Assignment 2

October 29, 2017

COMP 4107 Fall 2017
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NOTE: in our plots, the x-axis label should read "epoch" instead of "fold number". In each training epoch, we are using the entire training set using k-fold cross validation.

1 Running Our Code

To train and test our network, simply run one of the following commands. To avoid running the full 300 epochs, use the second command. The program required that the MNIST dataset is available in a directory named mnist-data. We have included that directory with the data inside it in as part of our submission.

```
# Run 300 epochs
python3 feedforward.py
# Run X epochs
python3 feedforward.py X
```

You will need to have numpy, scipy, and matplotlib installed.

2 Performing K-fold Cross Correlation

In our training process we used K-fold cross correlation to validate the accuracy of our classifier, ensuring we don't have significant generalization errors. It also helped us ensure we are not underfitting or overfitting. The following code demonstrates that we used K-fold cross correlation during training.

In each epoch, we use a different subset of training data to validate our classifier's output. The MNIST training set contains 60000 elements. Assuming we do 10-fold cross validation and we run 100 epochs: in the first epoch, we use the first 6000 elements of the training set for validation, and the remaining 54000 elements to train. In the second epoch, we use the second subset of 6000 elements for validation, and the remaining 54000 elements to train, and so on until we complete 100 epochs. After we use the final 6000 subset of training elements for validation, we roll back to using the first subset of 6000.

In [ ]:
```
num_folds = int(num_training_examples / net.batch_size)
train_error = []
test_error = []
cv_error = []

for k in range(num_folds):
    # Deep copy the training set because we want to manipulate it
    x = np.copy(train_x)
    y = np.copy(train_y).reshape(1, 60000)
    y_oh = np.copy(train_y_onehot)

    # Indices of the cross-validation (test) examples
    cv_start = net.batch_size * k
    cv_end = net.batch_size * (k + 1)

    # Get the cross-validation (test) examples
    cv_columns = [x for x in range(cv_start, cv_end)]
    cv_x = x[:, cv_columns]
    cv_y = y[:, cv_columns]
    cv_y_oh = y_oh[:, cv_columns]

    # Remove cross-validation (test) examples from the training set
    x = np.delete(x, cv_columns, 1)
    y = np.delete(y, cv_columns, 1)
    y_oh = np.delete(y_oh, cv_columns, 1)

    # Shuffle the training examples for better results
    x, y, y_oh = net.shuffle_training_set(x, y, y_oh)

    training_batches = filter(lambda fold: fold != k, range(num_folds-1))
    for i in training_batches:
        batch_start = net.batch_size * i
        batch_end = net.batch_size * (i + 1)

        x_batch = np.array([row[batch_start:batch_end] for row in x])
        y_batch = np.array([row[batch_start:batch_end] for row in y_oh])

        a, o = net._propagate(x_batch)
        d = net._backpropagate(y_batch)
        net._adjust_weights()
        net._adjust_biases()
```

3 Using Weight Decay for Regularization

We added a hyperparameter called weight_decay which we used to regularize our weights. The following code demonstrates that we used weight decay when updating our network's weights.

```
for weight, delta, activation in zip(self._w, self._d, self._a):
    regularization = (learn_rate * self.weight_decay) * weight
    new_w = weight - learn_rate * np.dot(delta, activation.T) - regularization
    new_weights.append(new_w)
```

The following code shows how we used the weight decay hyperparameter again when updating our network's biases:

```
for bias, delta in zip(self._b, self._d):
    regularization = (learn_rate * self.weight_decay) * bias
    new_b = bias - learn_rate * (np.sum(delta, axis=1)).reshape(bias.shape) - regularization
    new_biases.append(new_b)
```

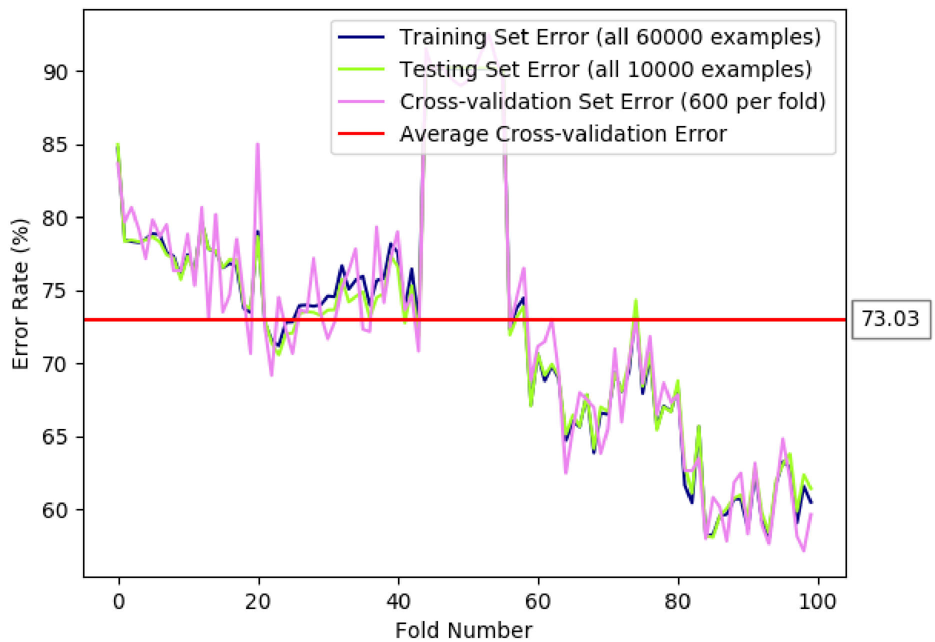
The use of weight decay helped reduce the generalization error by ensuring large weights do not get out of control.

4 Investigating Performance for Different Numbers of Hidden Layers

For each of the following experiements, we used:

- Learning Rate: **0.005**
- Weight Decay: **0.2**
- Number of folds: **100**
- Epochs: **100**

4.1 Zero Hidden Layers



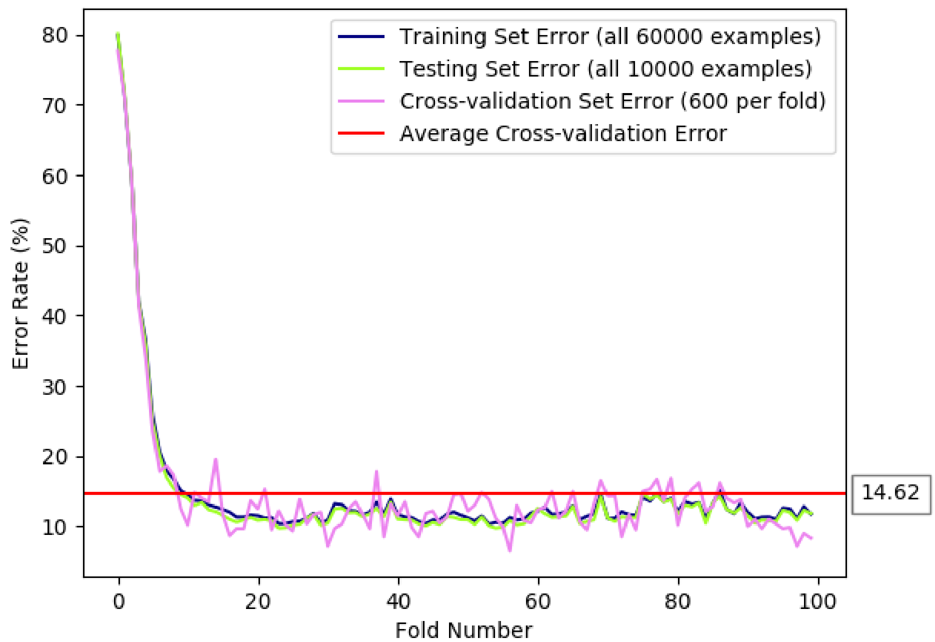
Architecture

- Input layer: 784
- Output Layer: 10

Result Average cross validation error: **73.03%**

First, we tried training a network with no hidden layers. The poor performance indicates that the 10-label MNIST classification problem warrants **at least** one hidden layer for accurate classification.

4.2 One Hidden Layer



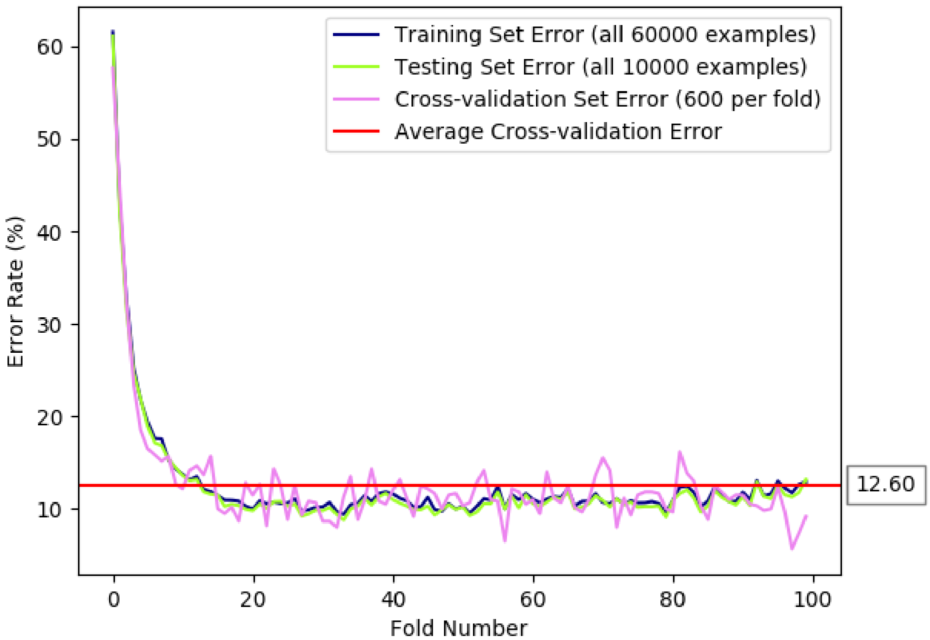
Architecture

- Input layer: 784
- Hidden Layer: 60
- Output Layer: 10

Result Average 100-fold cross validation error: **14.62%**

It looks like the classifier works reasonably well with only one hidden layer. Let's keep adding layers to see how it affects accuracy.

4.3 Two Hidden Layers

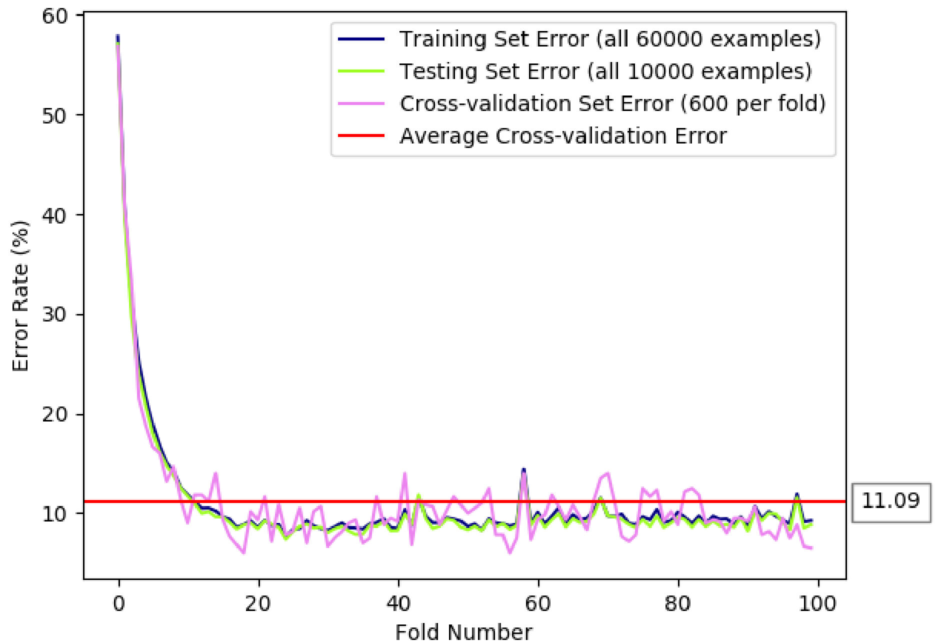


Architecture

- Input layer: 784
- Hidden Layer 1: 60
- Hidden Layer 2: 30
- Output Layer: 10

Result Average 100-fold cross validation error: **12.60%**
Adding a second layer helps a little bit without slowing down training very much.

4.4 Three Hidden Layers



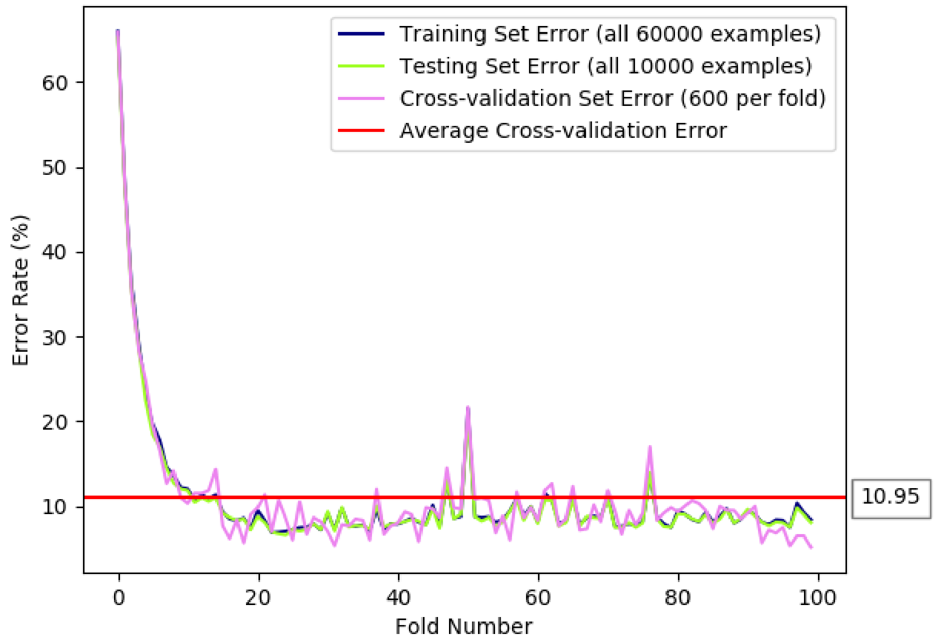
Architecture

- Input layer: 784
- Hidden Layer 1: 120
- Hidden Layer 2: 60
- Hidden Layer 3: 30
- Output Layer: 10

Result Average 100-fold cross validation error: **11.09%**

The third layer didn't help as much as adding the second layer did. Perhaps a third is unnecessary.

4.4.1 Four Hidden Layers



ff-numlayers-240-120-60-30.png

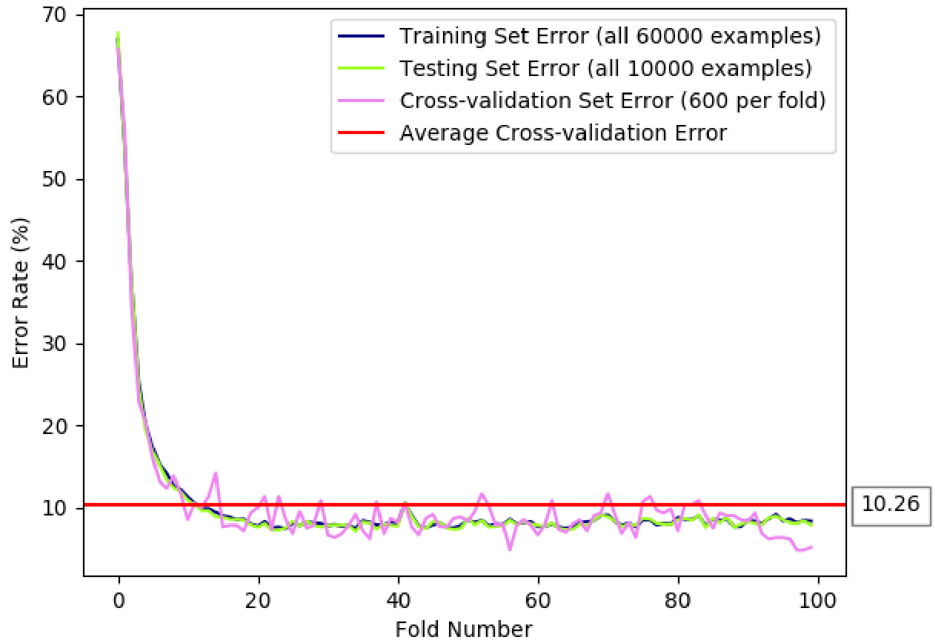
Architecture

- Input layer: 784
- Hidden Layer 1: 240
- Hidden Layer 2: 120
- Hidden Layer 3: 60
- Hidden Layer 4: 30
- Output Layer: 10

Result Average 100-fold cross validation error: **10.95%**

Again, the additional layer helped a little bit, but with each new layer, training time becomes longer. We haven't seen much improvement since having one hidden layer, so it looks like we must improve our classifier's accuracy through other means.

4.5 Conclusions



ff-numlayers-160-60-conclusion.png

Architecture

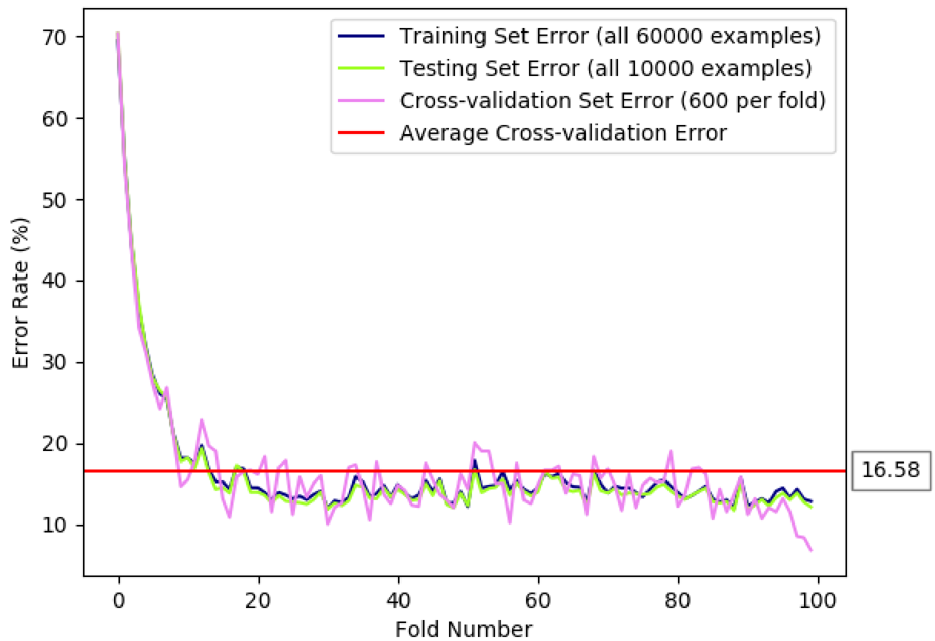
- Input layer: 784
- Hidden Layer 1: 160
- Hidden Layer 2: 60
- Output Layer: 10

Result 100-fold cross validation error: **10.26%**

After some experimentation, we settled on two hidden layers. It provided a good balance between accuracy and training speed. Now we'll try experimenting with the sizes of those two hidden layers to improve the accuracy of the classifier.

5 Investigating Performance for Different Sizes of Hidden Layers

5.1 Small Layers



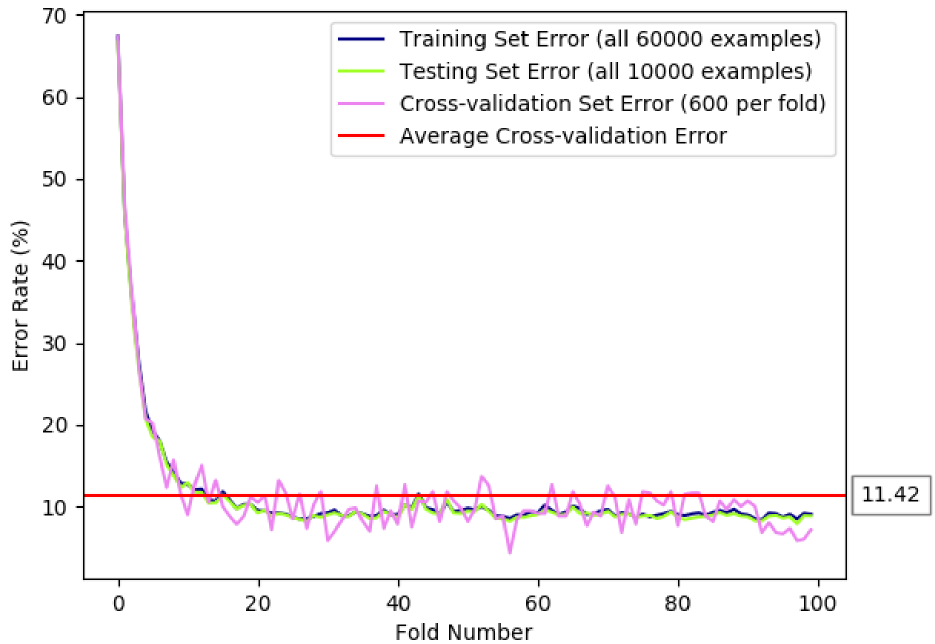
Architecture

- Input layer: 784
- Hidden Layer 1: 30
- Hidden Layer 2: 15
- Output Layer: 10

Result Average 100-fold cross validation error: **16.58%**

Making the layers very small doesn't give an awful classifier, but we can easily improve this.

5.2 Medium Layers

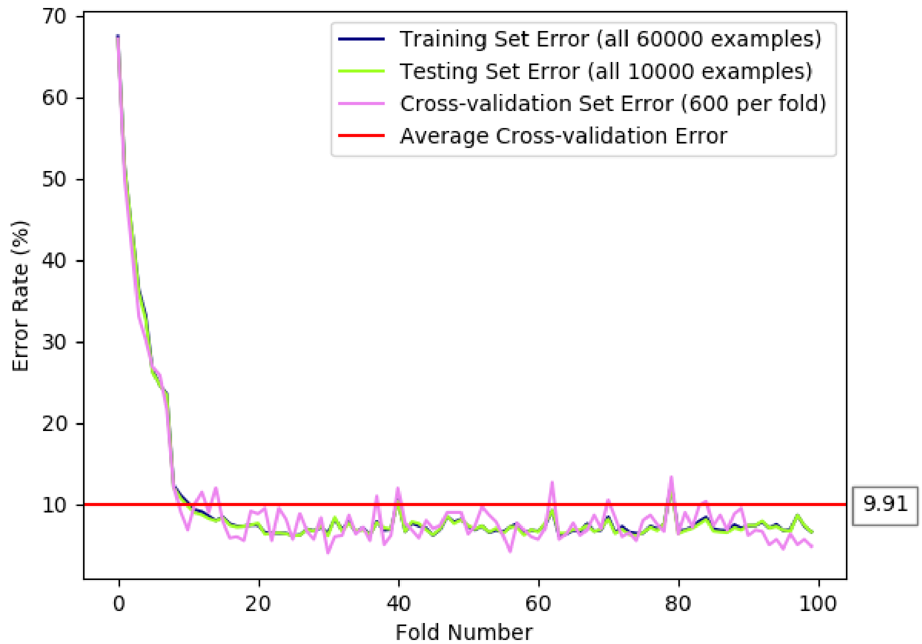


Architecture

- Input layer: 784
- Hidden Layer 1: 100
- Hidden Layer 2: 50
- Output Layer: 10

Result Average 100-fold cross validation error: **11.42%**
Making the layer bigger improved the accuracy of the classifier.

5.3 Large Layers

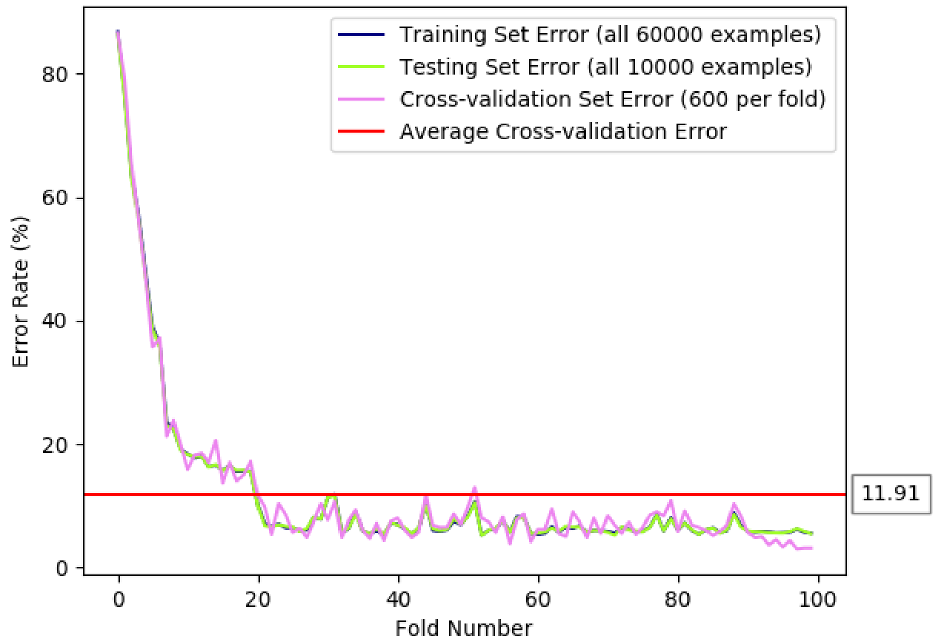


Architecture

- Input layer: 784
- Hidden Layer 1: 300
- Hidden Layer 2: 100
- Output Layer: 10

Result Average 100-fold cross validation error: **9.91%**
Making them even bigger helped even more.

5.4 Very Large Layers



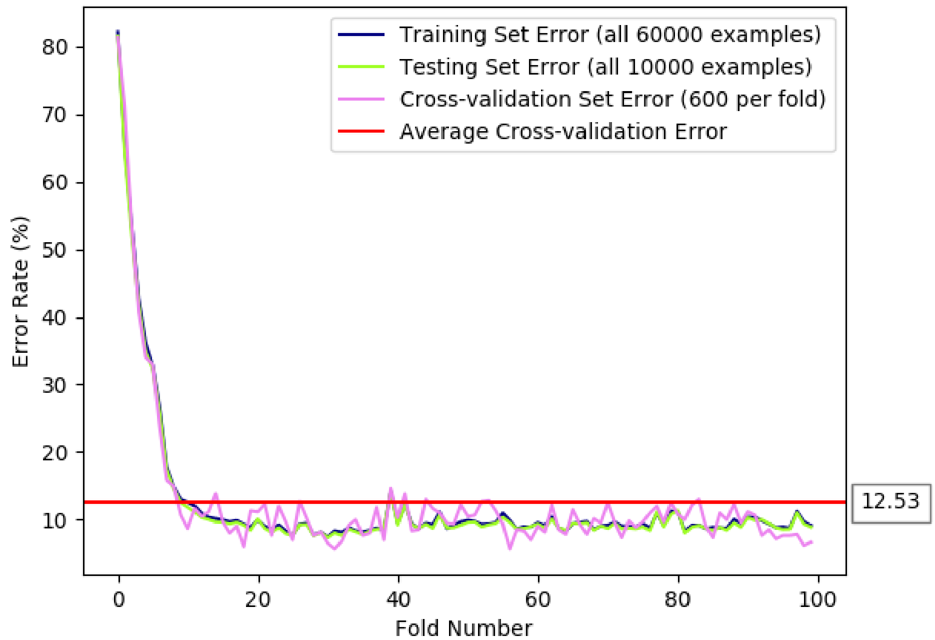
Architecture

- Input layer: 784
- Hidden Layer 1: 500
- Hidden Layer 2: 200
- Output Layer: 10

Result Average 100-fold cross validation error: **11.91%**

Making them too big has done the reverse. Let's make the layers smaller and try to find a good ratio between the two layers.

5.5 Larger Same Size Layers



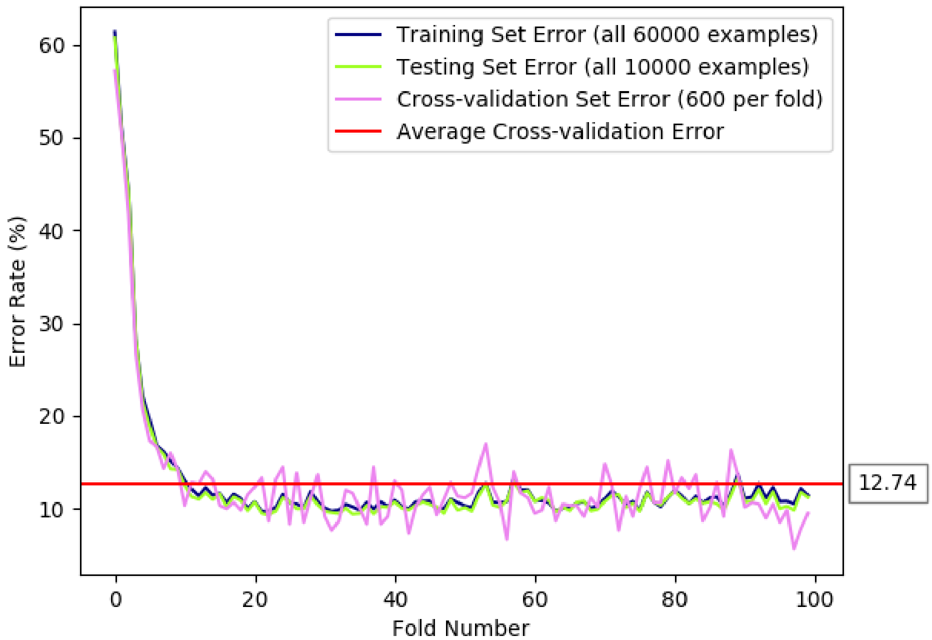
Architecture

- Input layer: 784
- Hidden Layer 1: 128
- Hidden Layer 2: 128
- Output Layer: 10

Result Average 100-fold cross validation error: **12.54%**

We tried using the same size for both hidden layer, but it didn't improve accuracy. Perhaps making both of them smaller would help.

5.6 Smaller Same Size Layers



ff-layersize-64-64.png

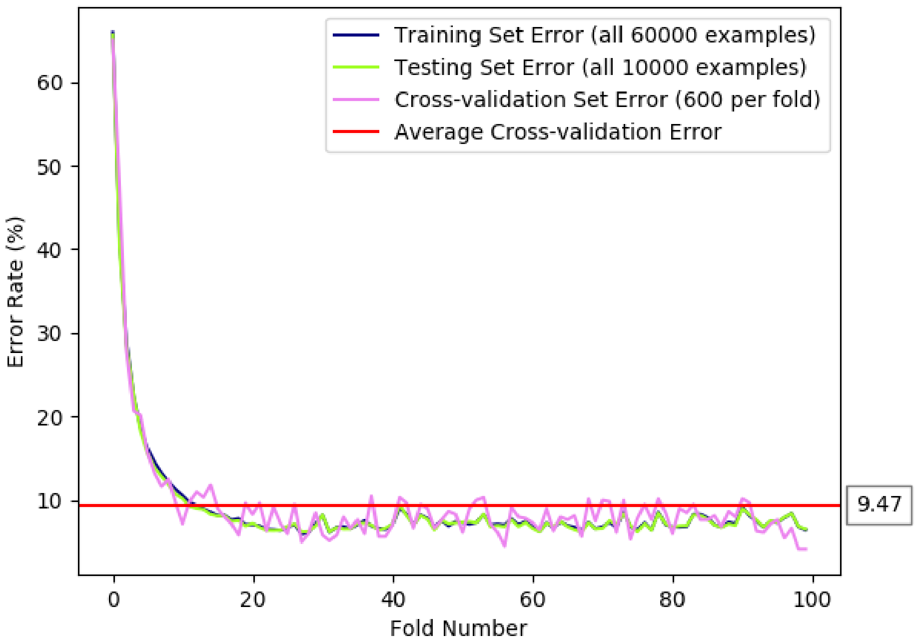
Architecture:

- Input layer: 784
- Hidden Layer 1: 64
- Hidden Layer 2: 64
- Output Layer: 10

Result Average 100-fold cross validation error: **12.74%**

We got a very similar result to using 128 for both hidden layers. Perhaps a wide difference between hidden layers would help.

5.7 Layers With Wide Size Difference



ff-layersize-300-40.png

Architecture

- Input layer: 784
- Hidden Layer 1: 300
- Hidden Layer 2: 40
- Output Layer: 10

Result 100-fold cross validation error: **9.47%**

It looks like a wide difference between the two layers helps the classifier perform better, but not better than we've perviously seen.

5.8 Conclusions

We learned a few things here:

- if the final hidden layer is too big, accuracy goes down
- layers should not be too small
- layers should not be too big
- layers should not be the same size
- layers should have sizeable size differences between them

6 Final Results + Statistical Analysis

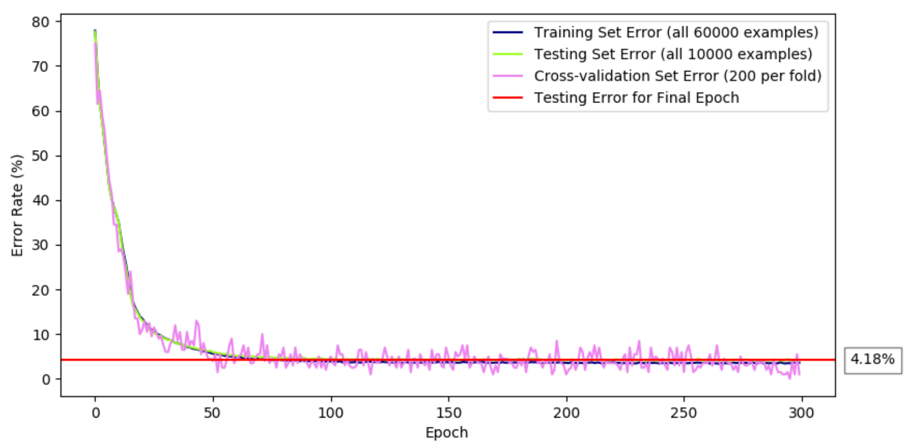
6.1 Our Best Performance

In this run, we had the following error rates:

- error when testing with the 10,000 element test dataset: **4.18%**
- average cross-validation error over 300 folds: **6.65%**
- average cross-validation error over final 30 folds: **2.67%**

We used the following hyperparameters to achieve this:

- MLP Architecture: **784, 160, 60, 10**
- Learning rate: **0.001**
- Weight Decay: **0.2**
- Batch Size: **200**
- Epochs: **300**



We will use this result in the following section for statistical analysis.

6.2 Mean Classification Accuracy

6.2.1 For the mean classification error rate using the 10000 element test set

We computed a **95%** confidence interval for the test error of the classifier after the final epoch. This value is the error rate when using the **10,000 element testing set**. We used the following code to compute the confidence interval. The code is part of our submitted program; it runs after every time training is complete and prints out the interval.

```
confidence = 0.95
samples = test_error[-int(epochs/10):-1]
mean = np.mean(samples)
sigma = np.std(samples)
interval = scipy.stats.norm.interval(confidence, loc=mean, scale=sigma)
```

Let X be a **random variable** representing the test error rate of the classifier. We used the test error rate for the a portion of epochs' test error rates—when the classifier has converged—as samples to construct the **probability distribution** for X. We did this to avoid the misleading values at the first few epochs. This does not damage our results; it simply makes the interval slightly larger. Considering the probability distribution, we are **95% confident** that the mean lies within the interval **(3.99%, 4.40%)**.

6.2.2 For the mean cross-validation error rate over 300 epochs

Using the **same code** and **same approach** as the subsection above, we computed the **95%** confidence interval for the mean cross-validation error over 300 epochs. We are 95% confident that the mean lies in the interval **(-12.08%, 25.38%)**

6.2.3 For the mean cross-validation error rate over the final 30 epochs

Since the above interval considers the values from the first few epochs, we will use values from after the weights have converged. Using the **same code** and **same approach** as the subsection above, we computed the **95%** confidence interval for the mean cross-validation error over 30 epochs. We are 95% confident that the mean lies in the interval **(3.30%, 5.30%)**

7 References

The work we provided in our submission is our own.

We referred to the following sources for simple matters like tweaking hyperparameters and choosing layer sizes. The professor stated in the forums that this is allowed: https://culearn.carleton.ca/moodle/mod/forum/discuss.php?d=296418

- https://stats.stackexchange.com/questions/29130/difference-between-neural-net-weight-decay-and-learning-rate
- https://github.com/andrewdyates/Radial-Basis-Function-Neural-Network
- https://florianmuellerklein.github.io/nn/
- https://github.com/FlorianMuellerklein/Machine-Learning/blob/master/MultiLayerPerceptron.py
- https://github.com/hdmetor/NeuralNetwork
- https://machinelearningmastery.com/report-classifier-performance-confidence-intervals/
- https://stackoverflow.com/questions/28242593/correct-way-to-obtain-confidence-interval-with-scipy
- https://machinelearningmastery.com/implement-backpropagation-algorithm-scratch-python/