Comparing Hamiltonian Monte Carlo and Elliptical Slice Sampling for constrained Gaussian distributions

732A76 Research Project Report

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1 Background

High-dimensional multivariate gaussian distribution is used in various models and applications. In some cases, we need to generate from a certain distribution which applies constraints to a multivariate Gaussian distribution (Gelfand et al. [1992] and Rodríguez-Yam et al. [2004]). Sampling from this distribution is still a challenging issue, particularly because it is not straightforward to compute the normalizing constant for the density function.

The gibbs sampler has proven to be a suitable choices to sample from truncated multivariate Gaussian distributions (Gelfand et al. [1992]). Recently, more sophisticated methods have been developed to generate samples from truncated multivariate Gaussian distributions. In this research project, two methods, namely Exact Hamiltonian Monte Carlo (Pakman and Paninski [2013]) and Analytic Elliptical Slice Sampling (Fagan et al. [2016]), will be compared.

2 Definitions

2.1 Truncated Multivariate Gaussian Distribution

The truncated multivariate Gaussian distribution is a probability distribution obtained from a multivariate gaussian random variable by bounding it under some linear (or quadratic) constraints.

Let $\mathbf{w}$ be a d-dimensional Gaussian random variable with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The corresponding truncated multivariate Gaussian distribution can be defined as

$$p(\mathbf{x}) = \frac{\exp\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\intercal \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\}}{\int_{\mathbf{F}\mathbf{x}+\mathbf{g}\geq 0} \exp\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\intercal \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\} d\mathbf{x}} \mathbb{1}(\mathbf{F}\mathbf{x}+\mathbf{g}\geq 0) \tag{2.1}$$

where $\mathbf{x}$ is a d-dimensional truncated Gaussian random variable, $\mathbb{1}$ is an indicator function, and $\mathbf{F}$ is an $m \times d$ matrix, which, together with the $m \times 1$ vector of $\mathbf{g}$, defines all m constraints of $p(\mathbf{x})$. We denote this as $\mathbf{x} \sim TN(\boldsymbol{\mu}, \boldsymbol{\Sigma}; \mathbf{F}, \mathbf{g})$. We can rewrite $p(\mathbf{x})$ as

$$p(\mathbf{x}) = \frac{1}{Z} \exp\left\{ -\frac{1}{2}\mathbf{x}^\mathsf{T}\boldsymbol{\Lambda}\mathbf{x} + \boldsymbol{\nu}^\mathsf{T}\mathbf{x}\right\} \mathbb{1}(\mathbf{F}\mathbf{x} + \mathbf{g} \geq 0) \tag{2.2}$$

References

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