lab02

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Exercise I

Exponential Growth w/

$$\lambda$$

Manipulating Parameter Values

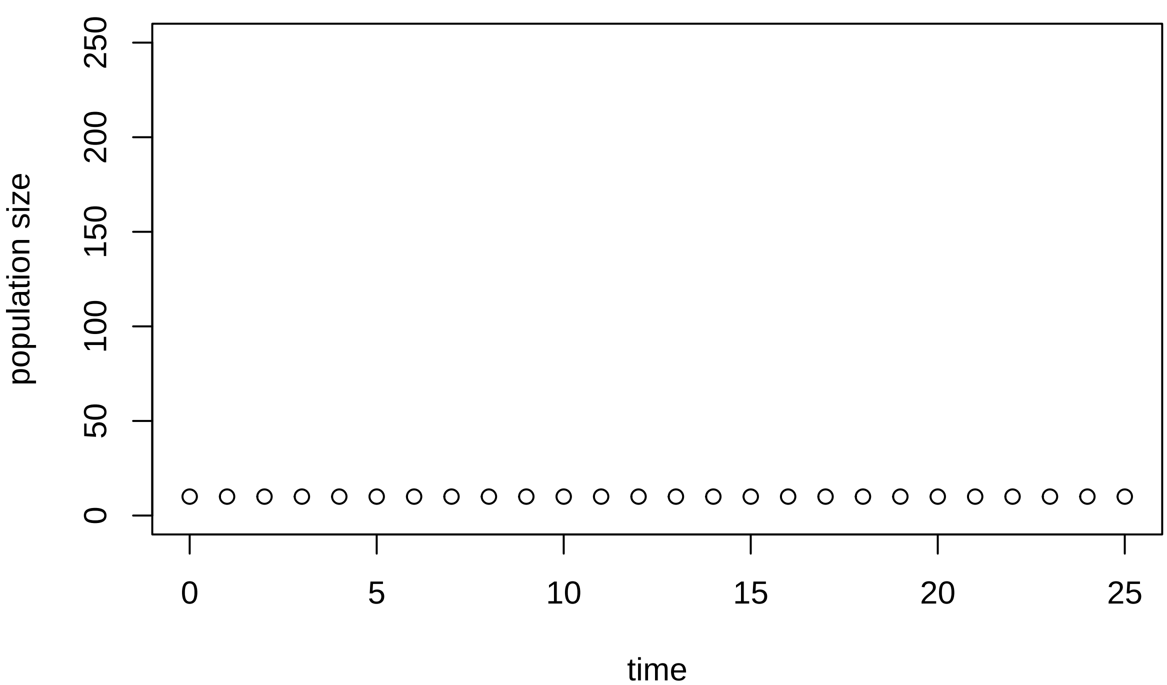
1. a)

$$\lambda = 1$$

At this value, there is no population growth or decay occurring, which is shown by the lack of change in population over time in the model. We see that the default parameter values has a lambda of 1.1, which shows a slight increase in population over time, but when changed to value 1, this growth halts. This results in a linear model rather than an exponential model. I also manipulated the x and y axes to better represent the data.

```
discreteExponential(N0=10, lambda=1, tEnd=25, axisNMax=250, autoScale=FALSE, useLogScale=FALSE)
```

```
## discreteExponential(N0=10, lambda=1.0, tEnd=25, axisNMax=250, autoScale=FALSE, useLogScale=FALSE)
```



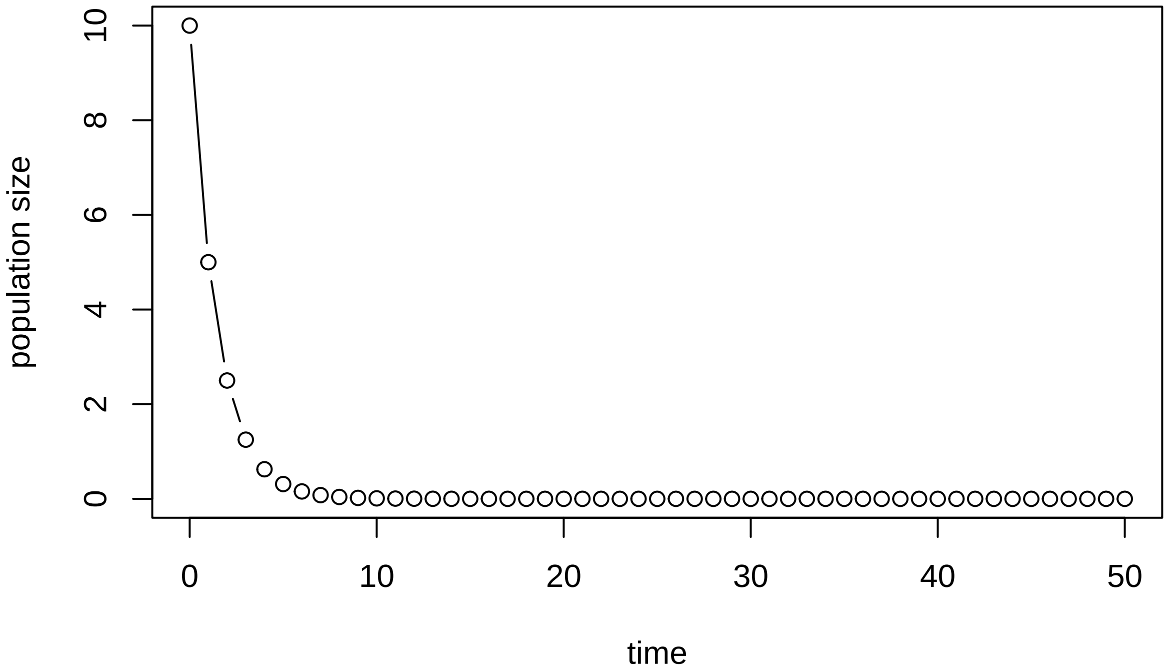
1. b)

$$\lambda < 1$$

We see that the population is exponentially decaying. A growth rate under 1 is a decaying population. In this example, I used a growth rate of 0.5 and manipulated the x and y axes from the default values to visualize the decay (i.e y axis at population size 10 over 50 time steps.)

```r
discreteExponential(N0=10, lambda=0.5, tEnd=50, axisNMax=10, autoScale=FALSE, useLogScale=FALSE)
```

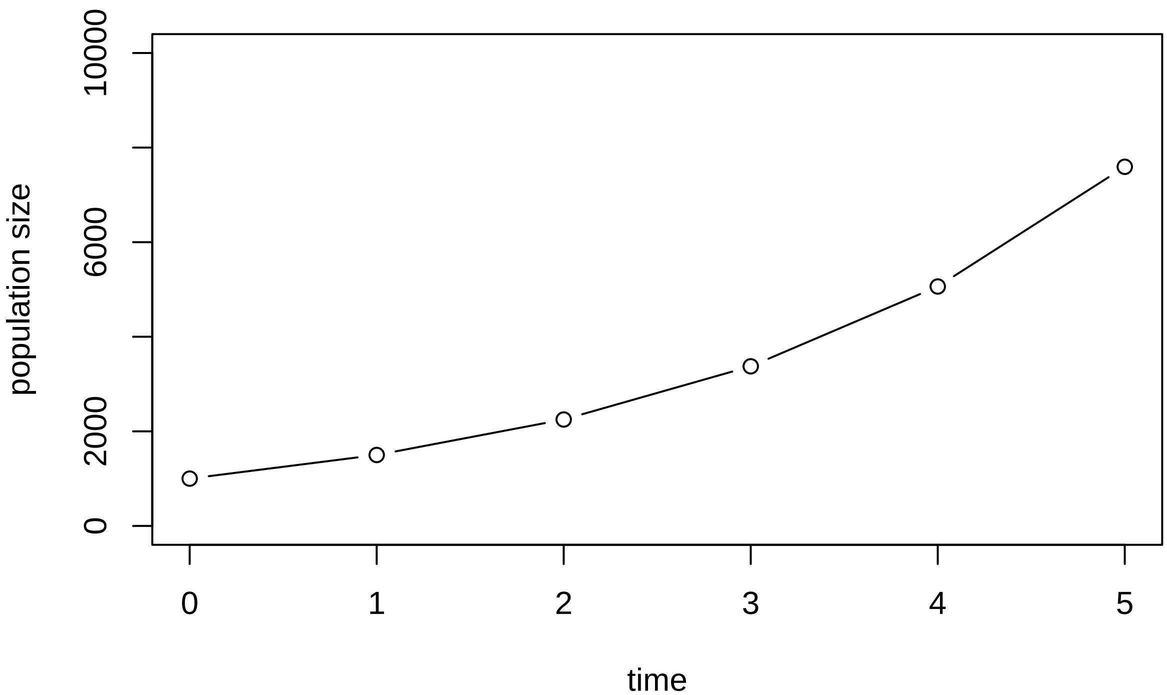
```
## discreteExponential(N0=10, lambda=0.5, tEnd=50, axisNMax=10, autoScale=FALSE, useLogScale=FALSE)
```



1. c) Changing the initial population size changes the y-intercept. For example, I used an initial population size of 1000, which changed the y-intercept to 1000. Decay and growth only depend on the growth rate, which I have set to grow at 1.5. I manipulated the x and y axes to better represent the data.

```r
discreteExponential(N0=1000, lambda=1.5, tEnd=5, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## discreteExponential(N0=1000, lambda=1.5, tEnd=5, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```



Exercise II

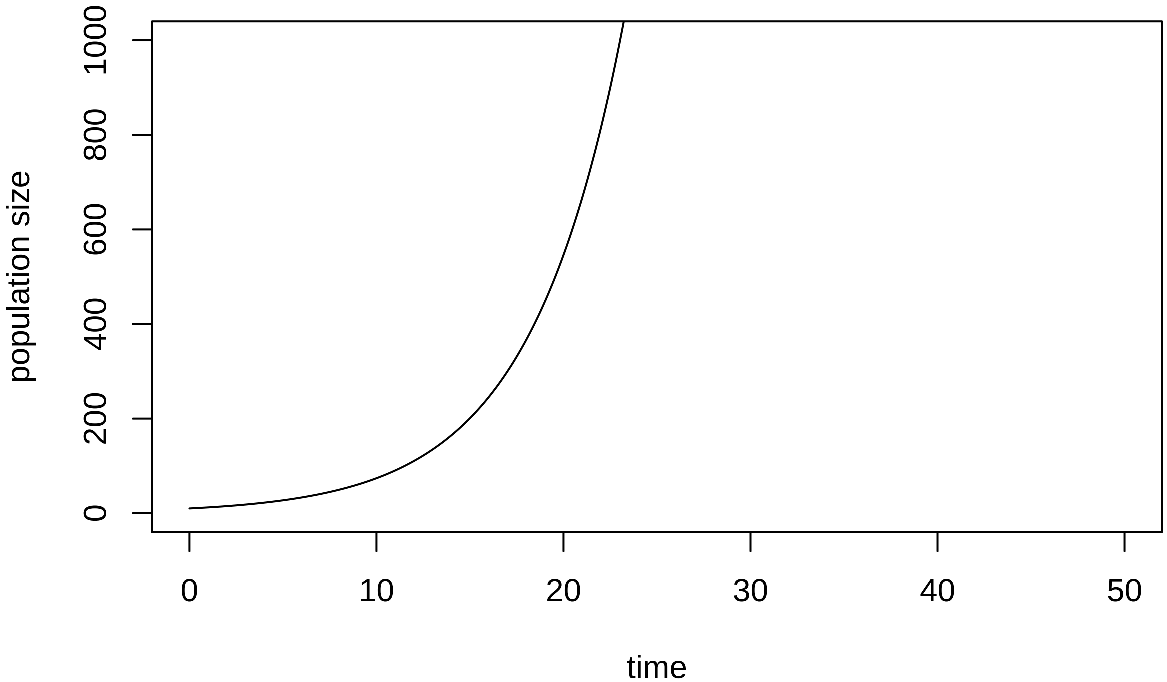
Continuous-Time Exponential Growth

Manipulating Parameter Values

2. a) In the continuous-time model of exponential growth, we can see that the changes in the population value are not denoted by exact points on the graph, rather they do not exist in this model and a smooth line is shown. Discrete time models and continuous time models are represented by differing representations of time-steps. Time is continuous in this model and we can clearly see an end to the population growth denoted by final t. This is where the line on the model stops.

```
continuousExponential(N0=10, r=0.2, tEnd=50, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```

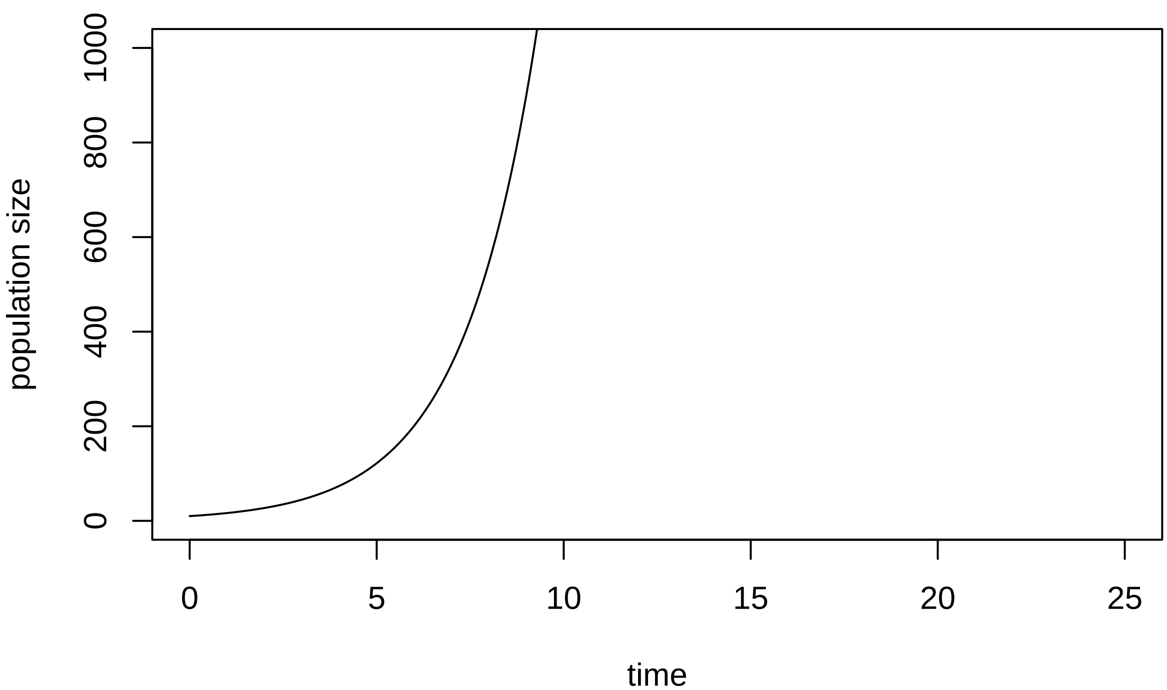
```
## continuousExponential(N0=10, r=0.2, tEnd=50, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```



2. b) In changing the value of r to 0.5, we see that the population grows much quicker. This also changes where the line is asymptotic on the x axis. As well, the line is much steeper.

```
continuousExponential(N0=10, r=0.5, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```

```
## continuousExponential(N0=10, r=0.5, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```



Exercise III

Closed-Form Equation for Discrete Growth and Continuous Growth

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- The discrete growth equation is given by:

$$N_t = \lambda^t N_0$$

- For no growth from the discrete model:

$$\lambda^t = 1$$

$$\lambda = 1$$

- The continuous growth equation is given by:

$$N_t = N_0 e^{rt}$$

- For no growth from the continuous model:

$$e^{rt} = 1$$

$$r = 0$$

So,

$$\lambda = 1, r = 0$$

- Derivation of the Relationship

$$\lambda^t = e^{rt}$$

$$ln(\lambda^t) = rt$$

$$t * ln(\lambda) = rt$$

Assuming t $>$ 0

$$ln(\lambda) = r$$

$$\lambda = e^r$$

So,

$$r = ln(\lambda)$$

and

$$\lambda = e^r$$

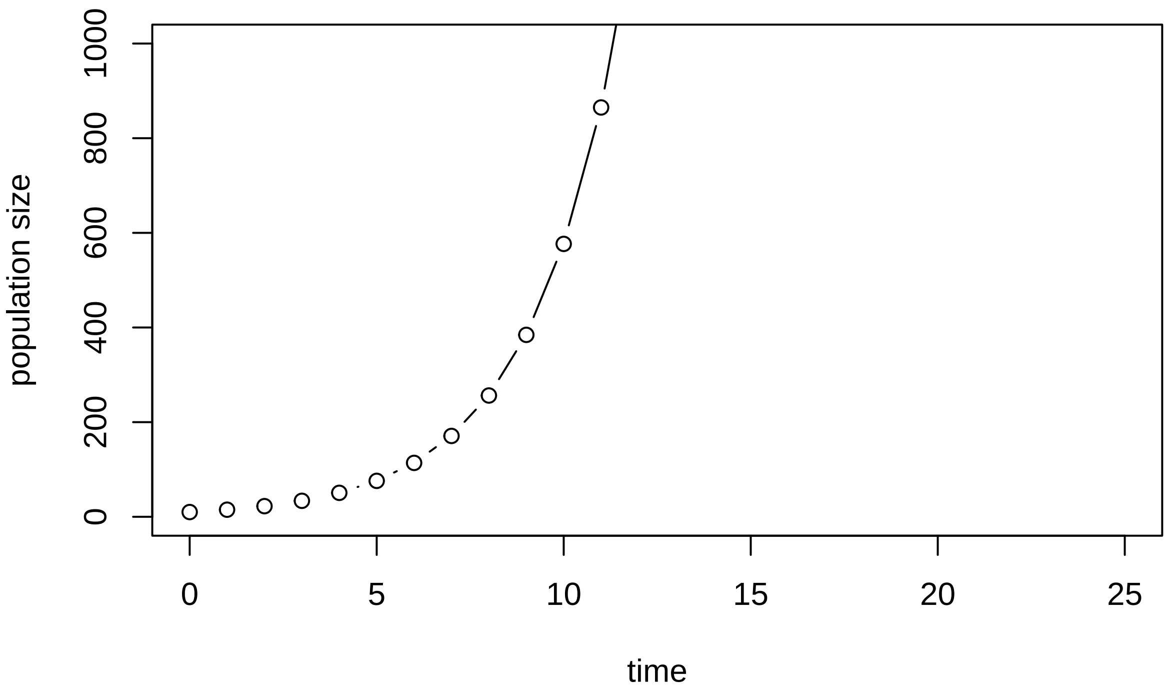
- **In conclusion, the mathematical relationship is: r is equal to the natural log of lambda.**

Simulating Discrete and Continuous Growth Processes

Plot 1 and 2: Discrete vs. Continuous Growth (lambda > 1, r> 0)

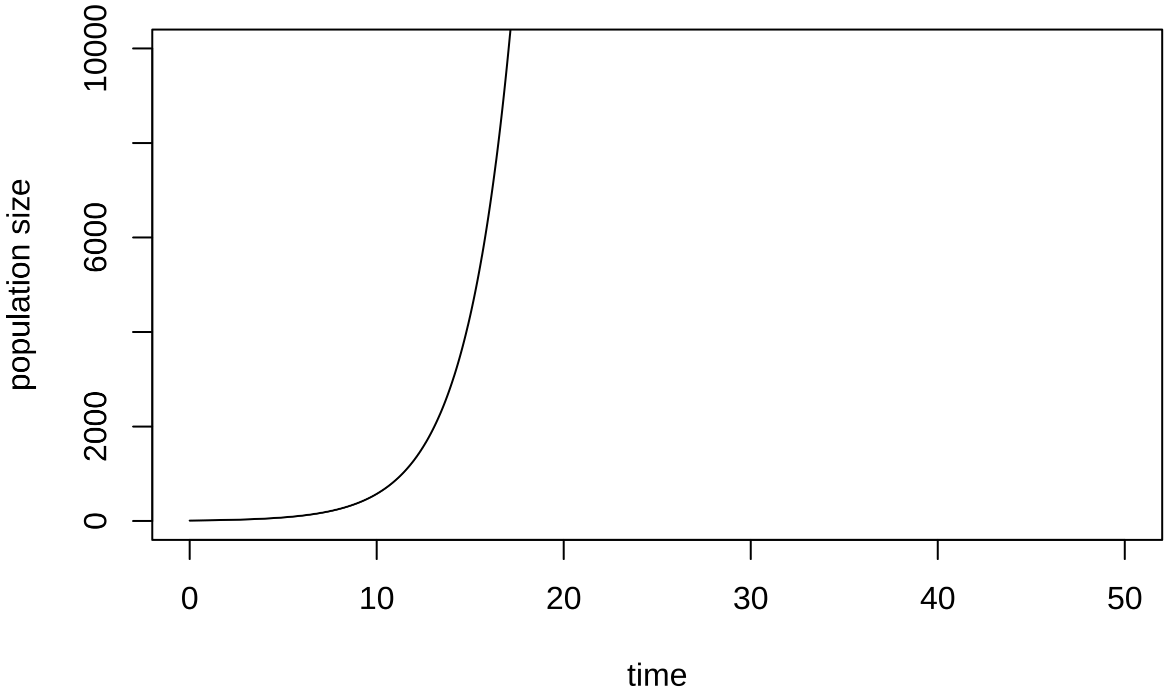
```
# Discrete Growth
discreteExponential(N0=10, lambda=1.5, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```

```
## discreteExponential(N0=10, lambda=1.5, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```




```
# Continuous-Time Model
continuousExponential(N0=10, r=0.405, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## continuousExponential(N0=10, r=0.4, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```



- In plots 1 and 2, I modeled a discrete-time plot with

$$\lambda = 1.5$$

  and

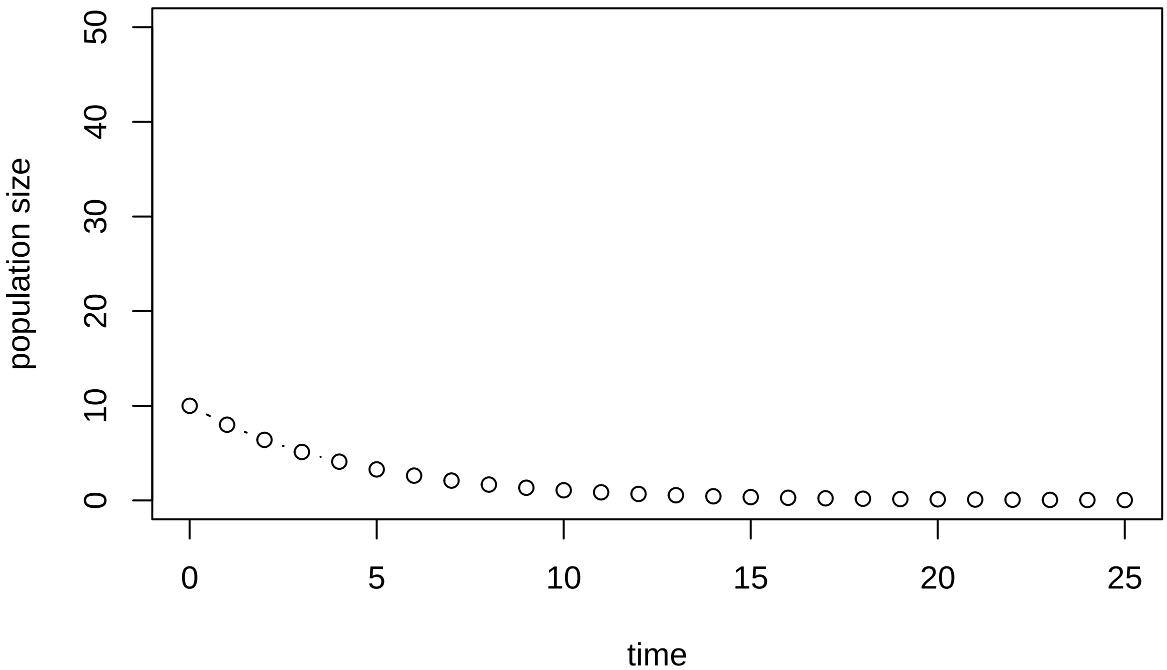
$$r = ln(1.5)$$

  to represent the relationship between the two parameters. In the plots, we can see that these two parameters result in similar growth.

Plot 3 and 4: Discrete vs. Continuous Decay (0 < lambda < 1, r < 0)

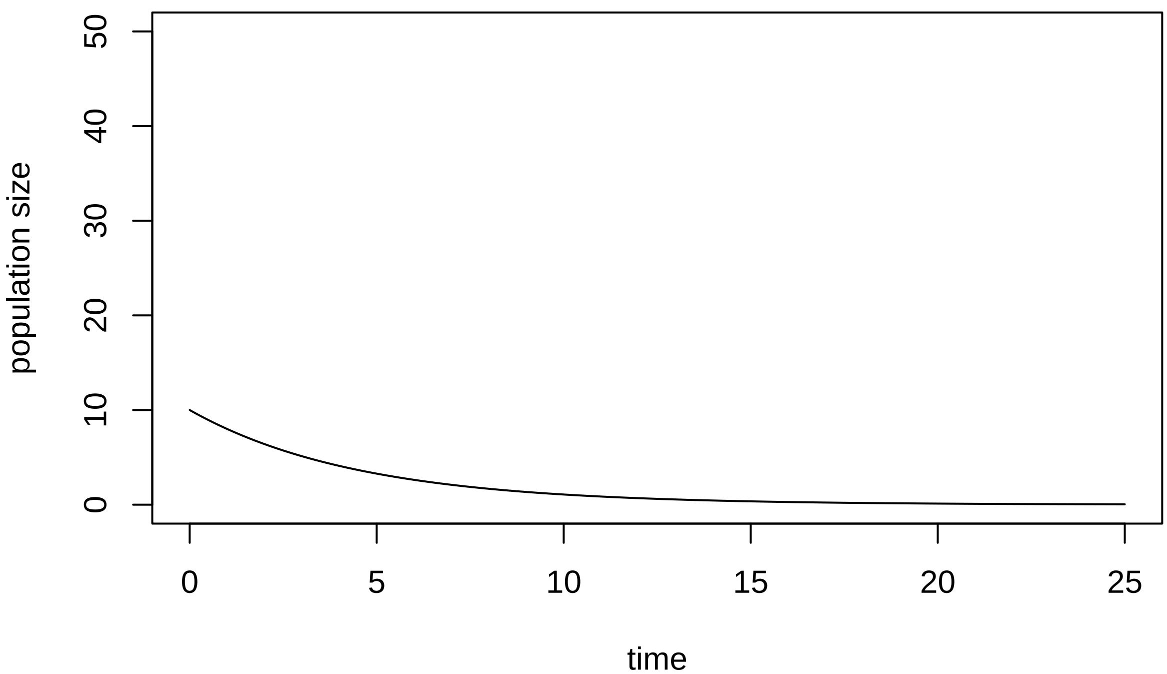
```
# Discrete Decay
discreteExponential(N0=10, lambda=0.8, tEnd=25, axisNMax=50, autoScale=FALSE, useLogScale=FALSE)
```

```
## discreteExponential(N0=10, lambda=0.8, tEnd=25, axisNMax=50, autoScale=FALSE, useLogScale=FALSE)
```



```r
# Continuous Decay
continuousExponential(N0=10, r=-0.223, tEnd=25, axisNMax=50, autoScale=FALSE, useLogScale=FALSE)
```

```
## continuousExponential(N0=10, r=-0.2, tEnd=25, axisNMax=50, autoScale=FALSE, useLogScale=FALSE)
```



- In plots 3 and 4, I modeled a discrete-time plot with

  $$\lambda = 0.8$$

  and

  $$r = ln(0.8)$$

  to represent the relationship between the two parameters. In the models, we can see that both predict similar population decline.

Exercise IV

Logistic Model in Continuous Time

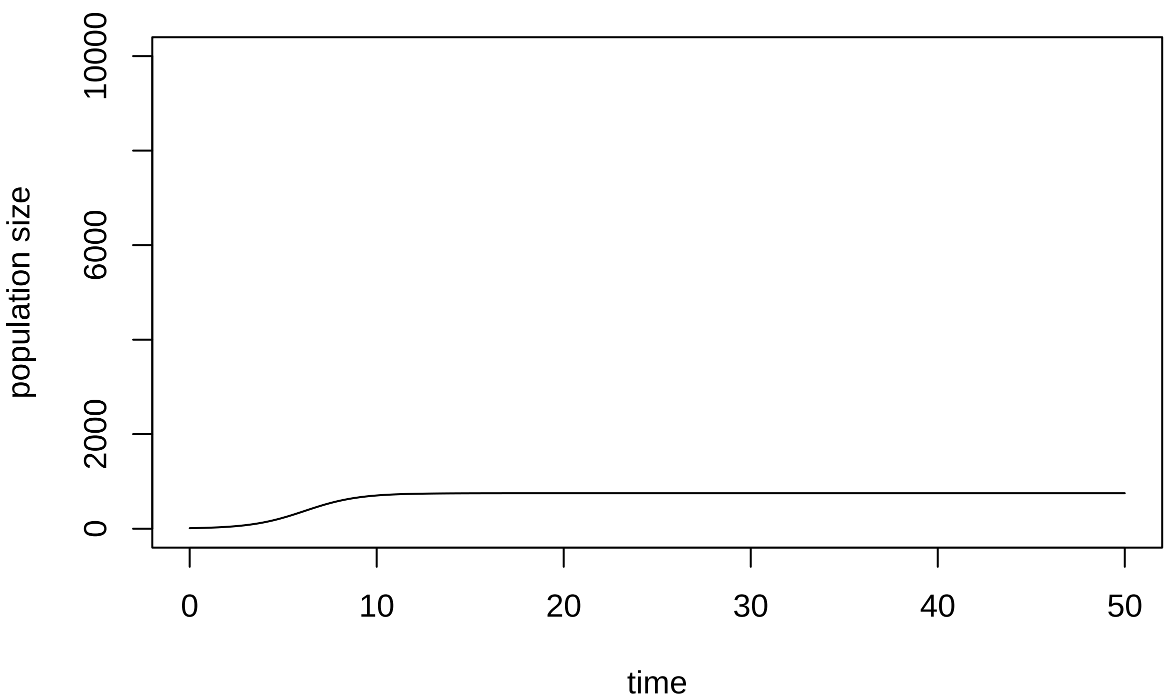
Comparing Behaviors of Logistic and Ricker Models

$$r_0 = 0.7$$

- Changing the growth parameter of the logistic model and Ricker model does not change the predictions of the two models at r = 0.7. This is represented by the logistic model having a continuous smooth line, while the Ricker model shows distinct data points at each time step.

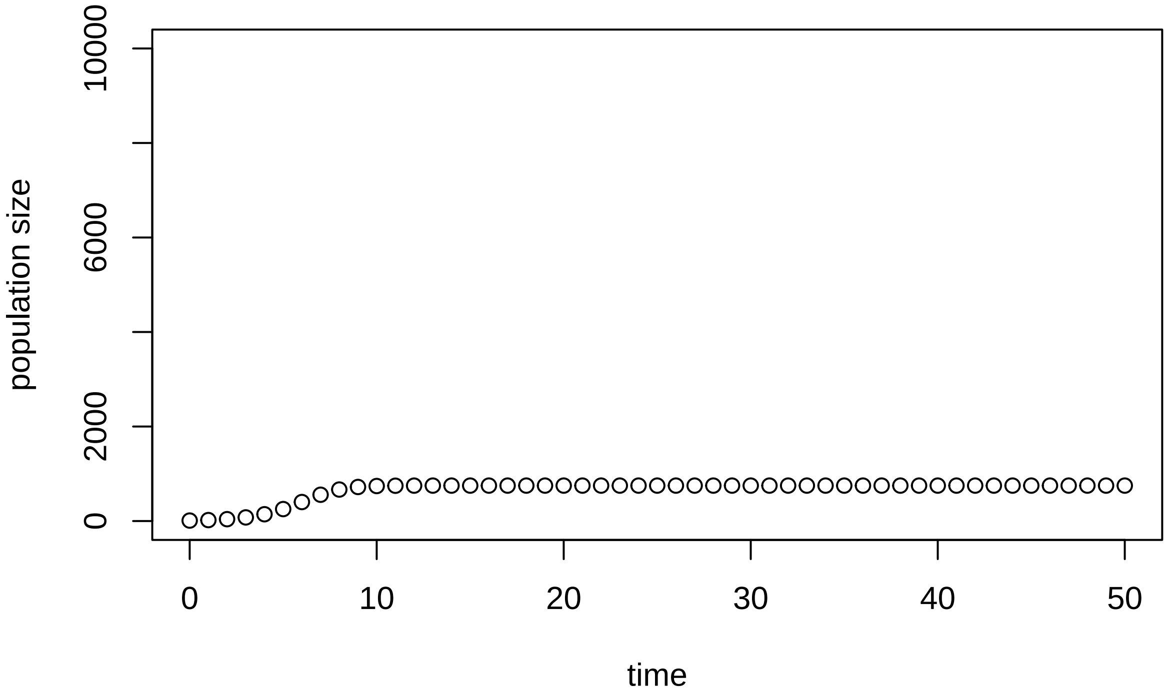
```
logistic(N0=10, r0=0.7, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## logistic(N0=10, r0=0.7, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```




```
ricker(N0=10, r0=0.7, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## ricker(N0=10, r0=0.7, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

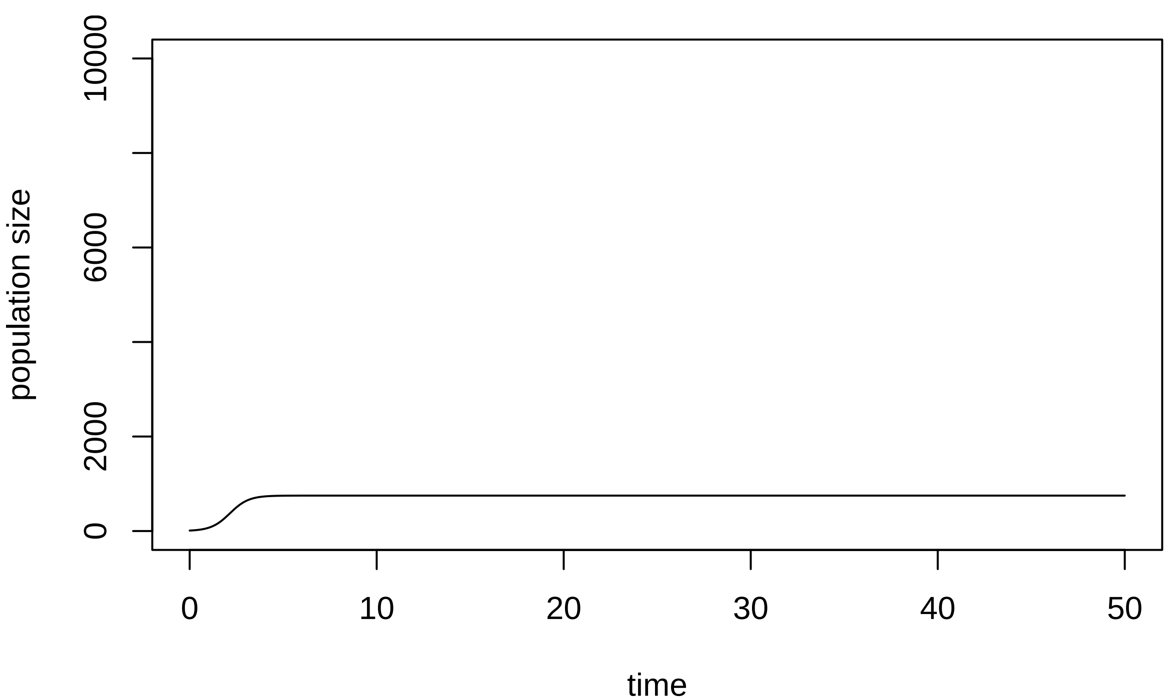


$r_0 = 2.0$

- We start to see differences in representations of the data at r = 2.0. The logistic model seems to lack the ability to represent the subtle oscillations that are shown in the Ricker model at each time step. The Ricker model provides a complexity the the logistic model is not showing.

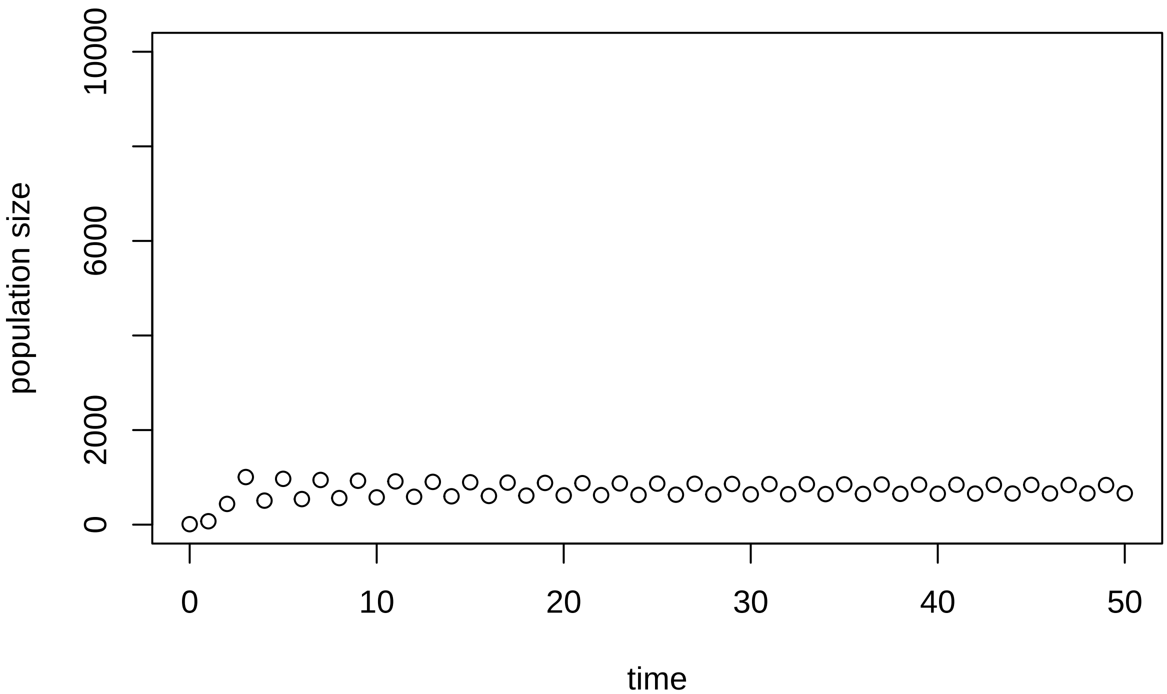
```
logistic(N0=10, r0=2.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## logistic(N0=10, r0=2.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```



```r
ricker(N0=10, r0=2.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## ricker(N0=10, r0=2.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

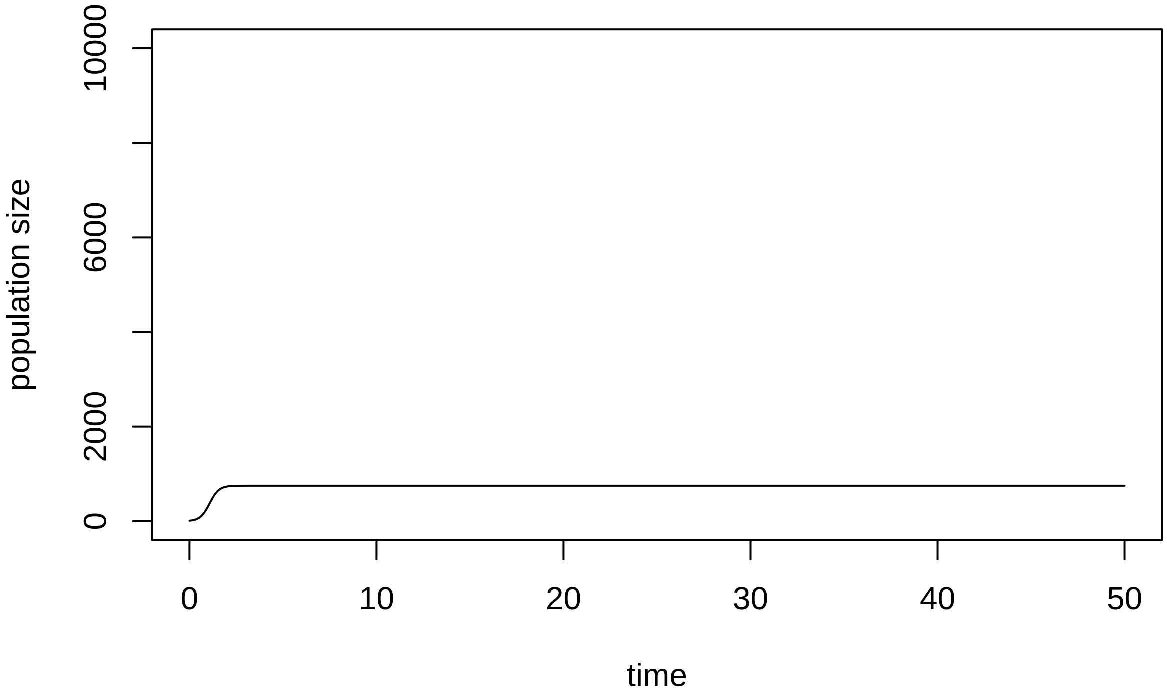


$r_0 = 4.0$

- In these plots, we see even more variation between the logistic and Ricker model. The oscillations are greater in the Ricker model, and the logistic model has not changed much from the previous differing r values.

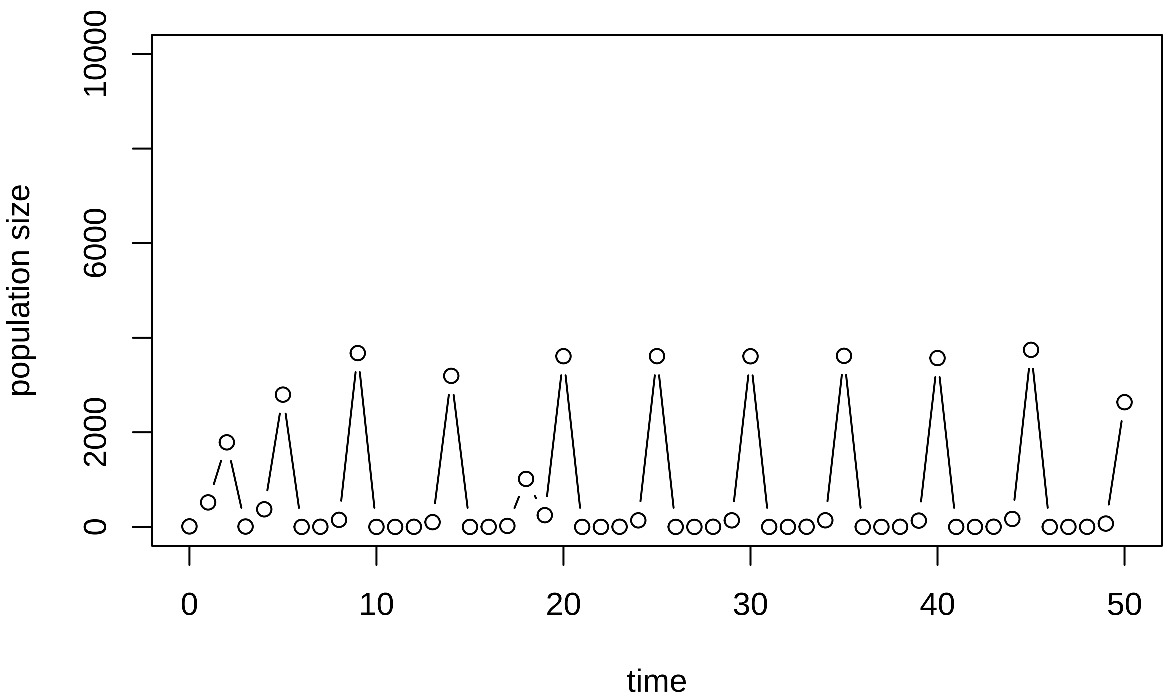
```r
logistic(N0=10, r0=4.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## logistic(N0=10, r0=4.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```




```r
ricker(N0=10, r0=4.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```

```
## ricker(N0=10, r0=4.0, K=750, tEnd=50, axisNMax=10000, autoScale=FALSE, useLogScale=FALSE)
```



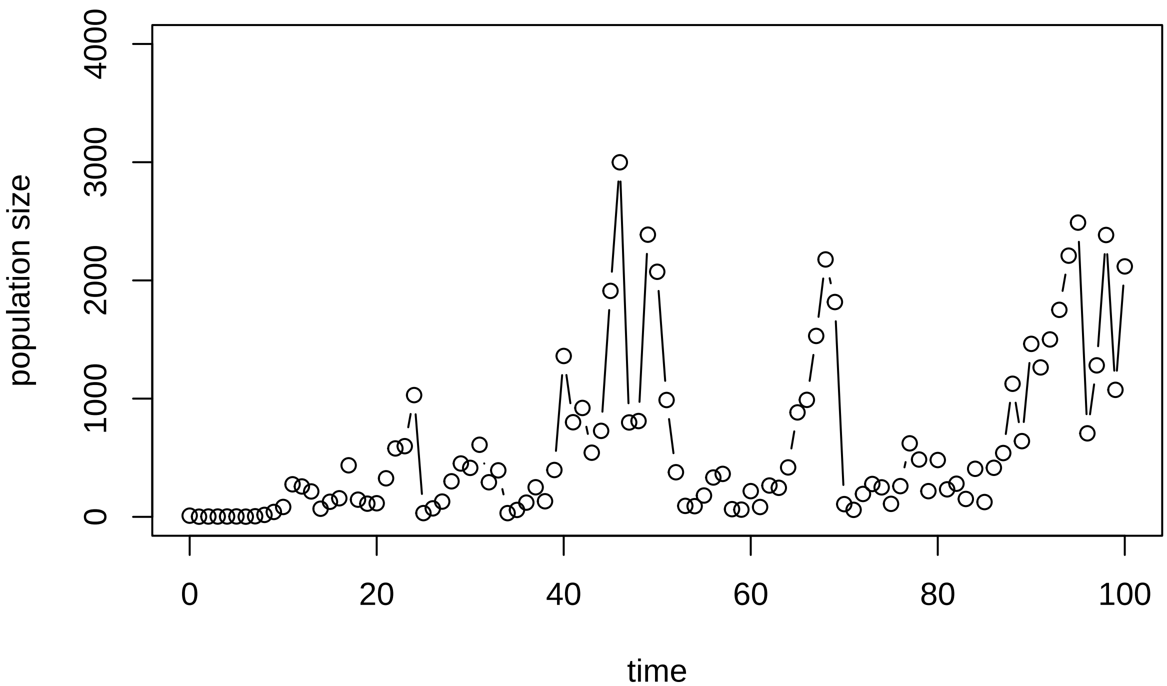
Exercise V

Applying a Stochastic Lambda

- In the stochastic model below, we see that a majority of the data points are clustered around each other and there seems to be somewhat of an oscillatory pattern. This population seems to be much more prone to stability. This reflects the uncertainty of the model and lack of predictive nature. In the deterministic model, there is much more spacing between the data points, and an oscillatory pattern is more prevalent and easily visualized. The carrying capacity governs both populations, but the deterministic model is much more unstable. We could say that the carrying capacity impacts the deterministic model population much more.

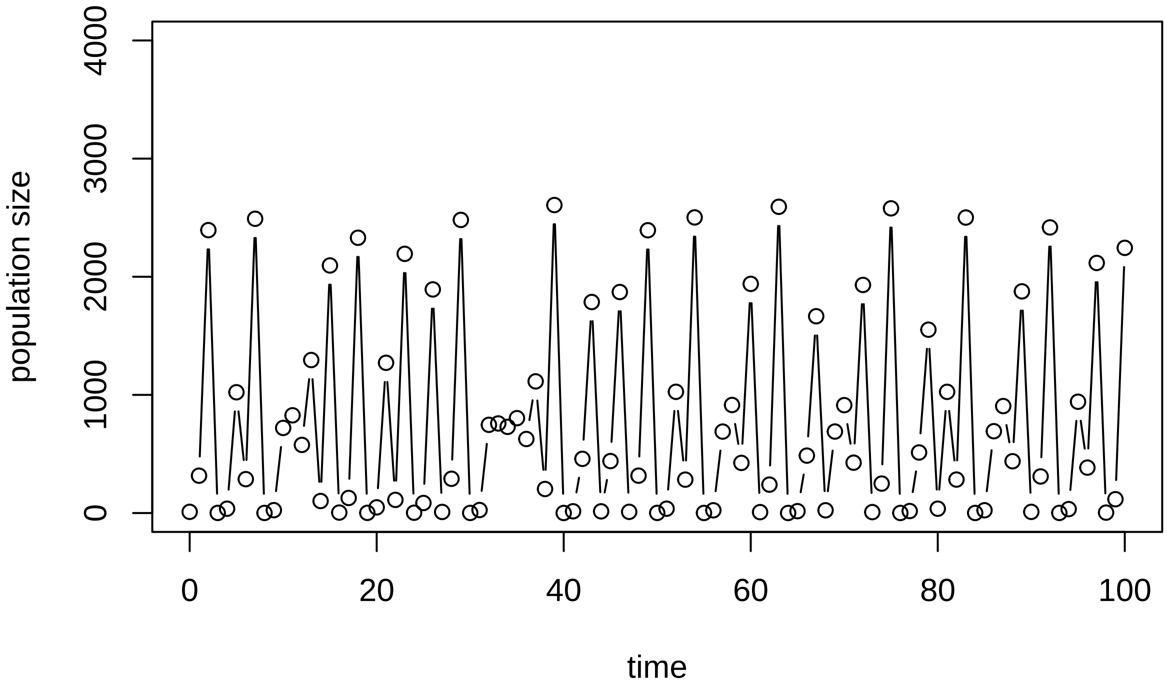
```
rickerStochastic(N0=10, r0=0.7, K=750, lambdaSD=1.0, tEnd=100, autoScale=FALSE, axisNMax=4000, useLogSca
```

```
## rickerStochastic(N0=10, r0=0.7, K=750, lambdaSD=1.0, tEnd=100, autoScale=FALSE, axisNMax=4000, useLog
```




```
ricker(N0=10, r0=3.5, K=750, tEnd=100, axisNMax=4000, autoScale=FALSE, useLogScale=FALSE)
```

```
## ricker(N0=10, r0=3.5, K=750, tEnd=100, axisNMax=4000, autoScale=FALSE, useLogScale=FALSE)
```



Exercise VI

Understanding Ricker Behavior

- It can be assumed that the Ricker model is meant to represent populations that breed at distinct times, while the continuous logistic model represents populations that continuously breed. The continuous logistic model would be useful to visualize growth in populations like humans, ape species, and rodent species since they exhibit continuous breeding patterns. The Ricker model would be useful to visualize growth in populations like bird species that breed during the spring, female deers that breed within a specific period of the year, and species that have distinct life stages like butterflies.

Exercise VII

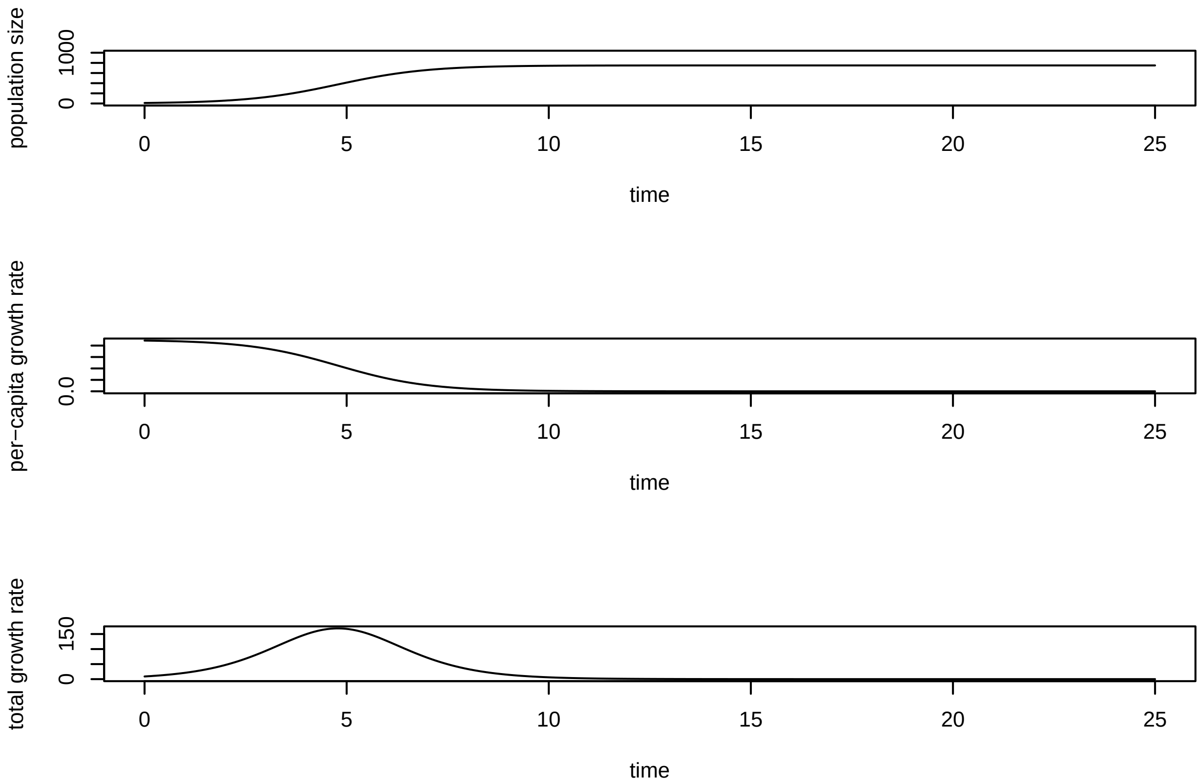
Logistic Equation with two extra graphs

```
7.

    a)  I used an N0 of 10, r0 of 0.9 and K of 750. The total growth
        rate is proportional to the population size. Also, the total
        growth rate is calculated by multiplying the per-capita
        growth rate by the population size.
```

```r
logisticThreePlots(N0=10, r0=0.9, K=750, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```

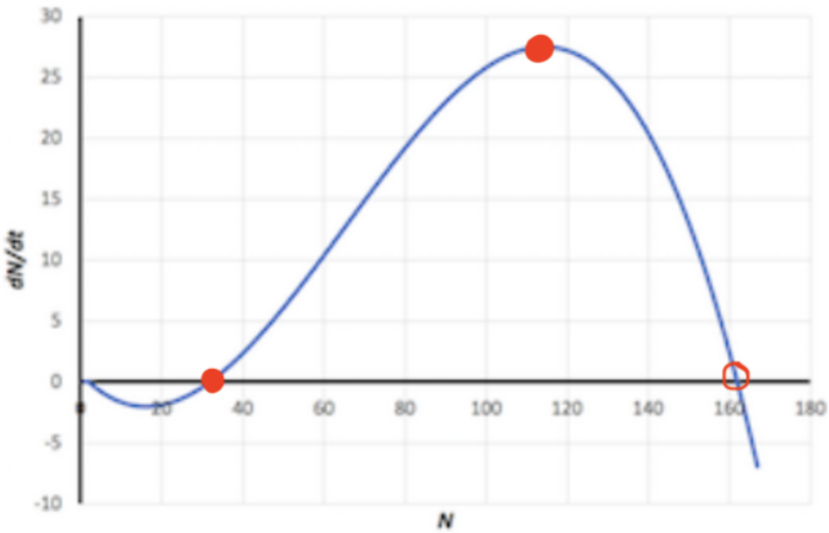
```
## logisticThreePlots(N0=10, r0=0.9, K=750, tEnd=25, axisNMax=1000, autoScale=FALSE, useLogScale=FALSE)
```



Exercise VIII

Plotting relationship between population growth and population density

8. a) This model seems to be representing a population that is regulated. This is a logistic model where we see initial rapid growth followed by a decrease when the population grows. This can be assumed because there is a clear plateau for the carrying capacity where the growth rate plummets at the population size grows to be too large. This is the maximum population size the environment can handle.
9. b) This model does not seem to exhibit an Allee effect, and rather logistic growth, because the population does not remain small with a low growth rate. The model represents “normal” conditions where there is negative correlation between the growth rate and the population density. The Allee effect would exhibit a positive correlation between growth rate and population density. A biological explanation for an Allee effect could be the lack of genetic diversity in a population causing a decrease in fitness as the population size decreases. This would put the population at risk of extinction.
10. c) Sketching the logistic model. In red are the 3 equilibrium points where there is no change in the population density or growth rate. The point where the population is small and has not started growing is stable. The point at the carrying capacity is a stable point. This is normal considering the population starts to decrease once it hit the maximum. The final point after the population declined is unstable and seeming to be near extinction because of the steep decrease after hitting the carrying capacity.


30
25
20
15
10
5
0
-5
-10
dN/dt
0
20
40
60
80
100
120
140
160
180
N