Gene theory on Prime topological spaces

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December 14, 2019

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1 Introduction

Although this article is not about knot theory, at first I give a conjecture in words of elementary knot theory. Then I, step by step, depict how the conjecture leads readers to a new frame work fundamental to topology and mathematics.

A notable thing is that this article somehow can be compared to biology, namely genomics whereas classical mathematics is somehow classical biology. Classical mathematics researches defined objects whereas this article mainly researches genes of objects, i.e., definitions of objects.

Currently, **all proofs are abstract** because almost all formal proofs of the theory are expected to require **computer-assisted proofs**; all definitions of objects must be input into computer programs.

Having said that, it is still true that the new theory is contributing sets of new fundamental notions to mathematics; an isomorphism between two sets as a generalization of homeomorphisms, a prime topological space, a prime set of sub spaces, a prime function, factorization of a non redundant logical expression; these are pairwise well related.

1.1 Notations

Blue texts indicate the words are new for readers; the words will be formally defined soon later. For example, prime topological space.

"$x \ {}_{and}\wedge \ y$" is almost equivalent to "$x \wedge y$" except that it is not promised to be commutative.

"$x \ {}_{or}\vee \ y$" is almost equivalent to "$x \vee y$" except that it is not promised to be commutative.

1.2 Conjecture

Conjecture 1.1. The following claim has no [1]disproof.

Take $\forall k_u$ as an [2]unknot.

$K := \{k \mid (k_u, k)$ are of a same ambient isotopy class$\}$.

$K_f := \{k \in K \mid f(k_u) = f(k)\ \}$.

Take $\forall (k_0, k_1) :\in K_f^2$.

There exists $\exists F$ as an ambient isotopy on $R^3 * [0, 1]$ such that as follows.

$F[1]$ [3]takes K_0 to K_1.

Take $\forall x :\in [0, 1]$, $\forall k_x$ such that $F[x]$ takes k_0 to k_x. Then $k_x \in K_f$.

Sub definitions with K as the domain:

- $j1(k) := \{j \mid$
  j is an orthogonal [4]projection of k onto some infinite plane $\}$.
- $j2(k) := \{j \in j1(k) \mid$
  $\neg\ (\exists p\ \ _{and}\wedge\ p \in \text{image}(j)\ \ _{and}\wedge\ \mid\ j^{-1}(p)\ \mid > 2)\ \}$.
- $j3(k) := \{n \mid$
  $\exists j\ \ _{and}\wedge\ j \in j2(k)\ \ _{and}\wedge\ n$ is the number of [5]double points on $j\ \}$.
- $f(k) := \{m \mid$
  m is the maximal number from $j3(k)\ \}$.

2 Subtext and definition

Take $\forall d$ as a logical expression. To study d, ideally d must be minimum by the text length. Though it is difficult to prove such a condition.

Meanwhile all d with some words removed is said a **subtext** of d. More formally, d must be expressed as a tree of logical operations. For example, $((x \wedge y) \vee ((\neg z) \vee (v \wedge w)))$ where variables also represent trees of logical operations. And removing a word corresponds to changing one term of a binary operation

[1] Probably it has no proof too.
[2] It can be any knot class.
[3] In other words, F takes K_0 to K_1.
[4] Hence, j is a function from k to an infinite plane.
[5] That is, the inverse image of a double point has exactly 2 distinct points of k; no matter the double point is a crossing or a tangent point.

to a unit term. For example, removing x from $(x \wedge y)$ results y through $(\top \wedge y)$; removing y from $(x \vee y)$ results x through $(x \vee \bot)$.

The original d is said **non redundant** if all its subtext is not equivalent to d.

Especially, if d is a definition then d is said **non redundant** if all its subtext does not define any equivalent entity as d.

In the rest, logical expressions are expected to be non redundant fundamentally, still practically preserving human readability.

3 Prime topological space

I claim that the conjecture is special because Euclidean spaces are prime topological spaces.

Definition 3.1 (Prime topological space)**.** Take $\forall X$ as a topological space. Then X is said **prime** if: $*1 \ {}_{and}\wedge \ *2$.

1. Let d be the definition of X. Take $\forall Y$ such that the definition of Y is a sub definition of d. If X is homeomorphic to some sub space of Y then (X, Y) are homeomorphic.

2. Take $\forall y$ as a non empty open set of X. Then y is not a countable set.

Proposition 1 (Prime topological space)**.** Take $\forall(n, R^n)$ such that $(n \geq 1 \ {}_{and}\wedge \ R^n$ is a Euclidean space of n-dimension). Then R^n is a prime topological space.

Proof. Refer to the definition of prime topological space. As **Assumption1**, this proposition fails. Hence $(*1 \ {}_{and}\wedge \ *2)$ of the definition fails with R^n in place of X. Though it holds as follows. Hence Assumption1 is false.

Consider $*1$ with R^n in place of X. It simply holds because we can find some definition d of R^n in accepted research papers of topology theory such that $*1$ holds for (d, R^n) in place of (d, X). In our favor, d is not referenced elsewhere in the definition.

Consider $*2$ with R^n in place of X. It simply holds as an accepted fact of elementary topology theory. If some non empty open set $\exists y$ of R^n is countable, then the well known formula fails. Namely, take $\forall p_1 :\in y$, then $\exists e :> 0$ such that: Take $\forall p_2 :\in \text{Space}(X)$, then $(\text{distance}(p_1, p_2) < e) \to (p_2 \in y)$. The formula fails because all open ball $\forall b$ with e as the radius is not countable in terms of Space(b).

"Space(X)", that is, the set of all points of X. □

4 Preliminary definitions

4.1 Isomorphism of memBers

Take $\forall m$. Then m is said a **memBer** if m is a member of some set.

Take $\forall \{c_i\}_{i \in [1,n] \subset \mathbb{N}}$ as a chain such that $c_i \in c_{i+1}$ if the indices are in the index set. Take $\forall i :\in [1,n]$, then c_i is said a **deep member** of c_n denoted as $c_i \in^{deep} c_n$. [6]Be careful.

Take $\forall m$ as a memBer, then Space(m) denotes the set of all points (p as each) such that $p \in^{deep} m$.

And m is said a **constant-memBer** if Space$(m) = \varnothing$; especially m is said an **empty constant-memBer** if all deep member of m is a set. Importantly, all number is a constant-memBer.

The **deep graph** of m is defined as the directed graph (V, E) on the set V of all deep members of m such that $E = \{(v1, v2) \in V * V \mid v1 \in v2\}$.

Ultimately, two memBers are said **isomorphic by** f if: (Their deep graphs are isomorphic by $\exists f$ as a graph isomorphism $_{and}\wedge$ relate-constant-memBer(f)).

That is, take $\forall L$ as a binary relation, then it is written as **relate-constant-memBer**(L) if: Take $\forall (x, y) :\in L$ such that either x or y is a constant-memBer, then $x = y$.

Although not necessary for this article, the following link proves that all homeomorphism of topological spaces is an isomorphism of memBers.

Link to the proof: All homeomorphism is an isomorphism of memBers.

4.2 Specifiable

Definition 4.1 (Specifiable relatively to)**.** Take $\forall (x, y)$ such that the definition of y entirely depends on x, then y is said **specifiable relatively to** x if it holds as follows.

Let d_x be the definition of x; let d_{y1} be the definition of y excluding d_x; let d_{y2} be a copy of d_{y1}. For d_x+d_{y1}+d_{y2} as a concatenated text, y of d_{y1} and y of d_{y2} are identical. ■

[6]There possibly exist multiple chains of set membership between (c_1, c_n).

For example, let x be a Euclidean space of 1-dimension; take $\forall y :\in \mathrm{Space}(x)$. For this case, d_x+d_{y1}+d_{y2} is that: let x be a Euclidean space of 1-dimension; take $\forall y :\in$ Space(x); $take \forall y :\in \mathrm{Space}(x)$. For this case, needless to say, y of d_{y1} and y of d_{y2} are not said identical; for some case they are identical and for some case they are not. So this y is not specifiable relatively to x.

For example, let x be a Euclidean space of 1-dimension; take $\forall y$ such that $y := \{(z1, z2) \mid (z1, z2) \in \mathrm{Space}(x)^2\ \}$. For this case, d_x+d_{y1}+d_{y2} is that: let x be a Euclidean space of 1-dimension; take $\forall y$ such that $y := \{(z1, z2) \mid (z1, z2) \in \mathrm{Space}(x)^2\ \}$; take $\forall y$ such that $y := \{(z1, z2) \mid (z1, z2) \in \mathrm{Space}(x)^2\ \}$. For this case, needless to say, y of d_{y1} and y of d_{y2} are identical. So this y is specifiable relatively to x.

4.3 Factor proposition

Definition 4.2 (Factor proposition)**.** Take $\forall p$ as a predicate written in a form of a finite sequence of **non redundant logical conjunctions**. Then all non empty sub sequence of p is said a **factor proposition** of p. ■

For example:

Let $p(y) :\equiv (y = \{1, 2\} \wedge y \in 2^{\mathbb{N}}\ )$.

Then p has no factor proposition because it is redundant.

For example:

Let $p(y) :\equiv (y = \{1, 2\})$.

Let $q(y) :\equiv (y \ni 1 \wedge y \ni 2 \wedge |y| = 2)$.

It is trivial that $p(y) \equiv q(y)$.

For example, $(y \ni 1 \wedge y \ni 2)$ is a factor proposition of $q(y)$.

In a broader sense, $(y \ni 1 \wedge y \ni 2)$ is a factor proposition of $p(y)$.

Other examples of factor propositions of $(y = \{1, 2\})$:

$(y \ni 1 \wedge y \ni 2 \wedge |y| = 2)$

$(y \ni 1 \quad \cancel{\wedge y \ni 2 \wedge |y| = 2} \quad)$.

$(\quad \cancel{y \ni 1 \wedge y \ni 2 \wedge} \quad |y| = 2)$.

$(\quad \cancel{y \ni 1 \wedge} \quad y \ni 2 \quad \cancel{\wedge |y| = 2} \quad)$.

$(y \ni 1 \wedge \quad \cancel{y \ni 2 \wedge} \quad |y| = 2)$.

4.4 Prime function

Definition 4.3 (Prime function)**.** Take $\forall f$ as a function. Then f is said a **prime function** if: (*1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *3).

1. The image of f is a set of non empty constant-memBers.

2. Take $\forall g$ such that (*s1 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *s6)).

3. Then (*s7 ${}_{and}\wedge$ ..... ${}_{and}\wedge$ *s9) holds.

s1. The definition of g is a sub definition of the definition of f.

s2. g is specifiable relatively to f.

s3. $\exists(x1, x2, x3) :\in \text{domain}(g)^3$.

s4. $x1$ is a non empty constant-memBer.

s5. $x2 \neq x3$.

s6. $g(x2) = g(x3)$.

s7. Let $p1(x) :\equiv (x = x2)$.

s8. Let $p2(x) :\equiv (g(x) = g(x2))$.

s9. $p2$ is a factor proposition of $p1$.

For example, let $f(x :\in \mathbb{R}) := \text{if } (x < 0 \ {}_{or}\vee\ x > 1) \text{ then } \top \text{ else } \bot$. Then *1 of the definition holds for f in place of f. Though this f is not a prime function. Because, for f, *2 holds whereas *3 dose not hold. Let us more formally define f as follows.

$g1(x :\in \mathbb{R}) := (x, (\text{if } x < 0))$.
$g2((x, b) :\in \text{image}(g1)) := (b \vee (\text{if } x > 1))$.
$f(x :\in \mathbb{R}) := g2 \circ g1(x)$.
That is, $f(x) = \{g2(z) \mid z \in g1(x)\ \}$.

For the new definition, *2 holds for $g2$ in place of g; for example, g2(-1,⊤)=g2(2,⊥). Meanwhile *3 fails for $g2$.

5 Prime set of sub spaces

I claim that the conjecture is special because all sets of sub spaces defined in the conjecture are prime sets of sub spaces of a prime topological space. Namely (K, K_f).

Definition 5.1 (Prime set of sub spaces)**.** Take $\forall X$ as a prime topological space. Let K_0 :=(the set of all sub spaces of X). Take $\forall f$ as a prime function on K_0 such that f is specifiable relatively to X.

Take $\forall K$ such that: $K \subset \text{domain}(f)$ $\;_{and}\wedge$ $\exists x1 :\in \text{domain}(f)$ $\;_{and}\wedge$ take $\forall x :\in \text{domain}(f)$ $\;_{and}\wedge$ $(x \in K) \equiv (f(x) = f(x1))$. Then K is said a **prime set of sub spaces of X.**

In addition, take $\forall[1, n] \subset \mathbb{N}_1$, take$\forall\{K_i\}_{i\in[1,n]}$ as a set of prime sets of sub spaces of X. Then $\bigcap_{i=1}^{n} K_i$ is also said a **prime set of sub spaces of X.** ■

Proposition 2. Refer to the conjecture. (K, K_f) of the conjecture are prime sets of sub spaces of R^3.

Proof for K. Refer to the conjecture. Let Def(prime set) denote the definition titled as "prime set of sub spaces".

- X:=R^3.
- K_0 :=(the set of all sub spaces of X).

K cam be redefined as follows.

- S_F :=(the set of all ambient isotopy on $X * [0, 1]$).
- $S_F := \{F[1] \mid F \in S_F\ \}$.
- S_F :=take $\forall h$ as a bijection from*to $\mathbb{R} * S_F$.
- $g1(k :\in K_0) := \{k\} * S_F$.
- $g2((k, r, F) :\in \bigcup \text{image}(g1)) :=$ (r,[7]if (F takes k_u to k)).
- $g3((r, b) :\in \text{image}(g2)) := b$.
- $g4(k :\in K_0) := (\ g3 \circ g2\ ) \overset{\in}{\circ}\ g1(k)$.

[8] Footnote on the new symbol.

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[7]That is, $\top$ or $\perp$.

[8]That is, (y=fun2 $\overset{\in}{\circ}$ fun1(x)) $\equiv$ ($y = \{\text{fun2}(z) \mid \exists z \in \text{fun1}(x)\ \}$).

- $f4(k :\in K_0) :=$ if $(g4(k) = g4(k_u))$.

  [9]Footnote.

- $K := \{k \in K_0 \mid \top = f4(k)\ \}$.

The antecedent of Def(prime set) holds for $(R^3, K_0, f4, K)$ in place of (X, K_0, f, K) as follows.

Requirement for X: To be a prime topological space.

In the previous proposition, R^n is said to be so.

Requirement for K_0: K_0 :=(the set of all sub spaces of X).

K_0 is defined to be so.

Requirement for f: f is a prime function on K_0 such that f is specifiable relatively to X.

In the definition of $f4$, it is clear that: $f4$ is a function on K_0 and is specifiable relatively to X,although some sub functions are not specifiable relatively to X.

$f4$ is a prime function as follows. Let Def(prime function) denote the definition titled as "Prime function".

*1 of Def(prime function) holds for $f4$. Namely image$(f4) = \{\top, \bot\}$.

Let us search for functions defined **either explicitly or implicitly** in the definition of $f4$ such that *2 of Def(prime function) holds with it in place of g. Be careful that *s2 of *2 of Def(prime function) says g must be specifiable relatively to X.

As you can see in the definition of $f4$, there exists no such sub definition.

Requirement for K: $K \subset$ domain(f) $_{and}\wedge$ $\exists x1 :\in$ domain(f) $_{and}\wedge$ take $\forall x :\in$ domain(f) $_{and}\wedge$ $(x \in K) \equiv (f(x) = f(x1))$.

Recall k_u defined in the conjecture. Needless to say, $f4(k_u) = \top$.

In fact, $K \subset$ domain$(f4)$ $_{and}\wedge$ $\exists k_u :\in$ domain$(f4)$ $_{and}\wedge$ take $\forall x :\in$ domain$(f4)$ $_{and}\wedge$ $(x \in K) \equiv (f4(x) = f4(k_u))$.

$\square$

Proof for K_f. Refer to the conjecture. Let Def(prime set) denote the definition titled as "prime set of sub spaces". K_f can be redefined as follows.

[9] $g4(k_u) = \{\top, \bot\}$.

- $j1(k :\in K_0) := \{j \mid$
  j is an orthogonal [10]projection of k onto some infinite plane $\exists P$ $\}$.

- $j2(j :\in \bigcup \text{image}(j1)) := \{\text{j}\}{*}\text{image}(j)$.

- $j3((j, p) :\in \bigcup \text{image}(j2)) := \{x \in \text{domain}(j) \mid j(x) = p\ \}$.

- $j4 := (\ \text{lambda}(x : |x|) \circ \text{collect}(S\text{: } |S| = 2) \circ j3\ \overset{\in}{\circ}\ j2\ )\ \overset{\in}{\circ}\ j1$.

- That is, $\text{lambda}(y : w(y)) \circ t(x) := w(t(x))$.

- That is, $\text{collect}(y : w(y)) \circ t(x) := \{y :\in t(x) \mid w(y) \equiv \top\ \}$.

- $f4 := \text{lambda}(S\text{: the maximum number from } S) \circ j4$.

- $K_f := \{k \in K_0 \mid f4(k) = f4(k_u)\ \}$.

The antecedent of Def(prime set) holds for $(R^3, K_0, f4, K)$ in place of (X, K_0, f, K) as follows.

Requirement for X: Omitted.

Requirement for K_0: Omitted.

Requirement for f: f is a prime function on K_0 such that f is specifiable relatively to X.

In the definition of $f4$, it is clear that: $f4$ is a function on K_0 and is specifiable relatively to X.

$f4$ is a prime function as follows. Let Def(prime function) denote the definition titled as "Prime function".

*1 of Def(prime function) holds for $f4$. Namely image$(f4) \subset \mathbb{N}$.

Let us search for functions defined **either explicitly or implicitly** in the definition of $f4$ such that *2 of Def(prime function) holds with it in place of g. Be careful that *s2 of *2 of Def(prime function) says g must be specifiable relatively to X.

As you can see in the definition of $f4$, there exists exactly one such sub definition. Namely the latter term of the composition of $f4$; lambda(S: the maximum number from S). For example, for two distinct inputs,({9,10},{8,9,10}), of the [11]lambda function, the outputs are equal.

[10]Hence, j is a function from k to an infinite plane.

[11]I do not claim that it is a lambda function in the formal sense.

Refer to *s7 to *s9 of *3 of Def(prime function).
Call the lambda function as $j5$.
Let $p1(x) :\equiv (x = x2)$.
Let $p2(x) :\equiv (j5(x) = j5(x2))$.
Then $p2$ is a factor proposition of $p1$.
Hence *3 holds for $j5$ in place of g.

Requirement for K: $K \subset \mathrm{domain}(f) \ {}_{and}\wedge\ \exists x1 :\in \mathrm{domain}(f) \ {}_{and}\wedge$ take $\forall x :\in \mathrm{domain}(f) \ {}_{and}\wedge\ (\ x \in K\ ) \equiv (\ f(x) = f(x1)\ )$.

Recall k_u defined in the conjecture.

In fact, $K \subset \mathrm{domain}(f4) \ {}_{and}\wedge\ \exists k_u :\in \mathrm{domain}(f4) \ {}_{and}\wedge$ take $\forall x :\in \mathrm{domain}(f4) \ {}_{and}\wedge\ (\ x \in K\ ) \equiv (\ f4(x) = f4(k_u)\ )$.

$\square$

6 To project symmetry

I claim that the conjecture is special because we now can replace the words "ambient isotopy" with the new words "to project symmetry" so that the new conjecture implies the first conjecture. And the new notion is far more fundamental than the notion of ambient isotopy.

Definition 6.1 (To project symmetry)**.** Take $\forall(m1, m2)$ as memBers. Then $m2$ is said to **project the symmetry** to $m1$ if *1 implies *2.

1 Take $\forall(d1, d2, d3)$ as deep members of $m1$ such that $((m2, d1, d3), (m2, d2, d3))$ are isomorphic).

2 $((m2, m1, d1, d3), (m2, m1, d2, d3))$ are isomorphic.

Now the first conjecture can be generalized as follows.

Conjecture 6.1.
Take $\forall(X, K)$ such that K is a prime set of sub spaces of a prime topological space X. Then X projects the symmetry to K.

Proposition 3. Conjecture 6.1 implies Conjecture 1.1.

Proof. Assume this proposition fails. Hence Conjecture 1.1 fails with an assumption that Conjecture 6.1 holds. First of all, by a previous proposition, the assumption of Conjecture 6.1 holds for (R^3,K_f) in place of (X, K). Let X denote R^3 of Conjecture 1.1. There exists a counterexample of Conjecture 1.1. Namely, $\exists(k1, k2) :\in K_f^2$ such that $(*1 \ {}_{and}\wedge \ *2)$.

1. Take $\forall F$ as an ambient isotopy on X*[0,1] such that $F[1]$ takes $k1$ to $k2$.
Then for some time point $\exists t :\in [0, 1]$, $F[t]$ takes $k1$ to $\exists k :\notin K_f$.

2. $(k1, k2)$ are disjoint.

Meanwhile there exists $\exists k3 :\in K_f$ such that $(*3 \ {}_{and}\wedge \ *4)$.

3. $*1$ fails for $(k1, k3)$ in place of $(k1, k2)$.

4. $(k1, k2, k3)$ are pairwise disjoint.

Recall K of Conjecture 1.1. That $(k1, k2, k3) \in K^3$ implies that $(X, k2)$ and $(X, k3)$ are isomorphic. Hence $(X, k2, k1)$ and $(X, k3, k1)$ are also isomorphic.

Conjecture 6.1 says that X projects the symmetry to K_f. Hence $(X, K_f, k2, k1)$ and $(X, K_f, k3, k1)$ are isomorphic. It contradicts to $(*1 \ {}_{and}\wedge \ \ldots\ldots \ {}_{and}\wedge \ *4)$. Hence the main assumption is false. □