Sarah Bazari BIOL564 Midterm Sheet

Discrete-Time Models- population size measured in discrete time steps, dimensionless

Recursion Equation: $N_{t+1} = f(N_t) = N_t F(N_t)$ **$f(N_t)$; net growth rate, current pop**   **$F(N_t)$; per-capita growth on avg. total pop.**

Net Growth Rate: the percentage increase or decrease of a population based on births and deaths within that timeframe, $f(N) = B - D$, **$f(N) \geq 0$ with $f(0) = 0$; for growth**

Net Per Capita Growth Rate: the average rate a population increases per individual, showing the overall change in population size *per individual* over time, **(Births - Deaths) / Total Population $F(N) \geq 0$**

	Behavior	Stability
$\lambda > 1$	Monotonic growth	Unstable
$0 < \lambda < 1$	Monotonic decay	Stable
$-1 < \lambda < 0$	Oscillatory decay	Stable
$\lambda < -1$	Oscillatory growth	Unstable

$\lambda = f'(0) = 1 + r > 1$	Monotonic unstable
$\lambda = f'(K) = 1 - r$	
if $0 < r < 1$	Monotonic stable
if $1 < r < 2$	Oscillatory stable
if $r > 2$	Oscillatory unstable

1. Set up axes with N_t on the horizontal axis and N_{t+1} on the vertical axis
2. Sketch the diagonal line $N_{t+1} = N_t$
3. Sketch the net growth function, $f(N_t)$
4. Choose an initial condition, N_0, and plot (N_0, N_0)
5. Since $N_t + 1 = f(N_t)$ move vertically (up or down) to the curve $f(N_t)$ and plot (N_t, N_{t+1})
6. Move horizontally to the diagonal line $N_{t+1} = N_t$ and plot (N_{t+1}, N_{t+1})
7. Repeat steps 5-6 until the long-time (**asymptotic**) behavior becomes clear

Exponential Growth Model (Density-Independent): Linear Population Model

$F(N)$ does not depend on population size.

Equation: $N_{t+1} = \lambda N_t$ **If λ > 1, population grows exponentially.**

Solution (for initial condition N0): $N_t = \lambda^t N_0$ **If 0 ≤ λ < 1, population declines.**

Net Per-Capita Growth Rate: $F(N) = 1 + b - d = \lambda$

b is the average number of surviving offspring per individual over a lifetime, $b \geq 0$ **Growth $\lambda \to \infty$ as $t \to \infty, \lambda > 1$**

d is the probability that a given individual dies during the year, $0 \leq d < 1$ **Decay $\lambda \to 0$ as $t \to \infty, 0 \leq \lambda < 1$**

λ is the net per capita growth rate/ratio, $\lambda \geq 1$

Problems: population approaches infinity as time approaches infinity, does not include density dependent factors

Improved Form: $\lambda = 1 + b - d \to \lambda = 1 + b(N) - d(N) \to$ resource competition

Logistic Growth Model (Density-Dependent): Nonlinear Population Model

Equation: $N_{t+1} = N_t + rN_t(1 - \frac{N_t}{K})$ *Net Per-Capita Growth Rate*: $F(N) = 1 + r(1 - \frac{N_t}{K})$ *Carrying Capacity:* $K = \frac{r}{d_2}$, $d_2 = \frac{r}{K}$

Intrinsic Growth Rate: max rate at which population can grow when no environmental limits, $r = b - d_1$

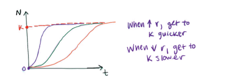
Growth rate slows down as N approaches K, At N = K, growth stops (dN/dt=0)

Bifurcation:point where stability of steady state changes, *period doubling bifurcation* where the cycles keep doubling until chaos and *trans-critical bifurcation* models where the trivial and carrying capacity steady states are interchanged with eachother. *Transcritical Bi:* zone= r<0, r>0, extinction and K against eachother

Equilibria: **Stable $f'(N^*) < 1$**   **Unstable $f'(N^*) > 1$**

Cobwebbing: determines if pops return to equilibrium of the form $N_{t+1} = f(N_t)$, *Logistic model*: $f'(N) = 1 + r - \frac{2rN_t}{K}$, reference tables of first 2 steady states

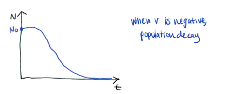
Continuous-Time Models: time variable no longer an integer, appropriate model for when there is constant growth (predator-prey, competition bacterial reproduction, disease models)



Equation: $N(t + dt) = N_t + bN(t)dt - dN(t)dt \to \frac{dN(t)}{dt} = rNt$, **as dt gets large → distance b/w Nt and N(t+dt) larger & dt smaller t→0**

Equilibria: $\frac{dN}{dt} = rN(1 - \frac{N}{K}) = f(N)$ set equal to 0

$N(t) = N_0 \cdot e^{rt}$	Behavior of Continuous Time
$r > 0,\ N(t) \to \infty$ as $t \to \infty$	
$r < 0,\ N(t) \to 0$ as $t \to \infty$	
$r = 0,\ N(t) = N_0$	



Condition for Stability: $f'(N) < 0$ **r→stable when r<0**   **when f'(K) → = -r → stable when r>0 unstable when r<0**

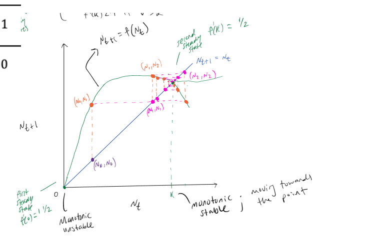
Differential Equation for Logistic Growth: $\frac{dN}{dt} = rN(1 - \frac{N}{K})$

Steady States: N*=0 (plug in 0, trivial state, unstable, extinction), N*=K (plug in 1, nontrivial state, stable equilibrium), occur when $N_t + 1 = N_t = N^*$, growth rate and decay rate are the same, $f(N) = N^* + rN^*(1 - \frac{N^*}{K}) = N^*$

	growth	decay
Discrete $\to N_t$	$\lambda > 1$	$\lambda < 1$
Continuous $\to N_t$	$r > 0$	$r < 0$

Perturbation: $n_{t+1} = N_{t+1} - N^*$

Increase in r → higher oscillations around K



Exponential Continuous Growth: separation variables, integration

Equation: $\frac{dN}{dt} = rN \to \frac{dN}{N} = rd_t$ *Integration)* $\int \frac{1}{N}(dN) = \int rdt \to ln(N) = rt + C$ *Solve N)* $e^{ln(|N|)} = e^{rt} \cdot e^{C} \to N(t) = Ae^{rt}$

Logistic Continous Time: $\frac{dN}{dt} = rN(1 - \frac{N}{K})$

Equilibria: $N^*,\ f(N^*) = 0$ **Stable $f'(N^*) < 0$**   **Unstable $f'(N^*) > 0$**

Allee Effects (Density Dependence at Low Population Sizes): per-capita growth rate decreases at low densities due to size (predator) 2) mating 3) dispersal pollination. ***Non-Critical Depensation*** (Weak Allee Effect) growth rate still positive at low N, ***Critical Depensation*** (Strong Allee Effect) growth rate negative at low N → possible extinction threshold. ***Allee Threshold***: population size below which extinction occurs, u with -K < u < K. No Allee Effect with ***compensation*** (growth rate highest at small pop size N)

Model: $\frac{dN}{dt} = rN(1 - \frac{N}{K})(\frac{N}{A} - 1)$

$$\frac{dN}{dt} = rN(1-K) - \overbrace{pN}^{\text{saturational predation}} = \overbrace{g(N)}^{\text{logistic}} - \overbrace{p(N)}^{\text{predation}} = f(N)$$

$$\frac{dN}{dt} = f(N) = \underbrace{N}_{\text{initial pop size}}\underbrace{F(N)}_{\text{net per capita}}$$

$$\frac{dN}{dt} = N(G(N) - P(N)) = NF(N) \Big\} \text{linearized } G(N) = r$$

$$G(N) = r(1-K)$$

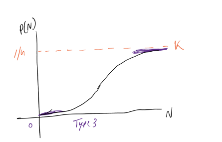
Chaos: Chaotic dynamics are deterministic (no random terms), aperiodic (no pattern), bounded (does not go to infinity), and depends on the initial conditions.

Predator-Prey/Pest Models: relationship between predator intake and prey density, pests lead to changes in steady states

Holling Type III Response: Functional Response model density-dependent predation rates, the intake of a consumer as a function of food density $p(N) = \frac{aN^2}{1+ahN^2}$, → Simplified $p(N) = \frac{a}{\frac{1}{N^2}+ah}$ a is attack rate, h is handling time (time consuming prey)

Spruce Budworm Model: $\frac{dN}{dt} = G(N) - P(N)$, #'s kill up to 80% of mature trees, examine when p(N) increases linearly w/ population size, Predation linear b/c predator limited in consumption, limited to amount budworms can eat→ **p(N) saturate at high budworm densities, when N is high impossible to consume all**

Budworms grow Logistically but also experience predation by birds at rate p(N): $\frac{dN}{dt} = rN(1 - \frac{N}{K}) - pN = g(N) - p(N) = f(N)$ $N \to \infty,\ p(N) = \frac{1}{h}$ $N \to 0,\ p(N) = 0$



Never reaches 0 or 1/h(K)   **Budworm rare → predation rate near 0**   **Budworm common → predation rate saturated (levels off at 1/h)**

Equilibria: **G(0)=0**   **G(N*)=0, N*=K**

Bifurcation: Bistability occurs when; $K_1 < K < K_2$ b/c 2 locally stable equilibrium separated by unstable equilibrium **; if K crosses K_2 budworm pop explodes; if K drops below K_1 pop collapses** *Hysteresis:* gradual jump to new equilibrium, current state of the population depends on the state of the past

Saddle-Node Bifurcation: if starts at N*3 or N*1=memory

Phase Line Diagram: plot $\frac{dN}{dt} = f(N)$ versus N and note when $f(N)$ is positive or negative

What implications might chaotic dynamics have for real populations?

Making predictions is challenging (ex: intro of species w hopes of controlling another pop. but it leads to an extinction of that species, ex: climate models)

What happens to the population size as K increases to a and then above K2?

Equilibria of N*1, vanishes and pop goes to N*3

What happens now to the population size as K decreases towards and then below K1?

N*3→N*1