Construction of an Ensemble of Logical Correctors on the Basis of Elementary Classifiers

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Abstract—A problem of constructing correct recognition algorithms on the basis of incorrect elementary classifiers is considered. A model of recognition procedures based on the construction of a family of logical correctors is proposed and analyzed. To this end, a genetic approach is applied that allows one, first, to reduce the computational cost and, second, to construct correctors with high recognition ability. This model is tested on real problems.

Keywords: logical data analysis, pattern recognition, covering of a Boolean matrix.

DOI: 10.1134/S1054661811040055

INTRODUCTION

The logical analysis of data in pattern recognition is based on distinguishing informative fragments in feature descriptions of objects—the collections of admissible values of features that allow one to distinguish between objects from different classes. These collections play a role of elementary classifiers. As a rule, the correctness of a recognition algorithm (an ability to correctly classify objects from a training sample) is guaranteed by the correctness of each of the elementary classifiers used [1–3]. For example, Kora-type algorithms use representative collections of classes as correct elementary classifiers. A representative collection ω of class K is a fragment of description of a learning object from K that is not contained in the descriptions of learning objects from other classes. The representative collection ω votes for the membership of a recognized object S in the class K, provided that the description of the object S contains ω.

In [4], the authors considered the construction of correct recognizing procedures based on arbitrary sets of admissible values of features, i.e., on elementary classifiers that are not necessarily correct. A (monotonic) Boolean function was proposed as a correcting function. It was shown that each correct class K corresponds to a covering of a Boolean matrix L_K that was specially constructed by a training sample and had very large dimensions even in the simplest case. The model of a recognition algorithm proposed in [4] is based on constructing the least complicated corrector of class K, i.e., a minimal covering of the matrix L_K. The problem of covering is one of the most computationally complex problems; therefore, we analyzed the possibility of reducing the enumeration that arises when constructing a minimal corrector.

In the present study, we analyze the problems of the practical application of the model proposed in [4]. To reduce its computational complexity, we apply a genetic approach. As the fitness function in the genetic algorithm, we use the cardinality of a corrector.

In addition, on the basis of a genetic approach, we synthesize an ensemble of correctors in which each corrector has good recognition ability. In this case, the fitness function is given by an estimate for the recognition ability of a corrector.

In each case, the classification of new objects is made on the basis of voting by the correctors constructed.

We tested the model of voting by logical correctors on real medical problems from the UCI repository [8] and from the internal repository of the Recognition system [9] and compared this model with the recognition algorithms from the Recognition system.

MODEL OF VOTING BY LOGICAL CORRECTORS

We consider a precedence recognition problem in the standard statement [1]. We analyze a set M of objects, of which it is known that it can be represented as a union of disjoint subsets (classes) $K_1, \ldots, K_l$. The objects of the set M are described by a collection of integer-valued features $x_1, \ldots, x_n$, each of which has a finite number of admissible values. As the source information, a set T of objects from M is given of which it is known to which classes they belong (the training

Received November 15, 2010

sample). Given a collection of values of the features $x_1, ..., x_n$, that describes a certain object S from M, it is required to find a class to which the object S belongs.

Let $H = \{x_{j_1}, ..., x_{j_r}\}$ be a collection of r different features, and let $\sigma = (\sigma_1, ..., \sigma_r)$, where σ_i is an admissible value of the feature x_{j_i}. The pair (H, σ) is called an elementary classifier. An elementary classifier (H, σ) generates a predicate $P_{(H,\sigma)}(S)$ that is defined on objects $S \in M$, $S = (a_1, ..., a_n)$, and is such that

$$P_{(H,\sigma)}(S) = \begin{cases} 1, & \text{if } a_{j_i} = \sigma_i, \quad i = 1, 2, \dots, r, \\ 0 & \text{otherwise.} \end{cases}$$

A collection $U = \{(H_1, \sigma_1), ..., (H_q, \sigma_q)\}$ of elementary classifiers is called a (monotonic) correct collection for a class $K \in \{K_1, ..., K_l\}$ if there exists a (monotonic) function F_K of logic algebra, that depends on q variables and is such that

$$F_K(P_{(H_1,\sigma_1)}(S), P_{(H_2,\sigma_2)}(S), \dots, P_{(H_q,\sigma_q)}) = \begin{cases} 1, & \text{if } S \in K \cap T, \\ 0, & \text{if } S \in \bar{K} \cap T, \end{cases}$$

(henceforth, $\bar{K} = \{K_1, ..., K_l\} \backslash \{K\}$).

The function F_K is called a (monotonic) corrector for the class K. Denote the binary collection $(P_{(H_1,\sigma_1)}(S), P_{(H_2,\sigma_2)}(S), ..., P_{(H_q,\sigma_q)}(S))$ by $\omega_U(S)$.

Let $a' = (a'_1, a'_2, ..., a'_n)$ and $a'' = (a''_1, a''_2, ..., a''_n)$ be binary collections. The expression $a' \geq a''$ denotes that $a'_i \geq a''_i$ for $i = 1, 2, ..., n$. Let $S', S'' \in M$ and U be a correct collection of elementary classifiers. When U is a monotonic correct collection of elementary classifiers, we set

$$\delta(S', S'', U) = \begin{cases} 1, & \text{if } \omega_U(S') \geq \omega_U(S''), \\ 0, & \text{otherwise.} \end{cases}$$

When U is not necessarily a monotonic correct collection of elementary classifiers, we set

$$\delta(S', S'', U) = \begin{cases} 1, & \text{if } \omega_U(S') = \omega_U(S''), \\ 0, & \text{otherwise.} \end{cases}$$

Let $W_K = \{U_1, U_2, ..., U_t\}$ be a set of (monotonic) correct collections of elementary classifiers of the class K. Then a class estimate for the recognized object S is given by

$$\Gamma(S, K) = \frac{1}{|T \cap K|} \sum_{U \in W_K} \sum_{S' \in T \cap K} \delta(S, S', U).$$

The object S belongs to that class for which this estimate is greater.

The following proposition is obvious.

Proposition 1. *The collection $U = \{(H_1, \sigma_1), (H_2, \sigma_2), ..., (H_q, \sigma_q)\}$ of elementary classifiers is monotonic and correct for the class K if and only if, for any two objects S' and S'' from the training sample such that $S' \in K$ and $S'' \notin K$, there exists an $i = \{1, 2, ..., q\}$ such that*

$$P_{(H_i,\sigma_i)}(S') = 1 \text{ and } P_{(H_i,\sigma_i)}(S'') = 0. \qquad (1)$$

The monotonicity condition in the above proposition can be removed if we replace condition (1) by

$$P_{(H_i,\sigma_i)}(S') \neq P_{(H_i,\sigma_i)}(S'').$$

Let U be a correct collection of elementary classifiers for the class K. The collection U is said to be dead-end if the condition $U' \subset U$ implies that the collection U' of elementary classifiers is not correct for K. The collection U is set to be minimal if there does not exist a correct collection of elementary classifiers for K of lower cardinality.

Let L be an arbitrary Boolean matrix. A collection H of columns of the matrix L is called a covering if the intersection of each row of the matrix L with at least one column entering H yields 1. A covering is said to be dead-end if none of its proper subsets is a covering. In addition, suppose given a weight vector $c = (c_1, ..., c_n)$ of columns of the matrix L. The weight of a covering is the sum of weights of its elements. A covering with the minimum weight is called a minimal covering. Note that, for a unit weight vector, the minimal covering is a covering containing the minimum number of columns.

A learning object $S' = (a'_1, ..., a'_n)$ is said to generate an elementary classifier (H, σ), $H = \{x_{j_1}, ..., x_{j_r}\}$, $\sigma = (\sigma_1, ..., \sigma_r)$, if $\sigma_i = a_{j_i}$ for $i = 1, 2, ..., r$. Consider the set $U_K = \{(H_1, \sigma_1), (H_2, \sigma_2), ..., (H_{N_K}, \sigma_{N_K})\}$ of all elementary classifiers generated by learning objects from the class K. The difference between the number of learning objects from K that generate an elementary classifier from U_K and the number of learning objects from $\bar{K}$ that also generate this elementary classifier is called the weight of this elementary classifier in the class K.

Let us assign to the pair of objects S' and S'' a row $B(S', S'') = (b_1, ..., b_{N_K})$ such that

$$b_j = \begin{cases} 1, & \text{if } P_{(H_j,\sigma_j)}(S') = 1 \text{ and } P_{(H_j,\sigma_j)}(S'') = 0, \\ 0, & \text{otherwise,} \end{cases}$$

$j = 1, 2, ..., N_K$. For the class K, we compose a Boolean matrix L_K consisting of all rows $B(S', S'')$ such that $S' \in K$ and $S'' \notin K$.

By construction, each column in L_K corresponds to a certain elementary classifier from the collection U_K. Suppose that a collection H of columns of the matrix L_K

corresponds to the collection U_H of elementary classifiers. One can easily verify the validity of Propositions 2 and 3 below.

Proposition 2. *The collection U_H of elementary classifiers is monotonic and correct for the class K if and only if H is a covering of the matrix L_K.*

Proposition 3. *The collection U_H of elementary classifiers is dead-end (minimal) and correct for the class K if and only if H is a dead-end (minimal) covering of the matrix L_K.*

When the correct collections of elementary classifiers are not necessarily monotonic, the set $U'_K = \{(H'_1, \sigma'_1), (H'_2, \sigma'_2), ..., (H'_N, \sigma'_N)\}$ of all elementary classifiers is formed by elementary classifiers generated by all learning objects. In this case, we construct a Boolean matrix L'_K consisting of rows $D(S', S'') = (d_1, d_2, ..., d_N)$ such that

$$d_j = \begin{cases} 1, & \text{if } P_{(H'_j, \sigma'_j)}(S') \neq P_{(H'_j, \sigma'_j)}(S''), \\ 0, & \text{otherwise}, \end{cases}$$

$j = 1, 2, ..., N$ for all pairs of objects $S' \in K$ and $S'' \notin K$.

To the columns of the matrices L_K and L'_K, we assign a weight equal to the weight of the elementary classifier that corresponds to this column.

Remark 1. A (monotonic) corrector for the class K can be constructed by using arbitrary recognition algorithms as the basic algorithms. Let $A_1, ..., A_q$ be an arbitrary collection of recognition algorithms. An algorithm A_i, $i = 1, ..., q$, generates a predicate $P_{A_i}(S)$ that is defined on objects $S \in M$ and is equal to 1 if the algorithm A_i assigns the object S to the class K and is equal to 0 otherwise. In this case, the concept of a (monotonic) correct collection of recognition algorithms is formed by analogy with a (monotonic) correct collection of elementary classifiers.

Remark 2. Proposition 1 implies that if $U = \{(H_1, \sigma_1), ..., (H_q, \sigma_q)\}$ is a correct collection of elementary classifiers, then the collection of features $H_1 \cup H_2 \cup ... \cup H_q$ is a test for the sample T.

In [4], a model of recognition procedures is proposed that is based on constructing a minimal covering of matrices L_K and L'_K. However, the number of elementary classifiers is large even for problems of small dimension. Therefore, the construction of a minimal correct collection of elementary classifiers with the use of the matrices L_K and L'_K requires considerable computational resources, which leads to the necessity of developing efficient methods for solving problems of algebra–logic correction of elementary classifiers.

In this paper, we consider a practically important and the least complicated case when an elementary classifier is constructed by a pair (x_j, a), where $j \in \{1, 2, ..., n\}$ and a is one of admissible values of a feature x_j. In [5, 6], Sotnezov applied a genetic algorithm to constructing a collection of elementary classifiers whose complexity is close to that of the minimal collection. He showed that a recognition algorithm based on voting by such a correct collection of elementary classifiers in most cases cannot guarantee an acceptable quality of recognition. Therefore, we propose a recognition algorithm that uses an ensemble of dead-end correct collections of elementary classifiers that, first, have high recognition ability and, second, require low computational power for their construction. To synthesize such an ensemble, we apply a genetic algorithm that is described in the following section.

CONSTRUCTION OF AN ENSEMBLE OF LOGICAL CORRECTORS ON THE BASIS OF A GENETIC APPROACH

The training sample is divided into two subsamples: the basic T_0 and a tuning T_1 subsample, by analogy with the method proposed in [7] for constructing a voting algorithm by representative collections of elementary classifiers. The sample T_0 is used for constructing matrices L_K or L'_K, while the sample T_1, for estimating the recognition quality of correct collections of elementary classifiers. The quality of recognition $\tau_K(U)$ by a correct collection U of elementary classifiers for the class K is estimated by

$$\frac{1}{|T_1 \cap K|} \sum_{S \in T_0 \cap K} \sum_{S' \in T_1 \cap \bar{K}} \delta(S, S', U).$$

Thus, an estimate for the quality of recognition of the correct collection U of elementary classifiers is given be the difference between the number of objects from class K correctly recognized by the collection U and the number of objects from other classes that are also assigned by the correct collection U of elementary classifiers to the class K.

To construct a set of correct collections U of elementary classifiers, we apply a genetic approach. The operation of a genetic algorithm resembles the development of a biological population in which each object is characterized by a collection of genes. The renewal of such a population with time occurs according to the law “the most adapted survives.” Moreover, new objects can be obtained by the operators of crossing and mutation, which combine the genes of the parents in a special way.

Let $L = (a_{ij})_{m \times n}$ be a Boolean matrix constructed by the sample T_0 and $c = (c_1, ..., c_n)$ be the weight vector of its columns. A covering of the matrix L will be represented as an integer vector of length m in which the ith component has the value of the number of a column that covers the ith row.

Using the greedy heuristics from [5, 6], we form an initial family of solutions $P = (Q_1, Q_2, ..., Q_N)$, which is called a population. The elements of the set P are called individuals.

For an individual Q_j, $j = 1, 2, ..., N$, from the population P, we define a fitness function $f(Q_j)$ that characterizes the quality of the solution obtained. An individual Q_j describes a covering of the matrix L, that, by construction, corresponds to a certain correct collection of elementary classifiers U_j. As the fitness function $f(Q_j)$, we use the quantity

$$\tau_K(U_j) - \min_{i \in 1, 2, ..., N} \tau_K(U_i) + 1.$$

Thus, the greater the fitness function of an individual, the greater the estimate for the recognition ability of the correct collection of elementary classifiers corresponding to this individual.

Each individual Q_j from population P is assigned a probability p_j, which is calculated by the formula

$$p_j = \frac{f(Q_j)}{\sum_{i=1}^{N} f(Q_i)}.$$

At the next step of the genetic algorithm, by the collection of probabilities $p_1, ..., p_N$, we determine two parent individuals $Q' = (q'_1, ..., q'_m)$ and $Q'' = (q''_1, ..., q''_m)$, from which, by the operation of crossing, we determine a descendant $Q = (q_1, ..., q_m)$ by the following rule.

Let f_1 and f_2 be the fitness degrees of the individuals Q' and Q'', respectively. Then

$$q_i = \begin{cases} q'_i, & \text{with probability } \dfrac{c_{q'_i} f_1}{c_{q'_i} f_1 + c_{q''_i} f_2}, \\ q''_i, & \text{with probability } \dfrac{c_{q''_i} f_2}{c_{q'_i} f_1 + c_{q''_i} f_2}, \end{cases} \quad (2)$$

$i = 1, 2, ..., m$.

In contrast to the most popular one-point and two-point crossovers, the crossing operator (2) takes into consideration the structures of the parent individuals and their fitness functions. The greater the fitness function of a parent individual, the higher the probability that its gene is copied into the gene of its descendant.

The application of the crossing operator alone to renew the population may give rise to individuals with roughly identical sets of columns. This means that the algorithm converges at a certain local minimum in the neighborhood of which all the new descendants are situated. To overcome local minima, we apply the operator of mutation, which randomly changes (mutates) a given number of genes in the description of the descendant. Since the effect of the mutation operator on an individual is especially strong when the search for an optimal solution is convergent, we propose that the number of mutated genes $k(t)$ should increase with the number of algorithm steps as

$$k(t) = K\left\{1 - \frac{1}{Ct + 1}\right\},$$

where t is the number of an algorithm step and K and C are variable parameters that characterize the number of mutated genes at the last step of the algorithm and the rate of variation of the number of mutated genes, respectively.

Suppose that the application of the crossing and mutation operators yields an integer vector Q corresponding to the covering H of the matrix L. If H is not an irreducible covering, then we apply the procedure of restoring the admissibility of the individual Q, which consists in the following. Let M_j be a set of rows of the matrix L that are covered by column j. For each column $j \in H$, we analyze the set of rows M_j covered by this column in order of decreasing weights. If each row from this set is covered by at least one column from the collection $H \backslash \{j\}$, then the column j is removed from the collection H.

The procedure of restoring the admissibility of the individual Q yields an individual Q^* that corresponds to an irreducible covering of the matrix L. The individual substitutes for one of the individuals from the population P, provided that there is no individual in P that describes the same covering of the matrix L as Q^* and that the set $R_{Q^*} = \{Q' \in P | f(Q') \le f(Q^*)\}$ of individuals of the population P is nonempty. For the substitution, we randomly choose an individual from the set R_{Q^*}. The first condition is needed to prevent the rise of identical individuals and, as a consequence, to prevent the biological degeneration of the population P. The second condition implies that the fittest individuals, i.e., the individuals with the highest recognition ability, fall into the population.

The genetic algorithm terminates when the population is renewed N_{max} times, i.e., when it results in N_{max} individuals that are more fitted than those in the original population. The number N_{max} is the input parameter of the genetic algorithm and is specified in advance.

RESULTS OF TESTING ON REAL PROBLEMS

To test the model of voting by a family of logical correctors, we developed four algorithms.

1. **Algorithm A1.** A family consists of N monotonic correct collections of elementary classifiers with high

Table 1. Characteristics of test problems

Problem	Number of features	Number of objects in the first class	Number of objects in the first class	Presence of blanks
A	24	51	237	Yes
B	19	51	218	Yes
C	35	38	107	Yes
D	9	626	332	No
E	16	168	265	No

recognition ability. To construct the family, we applied the genetic algorithm described in the previous section.

2. **Algorithm A2.** A family consists of N correct collections of elementary classifiers with high recognition ability that are not necessarily monotonic. To construct the family, we applied the genetic algorithm described in the previous section.

3. **Algorithm A3.** A family consists of a single monotonic correct collection of elementary classifiers whose cardinality is close to that of the minimal collection. To construct the family, we applied the genetic algorithm described in [5, 6].

4. **Algorithm A4.** A family consists of a single monotonic correct collection of elementary classifiers that is not necessarily monotonic and whose cardinality is close to that of the minimal collection. To construct the family, we applied the genetic algorithm described in [5, 6].

We tested algorithms A1 and A2 with $N = 200$ and algorithms A3 and A4 on real problems taken from the UCI repository [8] and from the internal repository of the Recognition system [9]. The characteristics of the problems are given in Table 1. We compared algorithms A1–A4 with the algorithms implemented in the Recognition system for the total percentage R_1 of recognition and for the percentage R_2 of recognition weighted with respect to classes, which were calculated as follows.

Let $\bar{T}$ be a test sample of objects and r_i and n_i be the number of correctly recognized objects and the total

Table 2. Results of prediction

Algorithm	Problem A	Problem B	Problem C	Problem D	Problem E
A1	84.91% (71.4, 88.2) 79.83%	85.56% (83.3, 86.1) 84.71%	85.45% (75.0, 91.4) 83.21%	99.2% (100, 97.9) 98.94%	96.22% (97.4, 95.34) 96.39%
A2	86.79% (38.1, 98.8) 68.46%	72.72% (40.0, 91.4) 65.71%	67.01% (88.9, 62.0) 75.46%	97.66% (98.8, 95.7) 97.26%	95.13% (96.1, 94.4) 95.27%
A3	67.20% (16.0, 80.0) 48.0%	63.64% (30.0, 82.8) 56.43%	54.64% (22.2, 62.0) 42.12%	98.96% (100, 97.2) 98.58%	90.81% (94.8, 87.9) 91.38%
A4	61.60% (16.0, 73.0) 44.50%	43.63% (15.0, 60.0) 37.50%	44.33% (16.7, 50.6) 33.65%	87.53% (90.6, 82.3) 86.42%	76.22% (88.1, 67.6) 77.95%
Error evaluation algorithm	82% (0.0, 100) 50%	81.4% (0.0, 100) 50.0%	72.7% (50.0, 85.7) 67.85%	72.7% (94.7, 34.8) 64.75%	88.6% (85.9, 90.7) 88.3%
Binary decision trees	82% (0, 100) 50.0%	81.4% (0, 100) 50.0%	80.0% (45.0, 100.0) 72.5%	63.4% (100, 0) 50.0%	90.8% (96.9, 87.6) 92.25%
Logical regularities of classes	76.0% (33.3, 85.4) 59.35%	75.3% (50, 81) 65.5%	54.5% (20.0, 74.3) 47.15%	99.5% (100, 98.6) 99.3%	93.0% (90.0, 95.2) 92.6%
Support vectors method	90% (66.7, 95.1) 80.9%	89.7% (50.0, 98.7) 74.35%	80.0% (50.0, 97.1) 73.55%	63.4% (100, 0.0) 50.0%	92.4% (89.9, 94.3) 92.1%
Stochastic test algorithm	88% (66.7, 92.7) 79.7%	87.6% (72.2, 91.1) 81.65%	76.4% (40, 97.1) 68.55%	73.5% (77.5, 66.7) 72.1%	95.1% (92.5, 97.1) 94.8%

number of recognized objects in the set $K_i \cap \overline{T}$, $i = 1, 2, \ldots, l$, respectively. Then

$$R_1 = \frac{\sum_{i=1}^{l} r_i}{\sum_{i=1}^{l} n_i}, \quad R_2 = \frac{1}{l}\sum_{i=1}^{l}\frac{r_i}{n_i}.$$

For each algorithm and each problem, Table 2 presents the total percentage of recognition R_1 (the first row in each cell), the percentage of recognition for each of the classes (the second row in each cell), and the weighted percentage of recognition R_2 (the third row in each cell).

In problems with low-valued data (problems D and E), the best results are obtained with algorithms A1 and "Logical regularities of classes." In these problems, algorithms A2 and A3 also show good results, which, however, are inferior to algorithm A1 in the parameters R_1 and R_2.

Problems A, B, and C are characterized by high-valued features, the presence of blanks in data, and a large difference in the number of learning objects between classes. Therefore, it is more reasonable to compare the algorithms with respect to the parameter R_2 to estimate their recognition quality. Algorithm A1 showed the best results with respect to the parameter R_2 in problems B and C and was just slightly inferior to the support vectors method on problem A. An advantage of algorithm A1 is that it considers each class separately; this allows one to obtain nearly the same quality of recognition for each class.

Algorithms A3 and A4 are inferior to A1 and A2 and to the algorithms from the Recognition system almost on all the problems. Thus, the algorithm for voting by a single correct collection of elementary classifiers that is close to the minimal collection in its complexity is not efficient.

CONCLUSIONS

We have analyzed and tested an algorithm for voting by a family of correct collections of elementary classifiers. We have shown that a family consisting of a single correct collection that is close to the minimal collection in its complexity cannot guarantee an acceptable quality of recognition. On the basis of a genetic approach, we have developed an algorithm for the synthesis of an ensemble of correct collections of elementary classifiers with high recognition ability that is comparable and, in some cases, is even superior to the classical recognition algorithms on real problems.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project no. 10-01-00770) and by a grant of the President of the Russian Federation for supporting leading scientific schools (project no. NSh-7950.2010.1).

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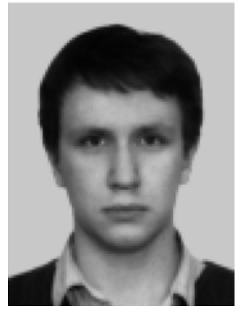

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