bgphy, a package for Bayesian analysis of mixed Gaussian phylogenetic models.

Bayu Brahmantio (bayubrahmantio@gmail.com)

The package at a glance

This package performs Bayesian inference of a class of phylogenetic comparative methods (PCM) which assumes Gaussian processes, specifically Brownian Motion (BM) or Ornstein-Uhlenbeck process (OU), as a model of univariate traits evolution on a phylogenetic tree. Moreover, it performs inferences of mixed Gaussian phylogenetic models (MGPM). In this setting, a phylogenetic tree can be partitioned into multiple different "evolutionary regimes" which consist of different processes along with different parameters.

The package uses PCMBase (Mitov et al. 2020) as the engine for high-speed likelihood calculations. A user-friendly interface for customizing priors and regime configurations is also provided. The Bayesian inference is done using importance sampling algorithm to compute statistics of the posterior distribution, which utilizes R's native parallel package to accelerate inference time.

Parameters notation

The BM and OU processes can be written by the following stochastic differential equations (SDEs):

- BM: $dx(t) = \sigma dW(t)$,
- OU: $dx(t) = -\alpha(x(t) \theta)dt + \sigma dW(t)$.

where x(t) is the trait at time t, W(t) is a Wiener process at time t. The parameter α can be interpreted as the strength of random fluctuation, while α is the strength of selection (or convergence rate) into an optimum value θ . Notice that, BM is a specific case of an OU process when $\alpha=0$. The ancestral state is also considered as a parameter, which is denoted by X_0 .

Given measurements on the tips \mathbf{x}_{tips} and a model \mathcal{M} with a certain regimes configurations and parameters Θ , the goal is to calculate the posterior distribution

$$P(\Theta|\mathbf{x}_{\text{tips}}) \propto P(\mathbf{x}_{\text{tips}}|\Theta)P(\Theta),$$

and compare different configurations, $\mathcal{M}_1(\Theta_1)$ vs $\mathcal{M}_2(\Theta_2)$ vs \cdots vs $\mathcal{M}_m(\Theta_m)$.

Installation

devtools::install_github("bayubeta/bgphy")

Getting started

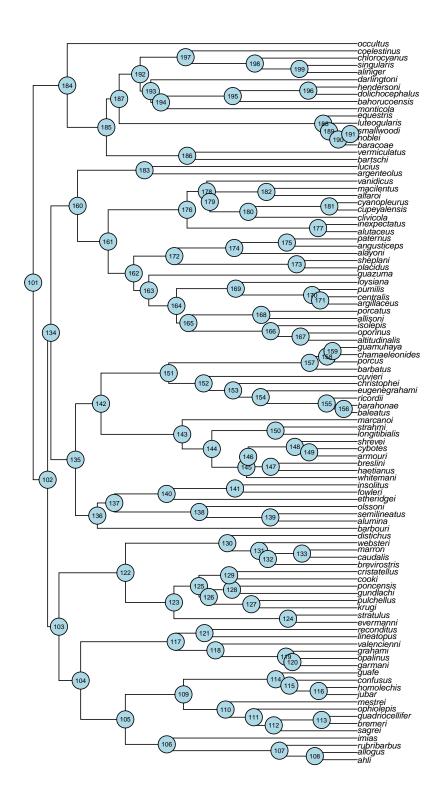
Loading the package:

```
library(bgphy)
library(ape)
```

Constructing a model using setModel()

Given a phylogenetic tree of class phylo and we can construct a model by using setModel() function by specifying the model types ("BM" or "OU") and their position in the tree. For example, we use the phylogenetic tree of Caribbean anoles (Mahler et al. 2013; retrieved from Bastide and Didier 2023). The tree is available within the package as lizardTree. Below is the tree plotted with its node numbers:

```
plot(lizardTree, no.margin = TRUE, cex = 0.7)
nodelabels(cex = 0.5, frame = "circle")
```



Global model

Assume that the traits are evolving under the same process throughout all parts of the tree (BM or OU). To construct such model we can run:

```
# Brownian Motion (BM)
BM <- setModel(tree = lizardTree, regime_names = "R1", modeltypes = "BM")
# Ornstein-Uhlenbeck (OU)
OU <- setModel(tree = lizardTree, regime_names = "R1", modeltypes = "OU")</pre>
```

This will return an object of class bgphy_model which contains the information of the model in PCMBase, the tree, and the default priors for the parameters of the model. The information of the models can be printed:

```
print(BM)
Regime
          Model
R1
          BM(sigma)
Priors:
X0 \sim normal(mean = 0, sd = 10)
sigma ~ halft(nu = 1, sigma = 3)
  print(OU)
Regime
          Model
R1
          OU(alpha, theta, sigma)
Priors:
X0 \sim normal(mean = 0, sd = 10)
alpha ~ halfnormal(sigma = 6.931472)
theta ~ normal(mean = 0, sd = 10)
sigma ~ halft(nu = 1, sigma = 3)
```

Mixed model

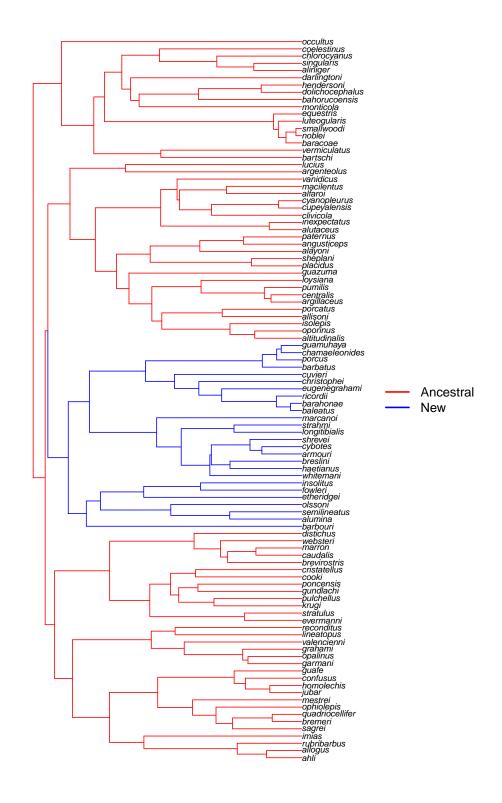
Suppose we are interested in splitting the tree into two regimes:

- Ancestral: starts from the ancestral node (101).
- New: starts from node 135.

Furthermore, the ancestral regime is assumed to follow a BM process while the New regime follows an OU process. The model is constructed as:

```
BMOU <- setModel(tree = lizardTree, regime_names = c("Ancestral", "New"),
                    modeltypes = c("BM", "OU"),
                    startNodes = list(Ancestral = "101", New = "135"))
  print(BMOU)
Regime
            Model
            ____
            BM(sigma_1)
Ancestral
            OU(alpha_2, theta_2, sigma_2)
New
Priors:
X0 \sim normal(mean = 0, sd = 10)
sigma_1 ~ halft(nu = 1, sigma = 3)
alpha_2 ~ halfnormal(sigma = 6.931472)
theta_2 ~ normal(mean = 0, sd = 10)
sigma_2 ~ halft(nu = 1, sigma = 3)
Using the function plot_regimes(), the regimes can be visualized on top of the tree:
```

```
plot_regimes(BMOU$tree, no.margin = TRUE, x.lim = c(0,1.6), cex = 0.7)
```



A more complicated configuration

Let's say that there are three regimes:

- R1: starts from the ancestral node (101), but includes the taxa ahli, lucius, and darlingtoni. Follows a BM process.
- R2: starts from node 104 and 160, follows an OU process.
- R3: starts from node 184, follows an OU process.

The model can be constructed as follows:

```
model3 <- setModel(tree = lizardTree, regime_names = c("R1", "R2", "R3"),</pre>
                   modeltypes = c("BM", "OU", "OU"),
                    startNodes = list(R1 = c("101", "ahli", "lucius", "darlingtoni"),
                                      R2 = c("104", "160"),
                                      R3 = c("184"))
print(model3)
```

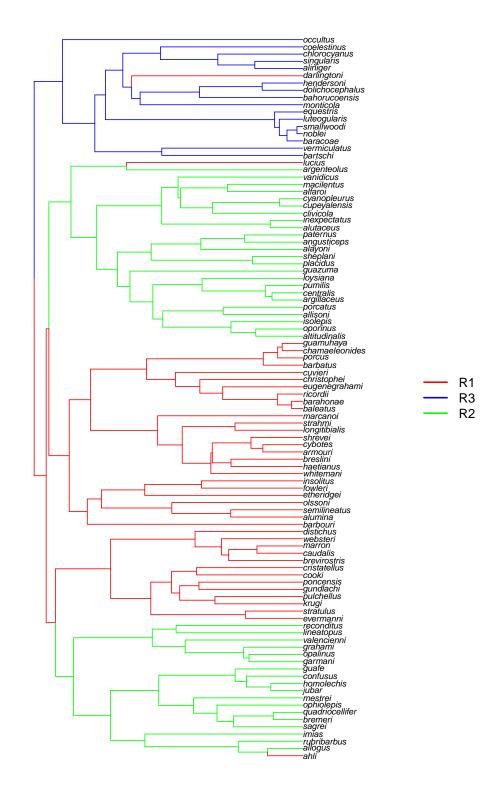
```
Regime
          BM(sigma 1)
R1
R2
          OU(alpha_2, theta_2, sigma_2)
R3
          OU(alpha_3, theta_3, sigma_3)
Priors:
XO \sim normal(mean = 0, sd = 10)
sigma_1 ~ halft(nu = 1, sigma = 3)
```

Model

```
alpha_2 ~ halfnormal(sigma = 6.931472)
theta_2 ~ normal(mean = 0, sd = 10)
sigma_2 ~ halft(nu = 1, sigma = 3)
alpha_3 ~ halfnormal(sigma = 6.931472)
theta_3 ~ normal(mean = 0, sd = 10)
sigma_3 ~ halft(nu = 1, sigma = 3)
```

And the regimes configuration can be shown by:

```
plot_regimes(model3$tree, no.margin = TRUE, x.lim = c(0,1.6), cex = 0.7)
```



Specifying the priors

By default, the priors for each parameters are given as follows:

- $X_0 \sim \text{Normal}(\mu = 0, \sigma^2 = 100),$
- $\alpha \sim \text{Half-Normal}(\sigma = \log(2)/(2 \cdot 0.05 \cdot \ell_T)),$
- $\theta \sim \text{Normal}(\mu = 0, \sigma^2 = 100),$
- $\sigma \sim \text{Half-t}(\nu = 1, \sigma = 3\ell_T),$

in which ℓ_T is the height of the tree.

We can customize the priors by changing the elements of the model\$priors by priorpdf objects provided by bgphy:

```
?bgphy::priorpdf
```

For example, we want to modify priors of the regime "New" in the BMOU model:

```
BMOU$priors$alpha_2 <- prior_halfnormal(sigma = 2)
BMOU$priors$theta_2 <- prior_normal(mean = 2, sd = 5)
BMOU$priors$sigma_2 <- prior_halft(nu = 1, sigma = 2)
print(BMOU$priors)

X0 ~ normal(mean = 0, sd = 10)
sigma_1 ~ halft(nu = 1, sigma = 3)
alpha_2 ~ halfnormal(sigma = 2)
theta_2 ~ normal(mean = 2, sd = 5)
sigma_2 ~ halft(nu = 1, sigma = 2)</pre>
```

Running the posterior inference

The main function of the package is the bgphy() function, which takes arguments:

- model: an object of class bgphy_model, constructed by using setModel().
- X: a data matrix of dimension $1 \times N$, where N is the number of tips of the tree. The column names of X must match the tip labels, that can be accessed by model\$tree\$tip.label.
- nsample: number of particles for the importance sampling method (the default is 10000).

- scale: scaling factor used for the estimated covariance matrix obtained from the Laplace approximation, defaults to 1.
- parallel: a Boolean variable determining the use of paralleled computing for the algorithm. By default is set to TRUE.

For this example, let us use simulated dataset XOU that is available in the package. This dataset contains 100 realizations of simulation under a certain OU process on the same tree that we currently use (Caribbean anoles). Since the data matrix needs to have column names that match the names on the tips of the tree, and since XOU consists of 100 realizations of the simulation, we can pick only one row for our example, and turn it into a format that the bgphy() accepts:

```
X <- matrix(XOU[1,], nrow = 1, dimnames = list(NULL, colnames(XOU)))</pre>
```

Let us run a Bayesian inference of the model specified by BMOU and using the data X, and using 100000 particles. The processing time of the algorithm is also recorded.

```
set.seed(1234, kind = "L'Ecuyer-CMRG")
start <- Sys.time()
post_BMOU <- bgphy(model = BMOU, X = X, nsample = 100000)
end <- Sys.time()</pre>
```

The posterior information can be seen by:

```
print(post_BMOU)
```

```
meanstd_errorstd2.5%50%97.5%X01.69698900.0372407250.203425971.26027331.72748332.0338315sigma_10.66076420.0005920780.047790010.57017360.65935620.7710253alpha_21.65391890.2452085961.032105450.57928171.46145794.1067720theta_21.46797450.0854524570.303331320.85512991.44941121.8781583sigma_21.21357160.0203990470.238760430.84379961.12516951.8217511
```

WAIC: 135.55799157259

Where mean denotes the posterior mean, as seen from each parameter, with std_error determines the standard error of the mean. The column std shows the standard deviation of the posterior distribution, while the last three columns describe the 2.5-th, 50-th (median), and 97.5-th percentile of the posterior distribution. WAIC is the widely applicable information criterion score (a.k.a. Watanabe-Akaike information score), where a lower value indicates a better fit to the data.

It takes around

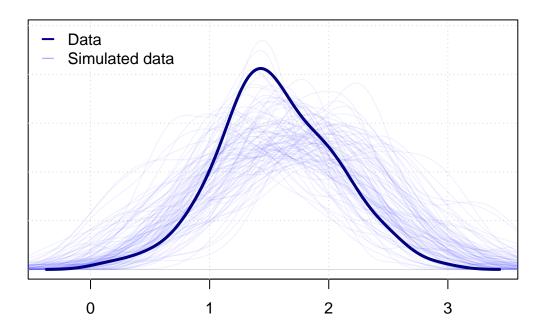
```
end - start
```

Time difference of 3.118615 mins

on a personal computer with 6-cores AMD Ryzen 5 PRO 4650U with Radeon Graphics 2.10 GHz processor, and 16 GB of RAM.

We can also see how the posterior distribution of our model fits the original data by simulating data from the posterior predictive distribution using post_pred_check():

```
post_pred_check(post_BMOU)
```



As we can see, the simulated data do not look very similar to the original data, since we assumed a mixed BM to OU model while the original data was simulated from a global OU model.

Let us now do the same procedure, but with an OU model defined in OU:

```
set.seed(1234, kind = "L'Ecuyer-CMRG")
post_OU <- bgphy(model = OU, X = X, nsample = 100000)</pre>
```

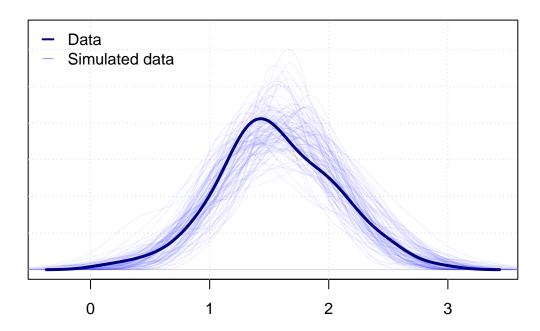
print(post_OU)

```
2.5%
                                                          50%
                                                                  97.5%
             mean
                    std_error
                                    std
XΟ
      -0.04072598 0.250516609 6.7173087 -13.8042433 0.118926 14.338151
alpha 2.68626668 0.033208425 0.6548028
                                           1.4919360 2.597807
                                                               4.209463
      1.72393757 0.022916946 0.5517664
theta
                                           0.5638046 1.688757
                                                               2.771920
sigma
      1.11785342 0.004589569 0.1323854
                                           0.8948859 1.106145
                                                               1.418570
```

WAIC: 123.546521423371

As we can see, the WAIC score is lower (better) than the previous mixed-model. The simulated data curves from the posterior predictive distribution also resemble the original data curve better, as shown on the following plot:

post_pred_check(post_OU)



References

- Bastide, Paul, and Gilles Didier. 2023. "The Cauchy Process on Phylogenies: A Tractable Model for Pulsed Evolution." Systematic Biology, August, syad053. https://doi.org/10.1093/sysbio/syad053.
- Mahler, D Luke, Travis Ingram, Liam J Revell, and Jonathan B Losos. 2013. "Exceptional Convergence on the Macroevolutionary Landscape in Island Lizard Radiations." *Science* 341 (6143): 292–95.
- Mitov, Venelin, Krzysztof Bartoszek, Georgios Asimomitis, and Tanja Stadler. 2020. "Fast Likelihood Calculation for Multivariate Gaussian Phylogenetic Models with Shifts." *Theoretical Population Biology* 131: 66–78.