Surface Properties

Surface properties describe reflection, transmission, absorption, and scattering of a surface.

Coincident Surfaces

TracePro models often incorporate geometry with coincident surfaces. This occurs for such systems where two objects have one common side, e.g., a cemented doublet. This section describes the behavior of TracePro with respect to Surface Properties applied to those coincident surfaces. The tree cases are:

1. No Surface Property (i.e., <None>) applied to either coincident surface - TracePro will calculate the reflectance and transmittance at this interface based on the Index of Refraction specified in the Material Properties for the two objects (the Fresnel reflection and transmission coefficients).
2. Surface Property applied to one coincident surface only while the other surface has the Surface Propery of <None> - TracePro will use the parameters of the single defined surface property to determine the flux and direction of the resulting rays.
3. Surface Properties applied to both coincident surfaces - Although this condition is somewhat nonsensical and is not recommended, TracePro will combine the two surface properties in accordance with the equations below:

$$R_c = R_1 + \frac{T_1^2 \cdot R_2}{1 - (R_1 \cdot R_2)}, \qquad (7.20)$$

$$T_c = \frac{T_1 \cdot T_2}{1 - (R_1 \cdot R_2)}, \text{ and} \qquad (7.21)$$

$$A_c = A_1 + \frac{T_1 \cdot ((A_1 \cdot R_2) + A_2)}{1 - (R_1 \cdot R_2)}, \qquad (7.22)$$

where R_1, T_1, and A_1 are the reflectance, transmittance, and absorptance, respectively, of the first coincident surface; R_2, T_2, and A_2 are the reflectance, transmittance, and absorptance, respectively, of the second coincident surface; and R_c, T_c, and A_c are the reflectance, transmittance, and absorptance, respectively, of the resulting combined property.

Note that other surface properties, such as the BSDF (BRDF or BTDF) are found in an analogous method.

BSDF

The Bidirectional Scattering Distribution Function (BSDF) is a measure of the light scattered from a surface in different directions. The BSDF is a function of both the incident direction and the scattering direction, hence the term bidirectional. Mathematically, the BSDF is defined as the scattered *radiance* per unit incident *irradiance*, or

$$\mathrm{BSDF}(\theta_i, \phi_i, \theta_s, \phi_s) = \frac{dL_s(\theta_s, \phi_s)}{dE_i(\theta_i, \phi_i)}. \quad (7.23)$$

Because radiance has units watts/m^2-sr and irradiance has units watts/m^2, the BSDF has units 1/sr (inverse steradians). Note that this equation for BSDF takes into account the projection of the surface “emitting” the scattered radiation, thus you can view Equation 7.23 as

$$\mathrm{BSDF} = \frac{P_{scat}/\Omega}{P_{inc} \cdot \cos\theta_{scat}} \quad . \quad (7.24)$$

where P_{scat} is the scattered power, P_{inc} is the incident power, Ω is the solid angle upon scattering, and θ_{scat} is the scatter angle from the normal to the scatter location. To remove this cosine dependence, you must post process in other software such as Excel.

In TracePro, the BSDF model is shift-invariant with respect to the incident direction. This property of BSDFs for polished surfaces was discovered by Harvey in his doctoral dissertation (See “Harvey-Shack BSDF” on page 7.16.). This means that the shape of the BSDF depends only on the difference between the specular direction and the scattered direction. This type of model is useful for a wide variety of surfaces, particularly optically polished surfaces.

The BSDF is really a generic term for scattering from surfaces. There are three specific types of BSDF:

- BRDF (Bidirectional Reflectance Distribution Function)
- BTDF (Bidirectional Transmittance Distribution Function)
- BDDF (Bidirectional Diffraction Distribution Function)

Harvey-Shack BSDF

In his dissertation (J. E. Harvey, “Light-Scattering Properties of Optical Surfaces,” Ph.D. Dissertation, U. Arizona, 1976) Harvey found that for many optical surfaces, the BSDF is independent of the direction of incidence if it is expressed as a function of direction cosines instead of angles. Harvey called this property “shift-invariant,” as in linear systems theory. Referring to Figure 7.4, β_0 is a projection onto the surface of the unit vector $\mathbf{r_0}$ in the specular direction, β is a projection onto the surface of the unit vector **r** in the scattering direction, and the magnitude of their difference, $|\beta-\beta_0|$, is the argument of the BSDF. Note that β and β_0 are <u>not</u> unit vectors. They are projections of unit vectors, so their lengths are less than or equal to one. The Harvey-Shack method gives a good model for the behavior of most optical surfaces, i.e. those for which:

- Scattering is due mainly to surface roughness
- Scattering (and thus surface roughness) is isotropic
- Surface roughness is small compared to the wavelength of light

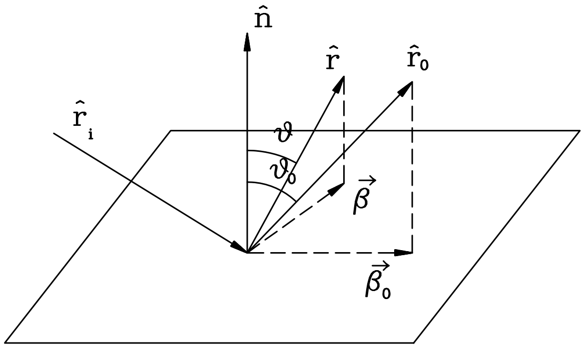

FIGURE 7.4 - Harvey-Shack BSDF: a shift-invariant BSDF representation

When measuring or evaluating the BSDF in the plane of incidence, i.e., when the scattering direction **r** lies in the same plane as the incident and specular directions $\mathbf{r_i}$ and $\mathbf{r_0}$, the value of $|\beta-\beta_0|$ reduces to $|\sin\theta - \sin\theta_0|$, where θ is the angle between the scattering direction and the surface normal, and θ_0 is the angle between the specular direction and the surface normal. For light incident normal to the surface, $\theta_0=0$ and so $|\beta-\beta_0|=\sin\theta$. Measurements are often made only in the plane of incidence, and many BSDF plots have $|\sin\theta - \sin\theta_0|$, $\sin\theta - \sin\theta_0$, $\sin\theta$, or θ as the horizontal axis.

ABg BSDF Model

The BSDF model used in TracePro is a quasi-inverse-power-law model called the ABg model. It is called the ABg model because of the three parameters in the following equation,

$$\mathrm{BSDF} = \frac{A}{B + \left|\vec{\beta} - \vec{\beta_0}\right|^g}, \tag{7.25}$$

where A, B, and g are parameters that can be used to fit the formula to measured data.

A typical ABg model BSDF, graphed on a log-log scale, is shown in Figure 7.5.

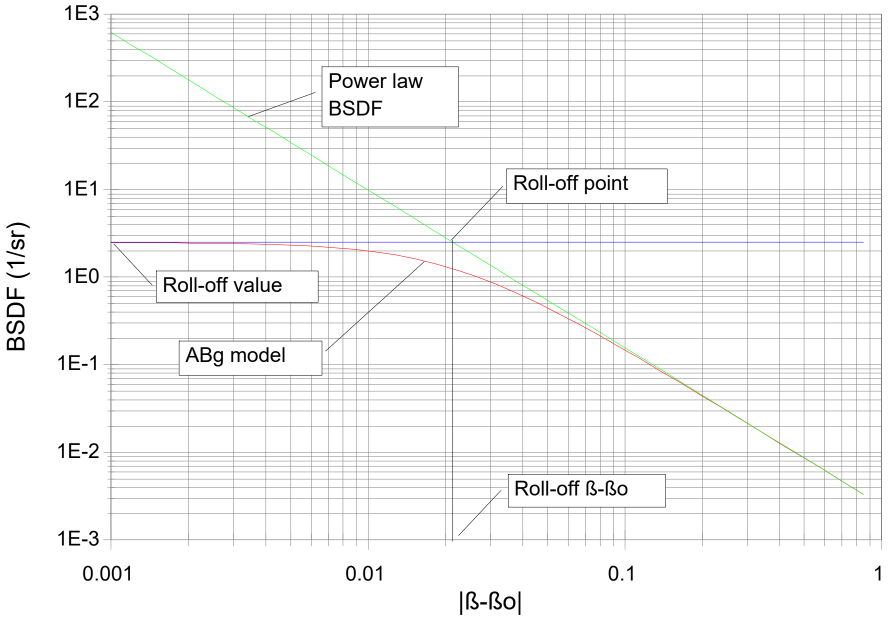

FIGURE 7.5 - ABg BSDF Model where A=0.0025, B=0.001, and g=1.8

Note the following properties of the ABg BSDF model:

- B must be greater than zero unless g is zero.
- If g is zero, it becomes a Lambertian BSDF with value A/(B+1), and total integrated scatter equal to πA/(B+1).
- If g is less than zero, the BSDF increases with $|\beta-\beta_0|$. Some surfaces display this behavior.

Most optically polished surfaces exhibit a BSDF with a shape that is well fit by this model. The model has a flat region at small values of $|\beta-\beta_0|$, a transition region around the roll-off point, and a region of nearly constant slope equal to (-g) at large values of $|\beta-\beta_0|$. Values of g typically are between 1 and 3 depending on substrate material, polishing method, and degree of polish. Values of B are typically 0.001 or smaller, and values of A vary widely. The value of the BSDF at $|\beta-\beta_0|$=0, where the curve "flattens out" is given by

$$\mathrm{BSDF}(0) = A/B. \tag{7.26}$$

The value of $|\beta-\beta_0|$ at which the curve rolls off, i.e., the roll-off $|\beta-\beta_0|$ in the curve above, determines the parameter *B*. *B* is related to the roll-off $|\beta-\beta_0|$ and g by

$$B = (\beta_{\mathrm{rolloff}})^g. \tag{7.27}$$

BRDF, BTDF, and TS

BSDF is an abbreviation for Bidirectional Scattering Distribution Function. BSDF is a generic term. Specific terms used for describing scattering are:

- BRDF — Bidirectional Reflectance Distribution Function

- BTDF — Bidirectional Transmittance Distribution Function

These quantities refer to reflective and transmissive scattering, respectively. When defining a Surface Property in TracePro, you can define both the BRDF and the BTDF.

A quantity associated with the BSDF is the Total Scatter, or TS. In TracePro, the TS is defined as the integral of the BSDF over all angles,

$$\mathrm{TS}=\int_0^{2\pi}\int_0^{\pi/2} \mathrm{BSDF}(\theta_i, \phi_i, \theta_s, \phi_s)\cos\theta\sin\theta(d\theta)d\phi\,, \tag{7.28}$$

where θ and ϕ are spherical polar coordinates defined with the surface normal as the z axis. For light incident normal to a surface, θ_i and ϕ_i are zero, and $\theta_s = \theta$ and $\phi_s = \phi$, and the TS is

$$\mathrm{TS}=\int_0^{2\pi}\int_0^{\pi/2} \mathrm{BSDF}(0, 0, \theta_s, \phi_s)\cos\theta\sin\theta(d\theta)d\phi\,. \tag{7.29}$$

In order for TracePro to conserve energy, all the light incident on a surface must be accounted for in a surface property. This means the sum of the coefficients for absorption, specular reflection and transmission, and scattering, must equal one,

$$a + R_{\mathrm{S}} + T_{\mathrm{S}} + R_{\mathrm{TS}} + T_{\mathrm{TS}} = 1\,, \tag{7.30}$$

where a = absorptance

R_s = specular reflectance

T_s = specular transmittance

R_{TS} = TS for reflection

T_{TS} = TS for transmission.

When you edit an existing Surface Property or add a new one, TracePro will not let you leave the Surface Property Editor if this conservation of energy equation is not satisfied. You have the option of having TracePro solve for any of these quantities in the editor.

Asymmetric BSDF

An asymmetric BSDF is one that is not cylindrically symmetric in direction cosine space. Asymmetric BSDFs in TracePro include the elliptical BSDF models, which have mirror symmetry about two orthogonal axes, the one-dimensional BSDF models, and the Asymmetric Table BSDF, which in general has no symmetry.

When you apply an asymmetric BSDF to a surface, a reference axis for the asymmetry must also be specified. There are two ways to do this in TracePro. You can either:

- Specify a fixed reference axis (use this for most asymmetric BSDFs).
- Specify that the reference axis “floats” with the plane of incidence.

This choice is controlled by the *Use fixed axis for zero-azimuth for asymmetric scatter* checkbox in the *Surface* pane of the **Apply Properties** dialog box. In most cases, you should check this check-box and enter the direction cosines of a reference axis in the Anisotropic Axis tab. The Anisotropic Axis is also used for the reference axis for the BSDF.

A useful hint to decide whether to use the *Use fixed axis for zero-azimuth for asymmetric scatter* option is as follows: If, when you shine light on your surface, the asymmetric scatter pattern rotates as you rotate the substrate, choose *Use fixed axis for zero-azimuth for asymmetric scatter*. If, on the other hand, the asymmetry of the scatter remains ***stationary*** as you rotate the substrate, you should ***uncheck*** the *Use fixed axis for zero-azimuth for asymmetric scatter* option.

When you apply a surface property with an elliptical BSDF, the x axis is the zero-azimuth axis, and the z axis is the surface normal. The y axis is determined by the right-hand rule.

The geometry for scattering is shown in Figure 7.6 for the *Use fixed axis for zero-azimuth for asymmetric scatter* option, and in Figure 7.7 for the unchecked option, in which the x axis or zero-azimuth axis always orthogonal to the plane of incidence. The plane of incidence is the plane containing the incident direction and the surface normal.

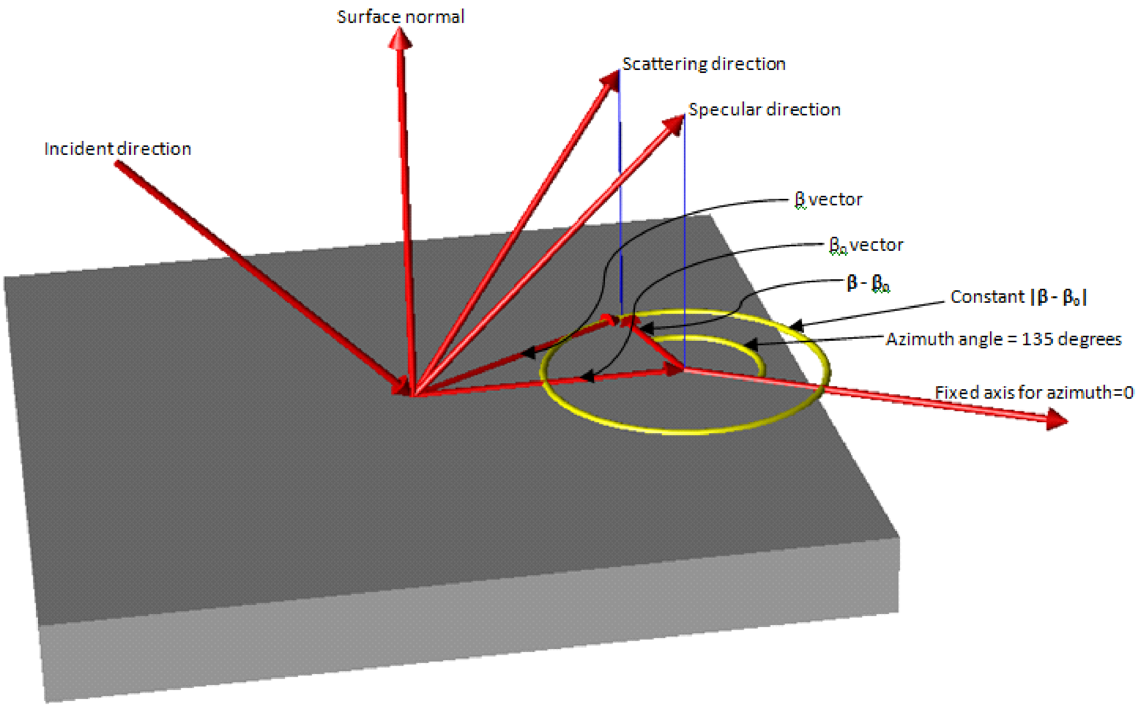

FIGURE 7.6 - Scattering geometry for the *Use fixed axis for zero-azimuth for asymmetric scatter* option.

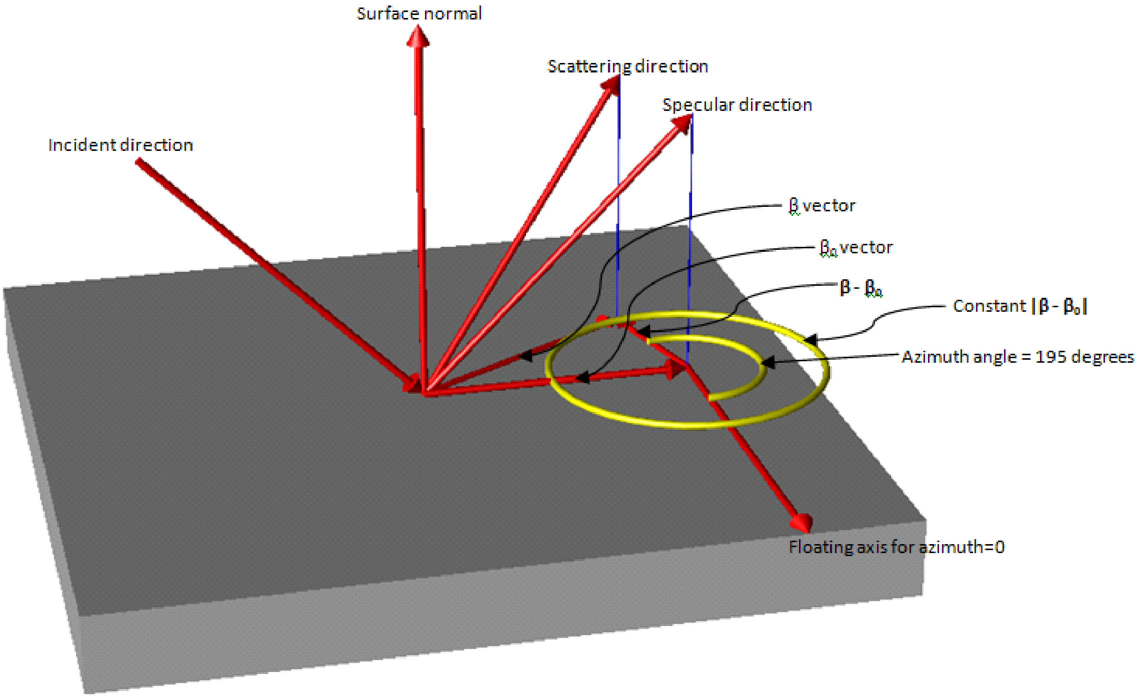

FIGURE 7.7 - Scattering geometry with the *Use fixed axis for zero-azimuth for asymmetric scatter* option *unchecked*.

Asymmetric Table BSDF

An asymmetric table BSDF model uses a table of BSDF values to specify the BSDF. The table is rectangular, i.e. you enter a list of $|\beta\text{-}\beta_0|$ values and azimuth values, and enter a BRDF and BTDF value for every combination of $|\beta\text{-}\beta_0|$ and azimuth. The beta coordinates are linear in plane of the surface as shown in Figure 7.6 and Figure 7.7. Therefore, the ring of constant $|\beta\text{-}\beta_0|$ represents one value of $|\beta\text{-}\beta_0|$, and the the azimuth angles for that $|\beta\text{-}\beta_0|$ value are different points spaced around the ring.

It is helpful to think of the $|\beta\text{-}\beta_0|$ and azimuth as plane polar coordinates, with $|\beta\text{-}\beta_0|$ as the radius coordinate. The plane for these coordinates is the tangent plane, the origin is the projection of the specular direction onto the tangent plane (i.e. the tip of the β_0 vector), and the azimuth=0 axis is the local x axis.

Elliptical BSDF

An elliptical BSDF is one that has coefficients that are elliptically interpolated, and therefore produces an asymmetric distribution of scattered light. An elliptical BSDF is one particular kind of asymmetric BSDF. The ABg model in TracePro is symmetric versus $|\beta-\beta_0|$ whereas the elliptical BSDF is asymmetric. Two elliptical BSDF models are available in TracePro: elliptical ABg and elliptical Gaussian.

When you create a surface property with elliptical BSDF in TracePro, the ellipse that determines the orientation of the scatter is defined by x and y axes. You must enter coefficients along for of the axes. The direction of the x axis is specified when apply the surface property to a surface. If you make an anisotropic surface property with an elliptical BSDF, one direction vector serves to orient both the anisotropic coefficients and the scattering ellipse. When you create an anisotropic surface property, you can add as many values of θ and ϕ as you wish to the property. For each temperature and wavelength, the surface property editor will create a table of coefficients for each pair of θ and ϕ. For each pair of θ and ϕ, you enter the peak BSDF and the x and y coefficients of the BSDF.

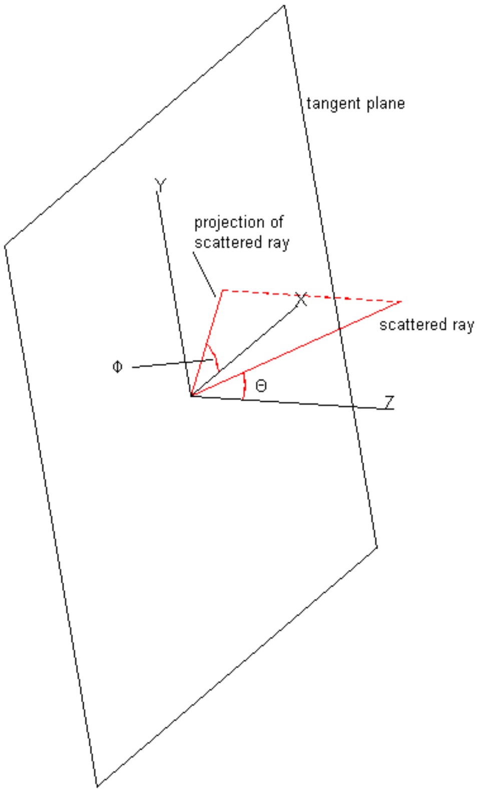

FIGURE 7.8 - Elliptical BSDF coordinates

Many surfaces exhibit asymmetric scattering behavior. Any surface that “looks different” from different directions is probably exhibiting asymmetric scattering. For example, a brushed or machined surface may scatter asymmetrically. Many composite materials that have fibers oriented in a particular direction exhibit asymmetric scattering. Some diffusers are designed to exhibit asymmetric scattering behavior, such as holographic diffusers.

Despite its versatility and generality, there are limitations to the capability of the TracePro elliptical BSDF. If you apply an elliptical BSDF surface property to a plane surface, the “grain” of the surface is fixed to one direction. To get the elliptical BSDF to be spatially dependent (i.e. to be different on different parts of the surface) you would have to break up the object on which the surface resides into smaller pieces.

However, you can model circular brush marks on a parabolic reflector, for example. Because the “azimuth = 0” axis that you enter is projected onto the tangent plane in order to calculate the azimuth angle, the orientation of the azimuth = 0 axis determines the symmetry of the elliptical BSDF. Suppose the axis of the reflector is along the z axis in the figure above. To model brush marks that go around the reflector, you would enter the azimuth=0 axis as (0, 0, 1) or along the z axis. To make brush marks parallel to the axis of the reflector, enter an azimuth=0 axis perpendicular to the reflector axis, e.g. (1, 0, 0) or (0, 1, 0). To make elliptical brush marks at a skew angle, enter an azimuth=0 axis that is neither parallel nor perpendicular to the reflector axis.

Elliptical ABg BSDF model

The elliptical ABg BSDF model is based on the ABg model, which is symmetrical. The elliptical ABg model is so called because you specify the axes of an ellipse for each coefficient, and TracePro fits an ellipse to these axes to determine the coefficients in other directions. The elliptical ABg model is determined by the following algorithm:

1. When you create a surface property with elliptical BSDF, you enter B and g coefficients for x and y axes.
2. When you apply the property, you specify the azimuth=0 axis. This becomes the local x axis for determining the coefficients.
3. During the ray trace, the local surface normal and the azimuth=0 axis are used to construct a local coordinate system. The surface normal is the local z axis and the azimuth=0 axis is the local x axis. If the azimuth=0 axis is not perpendicular to the surface normal, a new x axis is created by rotating the azimuth=0 vector in a plane containing it and the normal such the x axis is perpendicular to the normal (i.e. the local x axis lies in the tangent plane).
4. The direction of the scattered ray is projected onto the tangent plane and the azimuth angle ϕ is determined. See “Elliptical BSDF coordinates” on page 7.22.
5. The coefficients B' and g' are determined by making the x and y components axes of an ellipse,

$$\frac{1}{B'^2} = \frac{(\cos\phi)^2}{B_x^2} + \frac{(\sin\phi)^2}{B_y^2} \tag{7.31}$$

and

$$\frac{1}{g'^2} = \frac{(\cos\phi)^2}{g_x^2} + \frac{(\sin\phi)^2}{g_y^2} \tag{7.32}$$

6. The ABg BSDF is evaluated in the same way as for symmetrical ABg BSDF model, i.e.

$$\mathrm{BSDF} = \frac{A}{B' + \left|\vec{\beta} - \vec{\beta_0}\right|^{g'}} \tag{7.33}$$

7. Finally, the A coefficient is determined from

$$\mathrm{BSDF}(0) = \frac{A}{B'} \tag{7.34}$$

or

$$A = \mathrm{BSDF}(0) \cdot B' \tag{7.35}$$

where *BSDF*(0) is the peak BSDF. When you create an elliptical ABg BSDF model, then, for each row you enter the following coefficients:

- Peak BRDF
- BRDF B_x
- BRDF B_y
- BRDF g_x
- BRDF g_y
- Peak BTDF
- BTDF B_x
- BTDF B_y
- BTDF g_x
- BTDF g_y

Elliptical Gaussian BSDF

The elliptical Gaussian BSDF has a much simpler form. You enter the peak BSDF and the $1/e^2$ half-width in either direction. Then the BSDF has the form

$$BSDF = BSDF(0)e^{-2\left(\frac{x^2}{s_x^2}+\frac{y^2}{s_y^2}\right)} \tag{7.36}$$

where *BSDF*(0) is the peak BSDF. When you create an elliptical Gaussian BSDF model, then, for each row you enter the following coefficients:

- Peak BRDF
- BRDF s_x
- BRDF s_y
- Peak BTDF
- BTDF s_x
- BTDF s_y

The s_x and s_y values are the $1/e^2$ half-widths of the elliptical Gaussian BSDF in terms of β-β_0. For the case of normal incidence (β_0 = 0 deg), a β-β_0 value of 0.5 would equate to a $1/e^2$ half-width of 30 deg, sin(30)-sin(0) = 0.5.

1D ABg BSDF Model

The 1D ABg BSDF model is an asymmetrical BSDF model based on the ABg model, which is symmetrical. 1D BSDF models are used to model surfaces that scatter strongly in one plane, for example brushed or machined surfaces. The 1D

ABg model is a product of two orthogonal functions. Along the local x direction it is an ABg model, and along the local y direction it is a Gaussian.

1. When you create a surface property with 1D ABg BSDF, you enter B and g coefficients for the x axis, and a sigma value for the y axis.
2. When you apply the property, you specify the azimuth=0 axis. This becomes the local x axis for orienting the scatter pattern.
3. During the ray trace, the local surface normal and the azimuth=0 axis are used to construct a local coordinate system. The surface normal is the local z axis and the azimuth=0 axis is the local x axis. If the azimuth=0 axis is not perpendicular to the surface normal, a new x axis is created by rotating the azimuth=0 vector in a plane containing it and the normal such the x axis is perpendicular to the normal (i.e. the local x axis lies in the tangent plane).

When you create a 1D ABg BSDF model, then, for each row you enter the following coefficients:

- BRDF A
- BRDF B
- BRDF g
- BRDF sigma

1D Table BSDF Model

The 1D Table BSDF model is an asymmetrical BSDF model that uses a table of BSDF values along the +/-x direction, and a Gaussian along the y direction. 1D BSDF models are used to model surfaces that scatter strongly in one plane, for example brushed or machined surfaces. The 1D Table model is a product of two orthogonal functions. Along the local x direction it is a table of values, and along the local y direction it is a Gaussian.

1. When you create a surface property with 1D Table BSDF, you enter a table of values for the x axis, and a sigma value for the y axis.
2. When you apply the property, you specify the azimuth=0 axis. This becomes the local x axis for orienting the scatter pattern.
3. During the ray trace, the local surface normal and the azimuth=0 axis are used to construct a local coordinate system. The surface normal is the local z axis and the azimuth=0 axis is the local x axis. If the azimuth=0 axis is not perpendicular to the surface normal, a new x axis is created by rotating the azimuth=0 vector in a plane containing it and the normal such the x axis is perpendicular to the normal (i.e. the local x axis lies in the tangent plane).

The table of values along the x axis is versus positive and negative values of $(\beta-\beta_0)_x$ to enable asymmetry along the local x axis.