2$  then  $\prod_{i \in [2, n]} |C_i|$  else 0 ).  
 And all memBer  $m$  is said a **constant-memBer** if all deep member of  $m$  is not a point. And all memBer  $m$  is said an <sup>3</sup>**end-memBer** if  $m$  is either a constant-memBer or a point.  
 Needless to say all topological space is a memBer and all memBer  $m$  is expressed as a deep graph. To <sup>4</sup>resolve "deep graph", take  $\forall m$ , then the **deep graph** of  $m$  is defined as the directed graph  $(V, E)$  on the set  $V$  of all deep members of  $m$  such that  $E = \{(v1, v2) \in V * V \mid v1 \in v2\}$ .  
 Ultimately, two memBers are said **isomorphic** or **isomorphic by**  $f$  if (their deep graphs are isomorphic by  $f$  as a graph isomorphism and **relate-constant-memBer**( $f$ ). To resolve "relate-constant-memBer", take  $\forall L$  as a binary relation, then it is written as **relate-constant-memBer**( $L$ ) if (take  $\forall (x, y) : \in L$  such that either  $x$  or  $y$  is a constant-memBer, then  $x = y$ ).

Shifting to the notion of minors of memBers.

Take  $\forall (m1, m2)$  such that  $Space(m1) \subset Space(m2)$ .

Then  $m1$  is said a **minor** of  $m2$  if \*1 implies \*2.

1 Take  $\forall (d1, d2, d3)$  as deep members of  $m1$  such that  
 $((m2, d1, d3), (m2, d2, d3))$  are isomorphic).

2  $((m1, d1, d3), (m1, d2, d3))$  are isomorphic. ■

<sup>1</sup>The order implies that all member is smaller than the set.

<sup>2</sup>This word will not be used in the rest.

<sup>3</sup>This word will not be used in the rest.

<sup>4</sup>In this article, "to resolve" means to define the meaning of words after using the words.

## 4 Notations 45

Consider a proposition, e.g.,  $a$  and  $b$ . 46

And consider a proposition, e.g.,  $a \wedge b$ . 47

The two example propositions are unclear whether they are equivalent to each other. 48 49

In this article, the two are possibly different. 50

Speaking simply, " $a$  and  $b$ " are not checked by the author(me) if it can be commutative. 51 52

In this sense, " $a$  and  $b$ " is written as " $a \text{ and } \wedge b$ ". 53

And in this sense, " $a$  or  $b$ " is written as " $a \text{ or } \vee b$ ". 54

As a remark, I don't have any actual example of " $a$  and  $b$ " which is not commutative. 55 56 57

**Definition 4.1** (Restriction of binary relation). 58

Take  $\forall(L, X, Y)$  as a binary relation  $L$  and sets  $(X, Y)$ . 59

$L[X] := \{(x, y) \in L \mid x \in X\}$ . 60

$L[, Y] := \{(x, y) \in L \mid y \in Y\}$ . 61

## 5 Properties of equivalence relation 62

**Proposition 1** (Reflexive,symmetry,transitive properties). 63

The relation by isomorphisms of memBers has properties of reflexive, symmetry and transitive. 64 65

*Proof.* 66

- \*1 has been proved in graph theory. 67

- It is trivial that  $(*2 \text{ and } \wedge \dots \text{ and } \wedge *5)$  holds. 68

- Hence this proposition holds. 69

1 The relation by graph isomorphisms has properties of reflexive, symmetry and transitive. 70 71

2 Take  $\forall f_1, f_2, f_3$  as graph isomorphisms such that 72

$\text{domain}(f_2) = \text{image}(f_1) \text{ and } \wedge$  73

$f_3$  is the identity function on  $\text{domain}(f_3)$ . 74

3  $\text{relate-constant-memBer}(f_3) \text{ and } \wedge$  75

4	$\text{relate-constant-memBer}(f_1) \equiv \text{relate-constant-memBer}(f_1^{-1}) \text{ and } \wedge$	76
5	$(\text{relate-constant-memBer}(f_1) \text{ and } \wedge \text{relate-constant-memBer}(f_2)) \equiv$ $\text{relate-constant-memBer}(f_2 \circ f_1)$	77 78
	□	79

## 6 Homeomorphic topological spaces as isomorphic memBers 80 81

**Definition 6.1.** 82

Take  $\forall(m1, m2, c)$  such that ( 83

$c$  is a chain of set membership  $\text{ and } \wedge$  84

$m1$  is the <sup>5</sup>minimum member of  $c$  85

$m2$  is the <sup>6</sup>maximum member of  $c$ . 86

).

Then define  $(*1 \text{ and } \wedge \dots \text{ and } \wedge *5)$ . 87  
88

1  $m1$  is said a deep member of  $m2$ . 89

Hence all memBer is a deep member of itself. 90  
91

2  $|c| - 1$  is said a power of  $(m1, m2)$ . 92

3 It is written as  $m1 \in^{|c|-1} m2$ . 93

4 Let  $p$  be the maximum power of  $(m1, m2)$ . 94

Then  $\text{depth}(m1, m2) := p$ . 95

5 Let  $S := \{d \mid \text{there exists } \exists m \text{ such that } d = \text{depth}(m, m2)\}$ . 96

Then  $\text{depth}(m2) :=$ "the maixmum member of  $S$ ". 97

■ 98

**Definition 6.2** (Space of memBer). 99

Take  $\forall m$ . 100

Then define that 101

$\text{Deep}(m) := \{d \mid d \text{ is a deep member of } m \}$ . 102

$\text{Space}(m) := \{p \in \text{Deep}(m) \mid p \text{ is a point } \}$ . ■ 103

---

<sup>5</sup>No member of  $c$  is a member of  $m1$ .

<sup>6</sup>No member of  $c$  has  $m2$  as a member.

<b>Proposition 2</b> (Isomorphism of vertices).	104
Take $\forall(m1, m2, f, v1)$ such that (	105
$(m1, m2)$ are isomorphic by $f$ and $\wedge v1 \in Deep(m1)$	106
).	107
Then $v1, f(v1)$ are isomorphic by $f[Deep(v1)]$ .	108
<i>Proof.</i>	109
• Let $v2 := f(v1)$ .	110
• As <b>C1</b> , claim that $Deep(v2) \subset image(f[Deep(v1)])$ .	111
• Assume that the claim fails.	112
• There exists $\exists w2 : \in Deep(v2)$	113
as a minimum counterexample to *C1 compared by $depth(w2, v2)$ .	114
• It is trivial that $w2 \neq v2$ .	115
• There exists $\exists x2 : \in Deep(v2)$ such that $w2 \in x2$ .	116
• Hence $x2$ is not a counterexample to *C1	117
because $depth(w2, v2) < depth(x2, v2)$ .	118
• Hence There exists $\exists x1 : \in Deep(v1)$ such that $f(x1) = x2$ .	119
• Hence There exists $\exists w1 : \in x1$ such that	120
$(f(w1) = w2 \text{ and } \wedge w1 \in Deep(v1))$ . A contradiction.	121
• Hence The assumption on $(\neg *C1)$ is false.	122
• As <b>C2</b> , claim that $( Deep(v1) \subset image( f^{-1}[Deep(v2)] ) )$ .	123
• Though it is trivial that the same logic for the proof of *C1 proves *C2.	124
• Hence $Deep(v2) = image(f[Deep(v1)])$ .	125
• Hence $f[Deep(v1)]$ is a graph isomorphism	126
from $*to Deep(v1) * Deep(v2)$ .	127
• And it is trivial that	128
$relate\_constant\_memBer(f) \Rightarrow relate\_constant\_memBer(f[Deep(v1)])$ .	129
□	130

<b>Proposition 3</b> (Isomorphism of Spaces).	131
Take $\forall(m1, m2, f)$ such that $(m1, m2)$ are isomorphic by $f$ .	132
Then $f[Space(m1)]$ is a bijection from $*$ to $Space(m1) * Space(m2)$ .	133
<i>Proof.</i>	134
• Assume it is false.	135
• $image(f[Space(m1)]) \neq Space(m2)$ .	136
• $image(f[Space(m1)]) \not\subseteq Space(m2)$ or $\vee$	137
$image(f[Space(m1)]) \not\supseteq Space(m2)$ .	138
• Then there exists $\exists(m1, m2, f, p1, p2)$ as a counterexample such that	139
$(*A0 \text{ and } \wedge (*A1 \text{ or } \vee *A2))$ holds.	140
<b>A0</b> $(p1, p2) : \in Space(m1) * Space(m2)$ .	141
<b>A1</b> $f(p1) \notin Space(m2)$ .	142
<b>A2</b> $p2 \notin image(f[Space(m1)])$ .	143
• Assume $*A1$ holds.	144
• Then $f(p1)$ is either a constant-member ( or a non-constant-member as	145
a set).	146
• Though $f(p1)$ can not be a constant-member by that relate-constant-	147
member( $f$ ).	148
• Hence $f(p1)$ is a non-constant-member as a set.	149
• Though it contradicts to that $f$ is a graph isomorphism because $f(p1)$ has	150
edge to some its member.	151
• Hence the assumption of $*A1$ is false and $\wedge *A2$ holds.	152
• There exists $\exists c1 : \notin Space(m1)$ such that $f(c1) = p2$ .	153
• Hence $f^{-1}(p2) = c1$	154
• Though this condition has been denied in the disproof of $*A1$ .	155
• Hence the assumption of $*A2$ is false and $\wedge$ the main assumption is false.	156
□	157

<b>Proposition 4</b> (Pair of member's isomorphisms).	158
Take $\forall(I := \{1, 2, 3, 4\}, \{m_i\}_{i \in I}, f_{1,2}, f_{3,4})$	159
such that $(\ast 1 \text{ and } \wedge \dots \text{ and } \wedge \ast 4)$ holds.	160
Then $(\ast 5 \text{ and } \wedge \ast 6)$ holds.	161
<b>1</b> $(m1, m2)$ are isomorphic by $f_{1,2}$ .	162
<b>2</b> $(m3, m4)$ are isomorphic by $f_{3,4}$ .	163
<b>3</b> Let $f := f_{1,2} \cup f_{3,4}$ and let $f_s := f[Space(f)]$ .	164
<b>4</b> Then $f_s$ is a bijection.	165
<b>5</b> $f$ is a function.	166
<b>6</b> $f$ is a bijection.	167
<b>7</b> relate-constant-memBer( $f$ ).	168
	169
<i>Proof of <math>\ast 5</math>.</i>	170
• Let $(V, E)_{i \in \{1,2,3,4\}}$ be the deep graph of $m_i$ .	171
• Assume it is false.	172
• Then there exists $\exists((m1, m3), (m2, m4))$ as a minimum counterexample	173
by $depth((m1, m3))$ such that $f$ is not a function.	174
• Let us make sure that $f$ is a union of a set of bijections.	175
• There exists $\exists v : \in V_1 \cap V_3$ such that $ f[\{v\}]  \geq 1$ and $v \notin \{m1, m3\}$ .	176
• By the way, this proposition accepts the following $args_v$	177
in place of $(m1, m2, m3, m4, f_{1,2}, f_{3,4})$ .	178
• $args_v := ($	179
$v,$	180
$f_{1,2}(v),$	181
$v,$	182
$f_{3,4}(v),$	183
$f_{1,2}[Deep(v)],$	184
$f_{3,4}[Deep(v)]$	185
$).$	186

- In the rest, this  $args_v$  is proved to be a counterexample smaller than a 187  
minimal counterexample. 188
- As the first step, the such-that clause of this proposition holds for  $args_v$  189  
as follows. 190
- Equivalently  $( *1 \text{ and } \wedge \dots \text{ and } \wedge *4 )$  holds for  $args_v$  as follows. 191
- Assume  $*1$  fails for  $args_v$ . 192
- Hence  $(v, f_{1,2}(v))$  is not isomorphic by  $f_{1,2}[Deep(v)]$ . 193
- Though it contradicts to the proposition titled as "Isomorphism of ver- 194  
tices". 195
- Hence the last assumption is false. 196
- Hence  $*1$  holds for  $args_v$ . 197
- Hence  $*2$  holds for  $args_v$  because (for  $args_v$ ,  $*1$  and  $*2$  are logically equiv- 198  
alent). 199
- Assume  $*4$  fails for  $args_v$ . 200
- Let  $f_v := f_{1,2}[Deep(v)] \cup f_{3,4}[Deep(v)]$  and 201  
let  $f_{v,s} := f_{1,2}[Deep(v)][Space(f_v)] \cup f_{3,4}[Deep(v)][Space(f_v)]$ . 202
- Then  $f_{v,s}$  is not a bijection. 203
- Though it is false because  $f_{v,s} \subset f_s$ . Hence  $*4$  holds for  $args_v$ . 204
- Hence  $( *1 \text{ and } \wedge \dots \text{ and } \wedge *4 )$  holds for  $args_v$ . 205
- Moreover  $*5$  fails for  $args_v$  as follows. 206
- Assume  $*5$  holds for  $args_v$ . 207
- Then  $f_v$  is a function. 208
- Though 209  
 $v \in Deep(v)$  and 210  
 $f_v[\{v\}] = f[\{v\}]$  and 211  
 $|f_v[\{v\}]| = |f[\{v\}]| \geq 1$ . 212
- Hence  $*5$  fails for  $args_v$ . 213
- $args_v$  is a counterexample. 214



- And the size as a counterexample of  $args_v$  equals to  $depth((v, v))$ . 215
- Though  $depth((v, v)) < depth((m1, m3))$  <sup>7</sup>because 216  
 $depth((v, v)) = depth(v) + 2 < depth(m1) + 2 \leq depth(m1, m3)$ . 217
- Hence  $arg_v$  is a counterexample smaller than a minimum counterexample. 218
- Hence the main assumption is false. 219

□ 220

*Proof of \*6.* 221

- Consider the proposition  $*P_S$  titled as "Reflexive,symmetry,transitive prop-222  
erties". 223
- Consider the proposition  $*P_I$  titled as "Isomorphism of spaces". 224
- Then  $((*P_S \text{ and } \wedge *P_I) \text{ and } \wedge (*1 \text{ and } \wedge \dots \text{ and } \wedge *4))$  implies 225  
 $(*S1 \text{ and } \wedge \dots \text{ and } \wedge *S4)$ . 226

**S1**  $(m2, m1)$  are isomorphic by  $f_{1,2}^{-1}$  as an isomorphism. 227

**S2**  $(m4, m3)$  are isomorphic by  $f_{3,4}^{-1}$  as an isomorphism. 228

**S3** Let  $f_{-1} := f_{1,2}^{-1} \cup f_{3,4}^{-1}$  and let  $f_{s,-1} := f_{-1}[Space(f_{-1})]$ . 229

**S4** Then  $f_{s,-1}$  is a bijection. 230

- For  $*S4$ , it holds because 231  
(it is trivial that  $(f_{-1} = f^{-1} \text{ and } f_{s,-1} = f_s^{-1})$ . 232
- Moreover  $*5$  implies that  $f_{-1}$  is a function. 233
- Hence  $f^{-1}$  is a function. 234
- Hence  $*5$  implies that  $f$  is an injection. 235
- By the way,  $f$  is surjective because  $f$  is not defined the codomain. 236
- Hence  $f$  is a bijection. 237

□ 238

*Proof of \*7.* 239

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$${}^7(x, y) := \{\{x\}, \{x, y\}\}$$

- Assume it is false. 240
- There exists  $\exists(x, y) : \in f$  such that 241  
(either  $x$  or  $y$  is a constant-member)  $\text{ and } \wedge (x \neq y)$ . 242
- Though  $f = f_{1,2} \cup f_{3,4}$ . 243
- Hence  $(x, y) \in f_{1,2} \text{ or } \vee (x, y) \in f_{3,4}$ . 244
- There exists  $\exists g : \in \{f_{1,2}, f_{3,4}\}$  such that 245  
 $\neg(\text{relate-constant-member}(g))$ . 246
- It contradicts to  $(*1 \text{ and } \wedge *2)$ . 247
- The assumption is false. 248

□ 249

**Definition 6.3** (Constant space). 250

A constant space  $D$  is most likely a function to be used to state conditions on 251  
variables. 252

For example, let  $D$  be a function and let  $x, y, z : \in Z * Z * Z$  such that  $x = D(z)$  253  
and  $y = D(z)$ . 254

Then  $x = y$ . 255

In this case,  $D$  is used to make sure that variables hold equal values. 256

Be careful that all constant space is just a usual variable but a global constant. 257

**Proposition 5** (Isomorphism by member's isomorphisms). 258

Let  $*P_P$  denote the proposition titled as "Pair of member's isomorphisms". 259

Take  $\forall(S1, S2, f, F)$  as sets  $(S1, S2)$  such that  $(*A1 \text{ and } \wedge \dots \text{ and } \wedge *A7)$ . 260

Then  $(*10 \text{ and } \wedge \dots \text{ and } \wedge 12)$  holds. 261

**A1**  $| \text{ Deep}(\{S1, S2\}) | \leq \text{continuum}$ . 262

**A2**  $f$  is a bijection from  $*$ to  $S1 * S2$ . 263

**A3** There exists  $\exists D$  as a function and as a constant space. 264

**A4** Take  $\forall((m1, m2), (m3, m4)) : \in f^2$ . 265

**A5** There exists  $\exists f_{1,2}, f_{3,4}$  266

such that  $f_{1,2} = D((m1, m2)) \text{ and } \wedge f_{3,4} = D((m3, m4))$ . 267

<b>A5</b> Let $args := ($	268
$m1, m2, m3, m4,$	269
$f_{1,2},$	270
$f_{3,4}$	271
$).$	272
Then $*P_P$ accepts $args$	273
in place of $(m1, m2, m3, m4, f_{1,2}, f_{3,4}).$	274
<b>A6</b> $*P_P.( *1 \text{ and } \wedge \dots \text{ and } \wedge *4)$ holds for $args.$	275
<b>A7</b> Let $D_{1,2} := \{D((m1, m2)) \mid (m1, m2) \in f\}.$	276
Then $F = \text{union } D_{1,2}.$	277
<b>C10</b> $F[Space(F)]$ is bijective.	278
<b>C11</b> $F$ is a function.	279
<b>C12</b> $F$ is bijective.	280
<b>C13</b> $\text{relate-constant-memBer}(F).$	281
<b>C14</b> $(S1, S2)$ are isomorphic by $F \cup \{S1, S2\}.$	282
	■ 283
<i>Proof of *C10.</i>	284
• First of all, it is trivial that	285
$\text{domain}(F[Space(F)]) = Space(S1) \text{ and } \wedge$	286
$\text{image}(F[Space(F)]) = Space(S2).$	287
• Assume it is false.	288
• There exists $\exists(p1, p2) : \in Space(S1) * Space(S2)$ such that	289
$ F(p1)  \geq 1 \text{ or } \vee  F^{-1}(p2)  \geq 1.$	290
• Though it implies that the antecedent of this proposition have failed.	291
• Namely, there exists $\exists((m1, m2), (m3, m4))$	292
which has been taken as $\forall((m1, m2), (m3, m4))$ in *A4	293
such that, of *A6, $*P_P.( *4)$ have failed for $((m1, m2), (m3, m4)).$	294
• Hence the assumption is false.	295
	□ 296

*Proof of (\*C11 and  $\wedge$  \*C12 and  $\wedge$  \*C13).* 297

- First of all, consider the proposition titled as "Pair of member's isomorphisms". 298  
299
- The proposition implies that the antecedent of this proposition implies 300  
that \*A6 can be modified as the following \*A6 typed in red. 301
- That is, the original "\*4" has been replaced with "\*7". 302
- **A6**  $P_P.(^*1 \text{ and } \wedge \dots \text{ and } \wedge ^*7)$  holds for *args*. 303
- Call this modified antecedent as the modified antecedent. 304
- By the way, assume (\*C11 and  $\wedge$  \*C12 and  $\wedge$  \*C13) is false. 305
- (\*B1 or  $\vee$  \*B2) holds. 306
- **B1** There exists  $\exists(x1, x2) : \in S1 * S2$  such that 307  
 $|F(x1)| \geq 1 \text{ or } \vee |F^{-1}(x2)| \geq 1$ . 308
- **B2** There exists  $\exists f_{1,2} : \in D_{1,2}$  such that 309  
 $\neg \text{relate-constant-memBer}(f_{1,2})$ . 310
- Though it implies that the modified antecedent have failed. 311
- Namely, there exists  $\exists((m1, m2), (m3, m4))$  312  
which has been taken as  $\forall((m1, m2), (m3, m4))$  in \*A4 313  
such that, of \*A6, 314  
 $P_P.(^*5 \text{ and } \wedge ^*6 \text{ and } \wedge ^*7)$  have failed for  $((m1, m2), (m3, m4))$ . 315
- Hence the assumption is false. 316

□ 317

*Proof of \*C14.* 318

- Assume it is false. 319
- Let  $F_+ := F \cup \{S1, S2\}$ , Then (\*B1 or  $\vee$  \*B2) holds. 320
- As **B1**,  $(S1, S2)$  are not graph-isomorphic by  $F_+$ . 321
- As **B2**,  $\neg \text{relate-constant-memBer}(F_+)$ . 322
- Assume \*B2 holds. 323
- Hence  $\neg \text{relate-constant-memBer}(\{S1, S2\})$ . 324

- Hence there exists  $\exists(T1, T2) : \in \{(S1, S2), (S2, S1)\}$  such that 325  
 $T1$  is a constant-member  $\text{and} \wedge T2$  is not a constant-member. 326
- There exists  $\exists(c_1, p_2) : \in F$  such that 327  
 $(c_1 \text{ is a constant-member } \text{and} \wedge p_2)$  is not a point. 328  
By this contradiction, the assumption on \*B2 is false. 329
- Hence \*B1 holds. 330
- There exists  $\exists(v1, v2) : \in S1 * S2$  such that 331  
 $F(v1) \notin S2 \text{ or } \vee F^{-1}(v2) \notin S1$ . 332
- Though there exists  $\exists f_{1,2} : \in D_{1,2}$  such that ( 333  
 $(v1, F(v1)) \in f_{1,2} \text{ and} \wedge$  334  
 $f_{1,2}$  is a bijection from \*to Deep( $v1$ )\*Deep( $F(v1)$ ) 335  
 $).$  336
- Moreover  $F \supset f_{1,2}$ . 337
- Hence the assumption on \*B1 is false. 338
- The main assumption is false. 339

□ 340

**Definition 6.4** (Variations of Indexed set). 341

As you know, for example,  $\{x_i\}_{i \in \{1,2\}} := \{x_1, x_2\}$ , in mathematics. 342

In this article, 343

analogously,  $(x_i)_{i \in \{1,2\}} := (x_1, x_2)$ . 344

As an alternative simplified form,  $(x)_{i \in \{1,2\}} := (x_1, x_2)$ . 345

As one of many variations,  $(\{x\})_{i \in \{1,2\}} := (\{x_1\}, \{x_2\})$ . 346

As a comment, the order on the composed sequence should respect the most 347

natural order on the index set. 348

**Proposition 6** (Isomorphisms by spaces). 349

Take  $\forall(S)_{i \in \{1,2\}}, \forall(f, g)$  such that ( 350

$(S)_{i \in \{1,2\}}$  are isomorphic by  $f$  and also by  $g \text{ and} \wedge$  351

$f[\text{Space}(f)] = g[\text{Space}(g)]$  352

$).$  353

Then  $f = g$ . 354

*Proof.* 355

- Assume it is false. 356

- There exists  $\exists v_1 : \in \text{Deep}(S1)$  as a minimum counterexample 357  
 compared by  $\text{depth}(v_1)$  such that 358  
 $f(v_1) \neq g(v_1)$ . 359
- It is trivial that  $\text{depth}(v_1) > 0$ . 360
- Hence  $v_1$  is a set. 361
- $f[v_1] = g[v_1]$  because ( 362  
 take  $\forall w_1 : \in v_1$ , 363  
 then  $(\text{depth}(w_1) < \text{depth}(v_1) \text{ and } w_1 \text{ is not a counterexample})$  364  
 ). 365
- Hence  $f(v_1) = \text{image}(f[v_1]) = \text{image}(g[v_1]) = g(v_1)$ . 366
- The assumption is false. 367

□ 368

**Definition 6.5** (Isomorphism by spaces). 369

Take  $\forall (S)_{i \in \{1,2\}}, \forall (f, F)$  such that 370

$(S)_{i \in \{1,2\}}$  are isomorphic by  $F$  and  $\text{Space}(F) \subset f \subset F$ . 371

Then  $(S)_{i \in \{1,2\}}$  are also said isomorphic by  $f$ . ■ 372

**Proposition 7** (Homeomorphism as isomorphism). 373

As you know, the set theory defines that 374

$(x, y) := \{\{x\}, \{x, y\}\}$ . 375

Take  $\forall ((X, T))_{i \in \{1,2\}}, \forall H$  such that ( 376

$((X, T))_{i \in \{1,2\}}$  is a pair of topological spaces and 377

$H$  is a bijection from  $X_1$  to  $X_2$  and 378

$((X, T))_{i \in \{1,2\}}$  are homeomorphic by  $H$  379

). 380

Then  $(1 \text{ and } \dots \text{ and } 5)$  holds. 381

1.  $(X)_{i \in \{1,2\}}$  are isomorphic by  $H$ . 382

2. Take  $\forall (t_1, t_2) : \in T1 * T2$  such that  $t_2 = \text{image}(H[t_1])$ . 383

Then  $(t)_{i \in \{1,2\}}$  are isomorphic by  $H[t_1]$ . 384

3.  $(T)_{i \in \{1,2\}}$  are isomorphic by  $H$ . 385

4.  $(\{X\})_{i \in \{1,2\}}$  are isomorphic by  $H$ . 386

5.  $(\{X, T\})_{i \in \{1,2\}}$  are isomorphic by  $H$ . 387

6. $(\{\{X\}, \{X, T\}\})_{i \in \{1,2\}}$ are isomorphic by $H$ .	388
	■ 389
<i>Proof of *1.</i>	390
• Consider the proposition titled as "Isomorphism by member's isomorphisms".	391 392
• $(X)_{i \in \{1,2\}}$ are isomorphic by $H \cup \{(X1, X2)\}$ .	393
	□ 394
<i>Proof of *2.</i>	395
• Consider the proposition titled as "Isomorphism by member's isomorphisms".	396 397
• $(t)_{i \in \{1,2\}}$ are isomorphic by $H[t_1] \cup \{(t1, t2)\}$ .	398
	□ 399
<i>Proof of *3.</i>	400
• Consider the proposition titled as "Isomorphism by member's isomorphisms".	401 402
• Consider *2.	403
• Let $t_{1,2} := \{(t_1, t_2) \in T1 * T2 \mid t_2 = \text{image}(H[t_1])\}$ .	404
• $(T)_{i \in \{1,2\}}$ are isomorphic by $H \cup t_{1,2} \cup \{(T1, T2)\}$ .	405
	□ 406
<i>Proof of *4.</i>	407
• Consider the proposition titled as "Isomorphism by member's isomorphisms".	408 409
• Consider *1.	410
• $(\{X\})_{i \in \{1,2\}}$ are isomorphic by $H \cup \{(X1, X2), (\{X1\}, \{X2\})\}$ .	411
	□ 412
<i>Proof of *5.</i>	413

- Consider the proposition titled as "Isomorphism by member's isomorphisms". 414  
415
  - Consider \*1 and \*3. 416
  - $(\{X, T\})_{i \in \{1, 2\}}$  are isomorphic 417  
by  $H \cup \{(X1, X2), (T1, T2), (\{X1, T1\}, \{X2, T2\})\}$ . 418
- 419

*Proof of \*6.* 420

- Consider the proposition titled as "Isomorphism by member's isomorphisms". 421  
422
  - Consider \*4 and \*5. 423
  - $(\{\{X\}, \{X, T\}\})_{i \in \{1, 2\}}$  are isomorphic 424
  - by  $H \cup \{$  425  
 $(X1, X2),$  426  
 $(T1, T2),$  427  
 $(\{X1, T1\}, \{X2, T2\}),$  428  
 $(\{\{X1\}, \{X1, T1\}\}, \{\{X2\}, \{X2, T2\}\})$  429  
 $\}$ . 430
- 431