BIOL 564 Lab 5: Predator-Prey Dynamics

Senay Yitbarek

Spring 2025

I. Overview

In this lab you will be exploring the Lotka-Volterra predator-prey equations, including biologically more realistic variants that include density-dependent prey growth and predator satiation. You will primarily be analyzing equations by hand today. You will also explore predator prey dynamics using an interactive Mathematica simulation. You will hand in your write-up for this lab at the beginning of next week's meeting.

For the simulation portions of the lab, you will need access to Mathematica. You can download and install the program for free from UNC Mathematica on your personal computer (Warning: it is a large file, about 1 GB). Once you have installed the software, download the file `lab05_PredPrey_Mathematica.cdf` from the Lab 05: Predator-Prey directory, and open it. Note that the interactive Mathematica simulation uses terms rather than explicit parameters (the Parameters are buried within the code). Try to match the terms with the math as we proceed through the lab. (If you are curious about how the simulation works, you can open the source code `lab05_PredPrey_Mathematica_sourceCode.nb`.)

II. Density-Dependent Prey Growth

Recall the basic Lotka-Volterra predator-prey model covered in lecture:

$$\frac{dN}{dt} = rN - aNP$$

$$\frac{dP}{dt} = baNP - mP$$

where N and P are the prey and predator densities, r is the intrinsic growth rate of the prey, a is the per-capita rate of prey death per predator (the "attack rate"), b is the conversion efficiency of prey items into predators, and m is the per-capita predator mortality rate.

As you'll remember from class, we can solve for the prey and predator isoclines by setting these equations equal to zero and solving for P, which will give the predator abundance that results in a prey growth rate of zero, and for N, which will give the prey abundance that results in a predator growth rate of zero.

$$\frac{dN}{dt} = 0 = rN - aNP \quad \Rightarrow N = 0 \quad \text{or} \quad P = \frac{r}{a}$$

$$\frac{dP}{dt} = 0 = baNP - mP \quad \Rightarrow P = 0 \quad \text{or} \quad N = \frac{m}{ba}$$

As you should be able to see, these are just constants. This means that the prey isocline is simply a horizontal line on the P vs. N phase plot and the predator isocline is simply a vertical line, as you saw in lecture.

However, this basic formulation of the Lotka-Volterra predator-prey equation contains several unrealistic assumptions. To add some biological realism, we can add density dependence to the prey's dynamics using the logistic equation. Adding density dependence, the model becomes:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - aNP$$

$$\frac{dP}{dt} = baNP - mP$$

where K is the prey's carrying capacity.

Exercise 1.

In this exercise, you will analyze the Lotka-Volterra predator-prey model with prey density dependence.

a) What are the units of the parameter a? (Hint: first think about the units of $\frac{dN}{dt}$) (0.5pt)

b) Solve $\frac{dP}{dt} = 0$ and $\frac{dN}{dt} = 0$ in order to identify the isoclines for the predator and prey, respectively. Show your work. (1pt)

c) Plot the isoclines by hand in a phase plane (N vs. P). Identify the equilibrium points (there are 3), where $\frac{dN}{dt} = 0$ and $\frac{dP}{dt} = 0$. (0.5pt)

d) Label the regions of the phase plot with arrows indicating which direction the populations are changing in each of those regions. Label the isoclines themselves as well as their intercepts. (0.5pt)

e) Based on the direction of the arrows in your figure, label the equilibrium points as being stable or unstable. (0.5pt)

f) In Mathematica, choose parameter values to demonstrate this scenario. For now, set the predator's handling time to zero and competition between predators to zero. Keep θ at 1 to maintain classic logistic growth (*i.e.*, density dependent growth). Note: you can move the initial population values by dragging the yellow crosshairs around. Answer this question: How does changing 1) prey growth rate and 2) predator death rate alter dynamics? Please include a example screen shot of your plots in Mathematica. (2pts)

III. Predator Satiation

Another unrealistic assumption of the basic Lotka-Volterra predator-prey model is that the predation rate is independent of the prey density. Predators cannot, in reality, simply take a constant fraction of the available prey—there's a minimum "handling time" to process each prey item.

$$\frac{dN}{dt} = rN - \frac{aNP}{1 + ahN}$$

$$\frac{dP}{dt} = \frac{baNP}{1 + ahN} - mP$$

This type of relationship between prey density and predator feeding rate is known as a Type II functional response.

Exercise 2.

a) Using the same technique as in Ex. 2, identify the isoclines for the predator and prey, respectively, for the Lotka-Volterra model with predator satiation. Show your work. (1pt)

b) Plot the isoclines by hand in the phase plane and identify the equilibrium points. (0.5pt)

c) Label all regions of the plot with the direction of arrows in each region. (0.5pt)

d) Explain what happens to the prey isocline when a Type II functional response is added. What is one example of a biological mechanism leading to a Type II functional response in the predator population? (1.5pts)

e) Based on the direction of the arrows, label the equilibrium points as being stable or unstable. (0.5pt)

f) Explore these dynamics in Mathematica. Make sure you can translate between the parameters in the Mathematica simulation and the symbols used in our equations. Which parameters in the simulation are not included in the equations we saw in lecture? Please include a example screen shot of your plots in Mathematica. (1pt)

IV. Density-Dependent Prey Growth and Predator Satiation

What happens if we add both density-dependent growth of the prey and predator satiation? Combining the changes the we made above, we get:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \frac{aNP}{1 + ahN}$$

$$\frac{dP}{dt} = \frac{baNP}{1 + ahN} - mP$$

Exercise 3.

a) Solve for the prey and predator isoclines. Note that the prey isocline is now a parabola. Find expressions for where the isoclines intersect the N and P axes. Show your work. (1pt)

b) Plot the isoclines in the phase plane, label all the regions of the plot with arrows, and label the equilibrium points and their stability. (1pt)

c) Generalizing based on the results from the other two versions of the model, you might suspect that the coexistence equilibrium will be stable depending on where the predator isocline lies relative to the prey isocline. Describe these two scenarios: when do you think the equilibrium will be stable, and when will it be unstable? (1pt)

d) If you keep the parameter values constant and only change the starting point of the population, what happens to the long-term dynamics? How does this compare with the behavior of the basic version of the Lotka-Volterra equations, with no density dependence and no predator satiation? (1pt)

e) Discuss why the limit cycle behavior predicted in this model may not result in predator-prey coexistence in the real world (even when considering the assumptions of the predator and prey populations only interacting with each other, prey density dependent growth, and a type II predator functional response). Identify two key factors we discussed in class that this model does not incorporate (other than having to do with these assumptions just mentioned) that must be considered when translating its predictions to the real world.

f) In lecture we discussed a "paradox" that involves biological control, in which managers might try to reduce the abundance of a pest species, *e.g.*, the prey in the system, by causing the predator isocline to shift to the left. Explore with simulations what can happen when the predator isocline is shifted in this system. (Hint:

to start, set the prey growth rate to 0.1, conversion efficiency to 0.1, predator handling time to 1, predator death rate to 0.05. Then, start varying the appropriate parameters to change the predator isocline.) Please include a example screen shot of your plots in Mathematica.

i) What would it take, biologically speaking, to shift the predator isocline left or right? (1pt)

ii) What happens to both the predator and prey population sizes over time when the isocline is shifted? Why? Provide a description for when the predator isocline is shifted to the left, and another for when the isocline is shifted to the right. (2pts)