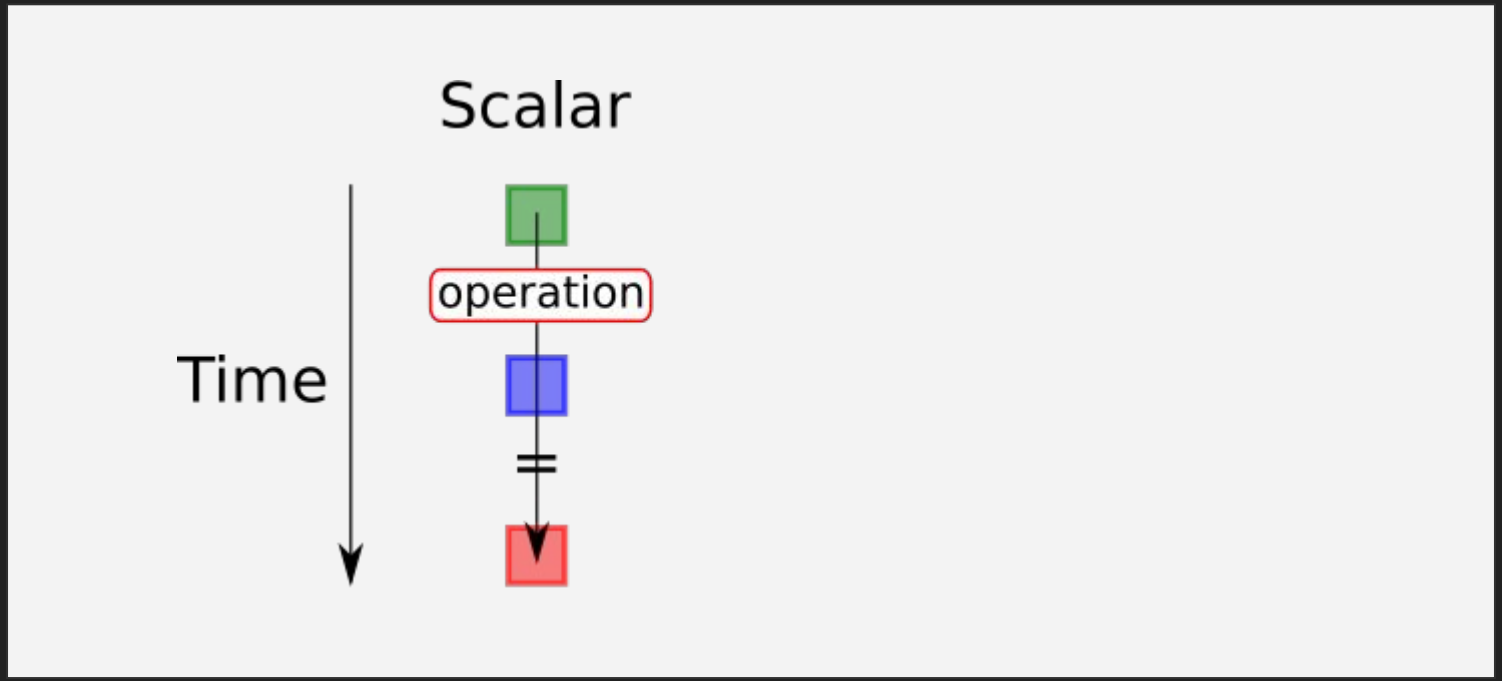
Introduction to (Unconventional) Vectorization

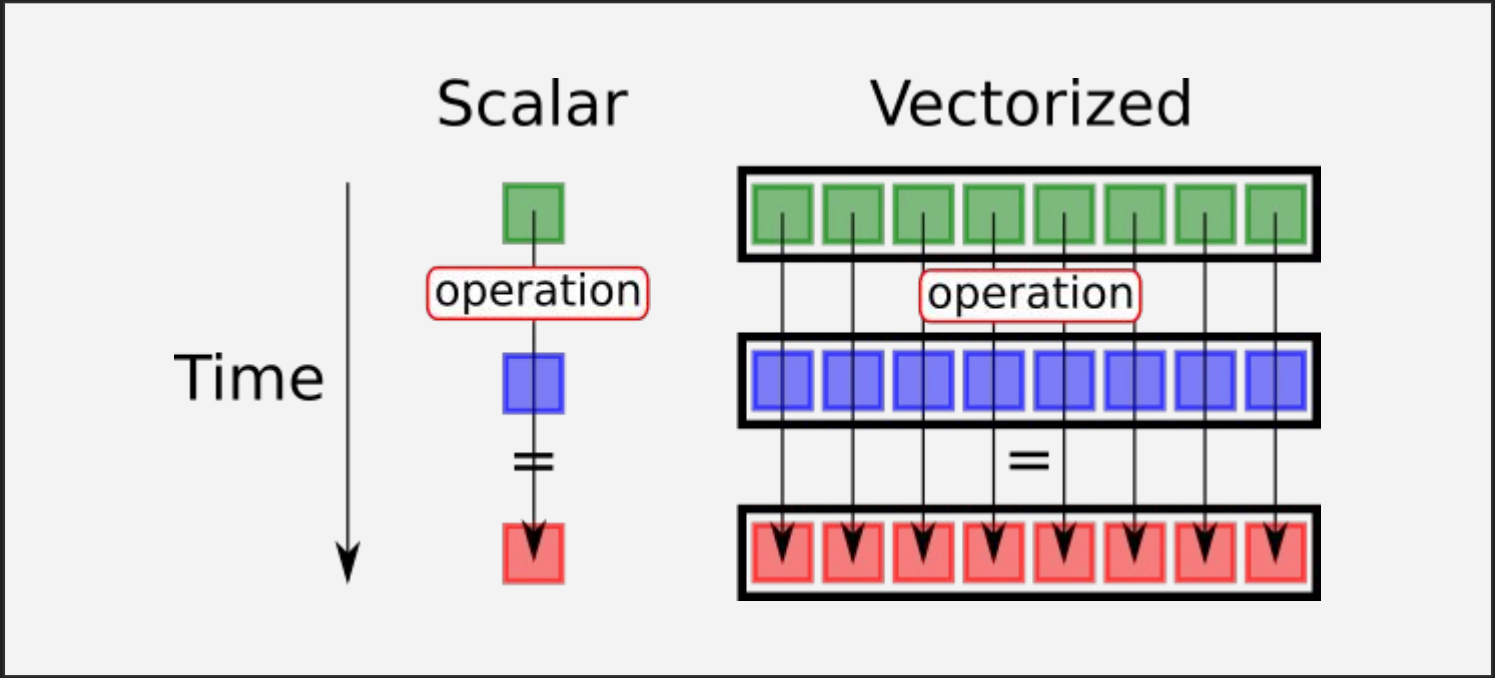
Stefanos Baziotis
NEC Deutschland
National and Kapodistrian University of Athens

stefanos.baziotis@gmail.com

What is Vectorization ?



What is Vectorization ?



Vectorized Code in a High-Level Language

```
int add(int a, int b) {
  return a + b;
}
```

Intrinsics

```
__m128i add4(__m128i a, __m128i b) {
  return _mm_add_epi32(a, b);
}
```

Intel Intrinsics Guide

Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  for (int i = 0; i < len; ++i)
    C[i] = A[i] + B[i];
}
```

Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  for (int i = 0; i < len; i += 4) {
    v1 = load 4 values from A
    v2 = load 4 values from B
    res = 4-packed add(v1, v2)
    store res (which is 4 values) in C
  }
}
```

Loading a Vector

```
_mm_loadu_si128((const __m128i*) m);
```

Loading a Vector

```
__m128i loadu_si128(void *m) {
  return _mm_loadu_si128((const __m128i*) m);
}
```

Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  for (int i = 0; i < len; i += 4) {
    __m128i v1 = loadu_si128(&A[i]);
    __m128i v2 = loadu_si128(&B[i]);
    __m128i res = add4(v1, v2);
    storeu_si128(&C[i], res);
  }
}
```

The Struggles of Vectorization

Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  for (int i = 0; i < len; i += 4) {
    __m128i v1 = loadu_si128(&A[i]);
    __m128i v2 = loadu_si128(&B[i]);
    __m128i res = add4(v1, v2);
    storeu_si128(&C[i], res);
  }
}
```

What if `len` is not a multiple of 4?

Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  int end = (len % 4 == 0) ? len : len - 4;
  for (int i = 0; i < end; i += 4) {
    __m128i v1 = loadu_si128(&A[i]);
    __m128i v2 = loadu_si128(&B[i]);
    __m128i res = add4(v1, v2);
    storeu_si128(&C[i], res);
  }
}
```

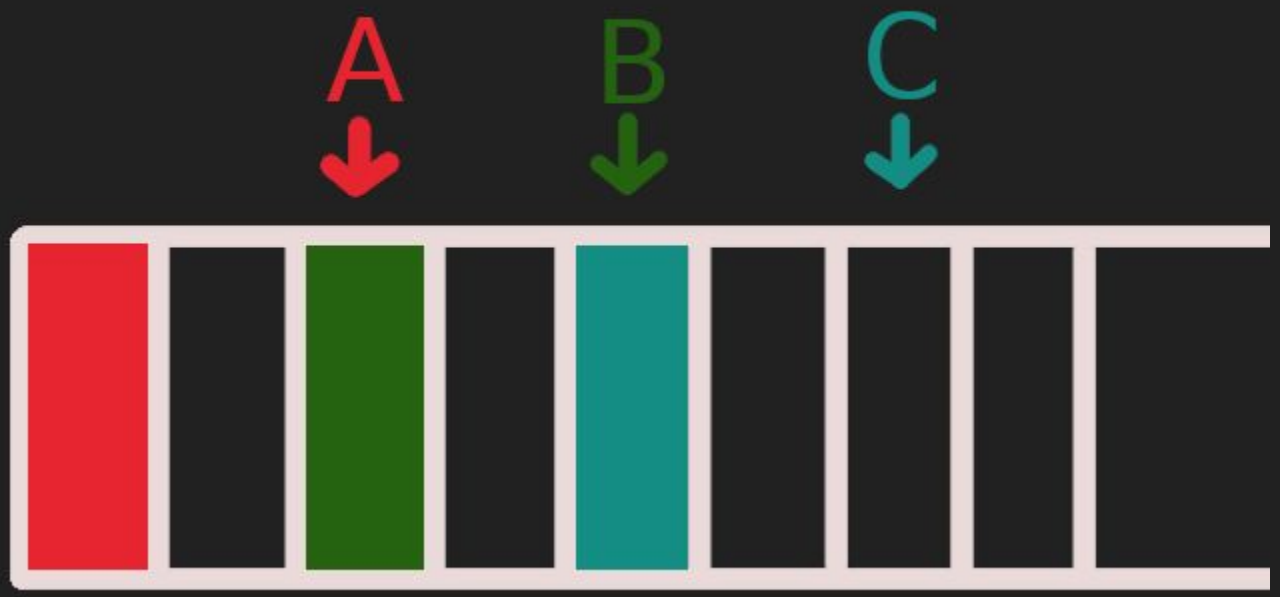
Vectorizing a Loop

```
void sum_arrays(int *A, int *B, int *C, int len) {
  int end = (len % 4 == 0) ? len : len - 4;
  int i = 0;
  for (; i < end; i += 4) {
    __m128i v1 = loadu_si128(&A[i]);
    __m128i v2 = loadu_si128(&B[i]);
    __m128i res = add4(v1, v2);
    storeu_si128(&C[i], res);
  }
  for (; i < len; ++i) {
    C[i] = A[i] + B[i];
  }
}
```

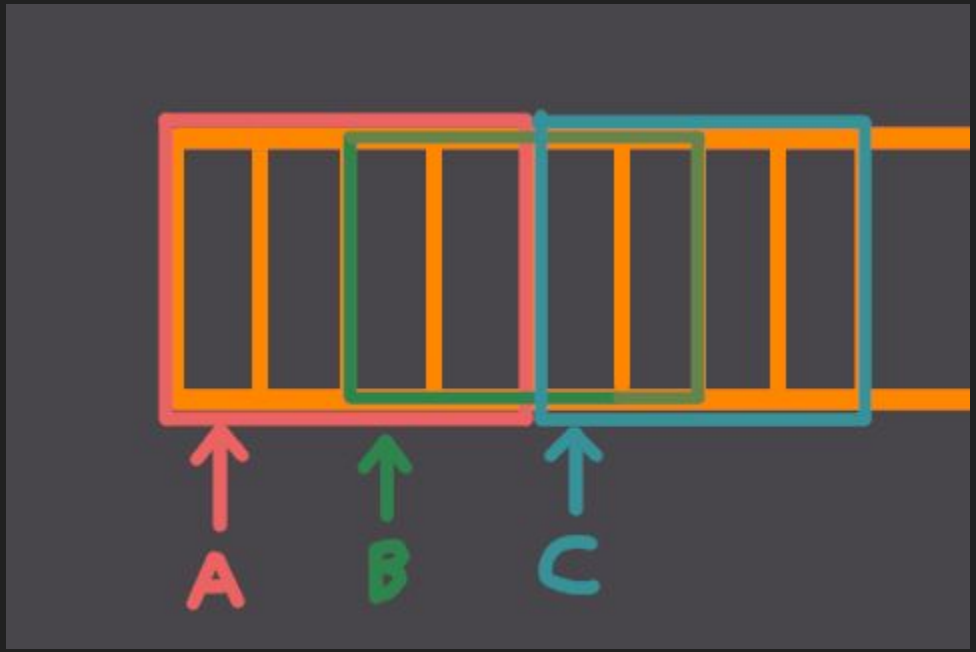
Aliasing

Aliasing

Aliasing



Aliasing



Aliasing

```
void sum_arrays(int *A, int *B, int *C, int len) {
  if (A + len > B)
    jump to the scalar version
  if (B + len > A)
    jump to the scalar version
  ...
}
```

Aliasing

```
void sum_arrays(int *restrict A,
                int *restrict B,
                int *restrict C,
                int len) {
  ...
}
```

Reductions

Reductions

```
new_value = operation(old_value, data);
```

Accumulation

```
int sum_of_array(int *a, int len) {
  int sum = 0;
  for (int i = 0; i < len; ++i)
    sum = sum + a[i];
}
```

Accumulation

```
int sum_of_array(int *a, int len) {
  int sum = 0;
  for (int i = 0; i < len; ++i)
    sum = sum + a[i];
}
```

new value

Accumulation

```
int sum_of_array(int *a, int len) {
  int sum = 0;
  for (int i = 0; i < len; ++i)
    sum = sum + a[i];
}
```

new value

old value

Accumulation

```
int sum_of_array(int *a, int len) {
  int sum = 0;
  for (int i = 0; i < len; ++i)
    sum = sum + a[i];
}
```

new value

old value

operation

Accumulation

```
int sum_of_array(int *a, int len) {
  int sum = 0;
  for (int i = 0; i < len; ++i)
    sum = sum + a[i];
}
```

new value

old value

operation

data

Associativity and Commutativity

```
new_value = operation(old_value, data);
```

Associativity

$$(((1 + 2) + 3) + 4) = (1 + 2) + (3 + 4)$$

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+

Associativity

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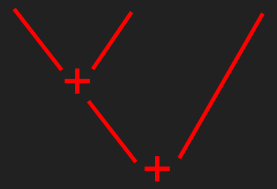
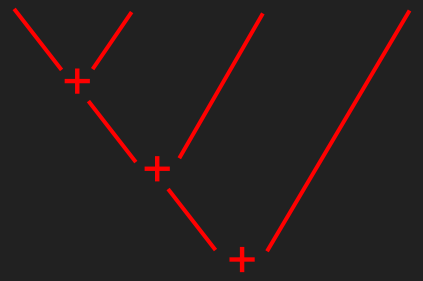


Associativity

$$(((1 + 2) + 3) + 4) = (1 + 2) + (3 + 4)$$



Associativity

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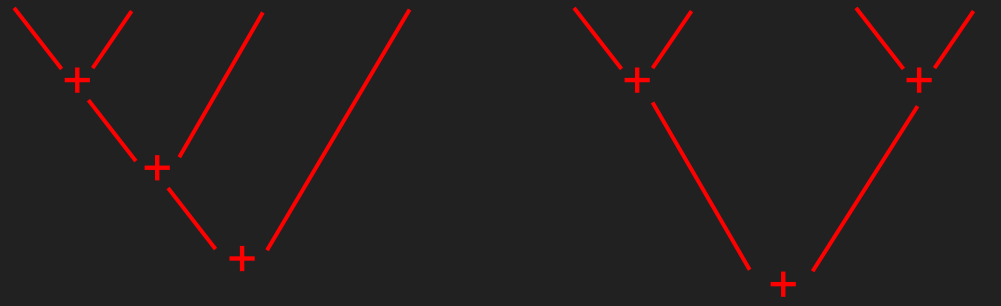

Commutativity

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

Commutativity

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

Commutativity

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

Associativity

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

Commutativity

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

Loading from Memory

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

Loading from Memory

1 | 2 | 3 | 4

+

5 | 6 | 7 | 8

Vectorizing Summanation

```
int sum_of_array(int *A, int len) {
  int end = (len % 4 == 0) ? len : len - 4;
  int i = ;
  __m128i acc = // Register with all lanes equal to 0;
  for (int i = 0; i < end; i += 4) {
    __m128i v = loadu_si128(&A[i]);
    acc = add4(v);
  }
  int final_result = add_all_lanes_epi32(acc);
  for (; i < len; ++i) {
    final_result += a[i];
  }
  return final_result;
}
```

Horizontal Add

```
int get_low_32_bits_si128(__m128i v) {
  return _mm_cvtsi128_si32(v);
}

int add_all_lanes_epi32(__m128i v) {
  v = _mm_hadd_epi32(v, v);
  v = _mm_hadd_epi32(v, v);
  return get_low_32_bits_si128(v);
}
```

Vectorizing Conditional Code

Invariant Condition

```
int a[VECTOR_WIDTH];
int b[VECTOR_WIDTH];
int out[VECTOR_WIDTH];
...
if (c) {
  for (int i = 0; i < VECTOR_WIDTH; ++i)
    out[i] = a[i] + b[i];
}
```

Invariant Condition

```
__m128i a_vector = _mm_loadu_si128(&a[0]);
__m128i b_vector = _mm_loadu_si128(&b[0]);
if (c) {
  __m128i res = add4(a_vector, b_vector)
  _mm_storeu_si128(&out[0], res);
}
```

Divergent Condition

```
int a[VECTOR_WIDTH];
int b[VECTOR_WIDTH];
int out[VECTOR_WIDTH];
...
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (c[i]) {
    out[i] = a[i] + b[i];
  }
}
```

Divergent Condition

```
int a[VECTOR_WIDTH];
int b[VECTOR_WIDTH];
int out[VECTOR_WIDTH];
...
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (c[i]) {
    out[i] = a[i] + b[i];
  }
}
```

Divergent!

Generic Form

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (cond[i]) {  // Divergent condition
    value[i] = some computation;
  } else {
    value[i] = some other computation;
  }
}
```

Predication

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  a[i] = some computation;
}

for (int i = 0; i < VECTOR_WIDTH; ++i) {
  b[i] = some other computation;
}
```

Predication

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  a[i] = some computation;
}

for (int i = 0; i < VECTOR_WIDTH; ++i) {
  b[i] = some other computation;
}

for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (cond[i]) {  // Divergent condition
    value[i] = a[i];
  } else {
    value[i] = b[i];
  }
}
```

Predication

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  a[i] = some computation;
}
```

1 instruction

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
  b[i] = some other computation;
}

for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (cond[i]) {  // Divergent condition
    value[i] = a[i];
  } else {
    value[i] = b[i];
  }
}
```

Predication

```
for (int i = 0; i < VECTOR_WIDTH; ++i) {
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for (int i = 0; i < VECTOR_WIDTH; ++i) {
  if (cond[i]) {  // Divergent condition
    value[i] = a[i];
  } else {
    value[i] = b[i];
  }
}
```

1 instruction!!

Outer-Loop Vectorization

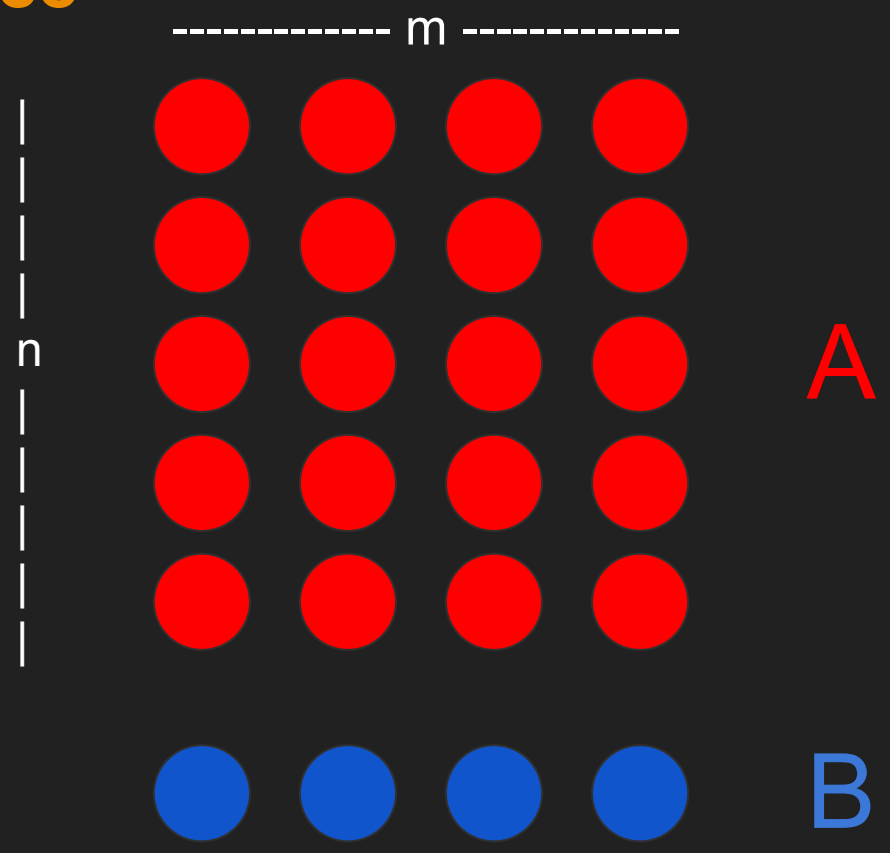
Simon Moll et al.

Nuzman, Zaks

Outer-Loop Vectorization

```
for (int i = 0; i < n; ++i) {
  int a = 0;
  for (int j = 0; j < m; ++j) {
    int v = A[j*m + i];
    a += v;
  }
  B[i] = a;
}
```

Iteration Space



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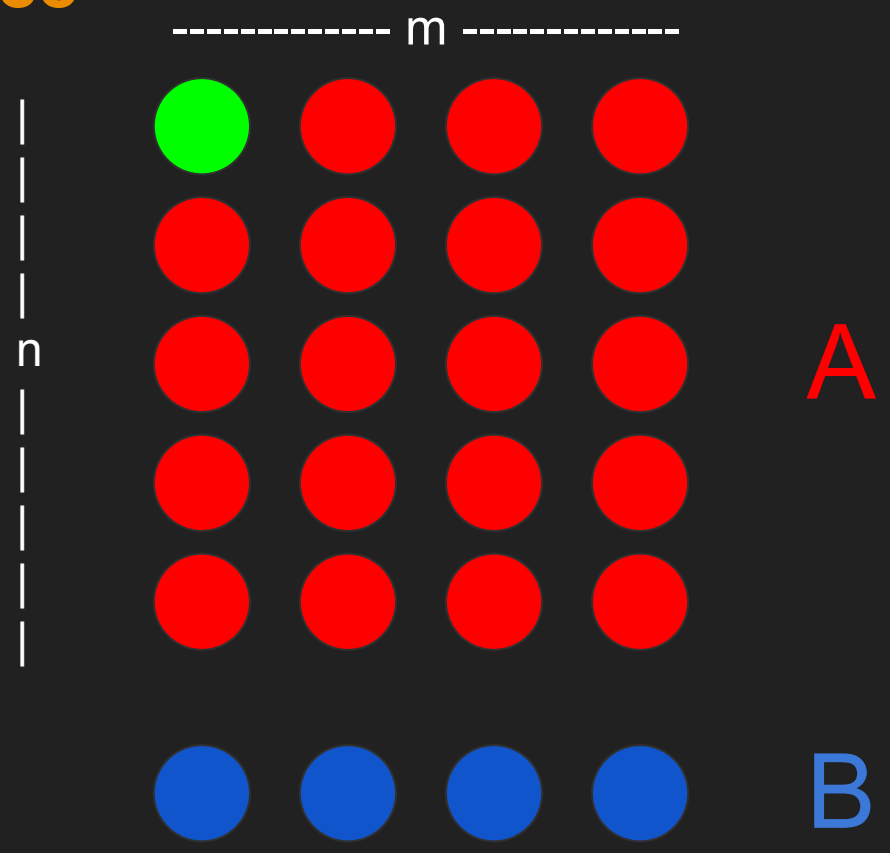

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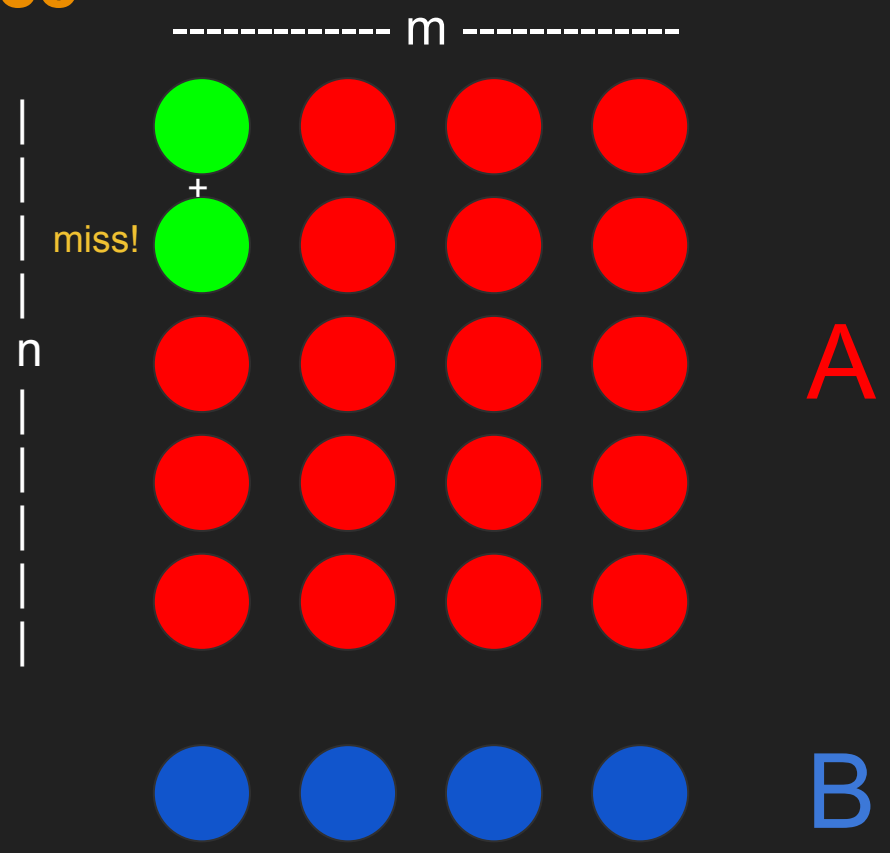

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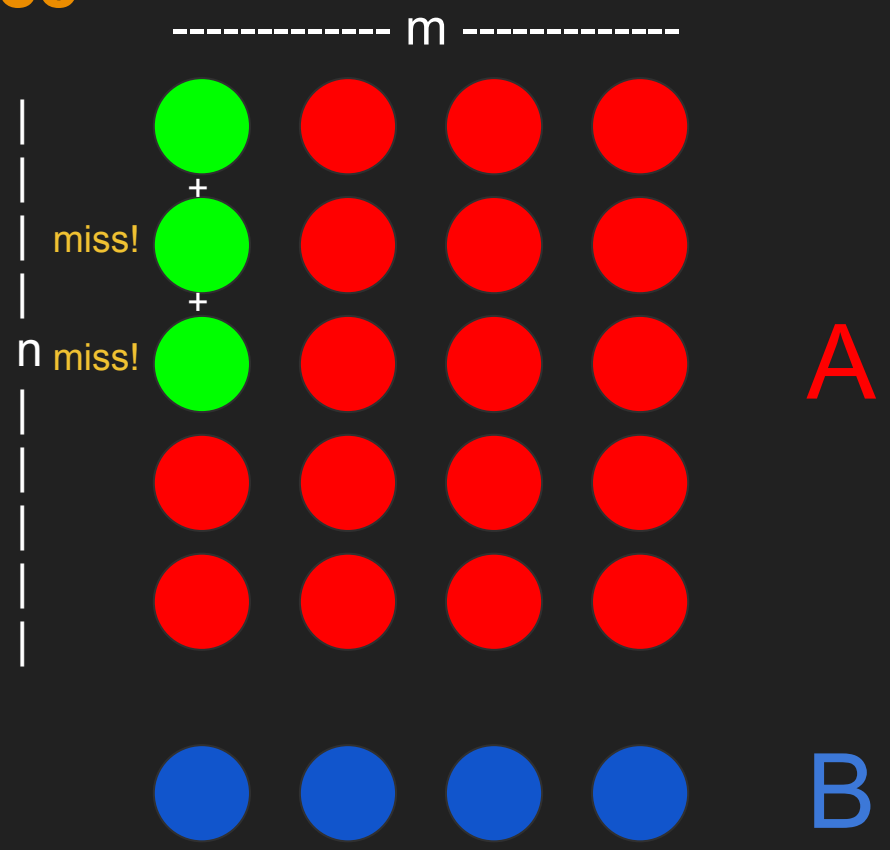

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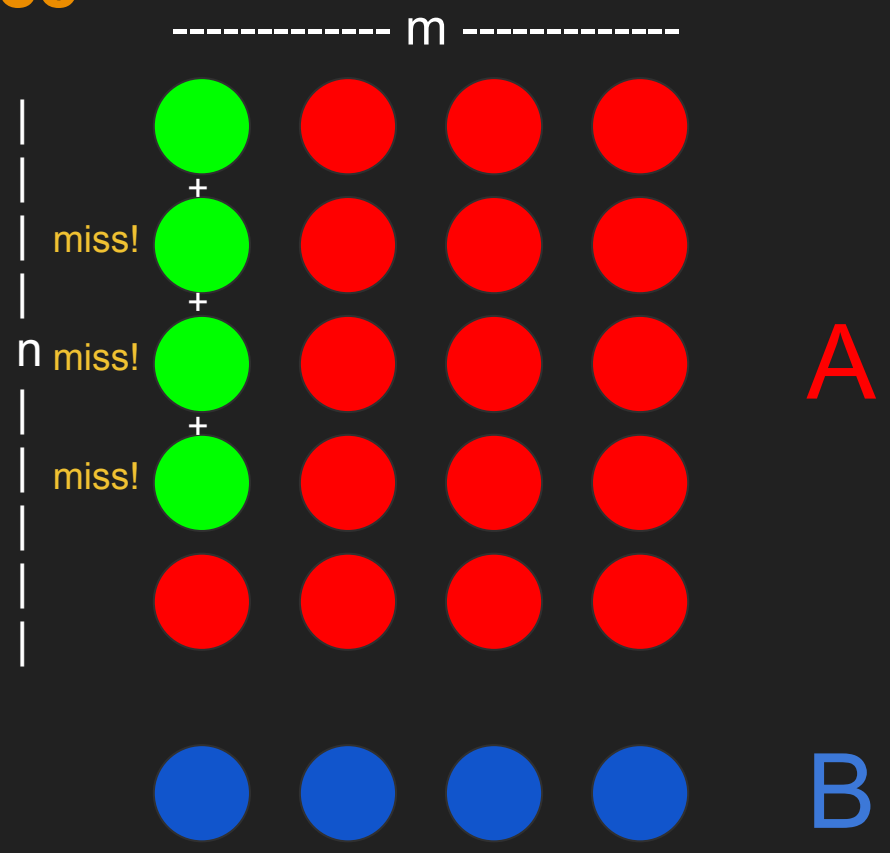
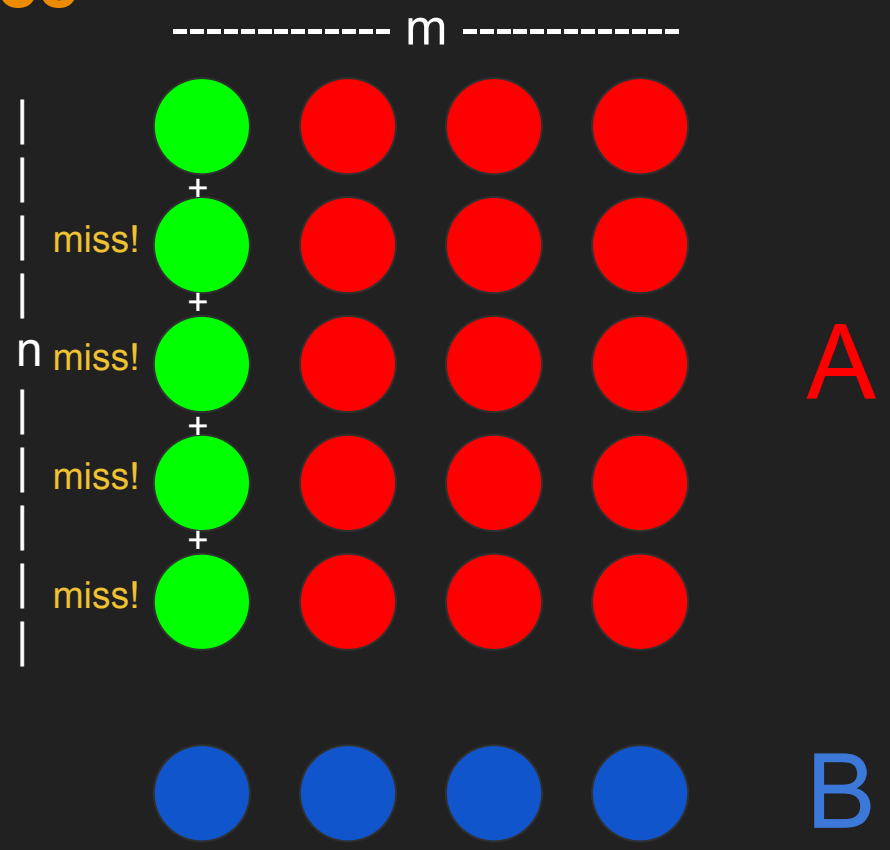


Iteration Space



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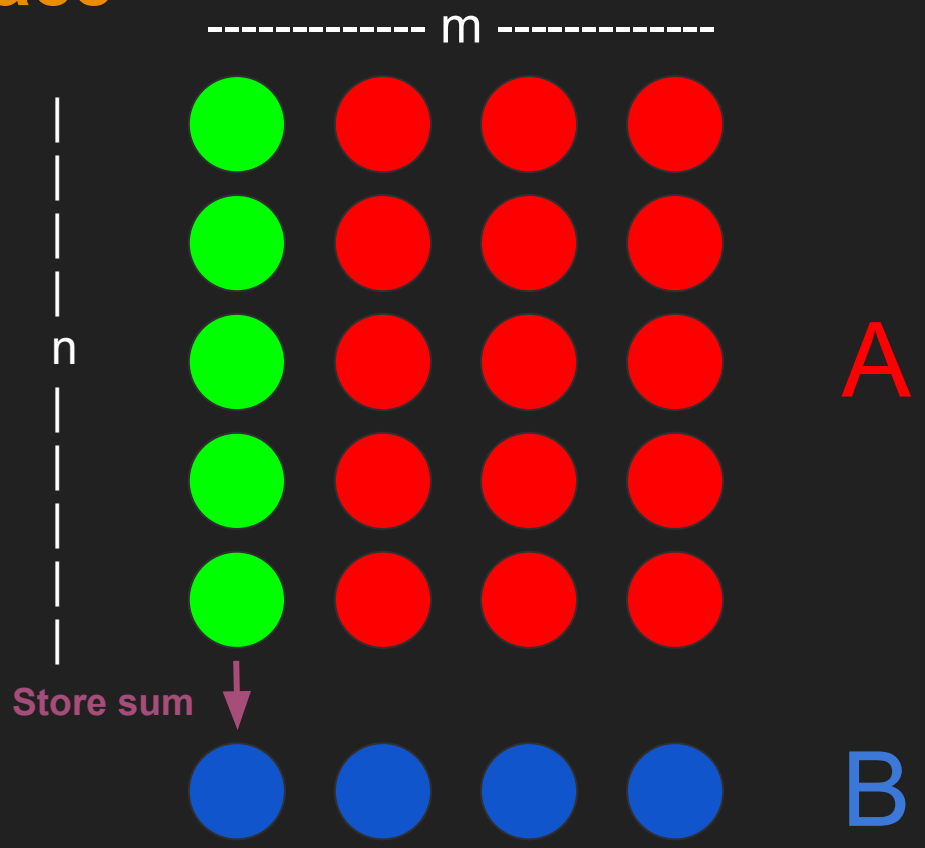

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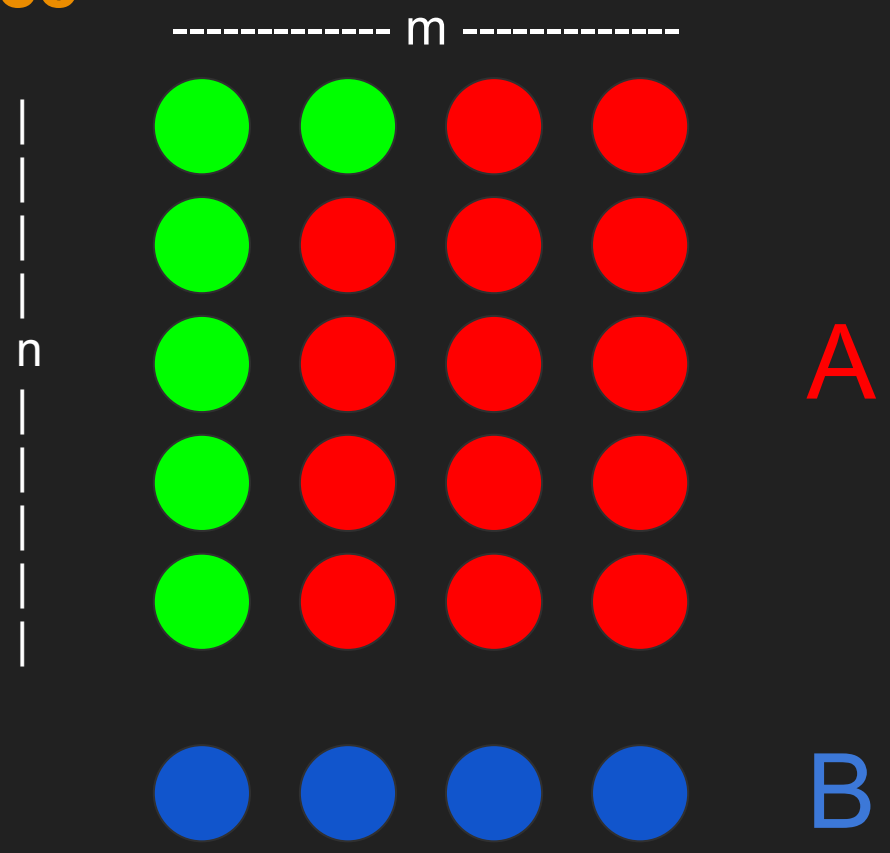
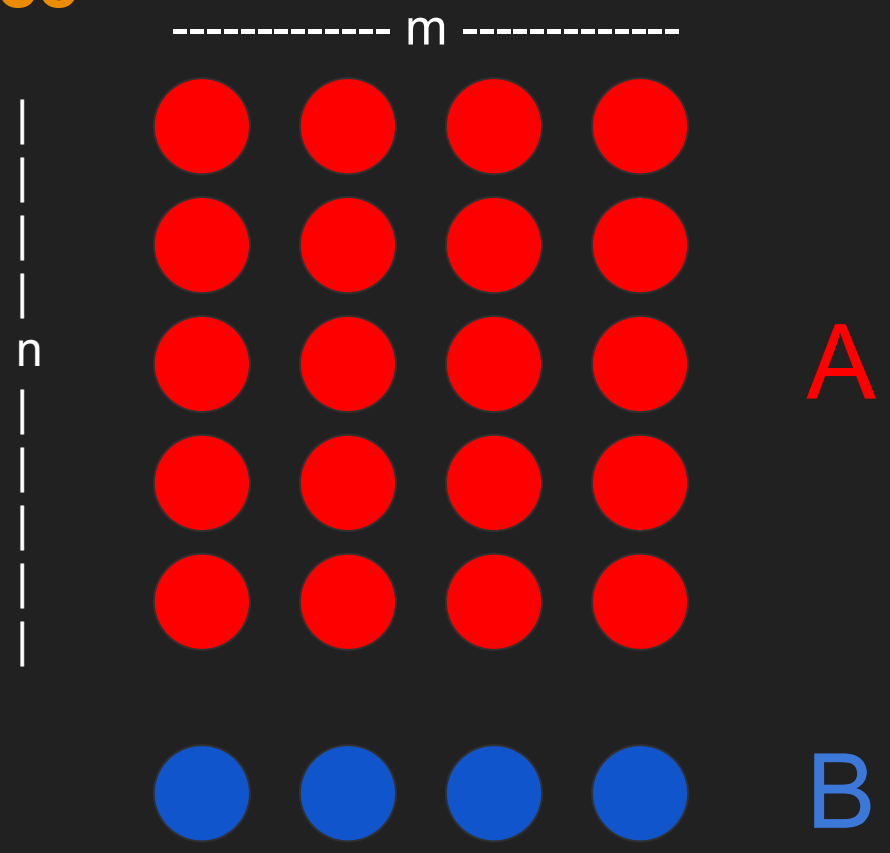


Loop Interchange?

```
… zero B;
for (int j = 0; j < m; ++j) {
  for (int i = 0; i < n; ++i) {
    int v = A[j*m + i];
    B[i] += v;
  }
}
```

Iteration Space



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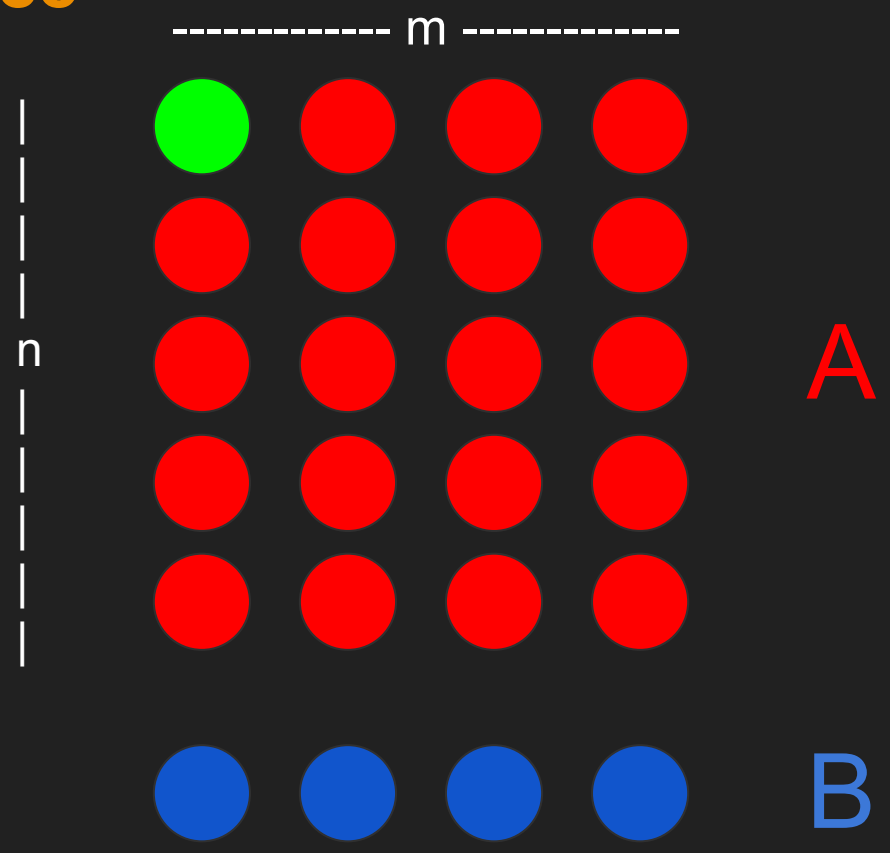

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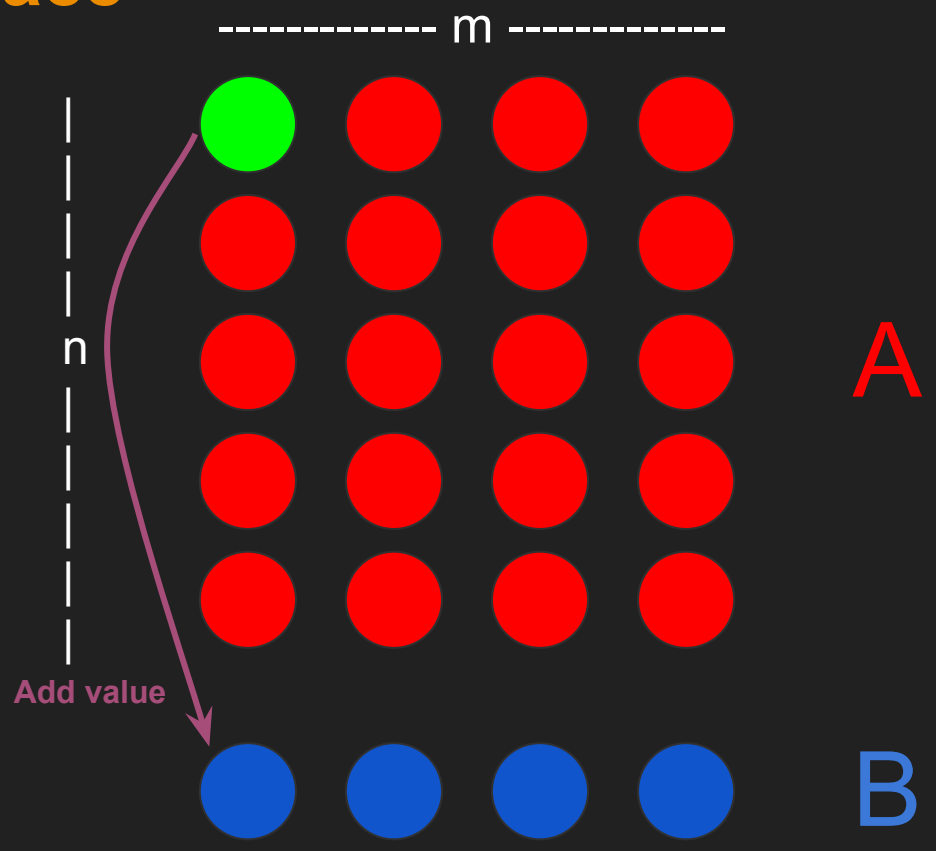

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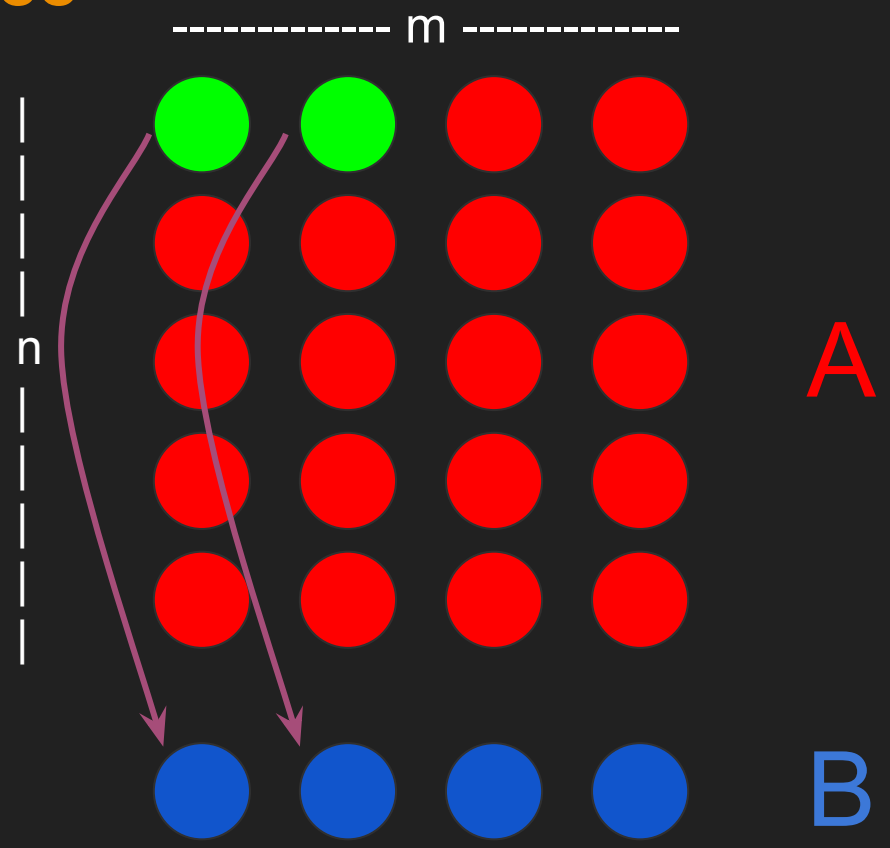

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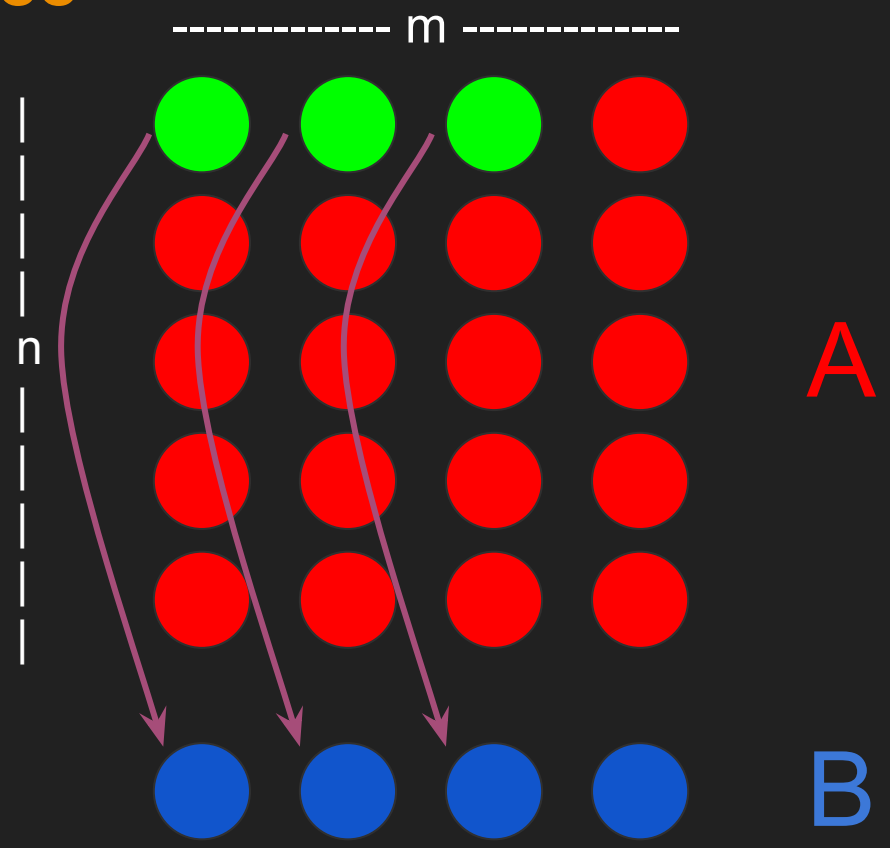

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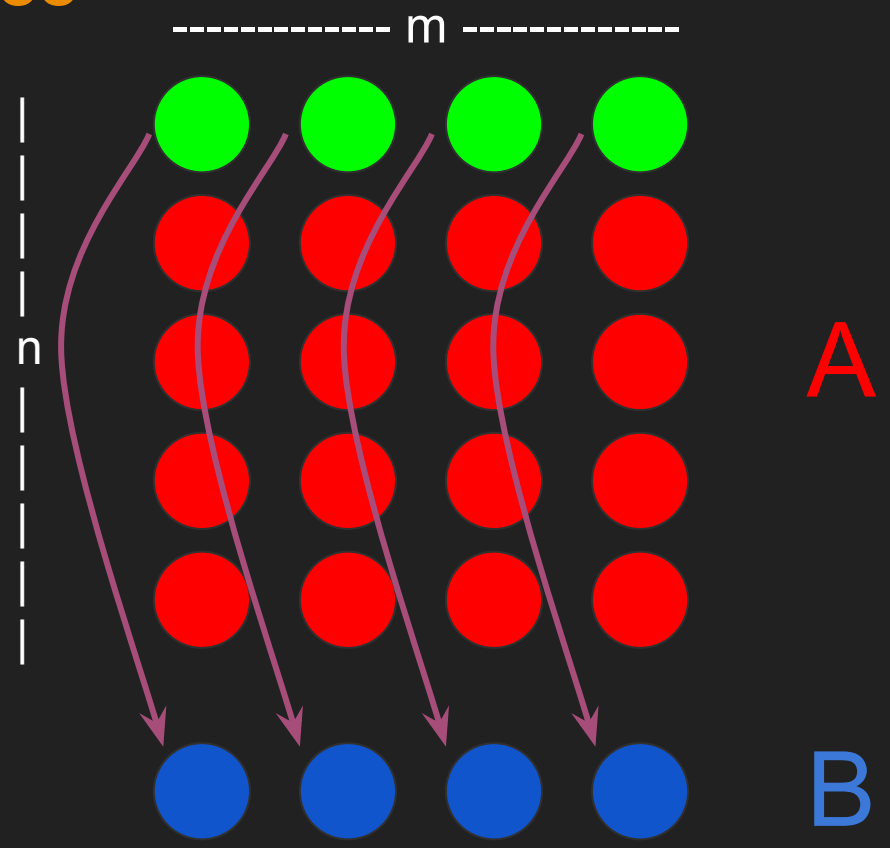

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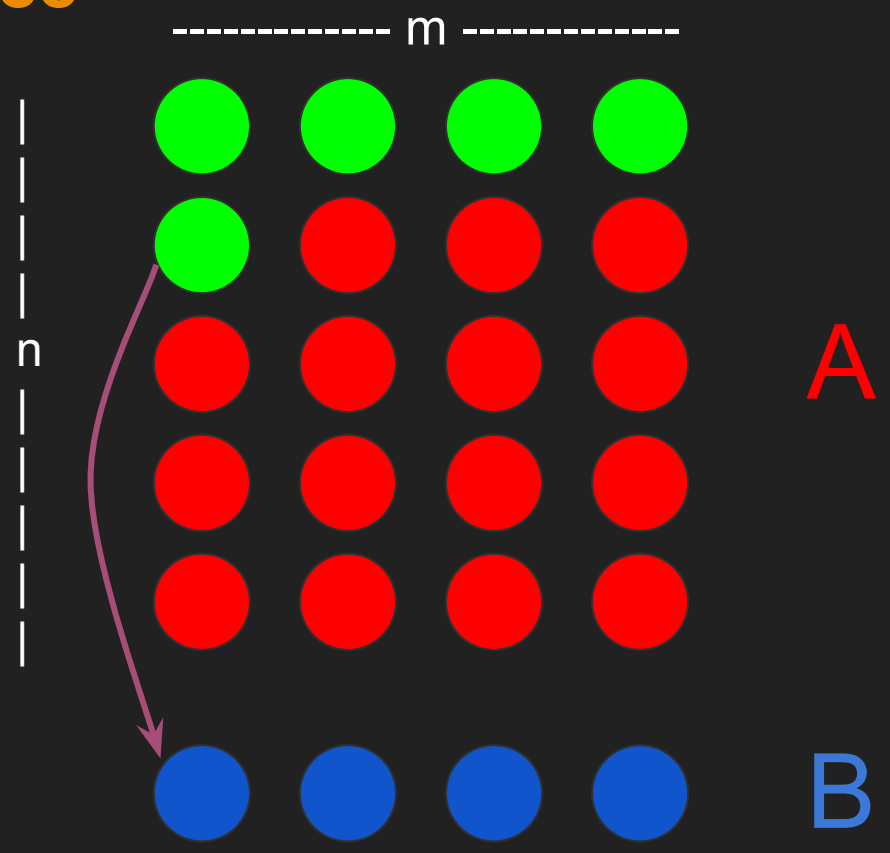


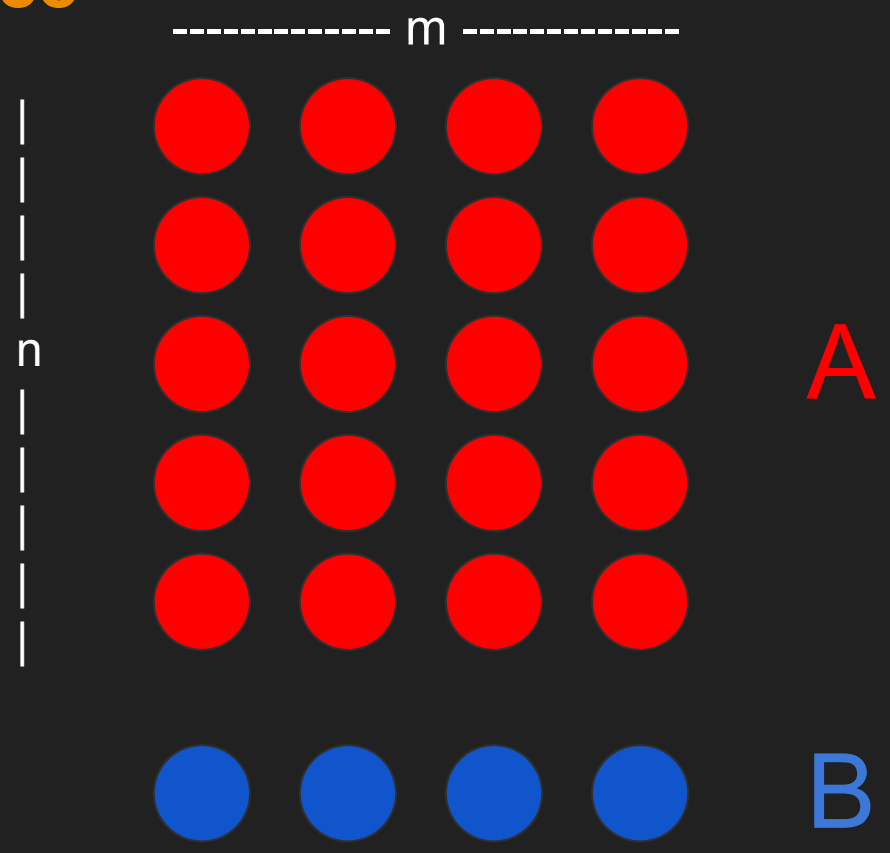
Outer-Loop Vectorization

```
for (int i = 0; i < n; ++i) {
  int a = 0;
  for (int j = 0; j < m; ++j) {
    int v = A[j*m + i];
    a += v;
  }
  B[i] = a;
}
```

Outer-Loop Vectorization

```
for (int i = 0; i < end; i += 4) {
  __m128i a = [0,0,0,0];
  for (int j = 0; j < m; ++j) {
    __m128i v = loadu_si128(A[j*m + i]);
    a = add4(a, v);
  }
  storeu_si128(&B[i], a);
}
… residual
```

Iteration Space



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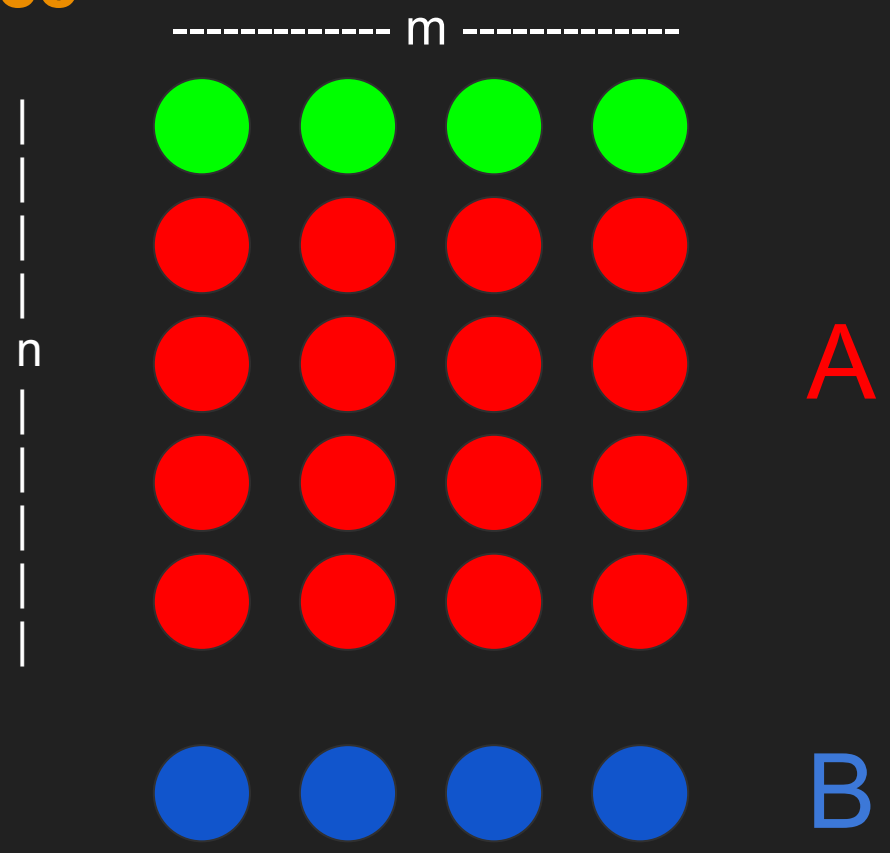

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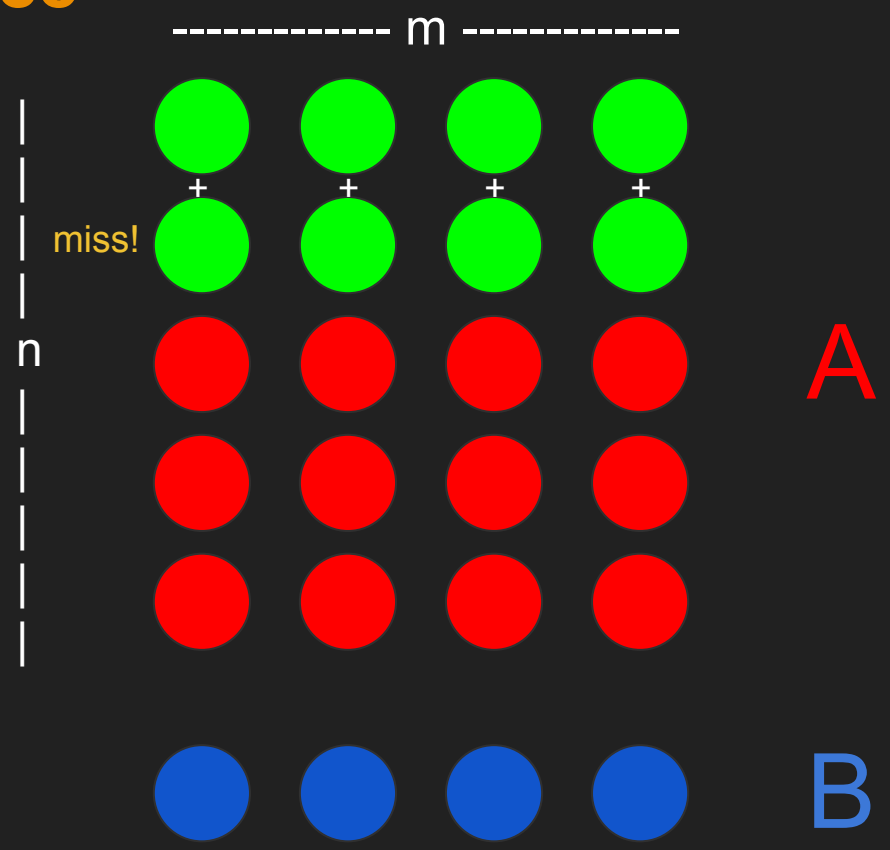

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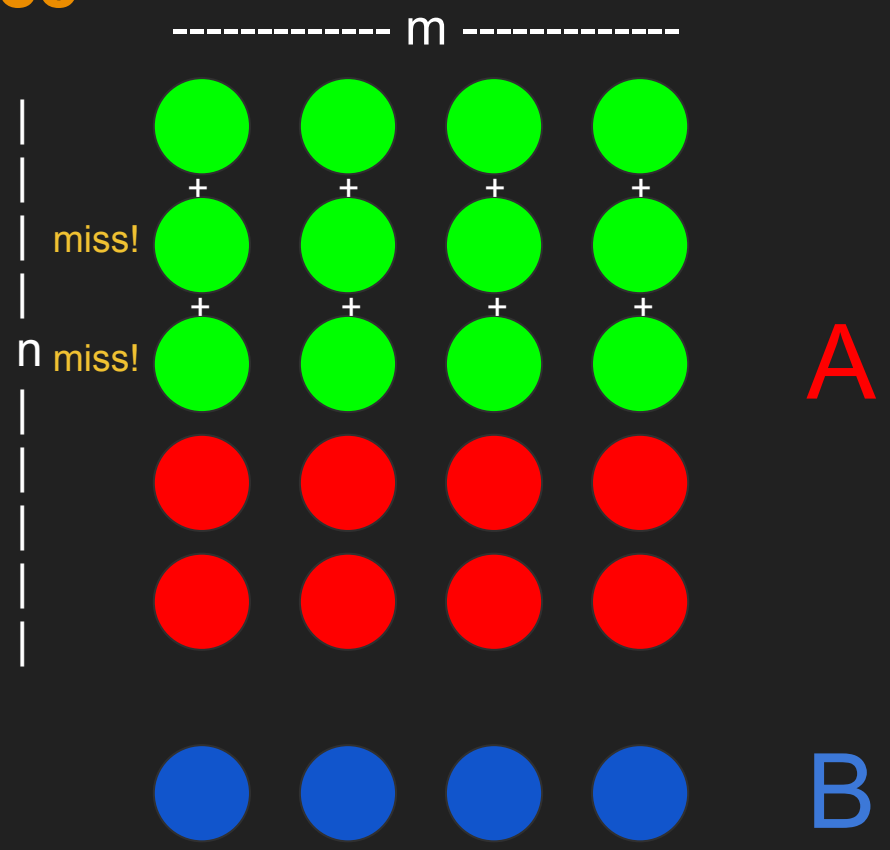

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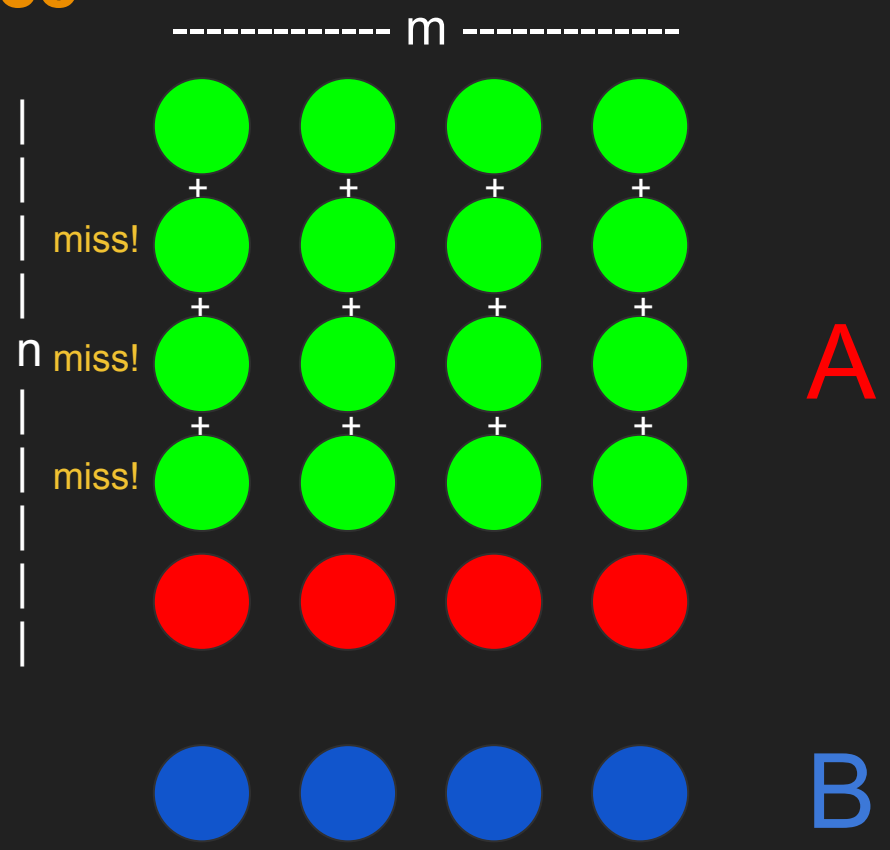

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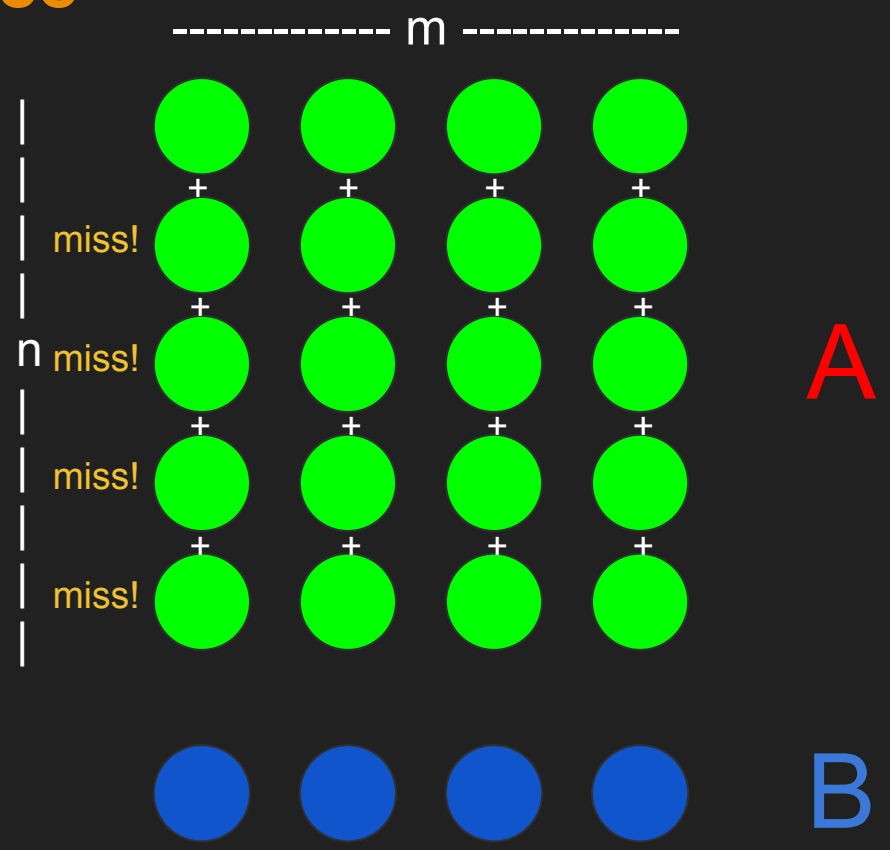

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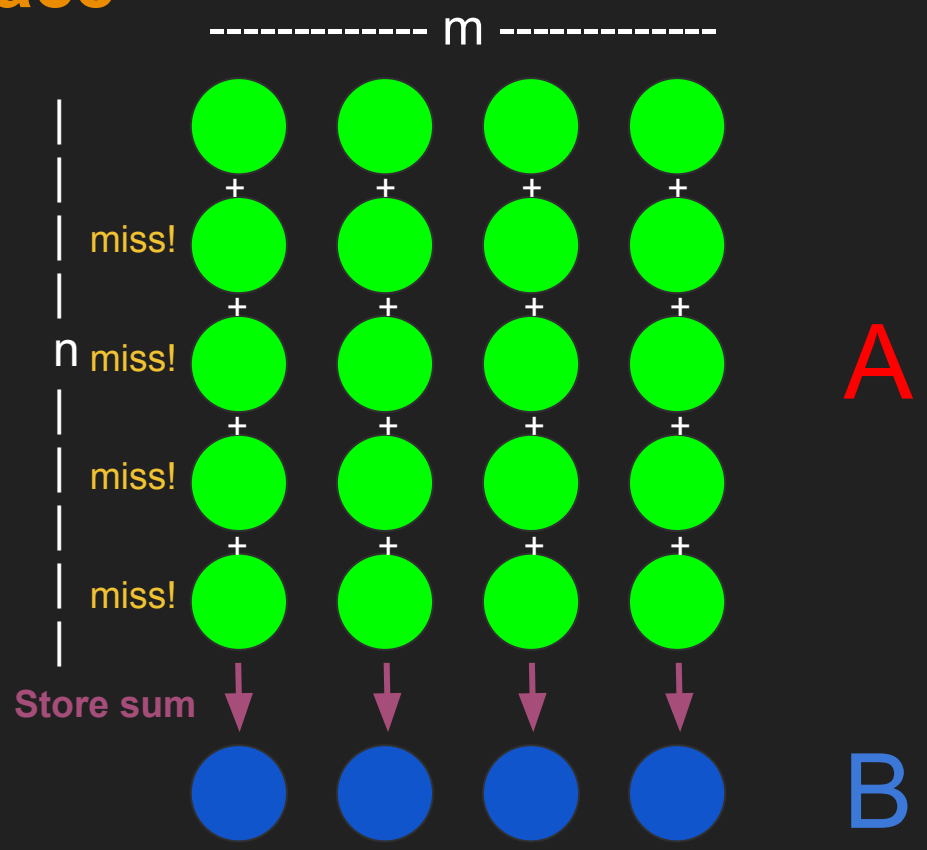

Conditional Code

```
for (int i = 0; i < n; ++i) {
  int a = 0;
  for (int j = 0; j < m; ++j) {
    bool p = C[j];
    if (p) {
      int v = A[j*m + i];
      a += v;
    }
  }
  B[i] = a;
}
```

Uniformity

A uniform value is characterized in relationto a loop and is one that does <u>not</u> vary *because of this loop*.

Divergence

```
Divergent = !Uniform
```

Uniformity in Innermost Loops

```
for (int i = 0; i < N; ++i) {
  int v = 1 + 2;
  a[i] = v;
}
```

Uniformity in Innermost Loops

```
for (int i = 0; i < N; ++i) {
  int v = 1 + 2;
  a[i] = v;
}
```

v is uniform and also loop-invariant

Hoist Out

```
int v = 1 + 2;
for (int i = 0; i < N; ++i) {
  a[i] = v;
}
```

Conditional Code

```
for (int i = 0; i < n; ++i) {
  int a = 0;
  for (int j = 0; j < m; ++j) {
    bool p = C[j];
    if (p) {
      int v = A[j*m + i];
      a += v;
    }
  }
  B[i] = a;
}
```

Conditional Code

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What is p in the i-loop?

Conditional Code

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What is p in the i-loop?

Uniform? ✓

Conditional Code

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    bool p = C[j];
    if (p) {
      int v = A[j*m + i];
      a += v;
    }
  }
  B[i] = a;
}
```

What is p in the i-loop?

Uniform? ✓

Loop-Invariant? ✗

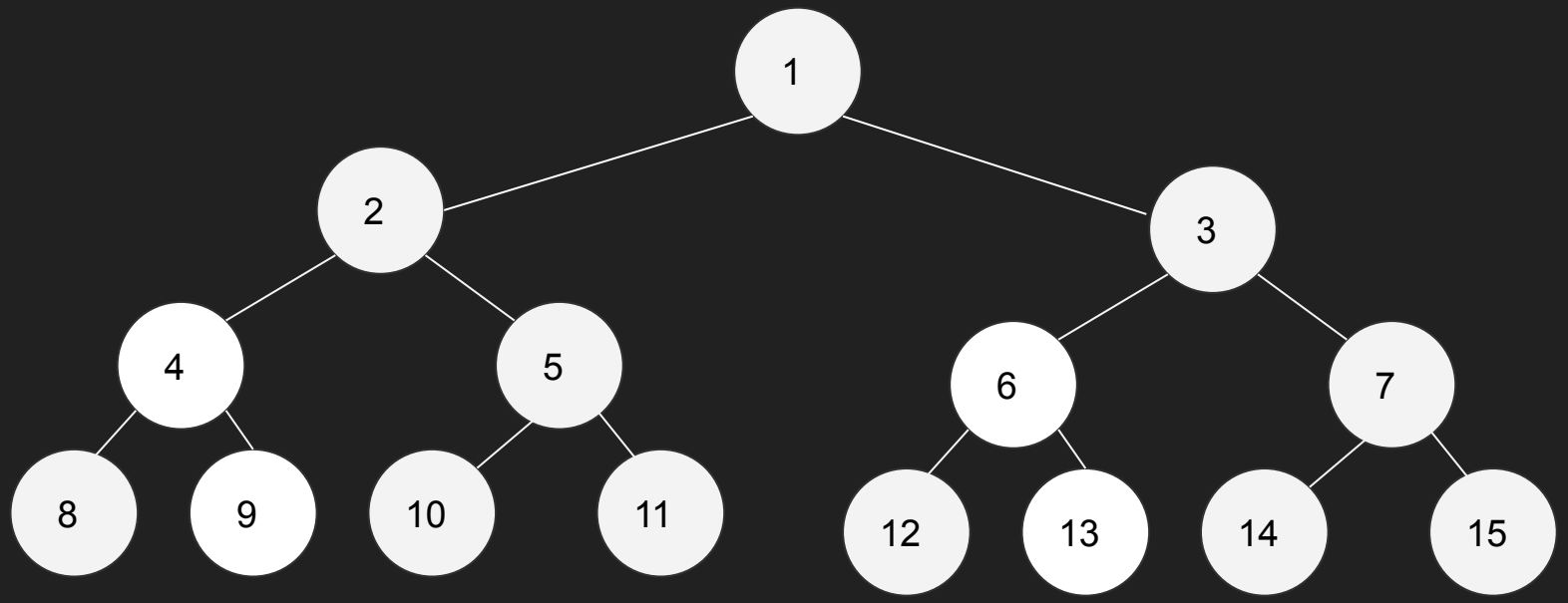
Uniformity Enables Outer-Loop Vectorization

```
for (int i = 0; i < end; i += 4) {  // vectorized
  __m128i a = [0,0,0,0];
  for (int j = 0; j < m; ++j) {
    bool p = C[j];
    if (p) {
      __m128i v = loadu_si128(A[j*m + i]);
      a = add4(a, v);
    }
  }
  storeu_si128(&B[i], a);
}
… residual
```

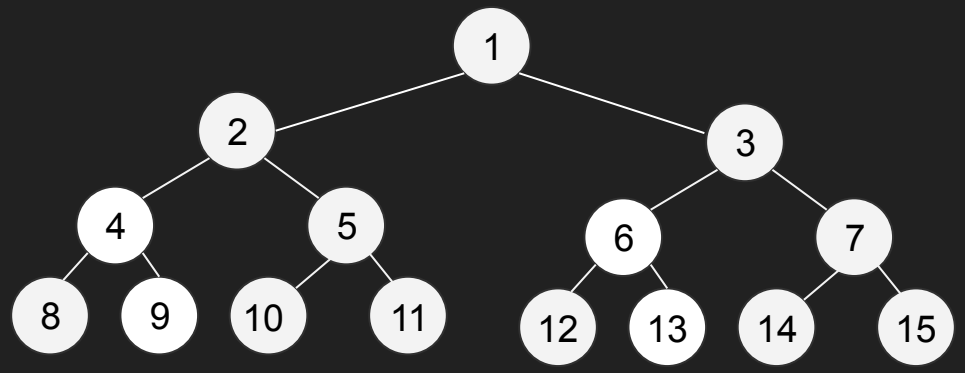
Recursive Tree Traversal Vectorization

Milind Kulkarni et al.

Tree



Tree



Nodes

	1	2	4	8	9	5	10	11	3	6	12	13	7	14	15
Traversals															
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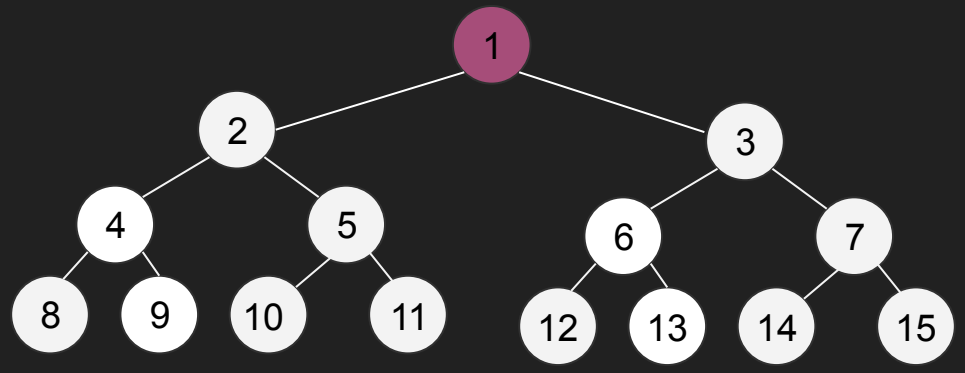
Tree



Nodes

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Traversals															
A															
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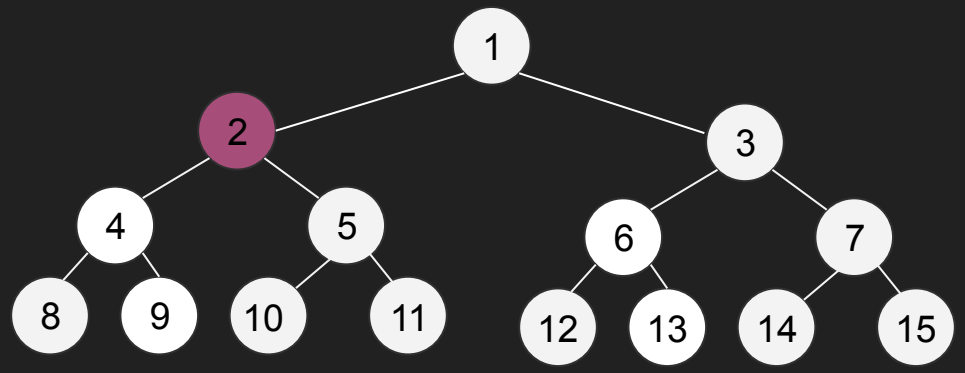
Tree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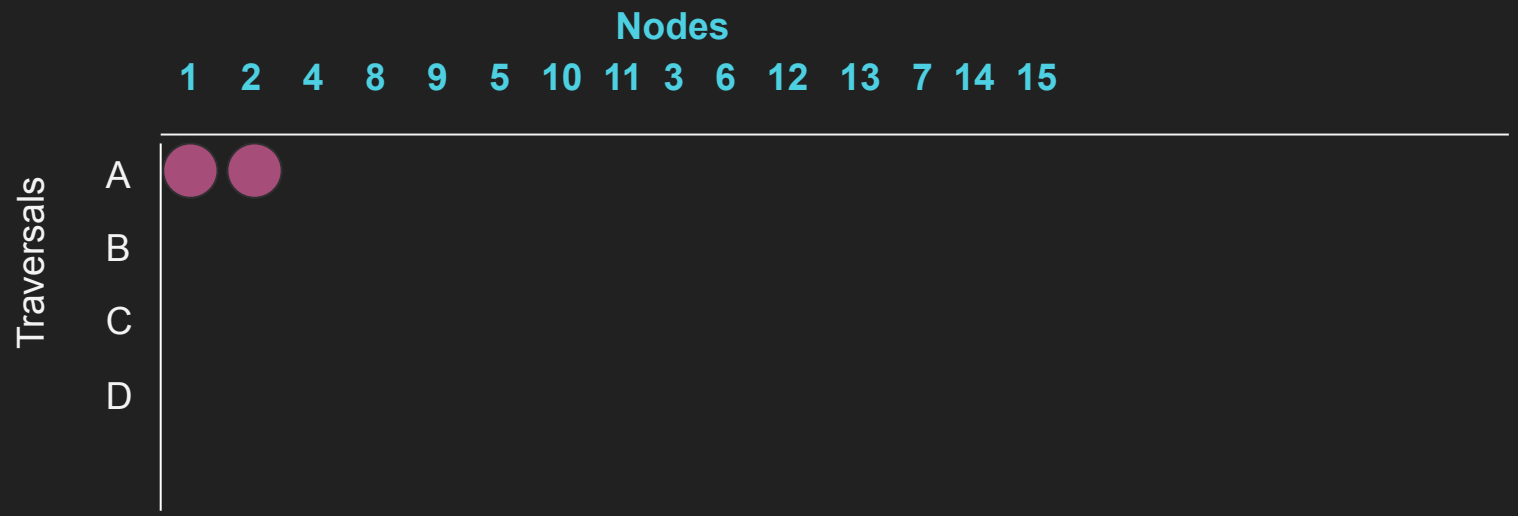
Nodes
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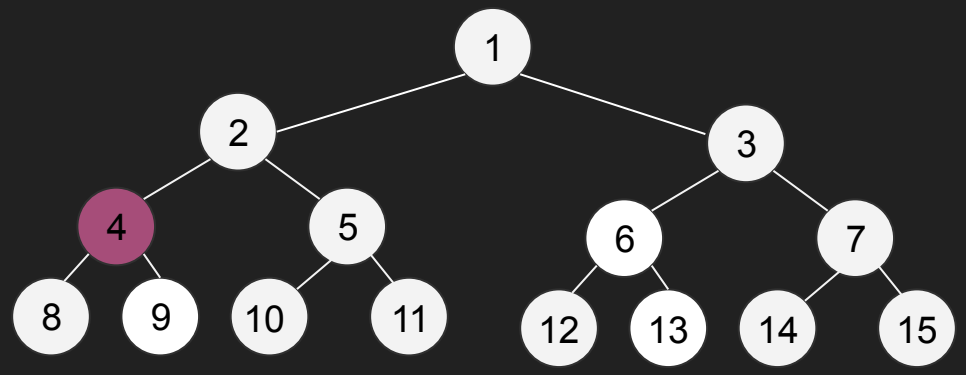
Tree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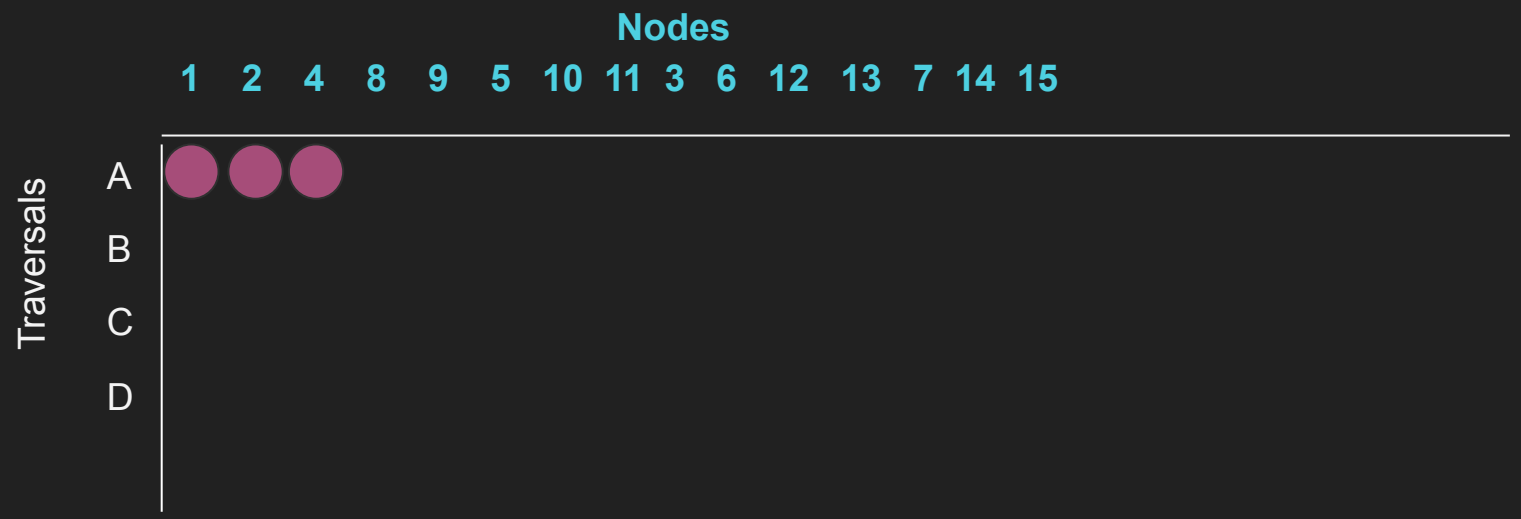
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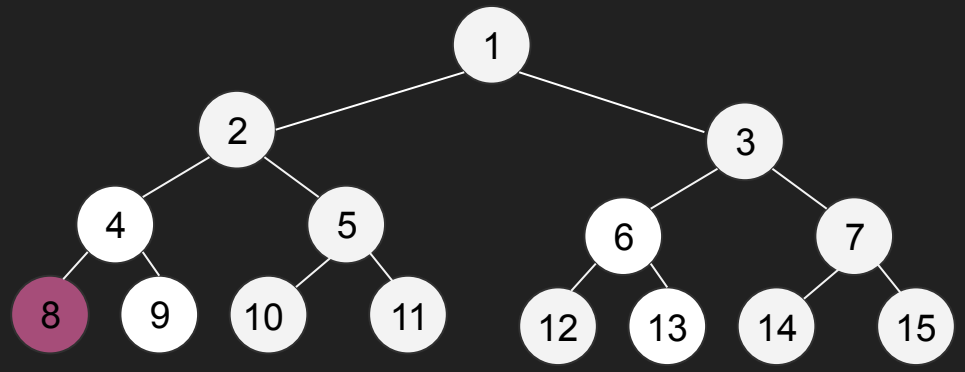
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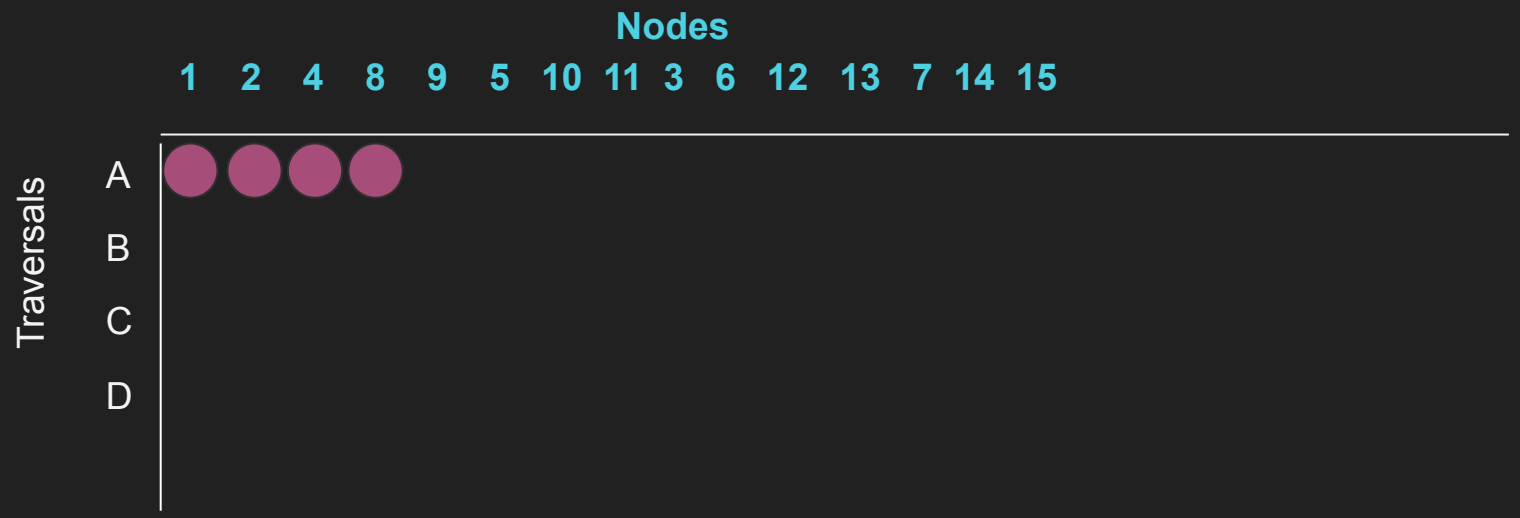
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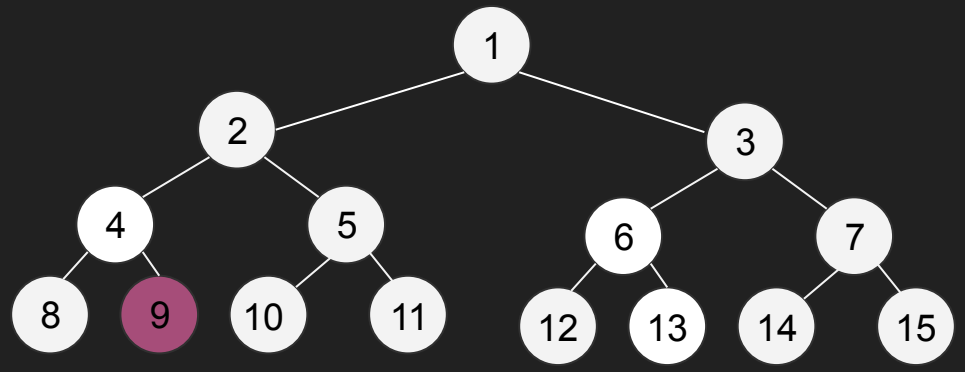
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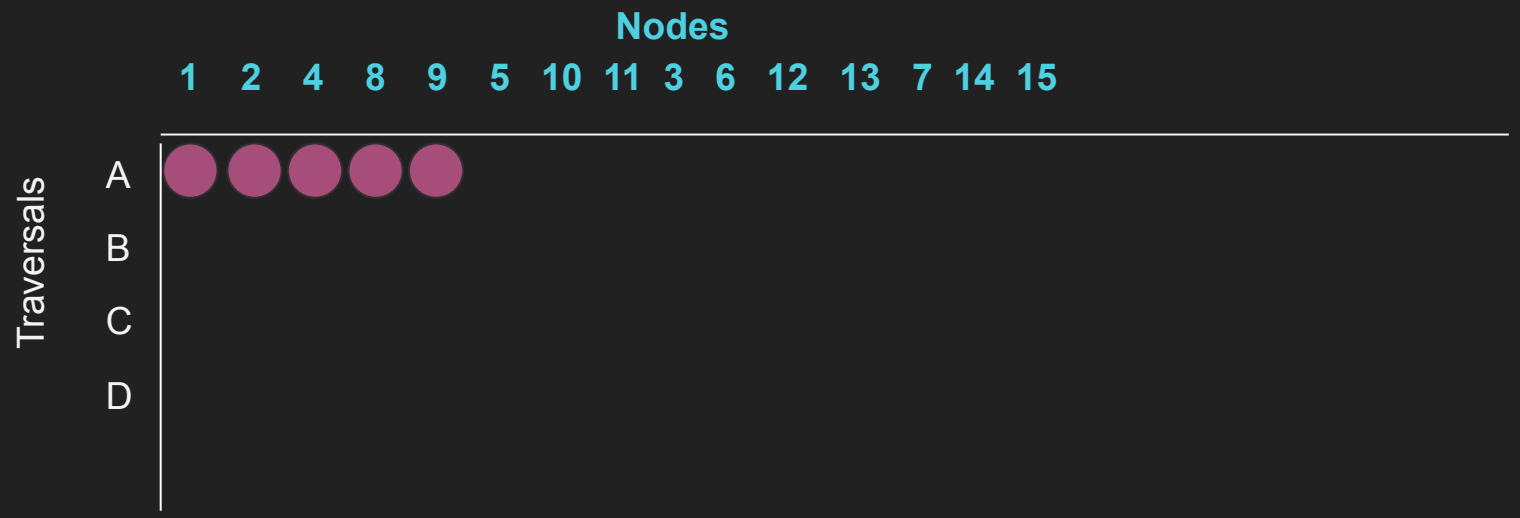
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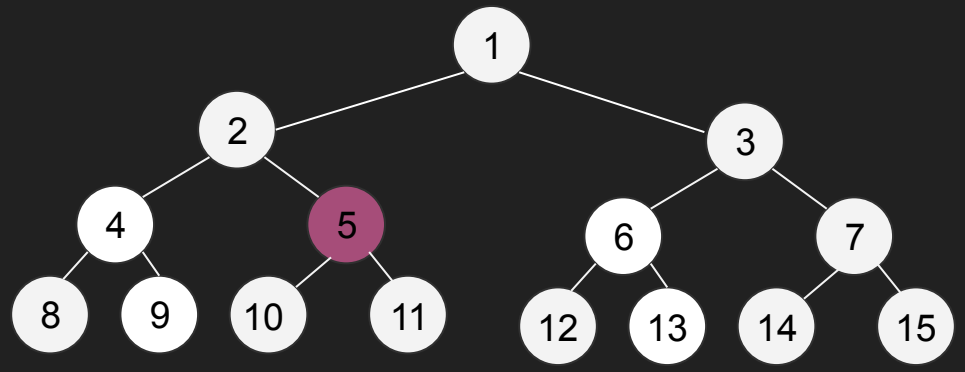
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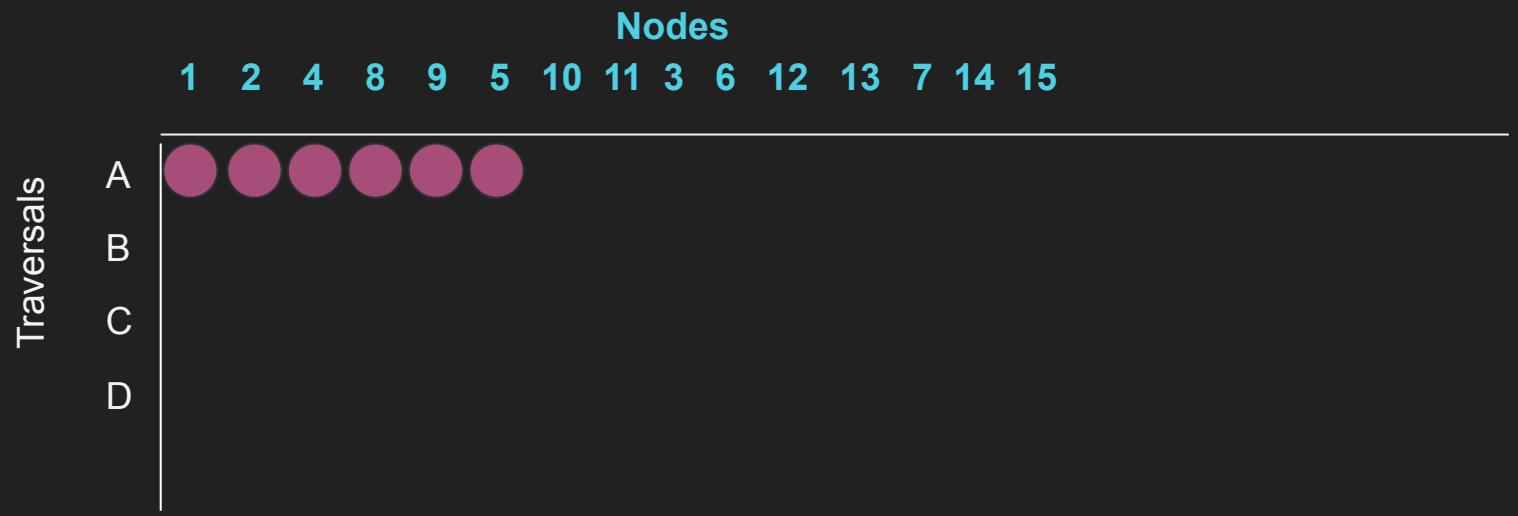
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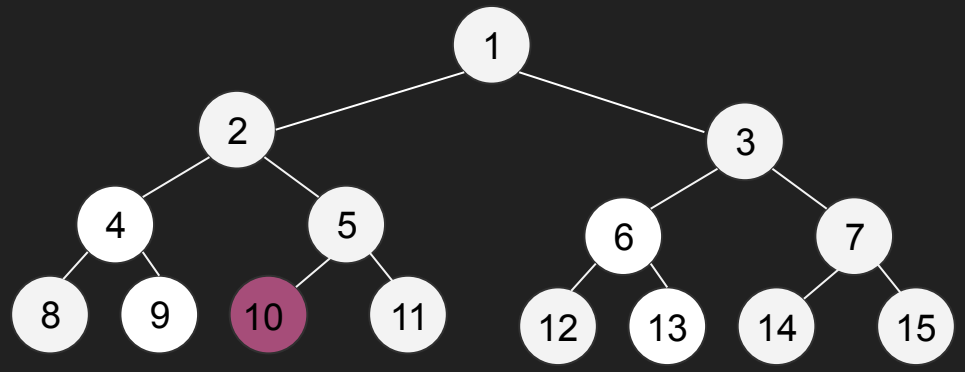
Tree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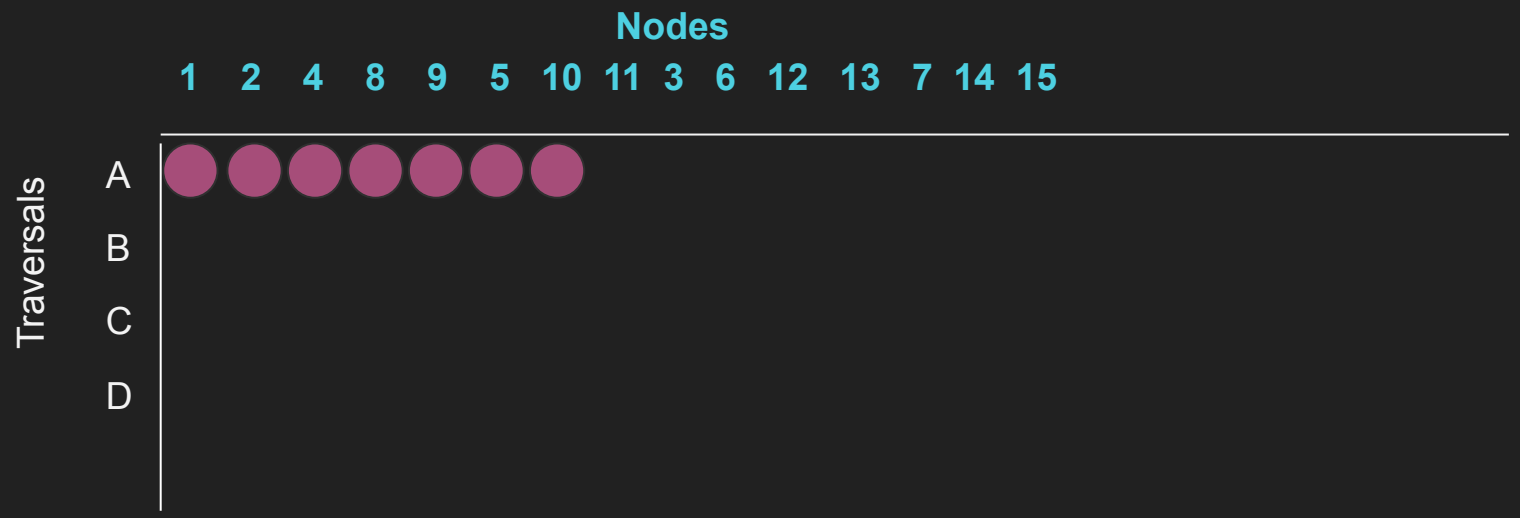
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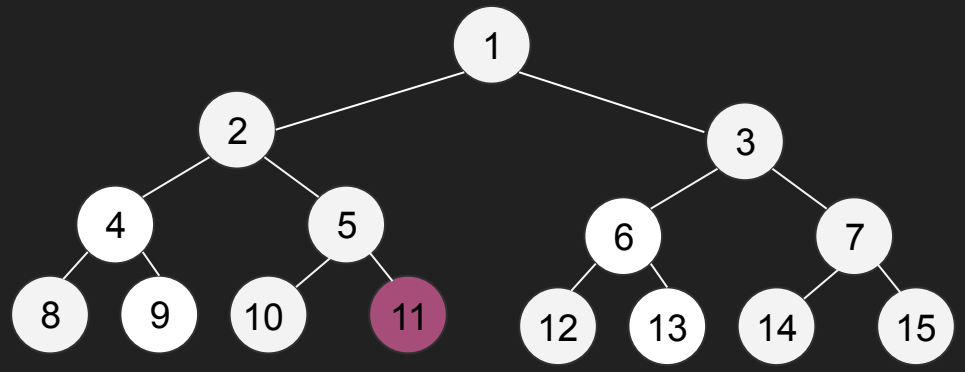
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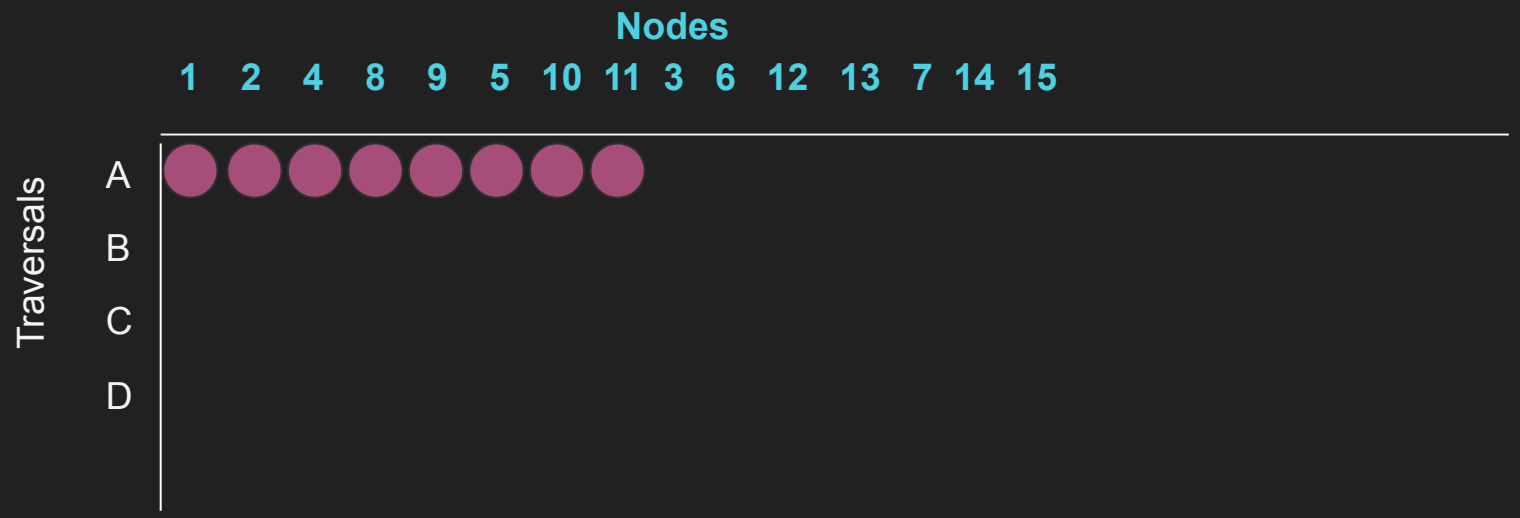
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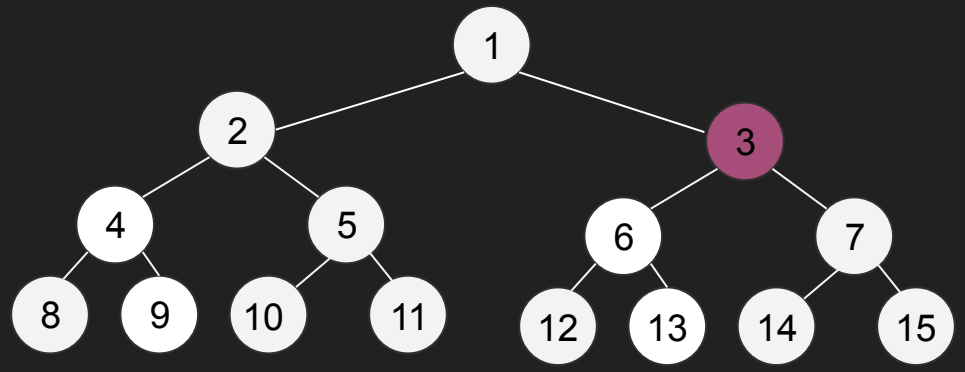
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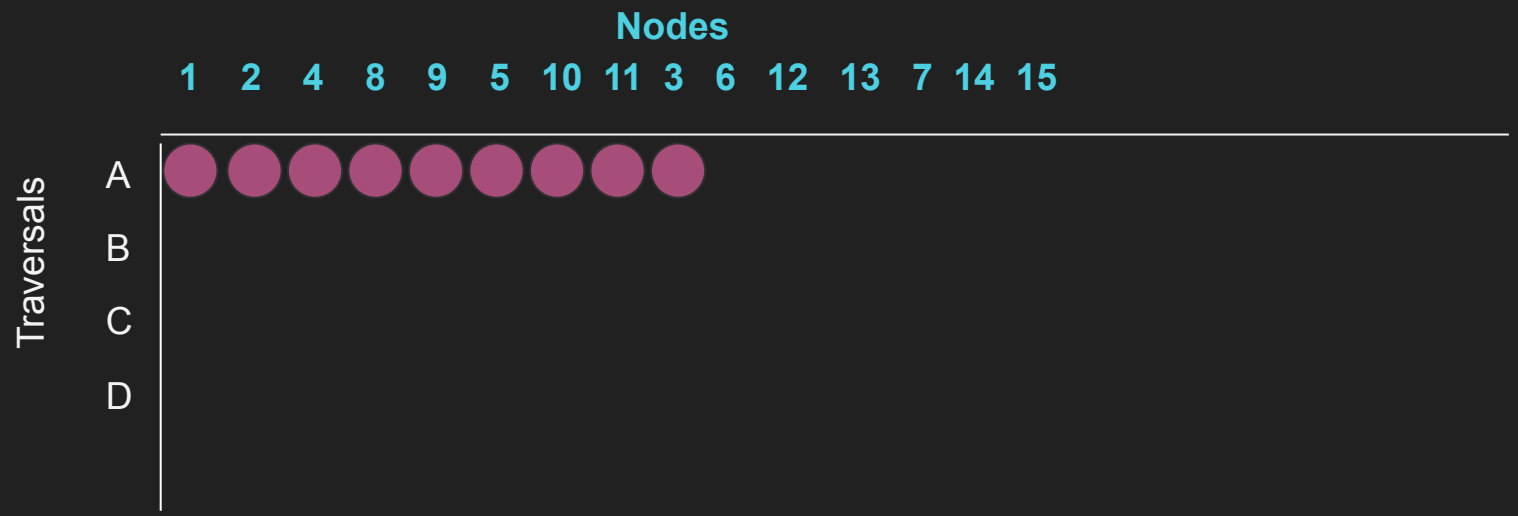
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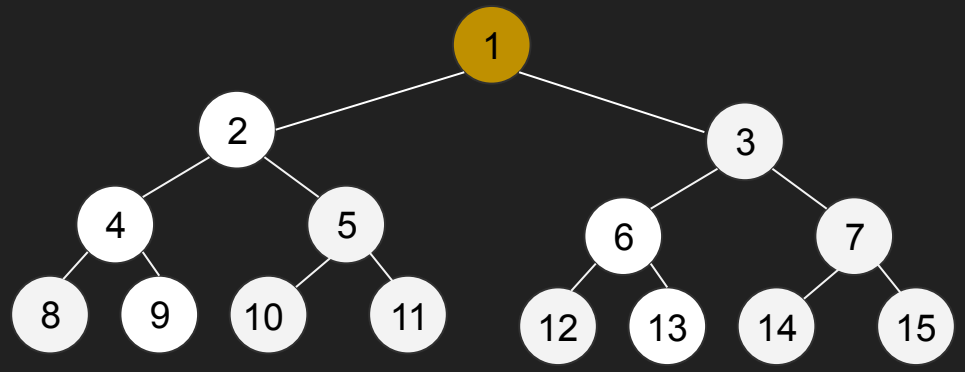
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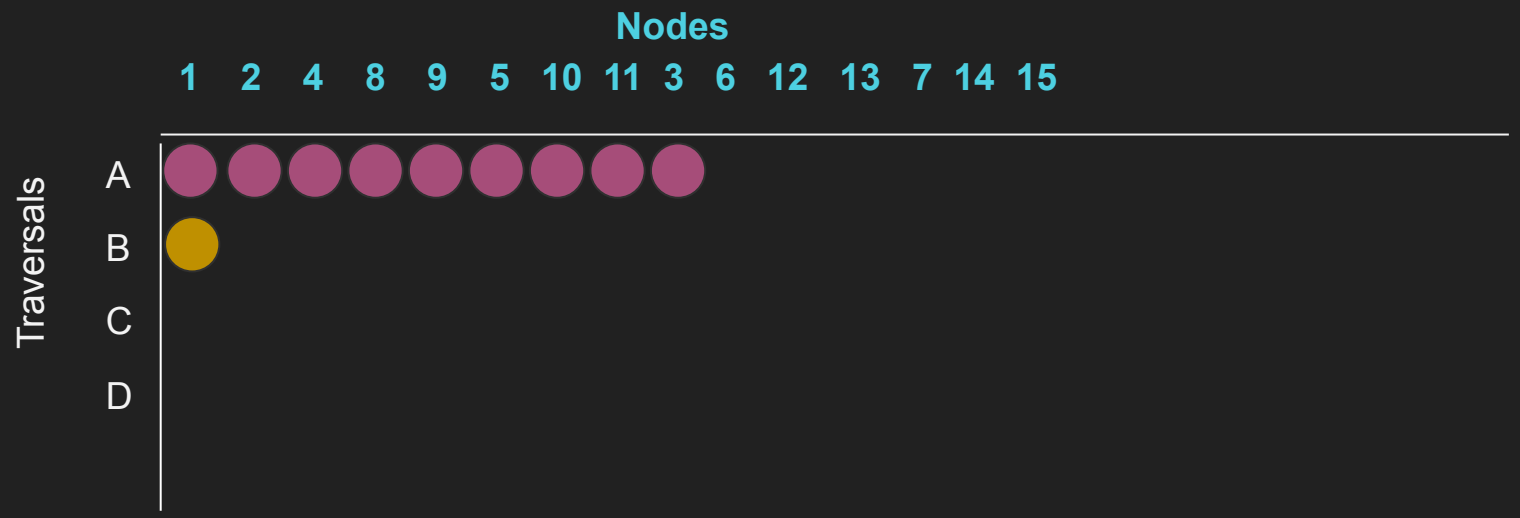
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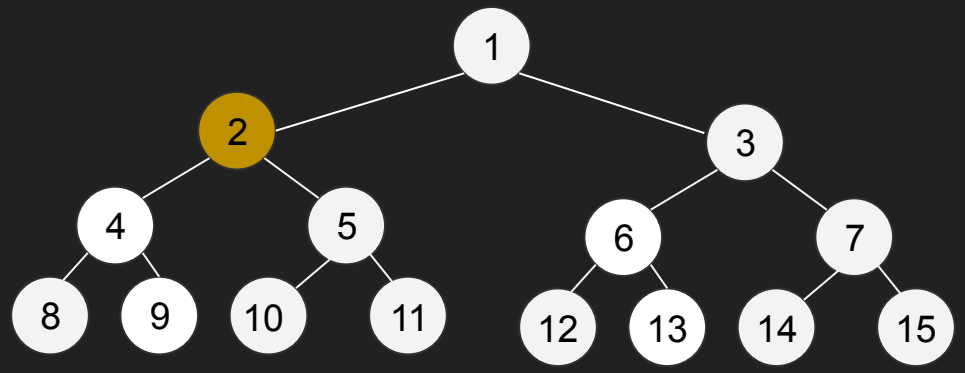
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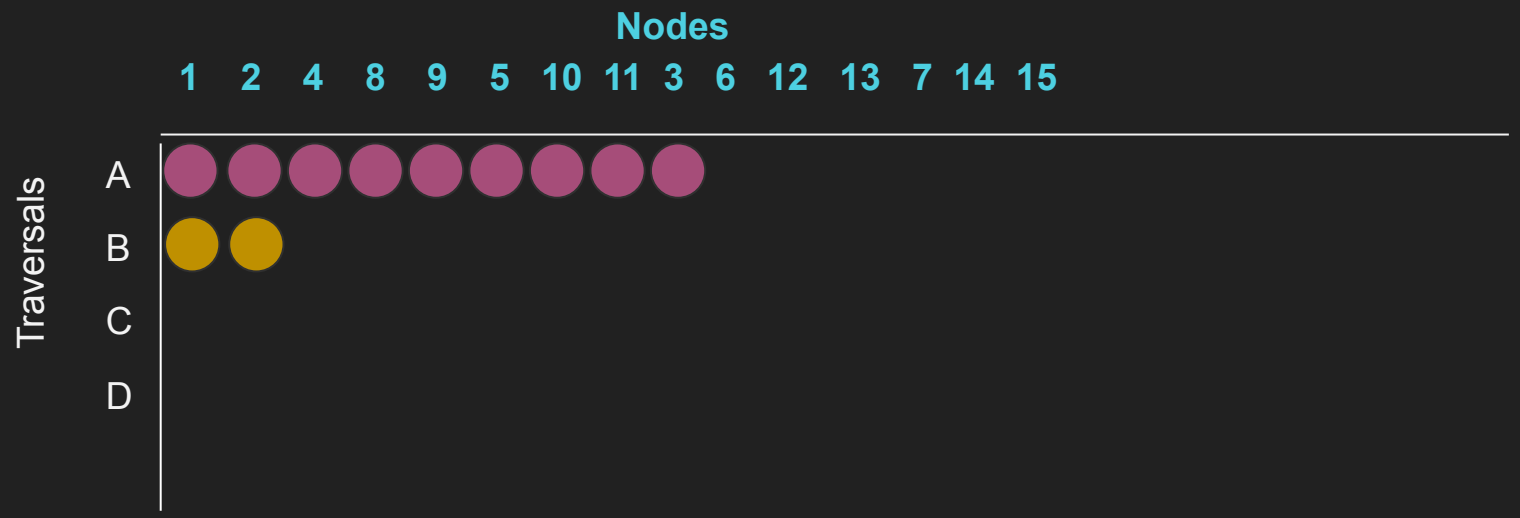
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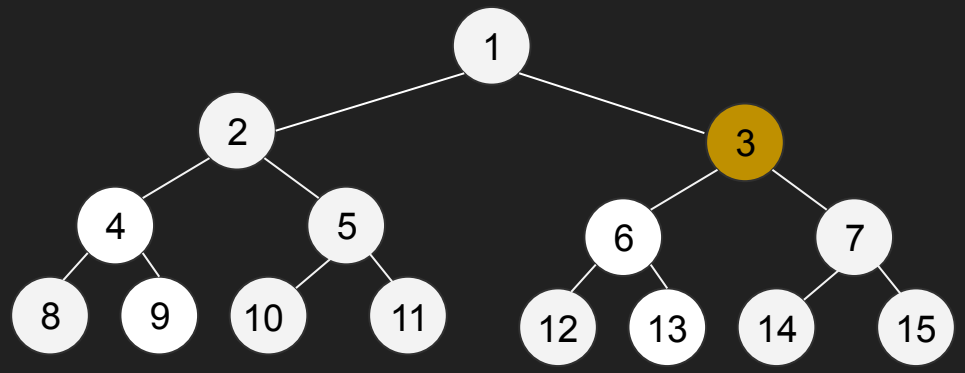
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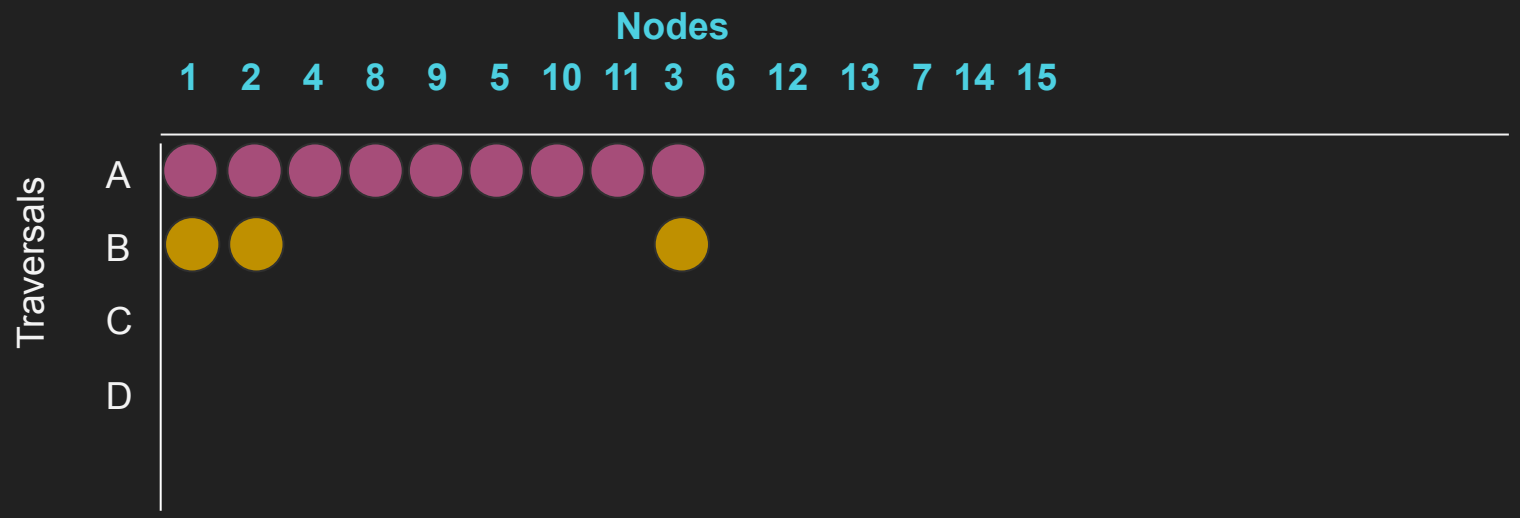
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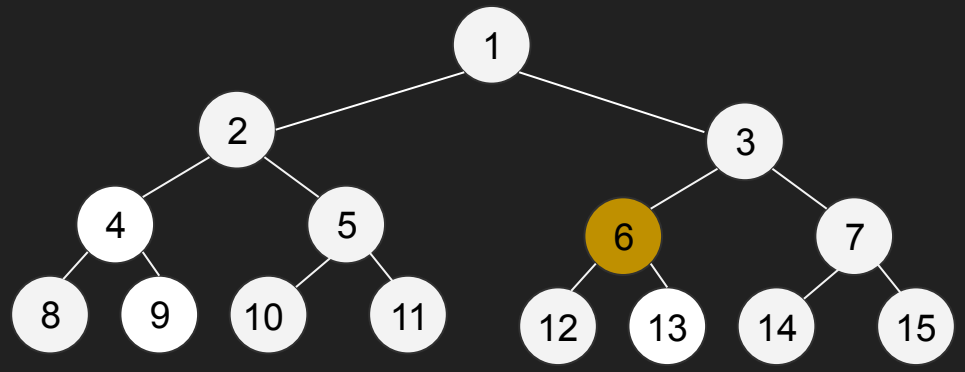
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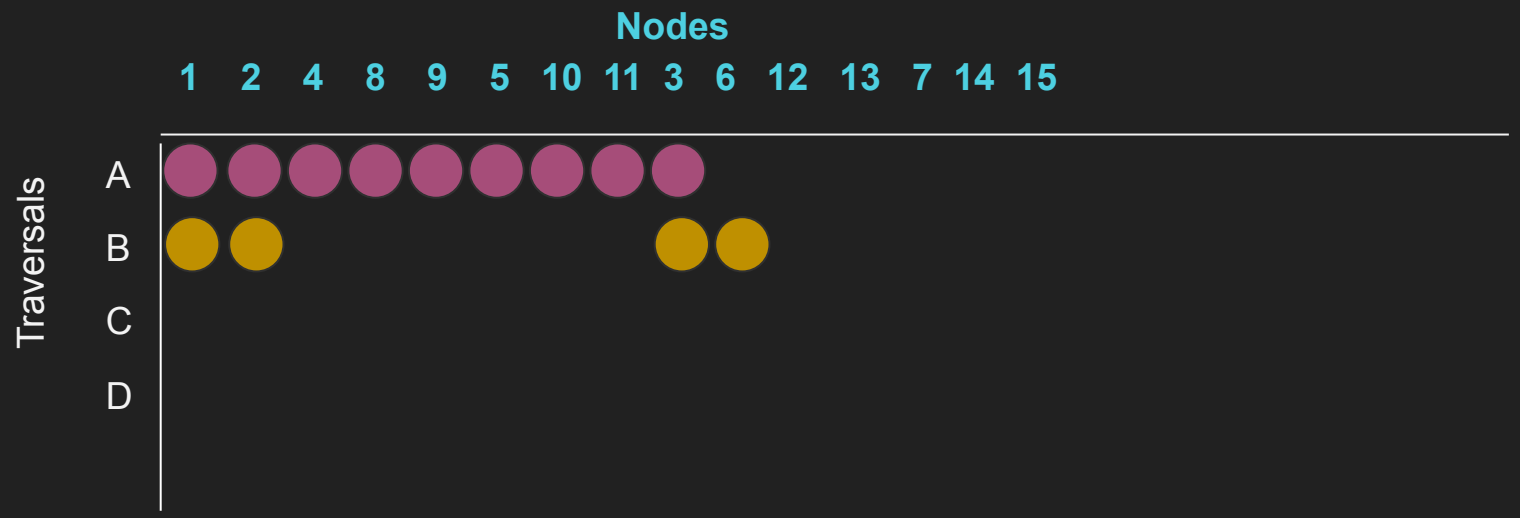
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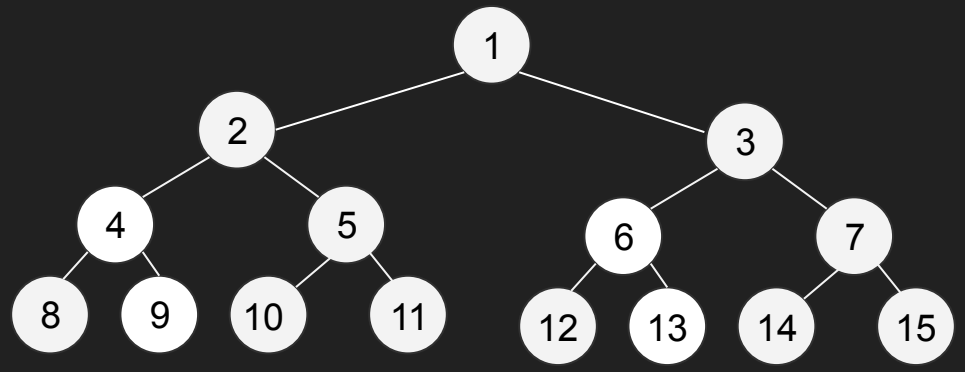
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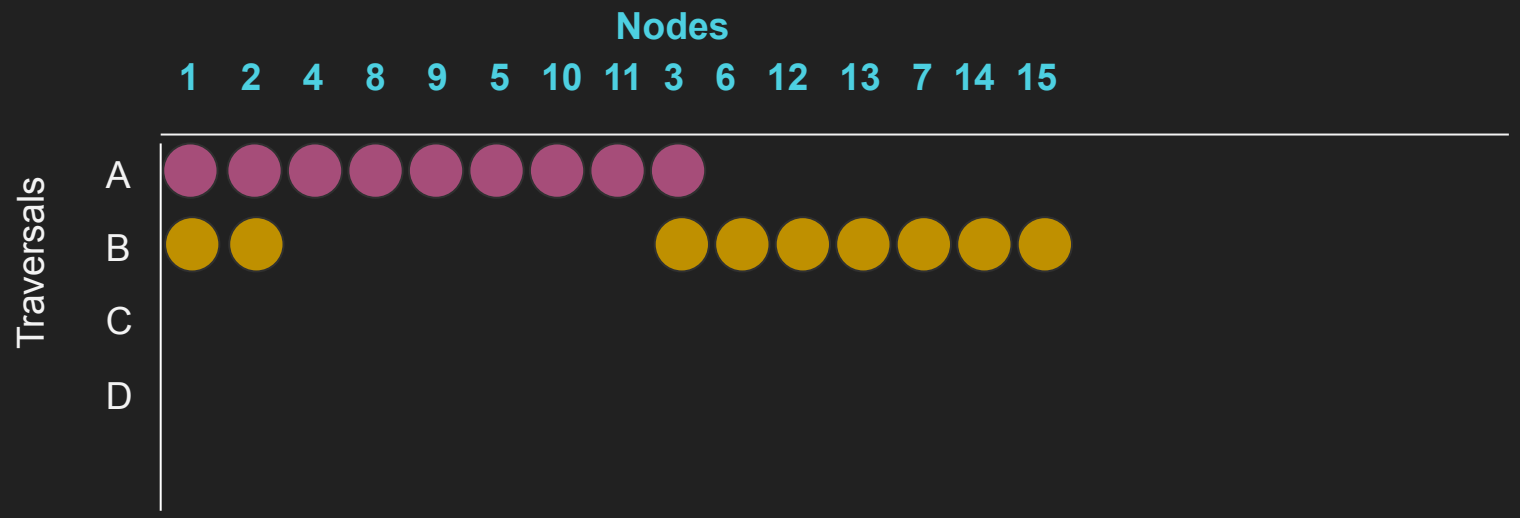
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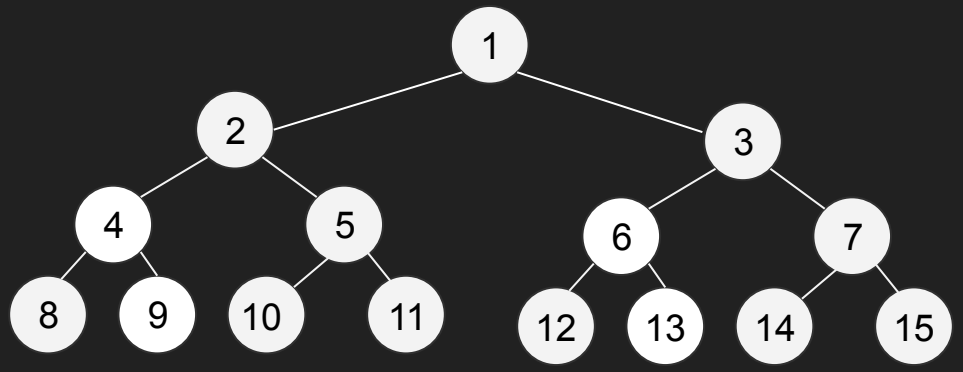
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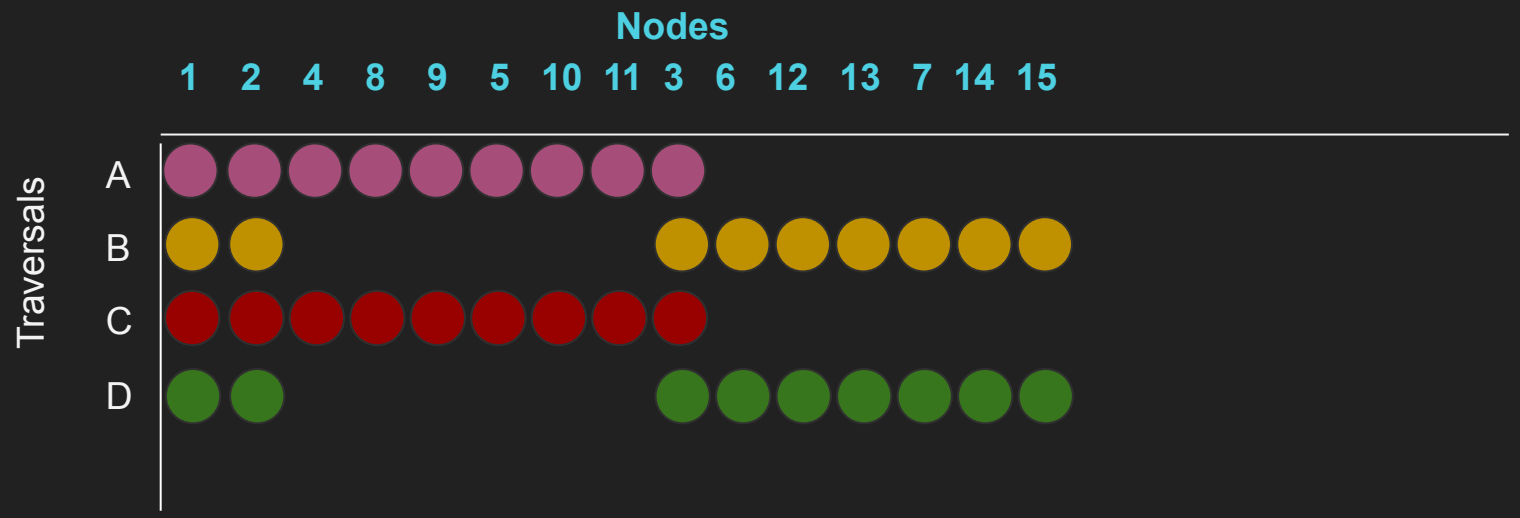
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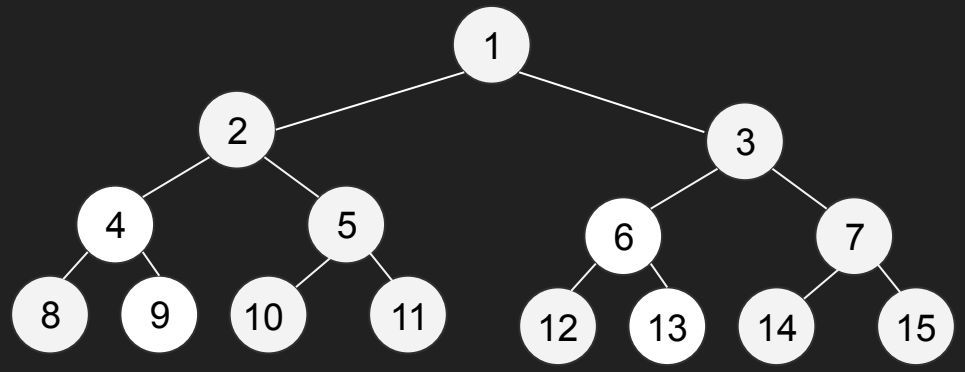
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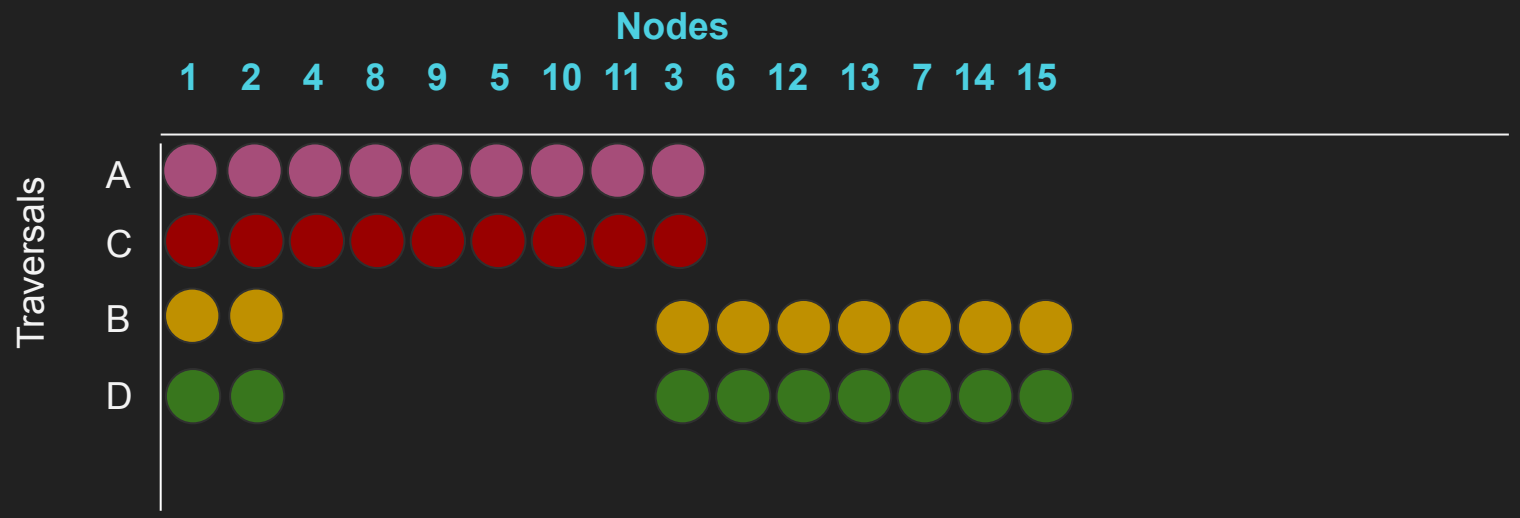
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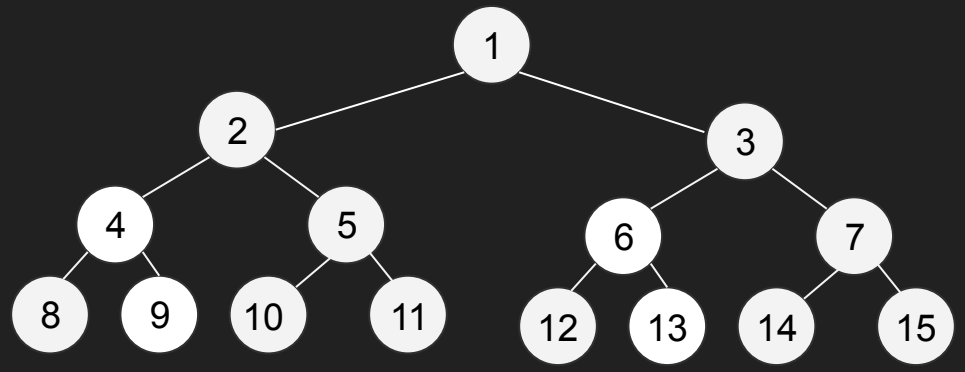
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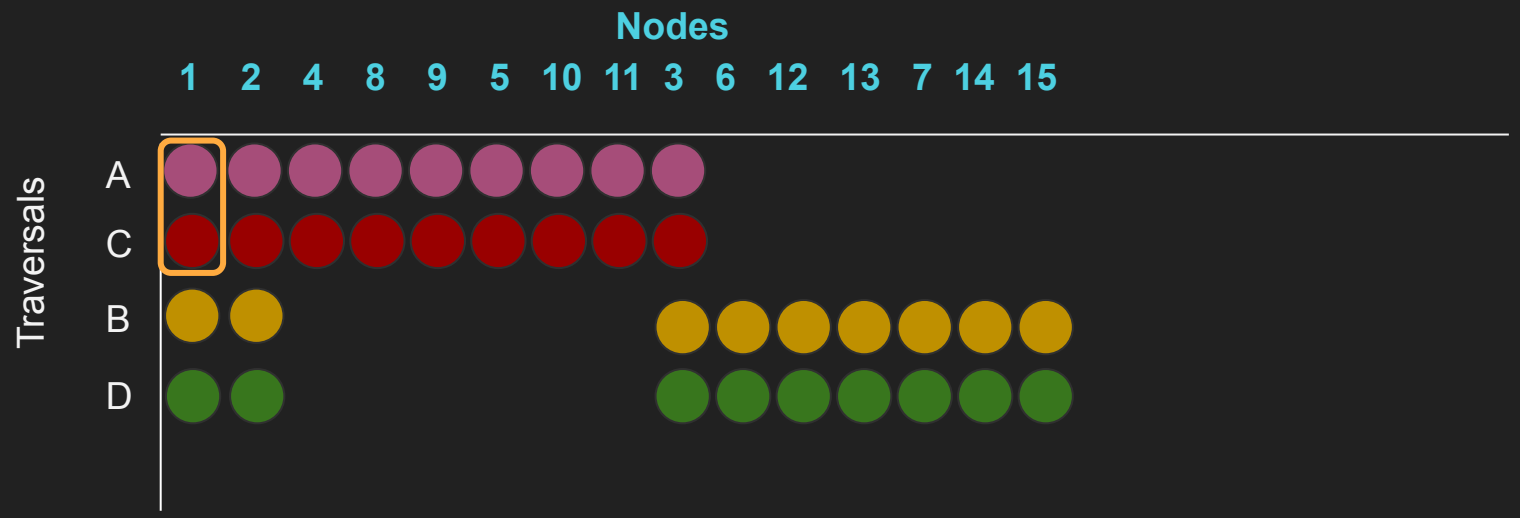
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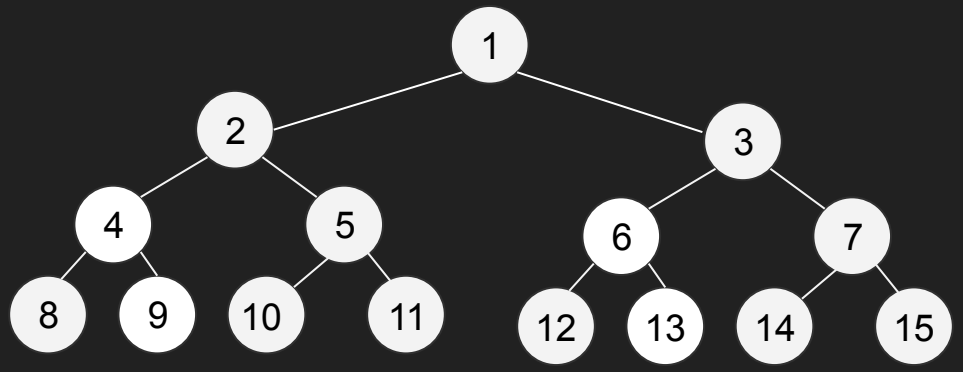
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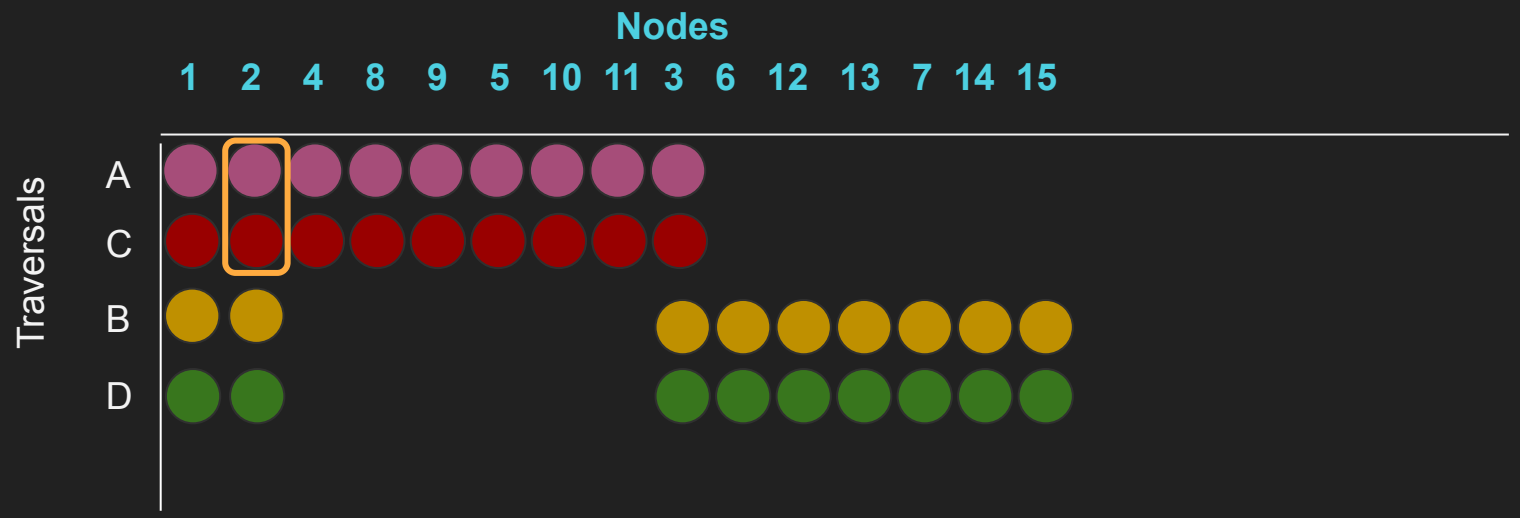
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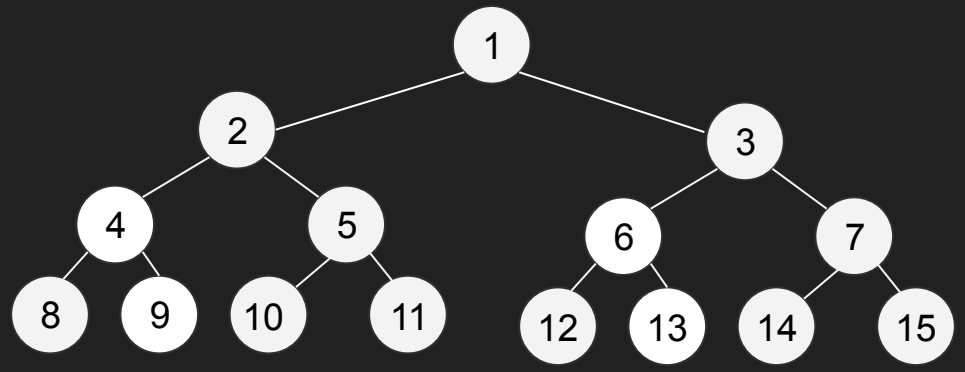
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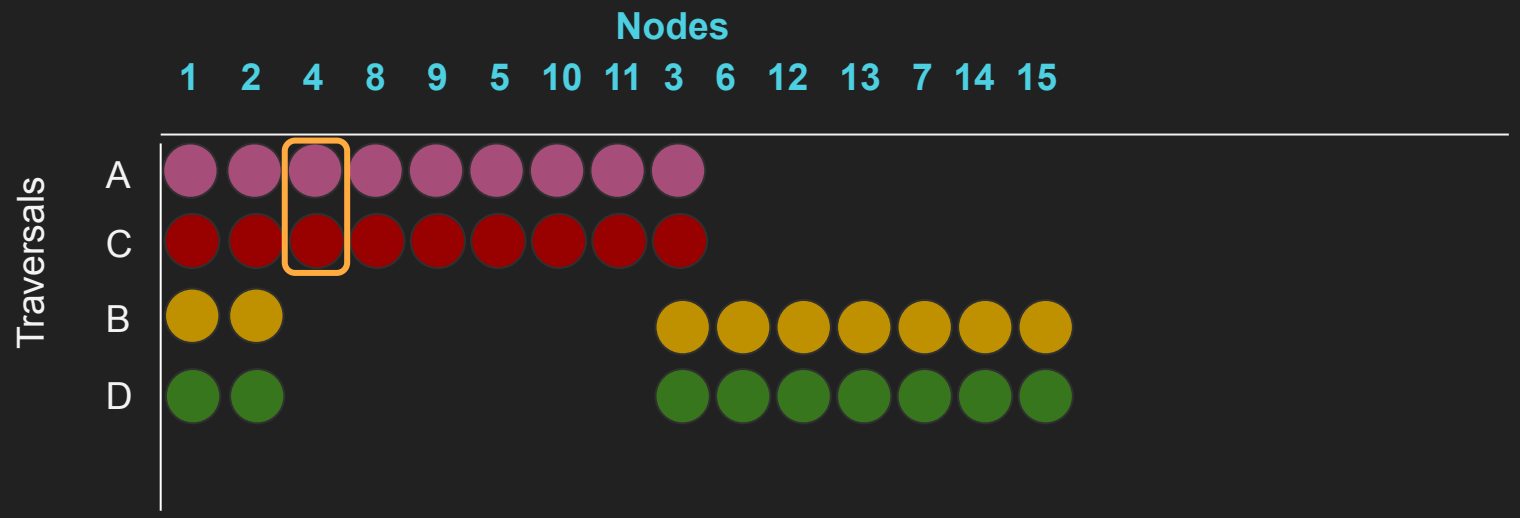
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Thank You!