Can School Choice Make the World (Cup) a Better Place?

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Abstract

The FIFA World Cup is the world's biggest sporting event when it comes around every 4 years. As such, the demand to acquire a ticket to watch a game in person is unfathomably high. Currently, FIFA uses an entirely random lottery to distribute tickets to fans. The demand is high enough that all tickets will be sold out, but it does not mean that most fans will end up with the tickets they want the most, and many will simply get tickets which are "good enough". In this paper, we seek to find what important properties of market design are fulfilled by the current ticket allocation method. Furthermore, we find that DAA with Multiple Tie Breaking, also known as the School choice mode, is a better and more robust mechanism for those getting tickets, that will satisfy more people by giving them a more preferred ticket. This proposed mechanism gives a high percentage of fans their first choice ticket, as well as being stable and strategyproof.

1 Market Description

FIFA organizes a wide range of soccer tournaments. We focus on ticket sales for prior editions of the FIFA Men's World Cup, as the ticket sales process for the upcoming 2026 event has yet to be announced (fans can only sign up for e-mail announcements [7]) and the organization has recently changed the tournament's format [8].

1.1 Two Allocation Sales Phases, One Last-Minute Phase

According to the 2022 tournament's website [4], ticket sales were organized into three phases:

Sales Phase (1). This was divided into two further sub-phases:

- ***Random Selection Draw sales period:*** From Jan 19th - Feb 8th of 2022, fans submit ticket applications at any time, without any preference given to earlier applications. According to FIFA, matches with excess demand were handled as follows: "*In cases where demand for a given match and ticket category exceeded the available ticket inventory for the domestic or international market, (a) random selection draw(s) took place to determine which applicants were allocated tickets. If such tickets were not oversubscribed, the relevant tickets were allocated to the respective applicants*" [4]. Fans are then given until March 21st to pay for their tickets in full.
- ***First Come First Serve sales period:*** From March 21st - March 29th of 2022, allocated tickets from the previous round that were not paid for and any unclaimed tickets became available. Ticket purchases are processed as real-time transactions, on a first-come first-served basis.

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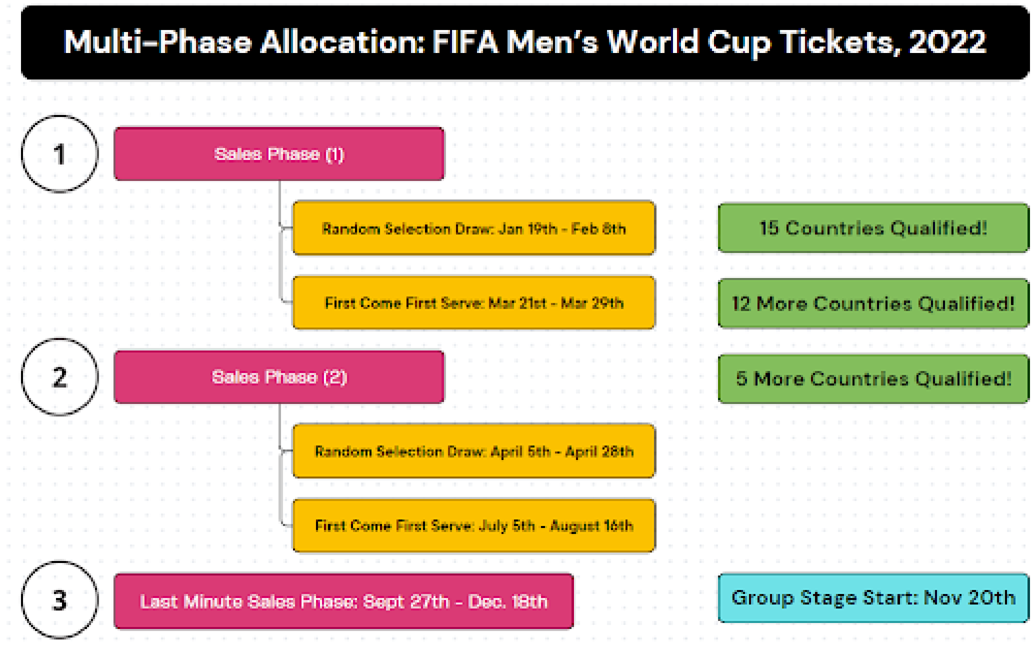

Figure 1: Diagram created by authors.

Sales Phase (2). Exact same procedure as Sales Phase (1), with the dates for Random Selection Draw being April 5th - April 28th 2022, and the dates for First Come First Served being July 5th - August 16th 2022.

Last-Minute Sales Phase. This period ran from September 27th, until the end of the competition on December 18th 2022. Any remaining tickets are allocated on a first-come, first-serve basis as a real-time transaction.

1.2 Information Updated Overtime - Which Teams Qualify?

The 31 teams taking part in the tournament (with Qatar being guaranteed a spot when they were announced as the host country in 2010), was decided via the qualification process, which spanned from October 2021 to June 2022. The majority of qualifying countries were determined by March 2022 [2]. Thus, some countries were guaranteed to play before Sales Phase (1) had even began, while others were only determined during Sales Phase (1) - sometimes on the last day. Only three teams (Wales, Australia, and Costa Rica) were determined during Sales Phase (2) [2]. The first match of the tournament's group stages began on November 20th [3], which coincides with the last-minute sales phase. See figure 1 for a visual depiction of this timeline.

1.3 Ticket Price Categories

According to FIFA's official website, ticket prices for each match are divided into four categories, with Category 1 being the most expensive tickets for seats in prime spots and Category 3 being the cheapest type of ticket available to all fans. Category 4 tickets are exclusively reserved for residents of Qatar [6]. A summary of average ticket cost by round and category for the 2022 tournament is below in figure 2:

Match	Cat. 1	Cat. 2	Cat. 3	Cat. 4
Opening Match	$618	$440	$302	$55
Group Matches	$220	$165	$69	$11
Round of 16	$275	$206	$96	$19
Quarterfinals Matches	$426	$288	$206	$82
Semifinals Matches	$956	$659	$357	$137
Third-Place Match	$426	$302	$206	$82
Final Match	$1607	$1003	$604	$206

Figure 2: Caption. Table obtained from: `https://www.nbcphiladelphia.com/news/sports/soccer/world-cup-2022/how-much-are-2022-fifa-world-cup-tickets-2/3427029/`

1.4 Nationality Quotas

FIFA centrally determines how many tickets to reserve for fans of participating nations, and entrusts national football leagues with distributing them to fans of that particular country [1]. For instance, U.S. Soccer received a designated allotment of tickets to distribute for matches in which the U.S. Men's National Team was competing [1]. There also exist "conditional tickets" for the Round of 16, Quarterfinals, Semifinals, and Final/Third Place Game, which are issued by the national leagues only if their team advances [1].

1.5 Process of Submitting Preferences

Currently, fans can submit a maximum of six (6) tickets per household per match, or sixty (60) tickets per household for the whole tournament. [1]

These preferences are submitted as indifferent, weak preferences, where FIFA assumes that you are okay with any of the tickets equally.

2 Problem Statement and Research Questions

Given that the World Cup is the world's biggest sporting event and millions of fans like to attend, we must question the properties of the ticket allocation scheme. We seek to determine the qualities of (a) the random lottery system and (b) the multi-phase setting.

Aside from equal treatment of equals, are there any other guarantees from the random lottery? Does the multi-phase allocation have the properties used in each phase?

While finding questions to these questions, we seek to propose a mechanism that improves the current scheme in areas like stability (closely related to fairness) or strategyproofness.

Therefore, we will ask:

- What are the properties of the current FIFA mechanism?
- Is there a difference or advantage in these properties with a multi-phase mechanism?
- Is there a known or new mechanism we can use to improve the allocation method for World Cup tickets?

3 General Model Settings

We will begin formalizing the general structure for our analysis of this market, including some simplifying assumptions that permit more elegant theoretical analysis and informative simulations.

3.1 General Assumptions

For our general setting, we will assume the following

1. *There a many more fans than tickets*
2. *Each agent gets at most one ticket*
3. *Unless stated otherwise, agents are assumed to report their true preferences*

3.2 Ticket

In this setting, the agents (fans) want tickets for the World Cup. Thus, we shall formally define what is a ticket.

Let G be the set of games in the tournament C be the set of seat categories (as described in the Market Description section)

Definition 1 *A **ticket section** is an element of* $G \times C$

That is, a ticket section is a tuple of size two where the first entry has the game $g \in G$ and the second entry has the category $c \in C$. For example, (g_1, c_1) is a ticket section, which has some capacity of n tickets.

Since all seats within a category are considered the same for each game when applying, we can assume that tickets have capacity. In practice, we can treat each ticket (an element of form (g_i, c_i)) as a school with capacity n, where each seat in the school is one ticket for a specific game, at a specific price category.

3.3 True Preferences

Considering that each fan knows (a) which games they would like to see and (b) the category preferred for each game given the implied cost, it is logical to model these agents as having *preferences* over these tickets. In particular, we assume that each fan $f_i \in \mathcal{F}$ has **strict** preferences over tickets of the form: $(\textit{game}, \textit{price})$. An illustrative example of this is provided below, using standard notation:

$$\begin{aligned} f_1 &: (g_4, c_4) \succ (g_4, c_1) \succ (g_4, c_2) \succ (g_7, c_3) \\ f_2 &: (g_1, c_1) \succ (g_{10}, c_1) \\ &\vdots \\ f_m &: (g_{15}, c_2) \succ (g_{15}, c_1) \end{aligned}$$

These preferences are submitted before each random selection draw, $k \in \{1, 2\}$.

3.4 FIFA Preferences

Given that FIFA does not use preferences as an order but as a binary variable where fans are indifferent over all reported ticket alternatives, we need to adapt our notation for this case. For the same preferences above, the FIFA preferences are

$$\begin{aligned} f_1 &: \{(g_4, c_4), (g_4, c_1), (g_4, c_2), (g_7, c_3)\} \\ f_2 &: \{(g_1, c_1), (g_{10}, c_1)\} \\ &\vdots \\ f_m &: \{(g_{15}, c_2), (g_{15}, c_1)\} \end{aligned}$$

Since we assume that agents report their preferences truthfully, if is reasonable to assume that they report all the tickets they would like to get and no other than those. Hence, the FIFA preferences can be derived from True preferences by replacing the ordering with indifference.

Notation will make clear which preferences are being used.

3.5 Multi-Phase Setting

To allow more flexibility and reflect more closely how FIFA allocates tickets, we have to consider how preferences can change over time. For this purpose, we use $k \in \{1, 2\}$ to represent each phase of the allocation.

3.5.1 Preferences

Now, we will define the notation for preferences in this case,

Hence, when we consider multi-phase allocations we consider write true preferences in the form of the following example.

$$\begin{aligned}
f_1^k &: (g_4, c_3) \succ (g_4, c_2) \succ (g_4, c_1) \succ (g_7, c_3) \\
f_2^k &: (g_1, c_3) \succ (g_{10}, c_3) \\
&\vdots \\
f_m^k &: (g_{15}, c_1) \succ (g_{15}, c_2)
\end{aligned}$$

FIFA Preferences are adapted similarly, by adding the k superscript.

3.5.2 Assignment Over Phases

Note that, any assignment mechanism for this setting must be run twice: once during the random selection process in Sales Phase (1), and once again during Sales Phase (2). By having two separate sets of preferences, we can model the "dynamic" nature of information update over time in the following ways:

1. **Fans leave the market:** If a fan receives their most preferred $(game, price)$ in Sales Phase (1), there is no incentive to stay in the market any longer. Thus, such a fan f_i would not submit a preference list for Sales Phase (2). *For simplicity, we further assume that a fan that has received* ***any*** *object on their preference list exits the market after Sales Phase (1), and does not participate in Sales Phase (2).*
2. **Fans enter the market:** Some fans may only be willing to enter the market if a certain country qualifies. Thus, such a fan f_i would not submit a preference list for Sales Phase (1), but can submit a preference list for Sales Phase (2).

Note that we assume that there are many more fans than tickets, thus resulting in scarcity. This directly implies that not all fans will receive a ticket in Sales Phase (1). They will all reapply during Sales Phase (2) with the same preferences.

4 Representation of Current FIFA Mechanism

To get a tractable model suited for analysis and comparison with other mechanisms, we have to bring the FIFA mechanism to the setting of market design.

4.1 RPSRD: Algorithm Representing Current Lottery System

In order to model the current way of ticket allocation, we will define a representation of the lottery system used by FIFA in the 2022 World Cup.

Given that we assume that each person can get at most one ticket to the World Cup, we need to adapt the lottery system to this constraint. We propose, first, a lottery over fans, where each individual fan then has a lottery over the tickets they applied to. This is essentially a modified version of the Random Priority Algorithm to model, in which tickets are assigned randomly to fans. Here, we first create a random ordering of fans. Next, fans get the chance to get a ticket in the created order, where for every fan's turn, they get a random available ticket from their indifferent preference list.

We will call this proposed algorithm **Random Priority Serial Random Dictator**, and define it below:

Algorithm 1 Random Priority Serial Random Dictator (FIFA Algorithm)

```
Input   : Set F of all fans
Input   : Set S of tickets with their capacities
Output  : Mapping μ of fans to their assigned ticket
μ ← empty mapping.
P(f) = fan f's preference list ∀f ∈ F
while S is not empty and F is not empty do
    f ← random fan from F
    while |P[f]| ≠ 0 do
        t ← random ticket from P[f]
        if t ∈ S then
            μ ← add mapping of (f, t)
            S[t]_capacity ← S[t]_capacity − 1
            Remove f from F
            if S[t]_capacity = 0 then
                Remove t from S
            break
        else
            Remove t from P(f)
return μ
```

From this point, we may refer to Random Priority Serial Random Dictator as FIFA Mechanism, for simplicity.

We expect the FIFA Mechanism to maintain properties of a lottery namely, a uniform distribution for true ranking of tickets for fans who were allocated a ticket, as shown in 7.

4.2 Efficiency of Single-Phase FIFA Mechanism

Claim 1 *In a single phase mechanism, where all the tickets are sold during the first phase, we can trivially show that the allocation will be efficient, given the constraints put forth by FIFA.*

That is to say, because the FIFA lottery assumes that people's preferences over tickets is indifferent, we know that the only way to make someone better off is to have a ticket after not having one, and someone not having a ticket is always made worse off.

Proof: To make an improvement, the market will always come down to two people: one who has a ticket and one who does not, as we assume all tickets are sold out in the first phase, and the only way to get a ticket if you do not have one is to get it from someone who does.

$$
\begin{aligned}
f_1 &: \{(g_1, c_3), (g_2, c_3), (g_3, c_3), (g_4, c_3)\} \\
f_2 &: \{(g_2, c_3), (g_3, c_3), (g_4, c_3), (g_5, c_3)\} \\
\mu(f_1) &= (g_2, c_3) \\
\mu(f_2) &= \emptyset
\end{aligned}
$$

We can see here if f_1 gave up their ticket to f_2, f_2 would be made better off, but f_1 would be strictly worse off.

Therefore, we can see that a single-phase mechanism is efficient under FIFA conditions. □

However, although FIFA simplifies preferences to be indifferent, we cannot assume that fans do not have strict preferences over games and categories.

4.3 Ex-Ante Inefficiency With Respect to True Preferences

Claim 2 *Random Priority Serial Random Dictator is Ex-Ante Inefficient*

Proof: True preferences:

$$f_1 : (g_1, c_1) \succ (g_1, c_2)$$
$$f_2 : (g_1, c_2) \succ (g_1, c_1)$$

with Random Priority Serial Dictator we get:

$$\beta' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

with FIFA preferences derived from true preferences and RPSRD we get the probabilistic matching:

$$\beta = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

Note that β stochastically dominates β'

Hence, RPSRD is not Ex-Ante efficient with respect to true preferences. $\square$

4.4 Tickets True Value

While the tickets have fixed prices according to their category, we know that a fan's value for the ticket may not be directly proportional to the price. For instance, a fan who wants a premium viewing experience might prefer more expensive tickets over cheaper ones. Therefore, we define the following utility function for fans:

For each fan f_i, at stage k, we define the following:

$$v_i^k : \{Games\} \times \{Category\} \longrightarrow \mathbb{R}^+$$

These functions map each tuple (g_j, c) to the true value that the agent assigns to that game, category combination. We will use the existence of this function to justify the strict preferences of each fan, as well as analyze the secondary market.

4.5 Secondary Market

Given that tickets are assigned in a probabilistic manner, we may have a final allocation where fan i has a ticket for (g, c) while fan j has no ticket. Further, suppose that after all stages we have $v^j((g, c)) > v^i((g, c))$. (We dropped the subscript for the phase as the valuation will not change as allocation phases are over.)

In this scenario, both i and j would benefit from i selling their ticket to j for some value p such that $v_j((g, c)) > p > v_i((g, c))$.

Hence, if fans are not satisfied with their ticket allocations, we should expect the emergence of a secondary market.

4.6 Strategyproofness

We can define a mechanism as strategyproof if is behaves as follows:

Definition 2 *A mechanism is strategyproof if for each participant it is a dominant strategy to report their true preferences (i.e., optimal regardless of the report of others).*

Claim 3 *The Current FIFA model, RPSRD with a maximum of 4 reported games, is not strategyproof*

Proof:

We can show the current FIFA model as not being strategyproof with a counter-example.

For the sake of simplicity in this argument, we will assume that the mechanism is the same, but one can only declare 2 games as ones they want. Additionally, there are only three games, games only have one price category, and one seat per game.

$$\begin{aligned} f_1 &: \{(g_1, \$), (g_3, \$)\} \\ f_2 &: \{(g_1, \$), (g_3, \$)\} \\ f_3 &: \{(g_1, \$), (g_3, \$)\} \\ f_4 &: \{(g_1, \$), (g_3, \$)\} \\ tickets &: \{(g_1, \$), (g_2, \$), (g_3, \$)\} \end{aligned}$$

Let us also assume that f_1 is indifferent between all 3 tickets available, but is only able to submit 2 tickets to his preferences.

Currently, each fan has a 50% chance to get a ticket with the FIFA mechanism (that is, to be selected in the random priority mechanism first or second).

Here we can see that this mechanism is not strategyproof, because f_1 reporting their true preferences is not the dominant strategy regardless of the rest of the fans.

Let us assume that f_1 knew what preferences the rest of the fans submitted. In that case, they would know that the rest of the fans have the same preferences. If f_1 were to know this, then their strategy would change to reporting their preferences as either :

$$f_1 : \{(g_1, \$), (g_2, \$)\}$$

or:

$$f_1 : \{(g_2, \$), (g_3, \$)\}$$

Where they would have a 100% chance of getting a ticket, as they would be the only one reporting $(g_2, \$)$, meaning no matter what order the fans are selected to get a ticket, f_1 would be the only one reporting that game, and therefore the only one with a chance to get it.

Therefore, we have shown that the current FIFA mechanism is not strategyproof. □

5 Analysis of Multi-Phase Mechanisms

Given that FIFA currently holds separate ticket allocation stages, we must analyze the properties of multi-stage mechanisms

5.1 Ex-Post Efficiency of Multi-Phase FIFA Mechanism

In a multi-phase FIFA Mechanism, not all tickets are sold during the first phase, and some are instead saved and distributed during the second phase. This is how FIFA allocates tickets, and this is due to there being some teams that are not qualified in the first phase, but are in the second.

As such, during each phase, we can treat the allocations performed during that phase as we did the allocations for the single-phase FIFA mechanism, in which for the constraints laid out by FIFA we have an efficient allocation. When looking at the whole allocation however, we can see that fans' preferences for games might change after a new country qualifies, and as such could lead to an inefficient allocation after both phases.

Claim 4 *In the multi-phase setting, with evolving preferences, the allocation may be ex-post-inefficient with respect to final preferences.*

Proof:

$$
\begin{aligned}
f_1^1 &: \{(g_1, c_4), (g_2, c_4)\} \\
f_2^1 &: \{(g_2, c_4), (g_3, c_4)\} \\
f_3^1 &: \{(g_2, c_4), (g_3, c_4)\} \\
\mu(f_1) &= (g_2, c_4) \\
\mu(f_2) &= (g_3, c_4) \\
\mu(f_3) &= \emptyset
\end{aligned}
$$

$$
\begin{aligned}
f_1^1 &: \{(g_1, c_4), (g_3, c_4)\} \\
f_2^1 &: \{(g_1, c_4), (g_2, c_4), (g_3, c_4)\} \\
f_3^1 &: \{(g_2, c_4), (g_1, c_4)\} \\
\mu(f_1) &= (g_2, c_4) \\
\mu(f_2) &= (g_3, c_4) \\
\mu(f_3) &= (g_1, c_4)
\end{aligned}
$$

□

Here, we can see that f_1's preferences change from the first to the second selling phase. As such, the tickets allocated during each round are efficient and individually rational. Through the two phases, however, we can see the allocation is not efficient (or individually rational).

We would not know what f_1's new preferences are, as they would not be reported, but here we see that we could have $f_1 \& f_3$ switch tickets, with f_2 being indifferent between $(g_2, \$)$ and $(g_5, \$)$, and f_1 being better off when having $(g_5, \$)$ after their preferences evolve for the second phase.

5.2 Inefficiency in a Multi-Stage Setting

While the previous result is trivial considering that the FIFA mechanism may produce an ex-post inefficient allocation for a single stage, we must highlight that this result is not derived from the nature of the algorithm used in the stages, but the nature of evolving preferences and multiple-phase allocation.

Claim 5 *With evolving preferences, in a multi-phase allocation setting, we may have an inefficient ex-post allocation with respect to final preferences regardless of the allocation algorithm.*

Proof: Suppose that after the first phase, we have $\mu^1(f_i^1) = (g_m, c_n)$. However, suppose that (g_m, c_n) is no longer acceptable for this fan, f_i^2, after the first phase. Then, since the assignment does not change for this fan, we find get an unstable, inefficient outcome with respect to final preferences. □

6 Discussion of Alternatives and Recommendation

Given the nature of the scenario, many-to-one allocation, there are some natural considerations options for algorithm choice. Moreover, we also have to consider the multi-stage and evaluate its positive and negative aspects.

6.1 One Phase vs Multi-Phase

First, we need to choose if we are to keep the multi-phase mechanism or favor a single phase instead. As we saw in our analysis of multi-phase mechanisms, they are not guaranteed to have stability or be efficient due to fans changing preferences between stages. Moreover, even if we disregard this problem, the nature of having multiple phases implies that we will not have global results for our allocation, as for example, efficiency in phase 1 and efficiency in phase 2, do not imply efficiency in the final allocation.

Therefore, our recommendation will be for single-phase mechanisms that take place after first the participants of the group stage are largely defined. We recommend this because at this stage fans have the information needed for group stage decisions and thus also have expectations for the knockout stage. Hence, by this moment, the variation of preferences should decrease significantly.

6.2 Properties of FIFA Mechanism

As we developed in previous sections, the FIFA Mechanism (RPSRD) does not have strong qualities. Despite having equal treatment of equals, there is no guarantee of efficiency, or strategyproofness given our setting. This is to be expected as its preferences are extremely limited and do not reflect true preferences. This loss of information from true preferences to FIFA preferences motivates looking for a new mechanism for this assignment problem.

6.3 "School Choice" Model with Weak Priorities

In terms of analyzing utility, we don't need to consider tickets as having "preferences", since they are objects. Thus, to run our assignment mechanisms, we assume the objects have "priorities" instead. Aside from nationality quotas, it does not make sense to give preferential treatment to some fans over others. This motivates using "weak priorities" in the following manner:

Games $g_j \in \mathcal{G}$ have **weak** priorities over fans.

$$g_1 : \{\text{"Fans with the nationality of the teams playing (up to 10\%)"}\}^{**} \succ \{\text{"Everyone Else"}\}$$

$$\vdots$$

$$g_{64} : \{\text{"Fans with the nationality of the teams playing (up to 10\%)"}\}^{**} \succ \{\text{"Everyone Else"}\}$$

6.4 Deferred Acceptance Algorithm with Random Tie Breaking

With DAA with Random Tie Breaking, fans propose to their preferred tickets in descending preference order. For Random Tie-Breaking, the indifference in priorities is broken before the algorithm is run, and results in a strict ordering over fans, globally across all preferences. This means that for all tickets, ties will be broken in favor of the same fan. It also means that the fan with the highest global priority will always win the tie for any ticket, and the one with the lowest global priority will always lose the tie for any ticket.

In this context, Random Tie Breaking makes little sense when compared to Multiple Tie Breaking. We would not want to employ a mechanism that gives one person an advantage every time. Instead, we would want to spread the odds and have every fan have the possibility of an advantage over another fan.

6.5 Probabilistic Serial Mechanism

Instead of the RPSRD, FIFA could use a Probabilistic Serial Mechanism to randomly assign tickets to fans. This would, however, require strict ordering of fans over their preferred games.

In a probabilistic serial mechanism, we have each fan "consume" their most preferred ticket that is not taken. The consumption is divided into everyone consuming a given ticket. Because there are several tickets in the same categories, we treat each ticket in this set as a separate object, and fans consume specific tickets rather than the whole category at once. These go in ascending order of tickets for each category, i.e. $t_{1,1} \succ t_{1,2}$...

This method would, by definition, not be strategyproof, therefore fans can try to game the system to their advantage. When looking for an alternative, we want a straightforward mechanism that fans don't have to worry about having to game the system, or having others game the system.

6.6 Proposed Improvement to Allocation

We will now propose what we see as the best improvement to the World Cup Tickets allocation process. In doing so, we seek to provide stability and strategyproofness, as well as maximize the amount of most-preferred tickets given to fans.

Firstly, we will go from a two-phase selling schedule to a single-phase schedule. This will only take place once every team and group is known. Fans will know when their country will play in the group stages, and which games they have a chance of playing in the knockout rounds, in order to be able to construct preferences accordingly.

6.6.1 Allocation Modeling with School Choice

Our allocation will consist of a School Choice model with games having weak priorities over fans. In these priorities, we assume that the rest of the ties are broken randomly, using a multiple tie-breaking approach.

With multiple tie-breaking ties are broken ahead of time but with a strict ordering over fans in each place where indifference is present. This means that two tickets might break the ties in favor of different fans. In practice, this means that each game ends up with different strict priorities over fans, randomly.

7 Simulation of Proposed Allocation Method

7.1 Assumptions Regarding Fan Decision-Making and Game Results

In our simulation model[5], we consider a tournament with n participating countries. In our model, we introduce a population of m individuals, each originating from one of these countries. Every individual is characterized by two main attributes:

1. **Country Ranking:** A personal ranking assigned to each country.
2. **Price Segment:** An indicator of the individual's financial capacity and their preference among ticket pricing categories.

These attributes are crucial for generating the preferences of fans regarding the different games in the tournament. The price segment specifically delineates into three distinct categories: high, medium, and low. For the purposes of this study, we omit the fourth category as discussed in Section 1. The price segment is represented by a random integer ranging from 1 to 9, where:

- Segment 1 fans are those only interested in the high price category.
- Segment 2 fans are those only interested in the medium price category.
- Segment 3 fans are those only interested in the lowest price category.
- Segment 4 fans are those interested in the high and medium price categories, preferring the high category more.
- Segment 5 fans are those interested in the high and medium price categories, preferring the medium category more.
- Segment 6 fans are those interested in the medium and low price categories, preferring the medium category more.
- Segment 7 fans are those interested in the medium and low price categories, preferring the low category more.
- Segment 8 fans are those interested in all price categories, prioritizing more affordable ones.
- Segment 9 fans are those interested in all price categories, prioritizing more expensive ones.

Additionally, each individual holds a ranking for every country, a random integer ranging from 1 to 10. For each individual, we assign a ranking of 10 to their own country. The preference level in the game is defined as the sum of the ranking of the two countries involved in the match.

To generate individual preferences regarding game attendance, we combine their price segments with their preference levels. Consider a hypothetical scenario involving three countries: Kazakhstan, Ecuador, and Mexico. Let's take the example of Alex, a fan hailing from Ecuador, with a price segment of 5 and country rankings of [6, 10, 4] for Kazakhstan, Ecuador, and Mexico, respectively. His preferences can be summarized as follows, in descending order of preference:

$$\begin{aligned} f^{Alex} : &(Kazakhstan - Ecuador, \$\$) \succ (Kazakhstan - Ecuador, \$\$\$) \\ &\succ (Mexico - Ecuador, \$\$) \succ (Mexico - Ecuador, \$\$\$) \\ &\succ (Mexico - Kazakhstan, \$\$) \succ (Mexico - Kazakhstan, \$\$\$) \end{aligned}$$

This hierarchy reflects Alex prioritization of attending games involving his home country, followed by matches between Kazakhstan and Mexico, with considerations for affordability reflected in his price segment.

Note that the preference generation methods above only works when fans know exactly who is playing whom, which is not the case when purchasing tickets for the Knockout Stage. We could simulate the preferences for the knockout stage by using the expected preference level for a game in the knockout stage.

Let $f(g, \textit{CountryA}, \textit{CountryB})$ represent the probability of *CountryA* playing against *CountryB* in game g of the Knockout Stage. It is posited that this probability distribution affects fan decisions when buying tickets in advance for Knockout Stage games.

For illustration, consider two groups:

- Group 1: Countries A, B, C, D
- Group 2: Countries E, F, G, H

Assuming fans are rational and aware of the probabilities of which countries will advance and face each other, they develop preferences for the games based on these probabilities. The preference level of game g for fan i is given by:

$$\sum_{g \in \{g_1, g_2, g_3\}} \sum_{\substack{\text{CountryA} \in \{A,B,C,D\} \\ \text{CountryB} \in \{E,F,G,H\}}} \left(r_i[\text{CountryA}] + r_i[\text{CountryB}]\right) f(g, \text{CountryA}, \text{CountryB})$$

where r_i represents the ranking of the countries by individual i.

For simplicity, we assume that the probability distribution function is uniform, meaning that all countries have equal chances.

7.2 Simulation Results

We carried out two studies using [5] to evaluate the FIFA model, designated as the Random Priority Serial Random Dictator (RPSRD), and the School Choice Deferred Acceptance Algorithm (DAA). In our experiments, we truncated preferences at either 4 or 10 schools or game-ticket pairs, providing a controlled comparison of the models.

We chose to omit simulations of the knockout stage games and related preferences. Including these stages would unnecessarily complicate the simulation without adding enough value. This decision is based on the assumption that the outcomes from the group stages would extend to the knockout stages, given that the inputs—game-ticket pairs (or schools) and their preferences—are clearly defined.

In the experiment where preferences are truncated at 4, we have 16 countries participating, with a total of 6,400 individuals (400 per country). Each game has a capacity of 40 seats per price category. We repeated this process 5 times and averaged the results, shown in Figure 3 and Figure 4.

In the experiment with a truncation at 8, the setup remains the same with 16 countries and 6,400 individuals. However, the capacity for each game is adjusted to 30 seats per price category. We repeated this process 5 times and averaged the results, shown in Figure 5 and Figure 6.

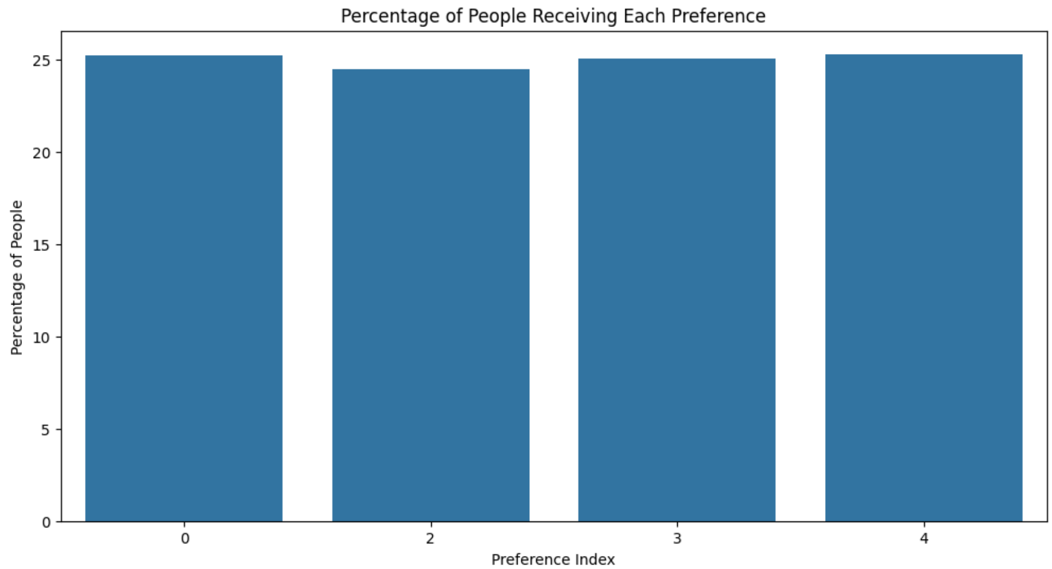

Figure 3: RPSRD | Percentage of People Receiving their i-th ranked preference

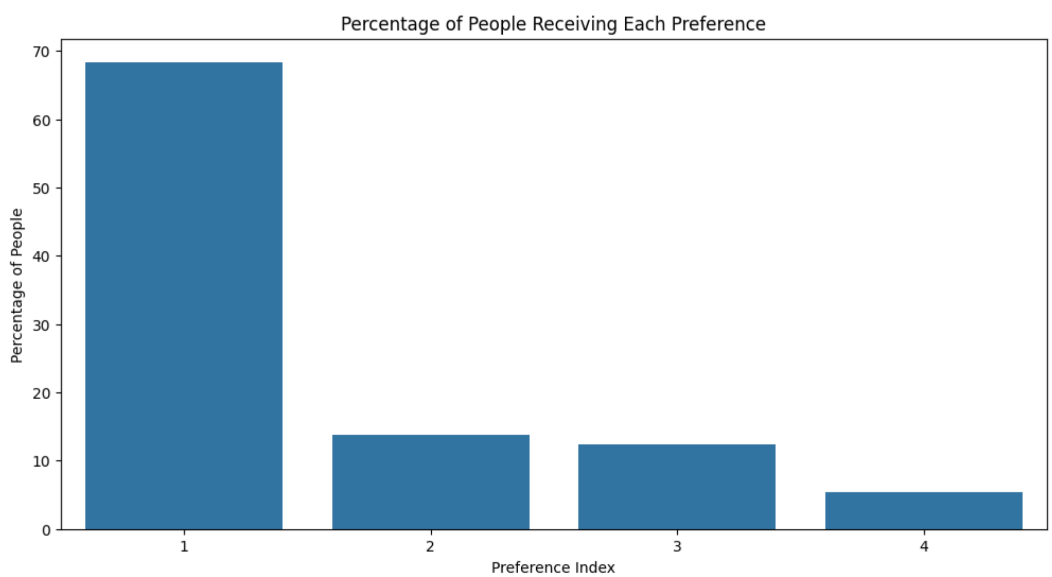

Figure 4: School Choice | Percentage of People Receiving their i-th ranked preference

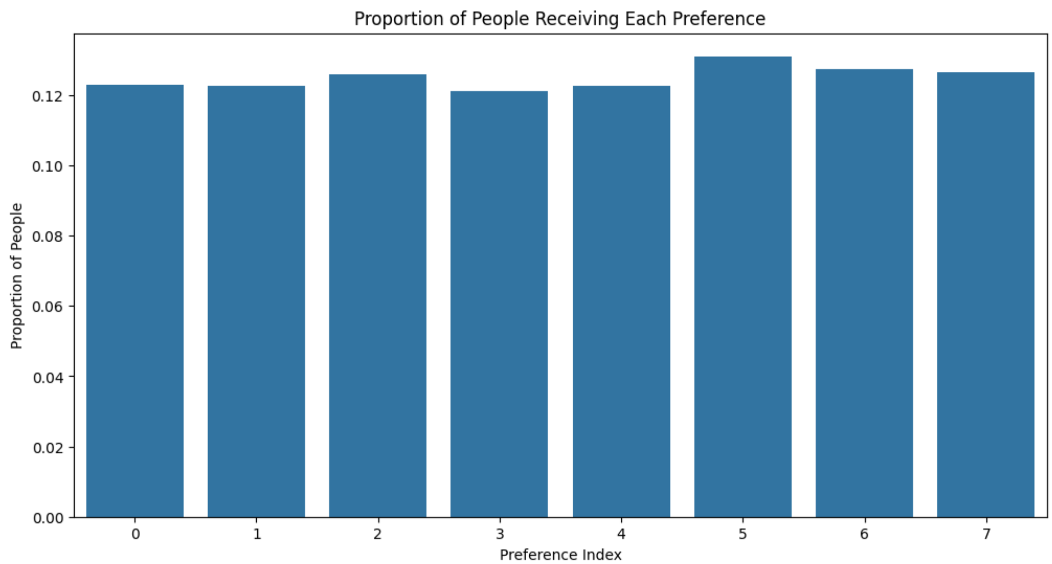

Figure 5: RPSRD | Percentage of People Receiving their i-th ranked preference

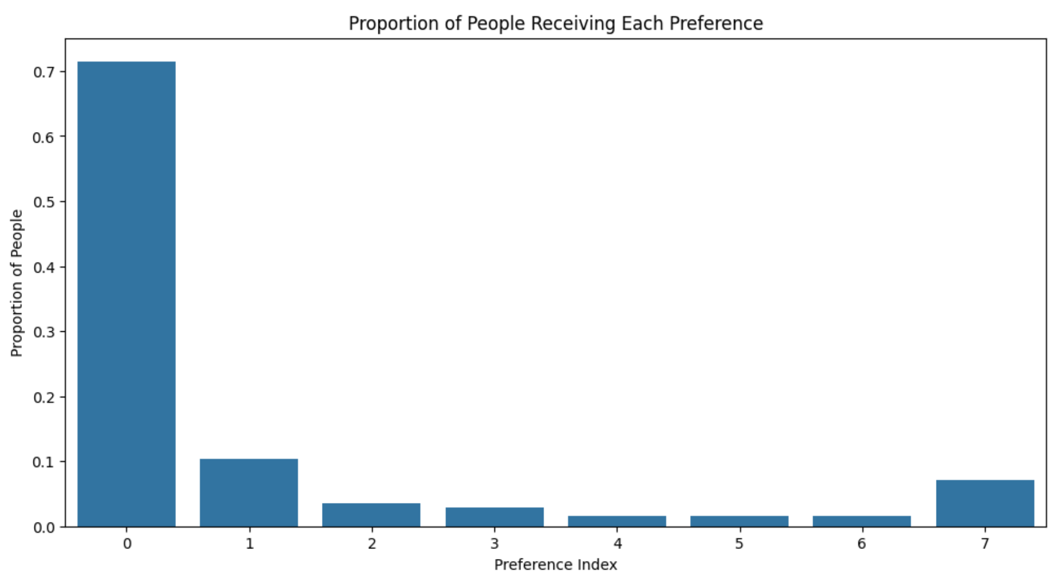

Figure 6: School Choice | Percentage of People Receiving their i-th ranked preference

7.3 Simulation Results Discussion

In both scenarios—whether truncating preferences to four or eight choices—the school choice model significantly outperforms the current FIFA allocation methods. Approximately 70% of participants receive their most preferred outcome with the school choice model, compared to only $\frac{100}{n}\%$ in a lottery-based system, where n is the number of reported preferences. Specifically, 25% and 12% of participants achieve their first choice with preferences truncated at four and eight, respectively. This highlights the superior performance of our proposed algorithm.

Similarly, the proportion of participants receiving their second-best option is consistent at about 12% for the school choice model in both cases. In contrast, these rates are 25% and 12% for preferences truncated at four and eight, respectively.

8 Conclusions

We have found (in 4, 5, and 7) the weak properties of the FIFA allocation mechanism. We also know that the School Choice model will yield stable and strategyproof results, which the FIFA mechanism does not.

Furthermore, we see that, in our simulations, the most popular result of the School Choice model is someone getting their most preferred ticket. Additionally, we see that every option but a fan's first choice is less popular with the School Choice model than it is with the FIFA mechanism.

Altogether, by using more information and a better algorithm, we are improving the welfare of fans by increasing their likelihood of getting their most preferred ticket, giving a fair environment (by the stability of the school choice model), and providing strategyproofness, which eliminates the need to being strategic when reporting preferences.

Therefore, we have provided more fans with more preferred tickets, as well as introduced robustness in ways like stability and being strategyproof. As such, we conclude that fans would be better off if tickets were to be assigned with a School Choice model, as opposed to the current FIFA mechanism.

9 Further Research

9.1 Impact of Improvement Cycles

Improvement cycles as part of the assignment mechanism will be controlled by us, the mechanism designers. Fans point at any ticket that matches their most preferred match and ticket category. If there are any cycles in this graph, we trade those tickets among those fans. This keeps going until we have no more cycles.

This would find an efficient allocation with the fans and tickets in the stable allocation given by the School Choice model. This would, however, make the allocation not strategyproof. This would be a tradeoff to explore in a longer analysis of every possible allocation method.

9.2 Nationality Quotas

When a portion of tickets are reserved for citizens of a country playing, does this lead to an efficient allocation? Additionally, is there a sweet spot for nationality quotas that can guarantee efficiency in the allocation? Lastly, does the introduction of the ability to resell tickets guarantee an efficient outcome?

In our research, we dismiss the allocation of tickets this way, assuming they are distributed in the best way the federation can come up with. In reality, the different ways in which federations distribute said tickets could be accounted for in an allocation model to answer the questions above.

9.3 Scalpers

Definition 3 *Let us define scalpers as agents who are not interested in the games, i.e. not fans, but are interested in purchasing tickets with the intent of making a monetary profit.*

In our research, we are concerned with how fans report preferences, and assume that those applying for tickets are fans. In reality, this is not the case. With this definition, what change to the allocation mechanism would make the system robust against resellers? Does the allocation system suffer when making it robust against scalpers?

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