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№1

a) $\cancel{O(n^4(n-i))=}$

$\cancel{T_1(n)\cdot T_2(n) = O(f(n)\cdot g(n))}$

~~$T_1(n)$ = n iterations~~ The number of iterations in the nested loop:

$\cancel{T_2(n)\ n,(n-1),(n-2)(n-3),\dots,1}$ $\underset{a_1}{n},(n-1),(n-2),\dots,\underset{a_n}{1}$

$$S = \frac{a_1+a_n}{2}\cdot n = \frac{n+1}{2}\cdot n = \frac{n(n+1)}{2}$$

$$\cancel{T_1(n)\cdot T_2(n) = O(f(n)\cdot g(n)) =}$$

$$= O\left(\frac{n(n+1)}{2}\right) = \cancel{O\left(\frac{n^2(n+1)}{2}\right)} = \cancel{O\left(\frac{n^3+n^2}{2}\right)} =$$

$$\cancel{= O(n^3+n^2)} = O\left(\frac{n^2+n}{2}\right) = O(n^2+n)$$

We have to consider the worst scenario:

$O(n^2)$

b) $O\left((\cancel{n-1})\cdot((n-1)+0)+\frac{n}{10}\right) = O\left(n\log(n) - \log n + \frac{n}{3}\right)$ ($\log n$)

The complexity for the worst-case scenario:

$\cancel{O(n\log(n))}$ $Max(O(n\log(n) - \log(n) + n)) =$

$= O(n\log(n))$

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~~2~~

~~b)~~

c) We have 3 scenarios:

1. $O(a+0+b) = O(a+b)$

0 - since $c<a$ is always false (from the condition if $a<b$ and $b<c \Rightarrow a<c$)

2. $O(b-c)$

3. $O(a+5-a) = O(5)$

Worst-case scenario is the 1st one ($O(a+b)$).

~~$O(a+b)$~~ we have 2 working conditions with complexity a and another one with complexity b. for loop ($O(a)$) works as well as else $(O(b))$ condition. So, the result complexity is: $O(a+b)$

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~ 2

a) $T(1) = 1$

$$T(n) = 100\,T\left(\frac{n}{10}\right) + n^2$$

1) Insertion.

$$T(n) = 100\,T\left(\frac{n}{10}\right) + n^2 =$$

$$= 100\left(100\,T\left(\frac{n}{100}\right) + \left(\frac{n}{10}\right)^2\right) + n^2 = 100^2\left(T\left(\frac{n}{100}\right)\right) + \cancel{11n^2} =$$

$$= 100^3\,T\left(\frac{n}{1000}\right) + \cancel{2} \cancel{11n^2 + 100n^2}\; n^2 + 10n^2 + 100n^2$$

$$= 100^4\,T\left(\frac{n}{10000}\right) + \cancel{3} \cdots + n^2 + 10n^2 + 100n^2 + 1000n^2$$

$$T_n = 100^i\,T\left(\frac{n}{10^i}\right) + \cancel{\ldots}\; n^2 + \cancel{(i-1)\,10n^2} \sum_{k=0}^{i} \cancel{k}\, 10^k n^2$$

~~2) Maximum recursion depth:~~

$$\cancel{\frac{n}{10^i} = 1}$$

$$\cancel{i = \log n}$$

$$= 100^2\,T\left(\frac{n}{100}\right) + 2n^2 =$$

$$= 100^3\,T\left(\frac{n}{1000}\right) + 3n^2 =$$

$$= 100^4\,T\left(\frac{n}{10^4}\right) + 4n^2 =$$

$$T_n = 100^i\,T\left(\frac{n}{10^i}\right) + i\,n^2$$

2) Maximum recursion depth, $\frac{n}{10^i} = 1$

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$i = \log(n)$

3) Insert i ($i = \log n$)

$$T_n = 100^{\log n} \cdot T\left(\frac{n}{10^{\log n}}\right) + \log(n) \cdot n^2 =$$

$$= \left(10^{\log n}\right)^2 \cdot T\left(\frac{n}{10^{\log n}}\right) + n^2 \log(n) =$$

$$= n^2 \cdot T(1) + n^2 \log n = n^2 + n^2 \log(n)$$

4) Asymptotic runtime complexity

$$f = O(g), \ \exists c \in \mathbb{R}^+, n_0 \in \mathbb{N}_0, \forall n \geq n_0 : f(n) \leq c\, g(n)$$

Assumption: $T(n) = O(n^2 \log(n))$

Proof:

let $c = 2$

$$n^2 + n^2 \log(n) \leq 2 n^2 \log(n)$$

$$n^2 \leq 2 n^2 \log(n) - n^2 \log(n)$$

$$n^2 \leq n^2 \log(n)$$

$$1 \leq \log(n)$$

$$10 \leq n \quad \checkmark$$

b) ~~1) Guess~~

~~$T(n) =$~~ 1) Guess

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b) $T(n) = 100\,T\left(\frac{n}{10}\right) + n^2$

$T(1) = 1$

$\rightarrow T(n) = O(n^2 + n^2 \log(n))$

2) Proof using induction:

Induction assumption: $T(n) = n^2 + n^2 \log(n)$

Induction base: $T(1) = 1^2 + \cancel{1^2 + \log(1)}\; 1^2 \cdot \log(1) =$

$= 1 \rightarrow$ ok

Induction step: insert induction assumption

$T(n) = 100\,T\left(\frac{n}{10}\right) + n^2 =$

$= 100\left(\left(\frac{n}{10}\right)^2 + \left(\frac{n}{10}\right)^2 \cdot \log\left(\frac{n}{10}\right)\right) + n^2 =$

$= 100\left(\frac{n^2}{100} + \frac{n^2}{100} \log\left(\frac{n}{10}\right)\right) + n^2 =$

$= n^2 + n^2 \cdot \log\left(\frac{n}{10}\right) + n^2 = 2n^2 + n^2 \cdot \log\left(\frac{n}{10}\right) =$

$= 2n^2 + n^2\left(\log(n) - \log(10)\right) = 2n^2 + n^2(\log(n) - 1) =$

$= 2n^2 + n^2 \log(n) - n^2 = \underline{n^2 + n^2 \log(n)}$

$\square$

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n^3

a) $T(n) = 8T\left(\frac{n}{2}\right) + n^3$

$a = 8$

$b = 2$

$f(n) = n^3$

$n^{\log_b a} = n^{\log_2 8} = n^3$

$f(n)$ compare with $n^{\log_b a}$

$f(n) = \Theta(n^{\log_b a}) = \Theta n$ (case 2)

$\rightarrow$ For $k = 0$, n and n are of the same order

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^3 \log n)$

b) $T(n) = T\left(\frac{n}{2}\right) + n \cdot \log n$

$a = 1$

$b = 2$

$f(n) = n \cdot \log n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n)$ compare with $n^{\log_b a}$ results in case 3:

$f(n) = \Omega(n^{(\log_b a) + \varepsilon})$ for $\varepsilon = 1$

f ~~$(n) = \Omega(n$~~

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Let $\varepsilon = \cancel{2}\frac{1}{4}$

$n \cdot \log n = \Omega \cancel{(n \log} (n^{\log_2 1 + \varepsilon})$

$n^{\log_2 1 + \frac{1}{4}} = n^{\frac{1}{4}}$

Compare $n^{\frac{1}{4}}$ with $\cancel{n \log n}$ $n \log n$,

$n \log n > n^{\frac{1}{4}}$, so $f(n) = \Omega(n^{(\log_b a) + \varepsilon})$

$a \cdot f(\frac{n}{b}) \leq c\, f(n)$, $c < 1$

$f(\frac{n}{2}) \leq c\, f(n)$

$\frac{n}{2} \log(\frac{n}{2}) \leq c \cdot n \cdot \log(n)$

$\frac{1}{2} \log(\frac{n}{2}) \leq c \cdot \log(n)$ ✓

$\frac{1}{2}$ and $c < 1$, but $\log(n) > \log(\frac{n}{2})$

$T(n) = \Theta(f(n))$

$T(n) = \Theta\, n \cdot \log(n)$

c) $T(n) = 3T(\frac{n}{3}) + \log(n)$

$a = 3$

$b = 3$

$f(n) = \log(n)$

$n^{\log_b a} = n^{\log_3 3} = \cancel{1}\, n$

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f(n) compare with $n^{\log_b a}$ results in

case 1:

let $\varepsilon = 1$

$f(n) = \log(1) = 0{,}1$

$n^{(\log_b a) - \varepsilon} = n^{1-1} = n^{0} = 1$ $\quad \underline{1 > 0{,}1}$

Thus, $\exists \varepsilon > 0: f(n) = O(n^{(\log_b a) - \varepsilon})$ holds.

$T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n)$

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