Exercise 1

a) **a has units of** $1/individuals \cdot time$

dN/dt : individuals/time

aNP: individuals/time (ends up being subtracted from growth)

N: prey density

P: predator density

b) **Solve for isoclines**

Prey isocline:

$$dN/dt = 0 \Rightarrow rN(1 - N/K) - aNP = 0$$

Factor N:

$$N[r(1 - N/K) - aP] = 0 \Rightarrow N = 0$$

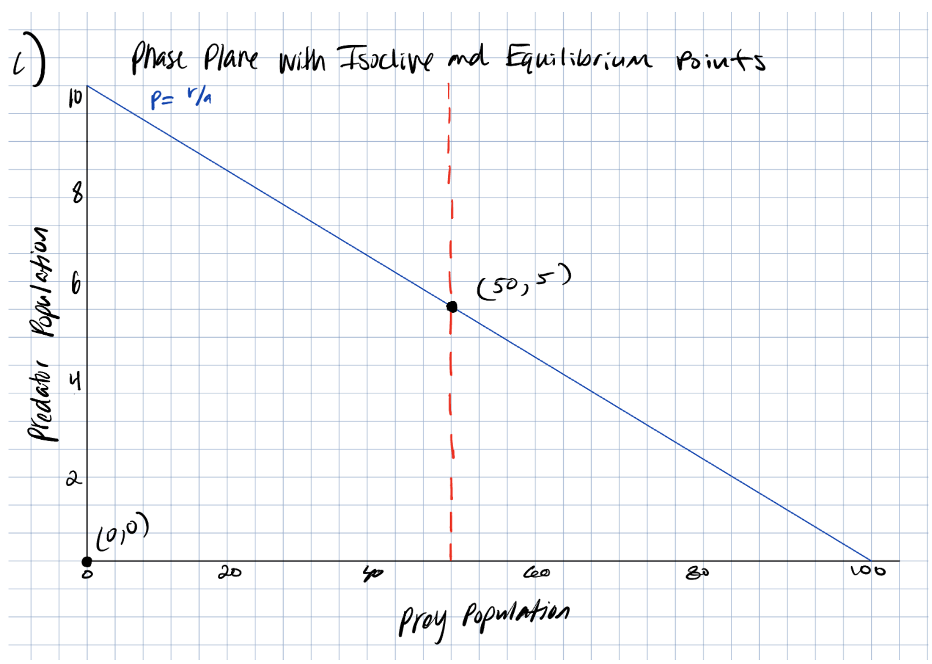
Solving for P:

$$P = r/a(1 - N/K)$$

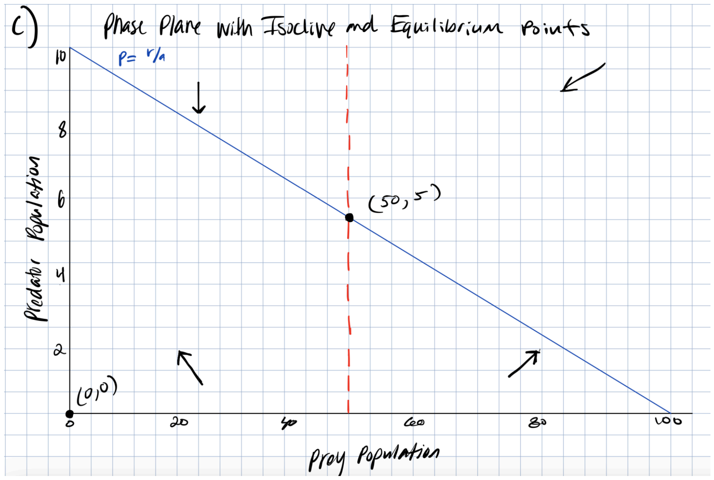
Predator isocline:

$$dP/dt = 0 \Rightarrow baNP - mP = 0 \Rightarrow P(baN - m) = 0 \Rightarrow P = 0 \text{ or } N = m/ba$$

c)



d)



e) Equilibrium stability

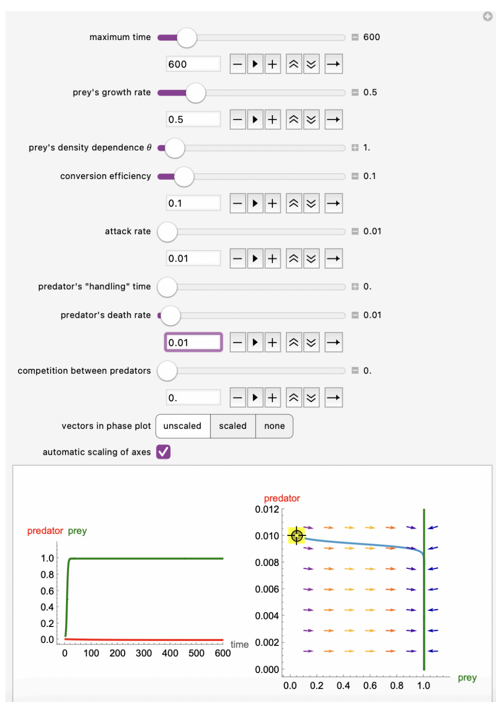
(0,0): unstable

(K, 0): semi-stable

$(m/ba,\ r/a(1 - m/baK))$: stable, spiral, oscillatory

f) In this simulation, I changed the parameter values to increase the predator death rate and lower the prey growth rate. The plot shows that:

- Predators go extinct: Predator density drops to zero fast, while prey stabilize near their carrying capacity.
- With no predator pressure, prey grows until density dependence limits the growth.
- The vector field shows arrows pointing downward and right, indicating a decreasing predator population at initial conditions.
- This explains that when predator death is too high or prey growth is too slow, predators cannot persist, and the system stabilizes at an equilibrium without predators.



Exercise 2

a)

2a) Solve for isocline

Prey isocline ($\frac{dN}{dt} = 0$)

$$rN = \frac{aNP}{1+ahN}$$

$$r = \frac{aP}{1+ahN}$$

$$1 + ahN = \frac{aP}{r} \Rightarrow ahN = \frac{aP}{r} - 1 \Rightarrow N = \frac{1}{ah}\left(\frac{aP}{r} - 1\right)$$

$$\boxed{N = \frac{1}{ah}\left(\frac{aP}{r} - 1\right)} \quad \text{or} \quad \boxed{P = \frac{r}{a}(1 + ahN)}$$

Predator isocline ($\frac{dP}{dt} = 0$)

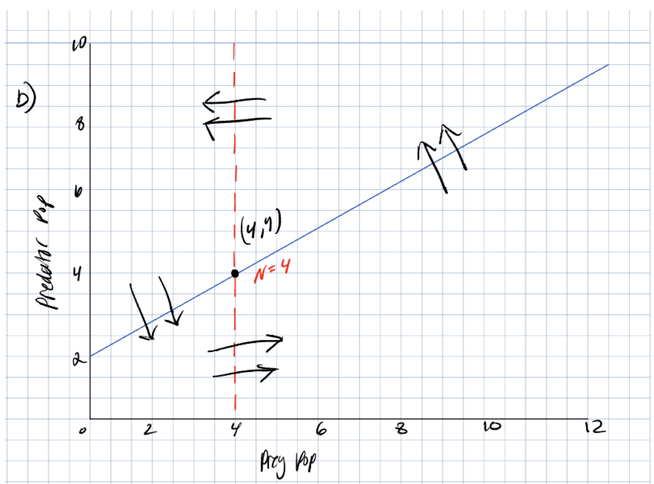
$$\frac{baNP}{1+ahN} = mP$$

$$\frac{baN}{1+ahN} = m$$

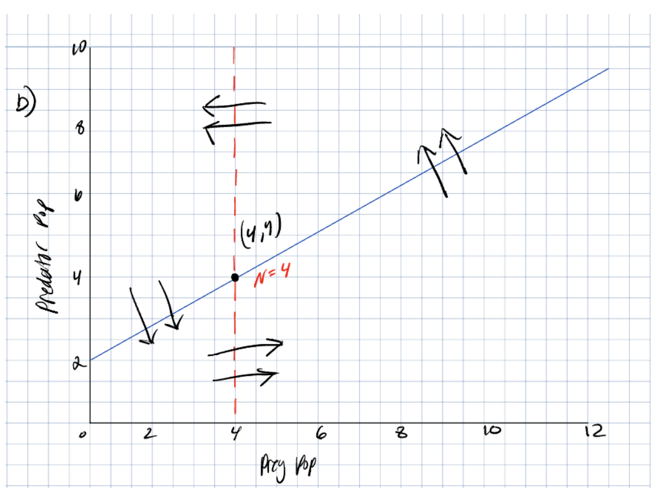
$$baN = m(1+ahN) \Rightarrow baN = m + mahN \Rightarrow baN - mahN = m \Rightarrow N(ba - mah)$$

$$\Rightarrow \boxed{N = \frac{m}{ba - mah}}$$

b)



c)



d) At low prey densities, predators can increase their consumption proportionally. But at high prey densities, predator handling time limits how many prey can be consumed. So predation saturates, and more predators are required to keep prey in check. There is a difference in the Lotka-Volterra model and Type II functional response. It goes from P=r/a to P=r/a(1+ahN). This linearly increases the function of N, which is why we see the line curve upward as prey density increases. This supports that in class, we learned that predators take time to handle and process each prey item, such as eating or digesting.

e) The coexistence equilibrium is stable. Populations are oscillating but settle into the stable equilibrium over time. It is a stable focus. In the phase plot, the vector arrows spiraled into the equilibrium point.

f)

- At these parameter settings, the predator cannot sustain itself. Prey are released from predator pressure and grow uncontrollably. Handling time (h = 0.5) slows predator feeding for predators to survive at the chosen death rate (m = 0.1). The simulation showed that the predator population declined to extinction while the prey reached a stable density. This occurred because the predator's handling time was too long to maintain a population given its death rate. The phase plane showed a directional downward flow across all regions, indicating predator decline.
- Prey growth rate = 1.0, attack rate = 0.5, conversion efficiency = 0.1, handling time = 0.5, predator death rate = 0.1, competition = 0.
- The resulting phase plane showed spiral trajectories converging on a stable coexistence point. The dynamics confirmed that increasing prey density requires disproportionately more predators due to predator handling time, demonstrating a classic Type II response.

Exercise 3

a) 3a) Solve for isoclines and intercepts.

$\frac{dN}{dt} = 0$

Prey isocline

$$rN\left(1-\frac{N}{K}\right) = \frac{aNP}{1+ahN}$$

$$r\left(1-N/K\right) = \frac{aP}{1+ahN}$$

$$r(1-N/K)(1+ahN) = aP$$

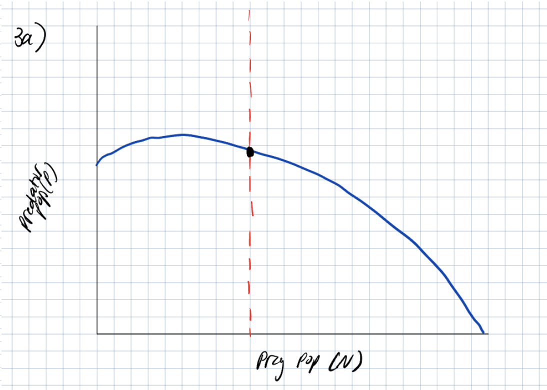
$$\boxed{P = \frac{r}{a}\left(1-\frac{N}{K}\right)(1+ahN)}$$

↳ Non linear curve!

Predator isocline (same as exercise 2)

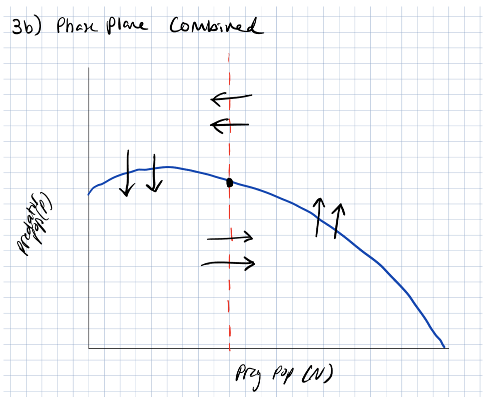
$$N = \frac{m}{ba - mah} \quad (\text{if } ba > mah)$$

- zero at $N = 0$ and $N = K$
- Positive in between
- $N = 0$ and $N = K$ as x-intercepts



- The coexistence equilibrium exists where the prey isocline curve crosses the predator isocline vertical line.
- **Intercepts for the prey isocline: N=0, (0, r/a), and (K, 0)**
- **Intercepts for the predator isocline: (N, 0)**

b)



The arrows show that the population spirals toward the equilibrium. This is damped oscillation and local stability patterns. Prey increase when below the isocline and decrease when above it. Predator increase when right of the predator isocline and decrease when left of it. In this case, the equilibrium point lies just left of the peak of the prey isocline, and the arrows indicate inward spiraling resulting in a stable node.

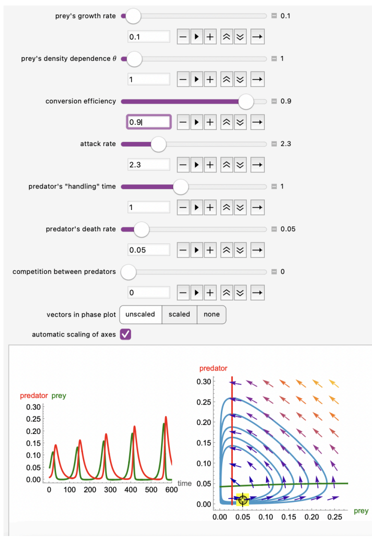
Above the prey isocline → prey decreases
Below the prey isocline → prey increases
Left of predator isocline → predator decreases
Right of predator isocline → predator increases

c) In general, the relative position of the predator isocline and the peak of the prey isocline determines the stability. In one scenario of stable coexistence, the predator isocline intersects the left peak of the prey isocline. This is where prey growth is self-limited due to the density dependence and pressures from predators. This would give a stable node point. In another scenario of unstable coexistence, the predator isocline intersects with the right declining side of the prey isocline. Here, prey growth is slowing and predator growth is not being regulated. This would lead to predator extinction.

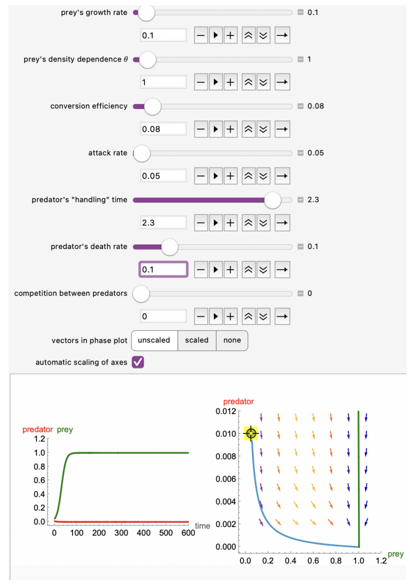
d) This model is a representation of density dependence in prey populations and predator satiation. Naturally, prey populations self-limit through resource depletion and predator feeding rates level off when prey are abundant. Different starting points lead to different paths but eventually converge to the same equilibrium node if a stable system. This is a self-correcting system due to logistic prey growth and predator saturation. In comparison with the Lotka-Volterra model, these models have cycles that orbit endlessly and do not settle. The dynamics never stabilize and any perturbation will shift the cycle. This is a more realistic model.

e) A realistic risk are extinction thresholds where if either population decreases too low and stochastic events occur, the population can be wiped out. That cycle minimum might fall below viable population levels. As well, environmental variability is a risk where seasonal shifts, climate events, and human activity can impact the stability predictions. Two factors that are not included in the models are delayed responses, in which predators respond to prey abundance in a delayed manner. An example could be reproduction time. In addition, spatial factors are not considered in these plots. For example, differing landscapes might effect movement between populations which can impact population sizes and increase instability.

f) left)



right)



- Left leaning predator isocline (b=0.9, a = 2.3, m = 0.05, h =1). Prey and predator show large oscillations, neither go extinct, but population sizes swing widely. System is unstable due to predator response, which is overly aggressive. Essentially, the predators are effective at limiting the prey populations. The prey is seen to rebound quickly, overshoot, and fuel the cycle. The predator does not stabilize at an equilibrium.

- Predator isocline shifted right (b=0.08, a=0.05, m=0.1, h=2.3) Predator crashes to extinction and the prey population increases and plateaus. The predator cannot sustain it's population due to low attack rates, high deaths, and long handling times. Since the isocline lies so far right, only extremely high prey densities could support predators. We see that prey growth slows as it approaches the carrying capacity which reflects density dependence. Eventually. the predators die out.

i) To shift the predator isocline left, increase b and a, and decrease m. To shift the predator isocline right, increase m and h, and decrease b and a.

ii) When shifted left, the population oscillates because the predator over-hunts the prey, which leads to unstable cycling. When shifted right, the predator population goes extinct because the predator cannot maintain itself and the prey evades population control by the predator.