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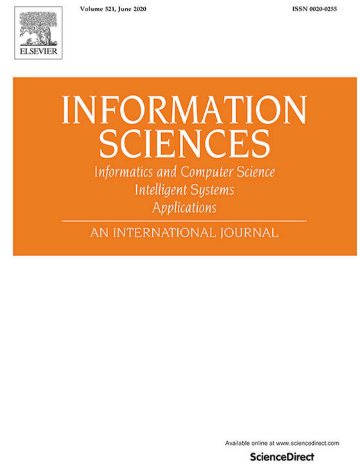

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[Title Page]

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Abstract

Rumors affect a variety of human affairs, which may cause a serious social disorder. In order to accurately simulate the propagation dynamics of rumors in real social networks and effectively ameliorate the harm of rumors to society, a new rumor propagation model is proposed in this paper. The model considers the influence of discussants, and divides the total population into four groups: ignorances, discussants, spreaders, and removers. The equilibria and basic reproduction number of the model are calculated, and the local and global asymptotic stability and the transcritical bifurcation of the equilibria are analyzed and proved. The theoretical analysis reveals the dynamic behavior and mechanism of rumor propagation. The parameters of the model are estimated by least-squares fitting, and the rumor spreading process is predicted according to the fitted parameters. The model is verified by using a rumor actual dataset. Simulation results show that the R-squared is 0.9544, which means that the proposed model can accurately simulate the rumor propagation in real social networks. By comparing the proposed model and the existing ones, the outcome indicates that discussants have an important impact on the rumor propagation process.

Keywords: Rumor; Propagation model; Stability; Parameter estimation; Bifurcation.

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1. Introduction

Rumors are defined as unauthenticated statements or reports related to common interests and are widely spread by various means of dissemination [24]. With the internet industry development, online social media, like WeChat, Twitter, and Facebook, have become important sources of information, providing tremendous convenience to our daily life. However, as online social media have a low threshold, large numbers of users and real-time dissemination of information, rumors in social media can soon trigger a mass effect, causing more users to trust rumors and propagate them. The "explosive" propagation of rumors in social media may cause serious harm to society, especially during epidemic outbreaks [20].

At the beginning of 2020, rumors about COVID-19 (Coronavirus Disease 2019) frequently appeared in social media, for example, "COVID-19 can spread between humans and pets", "Oseltamivir can help prevent COVID-19", and so on. These rumors were propagated on Microblog, WeChat, and other online social media, causing people to perform some irrational behavior, such as hoarding commodities and being overanxious. The spread of rumors can cause serious social harm, especially in social networks. To prevent the propagation of rumors and minimize their influence, it is necessary to research the rumor propagating process. As rumor propagation models are able to reveal the rules of rumor spreading, it is of significant theoretical and practical meaning to construct a rumor dynamics model, analyze the mechanism and rules of rumor propagation, and control the propagation of rumors.

An IDSRI (Ignorance–Discussant–Spreader–Remover–Ignorance) rumor propagation model is proposed in this paper. This study explores the spreading dynamics and mechanism of rumors in social networks. The major contributions in the paper are made as follows.

(i) A new rumor propagation model is presented by considering the impact of discussants in social networks. A discussant is assumed to know rumors, but not to spread them.

(ii) The dynamics behavior of the model is analyzed. The equilibria and basic reproduction number of the model are calculated, and the local and global asymptotic stability and the transcritical bifurcation of the equilibria are analyzed and proved. The theoretical analysis results reveal the mechanism of rumor propagation.

(iii) Unlike most existing studies, this paper employs a rumor actual dataset to simulate and verify the effectiveness of the model. The least-squares fitting is used to estimate all parameters of the model, which can accurately predict the rumor propagation process. The results show that the proposed model is effective and can be well applied to the rumor propagation in real social networks.

The remainder of this paper is organized as follows. Some related works are reviewed in Section 2. In Section 3, an IDSRI model is proposed. Section 4 calculates equilibria and the basic reproduction number, and analyzes the stability and transcritical bifurcation of our proposed model. Numerical simulations are presented to verify our model in Section 5. Section 6 gives a brief summary to conclude this paper.

2. Related Works

Spreading dynamics is a subject closely related to social life, which is widely used for epidemic propagation in social networks [2], virus propagation in communication networks [9], information propagation in social networks [25], influence spread in multiplex networks [35], malware propagation in cyber-physical systems [36], and opinion dynamics in cross-issue solidarity [26]. With the emergence of online social networks, a rumor propagation dynamics model has become one of the hot research topics [10].

The research of the rumor dynamics model started in the 1960s [5]. Deley and Kendall [6] first introduced the idea of infectious disease modeling to rumor spreading, and establish a classic D-K (Daley–Kendal) model. They divide the population into three categories, i.e., Ignorant who does not hear rumors, Spreader who propagates rumors, and Remover who has heard, but do not spread them. Maki and Thomson [19] put forward an M-K (Maki–Thompson) model based on the D-K model, and carried out mathematical theoretical analysis on the M-K model. These two models provide the theoretical basis for rumor propagation modeling, but they are not appropriate for the research of rumor propagation in a social network environment. The appearance of complex network makes it possible to simulate the spreading rule of rumors in different network topologies. Zanette [37,38] first studied rumor propagation in small-world networks and pointed out that the network topologies have a great influence on the rumor propagation.

In recent years, researchers have continuously improved the rumor dynamics model, which has made a lot of significant achievements in this research. Many rumor dynamics models have been presented, such as SIS (Susceptible–Infected–Susceptible) model [8], SIR (Susceptible–Infected–Removed) model [39], SIRS (Susceptible–Infected–Removed–Susceptible) model [11], 2SI2R (Spreader1–Spreader2–Ignorant–Stifler1–Stifler2) model [31], and SEIR (Susceptible–Exposed–Infected–Removed) model [28]. However, rumor propagation is a very complex process, which has the characteristics of complex and diverse contents, unpredictable process, enormous influence scope and serious social harm. Although the above models simulate the spreading process of different groups in social networks, they do not consider the influence of social activities and individual behavior on rumors.

In the studies of rumor dynamics models, researchers use some control mechanisms to analyze the process of rumors spreading. In [40], Zhao et al. studied an SIHR (Spreader–Ignorant–

Hibernator–Stiflers) model by adding remembering and forgetting mechanisms. As the remembering rate increases and the forgetting rate reduces, the densities of rumor spreaders increases. Li et al. [16] considered the effects of the rumor refuting mechanism and individual activity differences on rumor propagation. The rumor refuting mechanism can lessen the densities and duration of spreaders. Peng et al. [21] presented a node-level rumor spreading model considering a recommendation system, and rumor can fast spread under the recommendation mechanism. In addition, other mechanisms that can promote or suppress rumor propagation have been proposed, such as individual activity [13], influence mechanism [22], scientific knowledge [12], and counterattack mechanism [18]. By studying the above mechanisms and factors, a more comprehensive understanding and cognition of rumor spreading can be acquired.

Due to the factors of legal consciousness, family education and individual psychology, some people may not propagate them promptly after they contact with rumors, and regard them as doubters. An SEIR model with a hesitating mechanism for rumor spreading is studied, which can reduce the persistence and propagation of rumors [17]. Xia et al. [33] studied an SEIR model considering a hesitating mechanism, attractiveness and fuzziness of a rumor, and the results show that the clearer the rumor is, the less influential it will be. To enable the rumor spreading model to reveal more social rules, it can consider some social features, such as the role of memory, bandwagon effect, degree of trust, and subjective difference. Wang et al. [29,30] established a rumor spreading model based on information entropy by considering the above factors, which provided a generalizable framework for the study of rumor propagation in the real network.

Although the above models analyze the rumor spreading rule, but they do not use actual rumor datasets to verify their effectiveness. Chen et al. [4] used the Pheme dataset to verify their proposed SEIsIrR (Spreader–Exposed–Steady ignorant–Radical ignorant–Stifler) model in different network structures. They match the state of the model with the labels of the dataset for model fitting, and the results show that the dynamics of their model are consistent with real social networks. In addition, some papers do not study the influence of bifurcation on rumor spreading. The spread of rumors may give rise to static and dynamic bifurcations, such as transcritical bifurcation [14,15], backward bifurcation [42,43], and Hopf bifurcation [34,41,44].

Moreover, most studies assume that the rumors propagate in a closed system do not take into account population recruitment and exit. In fact, a social network is an open platform that needs to consider the impacts of changes in the number of nodes and individual activities. Thus, we not only consider dynamic population recruitment and exit, but also consider the transformation relationship between different types of nodes, especially the transformation between ignorances and removers. According to the above analysis, this paper studies a new model for rumor propagation in social networks, analyzes its stability and transcritical bifurcation, and estimates the parameters of the model with real data.

3. The IDSRI Model

Assume that a spreader propagates a rumor through direct touch with others. According to the characteristics of the propagation of rumors, the total population is divided into four categories, i.e., Ignorance (a person who does not know rumors), Discussant (a person who knows rumors but does not spread them, just participate in the discussion), Spreader (a person who knows rumors and spreads them), and Remover (a person who knows rumors and no longer spreads them). The densities of ignorances, discussants, spreaders, and removers are denoted by $I(t)$, $D(t)$, $S(t)$, and $R(t)$ at time t, respectively.

Since nodes in social networks have capable of propagation and recovery, this paper not only considers the transformation relationship between four types of nodes, but also adds features such as dynamic recruitment and exit. The other parameters are described as follows and shown in Table 1.

(1) Assume that a social network has a constant recruitment and exit rate. To keep the total number of nodes in a social network constant, the recruitment rate is set to be equal to the exit rate, denoted by μ. The newly recruited nodes are considered as I-state nodes, and a social network recruits new nodes with μ; the ignorance, discussant, spreader, and remover population exit at time t with μ.

(2) When a rumor appears in a social network, there will be some ignorances, discussants and spreaders in the system at time $t = 0$. An ignorance may have two states after contacting with a spreader. One becomes a discussant with the discussion rate β, and the other becomes a spreader with the infection rate α. In addition, an ignorance may become a remover who no longer spreads the rumor with the recovery rate δ. A discussant becomes a remover with the recovery rate λ, and becomes a spreader with the infection rate η. A spreader becomes a remover with the recovery rate γ. After a period of time t, a remover forgets the rumor with the forget rate ε and becomes an ignorance.

(3) All parameters are non-negative constants, and the range is between zero and one.

Table 1: Parameter description of the model.

Parameter	Physical interpretation	Parameter	Physical interpretation
$I(t)$	Ignorance compartment	β	Discussion rate of an ignorance
$D(t)$	Discussant compartment	ε	Forget rate of a remover
$S(t)$	Spreader compartment	δ	Recovery rate of an ignorance
$R(t)$	Remover compartment	η	Infection rate of a discussant
μ	Recruitment and exit rate	λ	Recovery rate of a discussant
α	Infection rate of an ignorance	γ	Recovery rate of a spreader

According to the above propagation rules, an Ignorance–Discussant–Spreader–Remover–Ignorance (IDSRI) model for rumor propagation is presented. The specific evolution process of different state nodes is shown in Fig.1.

$$\begin{cases} \dfrac{dI(t)}{dt} = \mu + \varepsilon R(t) - \alpha I(t)S(t) - \beta I(t)D(t) - (\delta+\mu)I(t) \\ \dfrac{dD(t)}{dt} = \beta I(t)D(t) - \eta D(t)S(t) - (\lambda+\mu)D(t) \\ \dfrac{dS(t)}{dt} = \alpha I(t)S(t) + \eta D(t)S(t) - (\gamma+\mu)S(t) \\ \dfrac{dR(t)}{dt} = \delta I(t) + \gamma S(t) + \lambda D(t) - (\varepsilon+\mu)R(t) \end{cases} \tag{1}$$

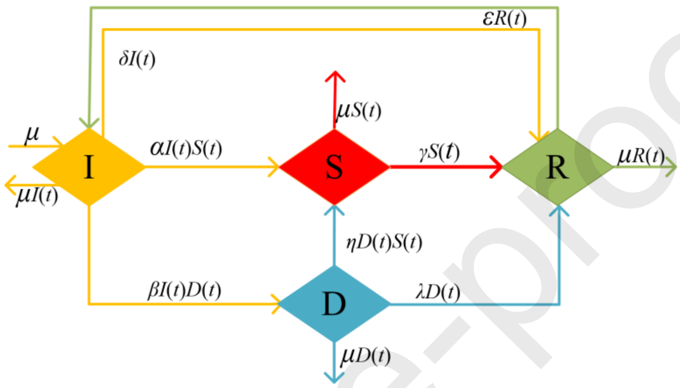


Fig. 1. The evolution process of different state nodes.

There are three basic propositions of system (1) as follows.

Proposition 1 There exists a solution $(I(t), D(t), S(t), R(t))$ to system (1).

Proof. The state space of system (1) is Ω = {($I(t)$, $D(t)$, $S(t)$, $R(t)$): $I(t) \geq 0$, $D(t) \geq 0$, $S(t) \geq 0$, $R(t) \geq 0$}. Let U = {($I(t)$, $D(t)$, $S(t)$, $R(t)$): $I(t) > 0$, $D(t) > 0$, $S(t) > 0$, $R(t) > 0$}. Then, U ⊆Ω is an open set.

System (1) can be written as $\frac{dx}{dt} = f$, where x = ($I(t)$, $D(t)$, $S(t)$, $R(t)$), and $f = (f_1, f_2, f_3, f_4)^{\mathrm{T}}$. The Jacobian matrix J of f can be obtained as follows.

$$J = \begin{pmatrix} -\alpha S - \beta D - \delta - \mu & -\beta I & -\alpha I & \varepsilon \\ \beta D & \beta I - \eta S - \lambda - \mu & -\eta D & 0 \\ \alpha S & \eta S & \alpha I + \eta D - \gamma - \mu & 0 \\ \delta & \lambda & \gamma & -\varepsilon - \mu \end{pmatrix}.$$

It is obvious that all the elements of the Jacobian matrix J are continuous. Then, $f : U \to R^4$ is a continuously differentiable map. Thus, there always exists a solution to system (1).

Proposition 2 If (I(0), D(0), S(0), R(0))>0, the solution $(I(t), D(t), S(t), R(t))$ of system (1) is non-negative for $t > 0$.

Proof. By contradiction, assume that there exists $t_1 > 0$ such that at least one of $(I(t_1), D(t_1), S(t_1), R(t_1))$ is not positive. By the continuity of the solution, there exists $t_0 \in (0, t_1]$ such that at least one of $(I(t_0), D(t_0), S(t_0), R(t_0))$ is equal to zero. Without loss of generality, we assume that t_0 is the minimal time with such a property.

If $I(t_0) = 0$, then $(D(t_0), S(t_0), R(t_0)) \geq 0$ is true, implying the truth of $\left.\frac{dI}{dt}\right|_{t=t_0} = \mu + \varepsilon R(t_0) > 0$. By the continuity of the solution, there exists $a > 0$ such that $I(t)$ is strictly monotone increasing in interval $(t_0 - a, t_0 + a)$. Let $t_2 \in (t_0 - a, t_0)$, then $S(t_2) < S(t_0) = 0$ holds. Since $S(0) > 0$, there exists $t_3 \in (0, t_2)$ such that $S(t_3) = 0$ by Bolzano's theorem, which contradicts the assumption of t_0.

If $D(t_0) = 0$, then $\left.\frac{dD}{dt}\right|_{t=t_0} = 0$ holds. It can be verified that $D(t) = 0$ for all $t \geq 0$ is the solution with initial value $D(0) = 0$. However, $D(0) > 0$ and $D(t_0) = 0$ contradict the uniqueness of the solution .

Similarly, it can illustrate that $S(t_0) = 0$ and $R(t_0) = 0$ lead a contradiction. Therefore, the solution $(I(t), D(t), S(t), R(t))$ of system (1) is non-negative for $t > 0$.

Proposition 3 The solutions of system (1) are uniformly bounded.

Proof. Assume that the size of the total individuals in online social networks is $N(t)$, namely

$N(t) = I(t) + D(t) + S(t) + R(t)$.

It is easy to know that $\frac{dN(t)}{dt} = \mu - \mu N(t)$ is true. Thus, $N(0) = I(0) + D(0) + S(0) + R(0)$ and $N(t) = (N(0) - 1)e^{-\mu t} + 1$ hold. Then we have $\lim_{t \to +\infty} N(t) = 1$. In conclusion, the positive variant set of system (1) is $\Gamma = \{(I(t), D(t), S(t), R(t) \in R_4^+ : I(t) + D(t) + S(t) + R(t) \leq 1\}$.

The densities of four state nodes satisfies normalized characteristic, i.e., $I(t) + D(t) + S(t) + R(t) = 1$. To reduce the dimension of system (1), the R-state nodes in the ordinary differential equations can be eliminated. The simplified model is as follows.

$$\begin{cases} \dfrac{dI(t)}{dt} = \mu + \varepsilon(1 - I(t) - D(t) - S(t)) - \alpha I(t)S(t) - \beta I(t)D(t) - (\delta + \mu)I(t) \\ \dfrac{dD(t)}{dt} = \beta I(t)D(t) - \eta D(t)S(t) - (\lambda + \mu)D(t) \\ \dfrac{dS(t)}{dt} = \alpha I(t)S(t) + \eta D(t)S(t) - (\gamma + \mu)S(t) \end{cases} \tag{2}$$

Since the solutions of system (1) are non-negative, system (2) satisfies $I(t) \geq 0$, $D(t) \geq 0$, $S(t) \geq 0$. Thus the state space of system (2) can be represented by $\Omega = \{(I(t), D(t), S(t)): I(t) \geq 0, D(t) \geq 0, S(t) \geq 0, I(t) + D(t) + S(t) \leqslant 1\}$.

4. Model Analysis

This section calculates the equilibria and the basic reproduction number, and performs the stability and the transcritical bifurcation analysis of system (2).

4.1 Equilibrium Existence

This part will calculate equilibria of the IDSRI model. Let the right end of system (2) be zero. The new equation can be written as

$$\begin{cases} \mu + \varepsilon(1 - I(t) - D(t) - S(t)) - \alpha I(t)S(t) - \beta I(t)D(t) - (\delta + \mu)I(t) = 0 \\ \beta I(t)D(t) - \eta D(t)S(t) - (\lambda + \mu)D(t) = 0 \\ \alpha I(t)S(t) + \eta D(t)S(t) - (\gamma + \mu)S(t) = 0 \end{cases} \tag{3}$$

By calculating equation (3), four non-negative equilibria are obtained:

Set $S = 0$, two rumor-free equilibria E_0 and E_1 are obtained.

(1) $E_0 = (I_0, 0, 0)$, where $I_0 = \dfrac{\varepsilon + \mu}{\varepsilon + \delta + \mu}$.

(2) $E_1 = (I_1, D_1, 0)$, where $I_1 = \dfrac{\lambda + \mu}{\beta}$, and $D_1 = \dfrac{-(\delta + \varepsilon + \mu)(\lambda + \mu) + \beta(\varepsilon + \mu)}{\beta(\eta + \lambda + \mu)}$.

Set $S \neq 0$, two rumor equilibria $E_2 = (I_2, 0, S_2)$ and $E_3 = (I_3, D_3, S_3)$ are obtained.

(3) $E_2 = (I_2, 0, S_2)$, where $I_2 = \dfrac{\gamma + \mu}{\alpha}$, and $S_2 = \dfrac{-(\delta + \varepsilon + \mu)(\gamma + \mu) + \alpha(\varepsilon + \mu)}{\alpha(\eta + \gamma + \mu)}$.

(4) $E_3 = (I_3, D_3, S_3)$, where $I_3 = \dfrac{\varepsilon\eta + \varepsilon\lambda + \eta\mu - \varepsilon\gamma}{\eta(\varepsilon + \delta + \mu) + \beta(\varepsilon + \gamma + \mu) - \alpha(\varepsilon + \lambda + \mu)}$,

$$D_3 = \frac{\gamma+\mu-\alpha I_3}{\eta} = \frac{-\alpha(\mu\eta+\varepsilon\eta+\varepsilon\lambda-\varepsilon\gamma)+(-\alpha(\lambda+\mu)+\beta(\gamma+\mu)+\eta(\delta+\mu)+\varepsilon(\eta+\beta-\alpha))(\gamma+\mu)}{\eta(-\alpha(\lambda+\mu)+\beta(\gamma+\mu)+\eta(\delta+\mu)+\varepsilon(\eta+\beta-\alpha))},$$

and $S_3 = \frac{\beta I_3-\lambda-\mu}{\eta} = \frac{\beta(\mu\eta+\varepsilon\eta+\varepsilon\lambda-\varepsilon\gamma)-(-\alpha(\lambda+\mu)+\beta(\gamma+\mu)+\eta(\delta+\mu)+\varepsilon(\eta+\beta-\alpha))(\lambda+\mu)}{\eta(-\alpha(\lambda+\mu)+\beta(\gamma+\mu)+\eta(\delta+\mu)+\varepsilon(\eta+\beta-\alpha))}$.

4.2 Basic Reproduction Number

The basic reproduction number in the rumor propagation model refers to the average number of ignorances influenced by each spreader during the period that all spreaders propagate a rumor. The basic reproduction number is calculated by using the next-generation matrix method [27]. Set $X=(D(t),S(t))^{\mathrm{T}}$, as seen in system (2). We have

$\frac{dX}{dt} = \mathcal{F}-\mathcal{V}$, where $\mathcal{F} = \begin{bmatrix} \eta D(t)S(t) \\ \alpha I(t)S(t)+\eta D(t)S(t) \end{bmatrix}$ and $\mathcal{V} = \begin{bmatrix} -\beta I(t)D(t)+(\lambda+\mu)D(t) \\ (\gamma+\mu)S(t) \end{bmatrix}$.

Let F and V represent the Jacobian Matrix of $\mathcal{F}$ and $\mathcal{V}$ at E_0, respectively. Then there are

$F = \begin{bmatrix} \eta S_0 & \eta D_0 \\ \eta S_0 & \alpha I_0+\eta D_0 \end{bmatrix}$, and $V = \begin{bmatrix} -\beta I_0+\lambda+\mu & 0 \\ 0 & \gamma+\mu \end{bmatrix}$.

Therefore, the matrix $K=FV^{-1}$ can be calculated as follows:

$$K = FV^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{\alpha I_0}{\gamma+\mu} \end{bmatrix}.$$

The spectral radius of the matrix $K=FV^{-1}$ is exactly equal to the basic reproduction number of the IDSRI model, i.e.,

$$R_0 = \rho(FV^{-1}) = \frac{\alpha(\varepsilon+\mu)}{(\gamma+\mu)(\varepsilon+\delta+\mu)}.$$

4.3 Equilibrium Stability

According to the Lyapunov theorem [32] and the Routh-Hurwitz stability criterion [7], the local asymptotic stability of the four equilibria are proved. Next, Castillo-Chavez and Song bifurcation theorem [3] is applied to verify that system (2) exhibits transcritical bifurcation at the rumor-free equilibrium.

Theorem 1. If $R_0<1$ and $\alpha > \frac{\beta(\gamma+\mu)}{\lambda+\mu}$, system (2) is locally asymptotically stable at the rumor-free equilibrium E_0.

Proof. The Jacobian matrix of system (2) at E_0 is expressed as follows:

$$J_{E_0}=\begin{pmatrix}-\varepsilon-\delta-\mu & -\varepsilon-\beta I_0 & -\varepsilon-\alpha I_0\\ 0 & \beta I_0-\lambda-\mu & 0\\ 0 & 0 & \alpha I_0-\gamma-\mu\end{pmatrix}$$

The eigenvalues for J_{E_0} are: $\kappa_1=-(\varepsilon+\delta+\mu)$, $\kappa_2=\beta I_0-(\lambda+\mu)$, and $\kappa_3=\alpha I_0-(\gamma+\mu)$.

Obviously, $\kappa_1<0$ holds. If $R_0<1$ and $\alpha>\frac{\beta(\gamma+\mu)}{\lambda+\mu}$, then we have $\kappa_3=(\gamma+\mu)(R_0^1-1)<0$ and $\kappa_2=\beta I_0-(\lambda+\mu)<\frac{\alpha(\lambda+\mu)}{\gamma+\mu}I_0-(\lambda+\mu)=(\lambda+\mu)(R_0^1-1)<0$.

Three eigenvalues of matrix J_{E_0} are negative real constants if $R_0<1$. According to the Lyapunov theorem, rumor-free equilibrium E_0 is locally asymptotically stable.

Theorem 2. If $R_0<1$ and $\alpha<\frac{\beta(\gamma+\mu)}{\lambda+\mu}+\frac{\eta(-\beta(\varepsilon+\mu)+(\varepsilon+\delta+\mu)(\lambda+\mu))}{(\eta+\lambda+\mu)(\lambda+\mu)}$, system (2) is locally asymptotically stable at the rumor-free equilibrium E_1.

Proof. The Jacobian matrix of system (2) at E_1 is expressed as follows:

$$J_{E_1}=\begin{pmatrix}-\varepsilon-\delta-\mu-\beta D_1 & -\varepsilon-\beta I_1 & -\varepsilon-\alpha I_1\\ \beta D_1 & \beta I_1-\lambda-\mu & -\eta D_1\\ 0 & 0 & \alpha I_1+\eta D_1-\gamma-\mu\end{pmatrix}$$

Construct a characteristic equation $\kappa^3+a_1\kappa^2+a_2\kappa+a_3=0$, where $a_1=\varepsilon+\delta+\beta D_1-\alpha I_1-\eta D_1+\gamma+2\mu$, $a_2=\beta D_1(\varepsilon+\beta I_1)+(\varepsilon+\delta+\mu+\beta D_1)(-\alpha I_1-\eta D_1+\gamma+\mu)$, and $a_3=\beta D_1(\varepsilon+\beta I_1)(-\alpha I_1-\eta D_1+\gamma+\mu)$.

Then $\Delta_1=a_1=\varepsilon+\delta+\mu+\beta D_1-\alpha I_1-\eta D_1+\gamma+\mu$, and

$$\begin{aligned}\Delta_2&=\begin{vmatrix}a_1 & 1\\ a_3 & a_2\end{vmatrix}=a_1a_2-a_3\\ &=\beta D_1(\varepsilon+\beta I_1)(\varepsilon+\delta+\mu+\beta D_1)+(\varepsilon+\delta+\mu+\beta D_1-\alpha I_1-\eta D_1+\gamma+\mu)(\varepsilon+\delta+\mu+\beta D_1)(-\alpha I_1-\eta D_1+\gamma+\mu)\end{aligned}.$$

If $R_0<1$ and $\alpha<\frac{\beta(\gamma+\mu)}{\lambda+\mu}+\frac{\eta(-\beta(\varepsilon+\mu)+(\varepsilon+\delta+\mu)(\lambda+\mu))}{(\eta+\lambda+\mu)(\lambda+\mu)}$, then $-\alpha I_1-\eta D_1+\gamma+\mu>0$ holds, implying the truth of $\Delta_1>0$ and $\Delta_2>0$. According to the Routh-Hurwitz stability criterion, system (2) is stable, and Theorem 2 is proved.

Theorem 3. If $R_0 > 1$ and $\alpha > \frac{\beta(\gamma+\mu)}{\lambda+\mu}$, system (2) is locally asymptotically stable at the rumor equilibrium E_2.

Proof. The Jacobian matrix of system (2) at E_2 is expressed as follows:

$$J_{E_2} = \begin{pmatrix} -\varepsilon-\delta-\mu-\alpha S_2 & -\varepsilon-\beta I_2 & -\varepsilon-\alpha I_2 \\ 0 & \beta I_2-\eta S_2-\lambda-\mu & 0 \\ \alpha S_2 & \eta S_2 & \alpha I_2-\gamma-\mu \end{pmatrix}$$

Construct a characteristic equation $\kappa^3 + b_1\kappa^2 + b_2\kappa + b_3 = 0$, where $b_1 = -\beta I_2 + \eta S_2 + \lambda + \varepsilon + \delta + 2\mu + \alpha S_2$, $b_2 = \alpha S_2(\varepsilon + \alpha I) + (-\beta I_2 + \eta S_2 + \lambda + \mu)(\varepsilon + \delta + \mu + \alpha S_2)$, and $b_3 = \alpha S_2(\varepsilon + \alpha I_2)(-\beta I_2 + \eta S_2 + \lambda + \mu)$.

Then $\Delta_1 = b_1 = -\beta I_2 + \eta S_2 + \lambda + \mu + \varepsilon + \delta + \mu + \alpha S_2$, and

$$\Delta_2 = \begin{vmatrix} b_1 & 1 \\ b_3 & b_2 \end{vmatrix} = b_1 b_2 - b_3$$
$$= \alpha S_2(\varepsilon + \alpha I_2)(\varepsilon + \delta + \mu + \alpha S_2) + (-\beta I_2 + \eta S_2 + \lambda + \mu)(\varepsilon + \delta + \mu + \alpha S_2)(-\beta I_2 + \eta S_2 + \lambda + \mu + \varepsilon + \delta + \mu + \alpha S_2).$$

If $R_0 > 1$ and $\alpha > \frac{\beta(\gamma+\mu)}{\lambda+\mu}$, then $-\beta I_2 + \lambda + \mu > 0$ is true; further $\Delta_1 > 0$ and $\Delta_2 > 0$ hold. According to the Routh-Hurwitz stability criterion, system (2) is stable, and Theorem 3 is proved.

Theorem 4. If $R_0 > 1$, and $\alpha < min\{\frac{\beta(\gamma+\mu)}{\lambda+\mu}, \frac{\beta(\gamma+\mu)(\varepsilon+\gamma+\mu)+\eta(\gamma+\mu)(\varepsilon+\delta+\mu)}{\eta(\varepsilon+\mu)+(\varepsilon+\gamma+\mu)(\lambda+\mu)}\}$, system (2) is locally asymptotically stable at the rumor equilibrium E_3.

Proof. The Jacobian matrix of system (2) at E_3 is expressed as follows:

$$J_{E_3} = \begin{pmatrix} -(\varepsilon+\delta+\mu)-\beta D_3-\alpha S_3 & -\varepsilon-\beta I_3 & -\varepsilon-\alpha I_3 \\ \beta D_3 & \beta I_3 - \eta S_3 - \lambda - \mu & -\eta D_3 \\ \alpha S_3 & \eta S_3 & \alpha I_3 + \eta D_3 - (\gamma+\mu) \end{pmatrix}$$

Construct a characteristic equation $\kappa^3 + c_1\kappa^2 + c_2\kappa + c_3 = 0$, where $c_1 = \alpha S_3 + \beta D_3 + \varepsilon + \delta + \mu$, $c_2 = \alpha S_3(\varepsilon + \alpha I_3) + \beta D_3(\varepsilon + \beta I_3) + \eta^2 S_3 D_3$, and $c_3 = \beta\eta D_3 S_3(\varepsilon + \alpha I_3) - \alpha\eta D_3 S_3(\varepsilon + \beta I_3) + \eta^2 D_3 S_3(\varepsilon + \delta + \mu + \alpha S_3 + \beta D_3)$.

Then $\Delta_1 = c_1 = \alpha S_3 + \beta D_3 + \varepsilon + \delta + \mu$, and

$$\Delta_2 = \begin{vmatrix} c_1 & 1 \\ c_3 & c_2 \end{vmatrix} = c_1 c_2 - c_3$$
$$= (\beta D_3 S_3(\alpha - \eta) + \alpha^2 S_3^{\ 2})(\varepsilon + \alpha I_3) + (\alpha D_3 S_3(\beta + \eta) + \beta^2 D_3^{\ 2})(\varepsilon + \beta I_3) + (\varepsilon + \delta + \mu)(\alpha S_3(\varepsilon + \alpha I_3) + \beta D_3(\varepsilon + \beta I_3)).$$

If $\alpha < \frac{\beta(\gamma+\mu)}{\lambda+\mu}$, then we have $\alpha S_3 + \beta D_3 = \frac{-\alpha(\lambda+\mu)+\beta(\gamma+\mu)}{\eta} > 0$; further $\Delta_1 > 0$ holds. If $R_0 > 1$ and $\alpha < \frac{\beta(\gamma+\mu)(\varepsilon+\gamma+\mu)+\eta(\gamma+\mu)(\varepsilon+\delta+\mu)}{\eta(\varepsilon+\mu)+(\varepsilon+\gamma+\mu)(\lambda+\mu)}$, then we have $D_3 > 0$, $S_3 > 0$, and $\beta D_3 S_3(\alpha-\eta)+\alpha^2 S_3{}^2 > 0$, leading to $\Delta_2 > 0$. According to the Routh-Hurwitz stability criterion, system (2) is stable, and Theorem 4 is proved.

Theorem 5. If $R_0 = 1$, system (2) has transcritical bifurcation at the rumor-free equilibrium E_0.

Proof. In system (2), if $R_0 = 1$, then the eigenvalue κ_3 is equal to zero, the rumor-free equilibrium E_0 changes from stable state to unstable state, and E_0 is a non-hyperbolic equilibrium.

Denote by $\vec{\omega} = (\omega_1, \omega_2, \omega_3)^{\mathrm{T}}$ a right eigenvector and $\vec{v} = (v_1, v_2, v_3)^{\mathrm{T}}$ a left eigenvector. They follow $J_{E_0} \cdot \vec{\omega} = 0$ and $\vec{v} \cdot J_{E_0} = 0$, i.e.,

$$\begin{cases} -(\varepsilon+\delta+\mu)\omega_1 - (\varepsilon + \frac{\beta(\varepsilon+\mu)}{\varepsilon+\delta+\mu})\omega_2 - (\varepsilon + \frac{\alpha(\varepsilon+\mu)}{\varepsilon+\delta+\mu})\omega_3 = 0 \\ (-(\lambda+\mu) + \frac{\beta(\varepsilon+\mu)}{\varepsilon+\delta+\mu})\omega_2 = 0 \\ (-(\gamma+\mu) + \frac{\alpha(\varepsilon+\mu)}{\varepsilon+\delta+\mu})\omega_3 = 0 \end{cases} \quad (4),$$

and

$$\begin{cases} -(\varepsilon + \frac{\beta(\varepsilon+\mu)}{\varepsilon+\delta+\mu})v_1 + (-(\lambda+\mu) + \frac{\beta(\varepsilon+\mu)}{\varepsilon+\delta+\mu})v_2 = 0 \\ -(\varepsilon+\delta+\mu)v_1 = 0 \\ -(\varepsilon + \frac{\alpha(\varepsilon+\mu)}{\varepsilon+\delta+\mu})v_1 + (-(\gamma+\mu) + \frac{\alpha(\varepsilon+\mu)}{\varepsilon+\delta+\mu})v_3 = 0 \end{cases} \quad (5).$$

By calculating equations (4) and (5), it can be obtained that $\omega_1 = -(\frac{\alpha(\varepsilon+\mu)+\varepsilon(\varepsilon+\delta+\mu)}{(\varepsilon+\delta+\mu)^2})$, $\omega_2 = v_1 = v_2 = 0$, and $\omega_3 = v_3 = 1$.

Let $m = \sum_{k,i,j=1}^{3} v_k \omega_i \omega_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(P_0, \alpha^*)$ and $n = \sum_{k,i=1}^{3} v_k \omega_i \frac{\partial^2 f_k}{\partial x_i \partial \alpha}(P_0, \alpha^*)$.

Then there are $m = v_3 \omega_1 \omega_3 \alpha = -\alpha(\frac{\alpha(\varepsilon+\mu)+\varepsilon(\varepsilon+\delta+\mu)}{(\varepsilon+\delta+\mu)^2}) < 0$ and $n = v_3 \omega_3 I_0 = \frac{\mu+\varepsilon}{\mu+\delta+\varepsilon} > 0$.

According to the Castillo-Chavez and Song bifurcation theorem, the transcritical bifurcation at $R_0 = 1$ of system (2) is proved.

Theorem 6. The rumor-free equilibrium E_0 is globally asymptotically stable if $R_0 < 1$ and

$\alpha > \frac{\beta(\gamma+\mu)}{\lambda+\mu}$.

Proof. Consider a Lyapunov function with undetermined coefficients:

$$V(D(t),S(t)) = \frac{1}{2}(k_0 D(t)^2 + k_1 S(t)^2).$$

where k_0 and k_1 are positive constants to be determined. It is clear that V is positive definite. Calculating the time derivative of V along system (2), we have

$$\begin{aligned}\frac{dV}{dt} &= k_0\beta I(t)D(t)^2 - k_0\eta S(t)D(t)^2 - k_0(\lambda+\mu)D(t)^2 + k_1\alpha I(t)S(t)^2 + k_1\eta D(t)S(t)^2 - k_1(\gamma+\mu)S(t)^2 \\ &= k_0\beta[I(t)-1]D(t)^2 + k_0\beta D(t)^2 + k_1\alpha[I(t)-1]S(t)^2 + k_1\alpha S(t)^2 + k_1\eta[S(t)-1]D(t)S(t) + k_1\eta D(t)S(t) \\ &\quad - k_0\eta S(t)D(t)^2 - k_0(\lambda+\mu)D(t)^2 - k_1(\gamma+\mu)S(t)^2 \\ &= k_0\beta[I(t)-1]D(t)^2 + k_1\alpha[I(t)-1]S(t)^2 + k_1\eta[S(t)-1]D(t)S(t) + k_1\eta D(t)S(t) - k_0\eta S(t)D(t)^2 \\ &\quad - k_0(\lambda+\mu-\beta)D(t)^2 - k_1(\gamma+\mu-\alpha)S(t)^2.\end{aligned}$$

Let $k_0 = \frac{\eta^2}{\lambda+\mu-\beta}$ and $k_1 = \gamma+\mu-\alpha$. Then

$$\begin{aligned}\frac{dV}{dt} &= \frac{\eta^2}{\lambda+\mu-\beta}\beta[I(t)-1]D(t)^2 + (\gamma+\mu-\alpha)\alpha[I(t)-1]S(t)^2 + (\gamma+\mu-\alpha)\eta[S(t)-1]D(t)S(t) \\ &\quad + (\gamma+\mu-\alpha)\eta D(t)S(t) - \frac{\eta^3}{\lambda+\mu-\beta}S(t)D(t)^2 - \eta^2 D(t)^2 - (\gamma+\mu-\alpha)^2 S(t)^2 \\ &= \frac{\eta^2}{\lambda+\mu-\beta}\beta[I(t)-1]D(t)^2 + (\gamma+\mu-\alpha)\alpha[I(t)-1]S(t)^2 + (\gamma+\mu-\alpha)\eta[S(t)-1]D(t)S(t) \\ &\quad - \frac{\eta^3}{\lambda+\mu-\beta}S(t)D(t)^2 - [\eta D(t) - (\gamma+\mu-\alpha)S(t)]^2 - (\gamma+\mu-\alpha)\eta D(t)S(t).\end{aligned}$$

If $R_0 < 1$, then $\alpha < \gamma+\mu$ holds, leading to $\gamma+\mu-\alpha > 0$. If $\alpha > \frac{\beta(\gamma+\mu)}{\lambda+\mu}$, then $\beta < \lambda+\mu$ holds, implying the truth of $\lambda+\mu-\beta > 0$. It can be verified that $\frac{dV}{dt} \le 0$. Moreover, $\frac{dV}{dt} = 0$ is true if and only if $D(t) = 0$ and $S(t) = 0$. According to the LaSalle invariant principle [23], we have $V(D(t),S(t)) \to \infty$ as $D(t) \to \infty$ or $S(t) \to \infty$. Thus, Theorem 6 is proved.

5. Numerical Simulation and Discussions

5.1 Stability Simulation and Analysis

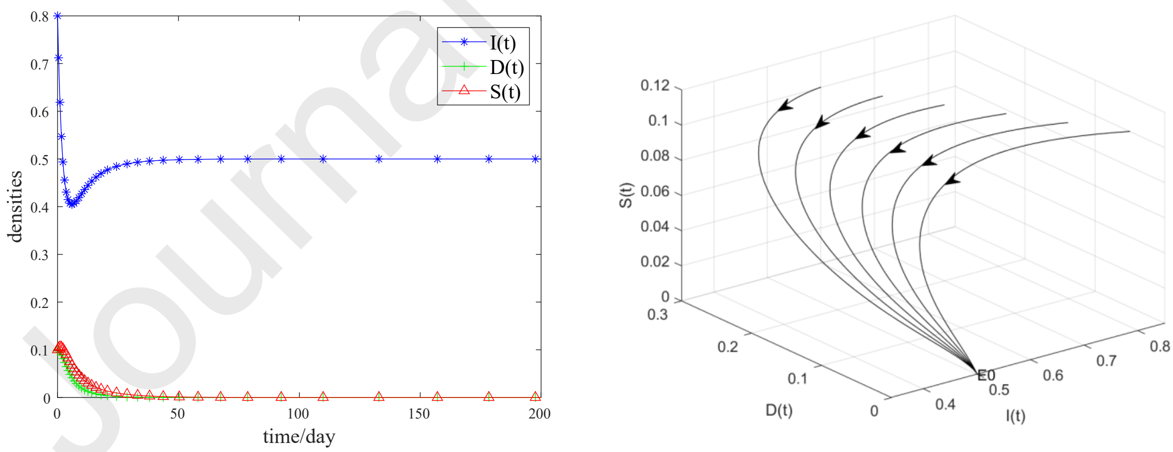
5.1.1 Stability Simulation and Analysis of Rumor-free Equilibria E_0 and E_1

By using two sets of data in Table 2, this part simulates the local asymptotic stability of E_0 and E_1. The numerical experiments include the densities evolution process of ignorances, discussants, and spreaders with the initial value (I(0), D(0), S(0)) = (0.8, 0.1, 0.1), and the trajectories change process of ignorances, discussants, and spreaders with the six initial conditions. The six initial values are (I(0), D(0), S(0)) = {(0.6, 0.3, 0.1), (0.65, 0.25, 0.1), (0.7, 0.2, 0.1), (0.75, 0.15, 0.1), (0.8, 0.1, 0.1), (0.85, 0.05, 0.1)}.

Table 2: Parameter for the local stability of E_0 and E_1.

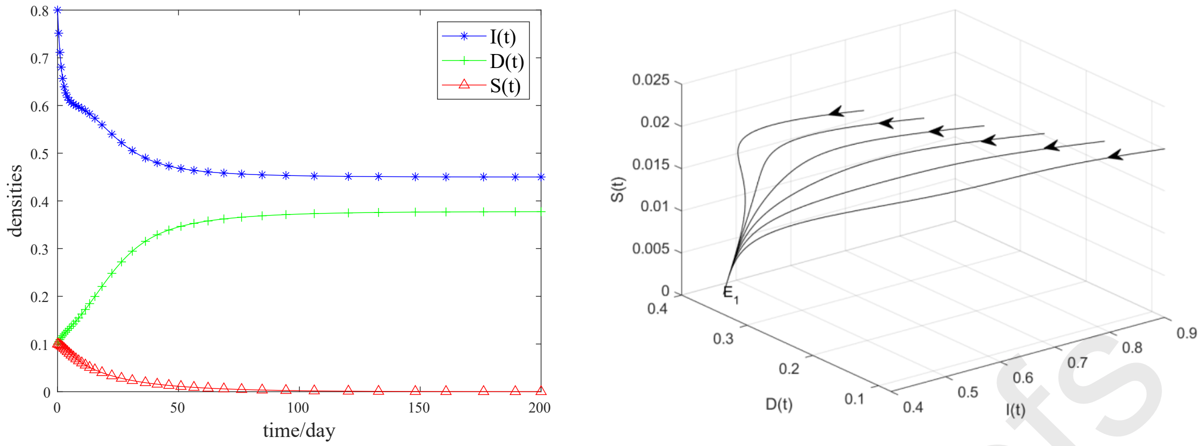
Parameter	α	β	γ	ε	η	λ	δ	μ
Data 1	0.6	0.5	0.3	0.12	0.2	0.3	0.2	0.08
Data 2	0.3	0.4	0.2	0.4	0.3	0.1	0.1	0.08

From Theorems 1 and 2, system (2) is locally asymptotically stable at E_0 and E_1 if $R_0 < 1$. By considering the system with parameters in data 1 of Table 2, $R_0 \approx 0.789 < 1$ can be obtained. Fig. 2 shows that there is a rumor-free equilibrium E_0 = (0.5, 0, 0) when system (2) reaches a steady state. By considering the system with parameters in data 2 of Table 2, the basic reproduction number is $R_0 \approx 0.887 < 1$. There exists another rumor-free equilibrium E_1 = (0.45, 0.38, 0) when system (2) reaches a steady state in Fig. 3.



(a) (b)

Fig. 2. At E_0: (a) the densities evolution plot of ignorances, discussants, and spreaders and (b) the phase diagram of six different initial values.



(a) (b)

Fig. 3. At E_1: (a) the densities evolution plot of ignorances, discussants, and spreaders and (b) the phase diagram of six different initial values.

Fig. 2(a) shows that the densities of discussants and spreaders drop down to zero; the densities of $I(t)$ decrease sharply first, and then increase until they reach a balance. Fig. 3(a) illustrates that if the densities of discussants continue to increase, the densities of spreaders will gradually decrease. In this state, rumor spreaders will gradually decrease until they die away. Figs. 2(b) and 3(b) show that system (2) will eventually stable in a state without rumors no matter how the initial value changes.

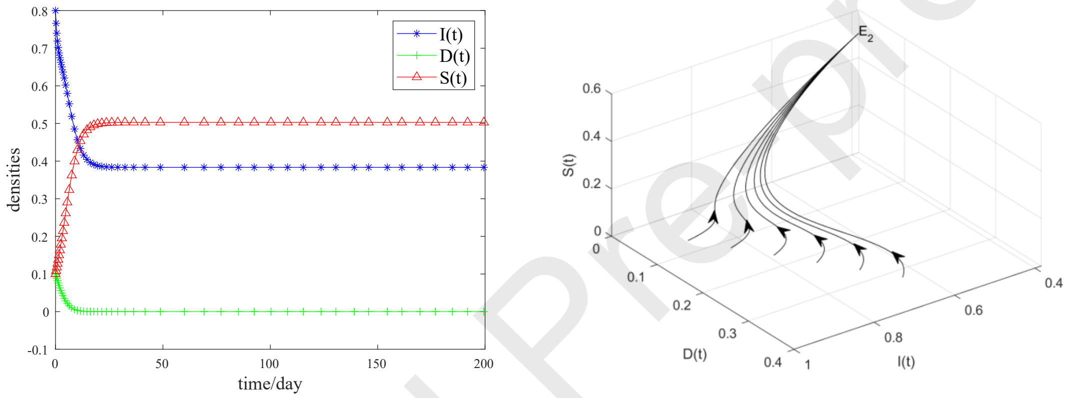
5.1.2 Local Stability Simulation and Analysis of Rumor Equilibria E_2 and E_3

This part will simulate the stability of the rumor equilibria E_2 and E_3 by using the two sets of data in Table 3. The densities of ignorances, discussants and spreaders with the initial value ($I(0)$, $D(0)$, $S(0)$) = (0.8, 0.1, 0.1), and the trajectories of ignorances, discussants and spreaders with six initial value conditions are obtained in Figs. 4 and 5. The six initial values are ($I(0)$, $D(0)$, $S(0)$) = {(0.9, 0.08, 0.02), (0.85, 0.13, 0.02), (0.8, 0.18, 0.02), (0.75, 0.23, 0.02), (0.7, 0.28, 0.02), (0.65, 0.33, 0.02)}.

Table 3: Parameter for the local asymptotic stability of two rumor equilibria.

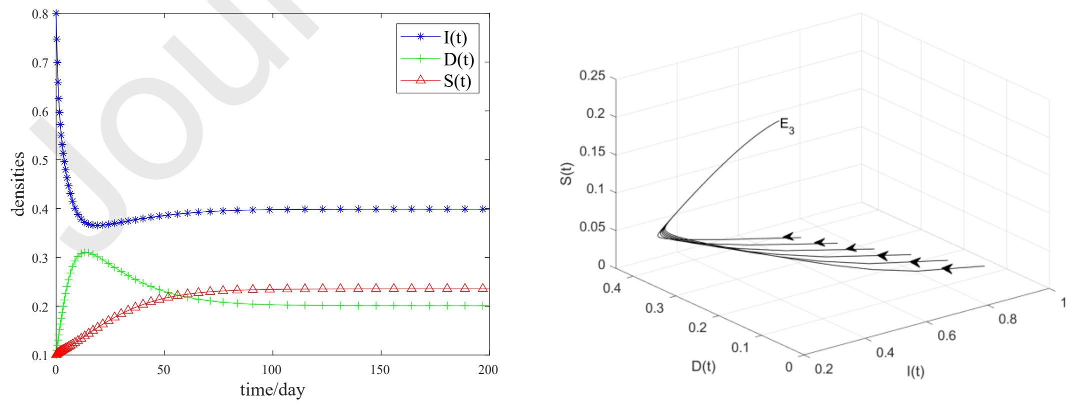
Parameter	α	β	γ	ε	η	λ	δ	μ
Data 1	0.6	0.3	0.15	0.92	0.5	0.3	0.1	0.08
Data 2	0.2	0.85	0.05	0.72	0.25	0.2	0.2	0.08

According to Theorems 3 and 4, system (2) is locally asymptotically stable at E_2 and E_3 if $R_0 > 1$. By considering the system with the parameters in data 1 of Table 3, we obtain $R_0 \approx 1.607 > 1$. Fig. 4 demonstrates that Theorem 3 holds, and there exists a rumor equilibrium E_2 = (0.38, 0, 0.5). Fig. 5 describes system (2) stable at E_3 = (0.4, 0.2, 0.23) by considering the system with the parameters in data 2 of Table 3, and Theorem 4 holds.



(a) (b)

Fig. 4. At E_2: (a) the densities evolution plot of ignorances, discussants, and spreaders and (b) the phase diagram of six different initial values.



(a) (b)

Fig. 5. At E_3: (a) the densities evolution plot of ignorances, discussants, and spreaders and (b) the phase diagram of six different initial values.

Figs. 4(a) and 5(a) show that spreaders will keep increasing and eventually reach a steady state. In this state, rumors will always exist and seriously affect our opinion. Figs. 4(b) and 5(b) demonstrate that the densities of spreaders are irrelevant to the initial values, but depend on the basic reproduction number.

5.2 Transcritical Bifurcation Simulation and Analysis

According to Theorem 5, system (2) exhibits transcritical bifurcation at the rumor-free equilibrium E_0 if $R_0 = 1$. By considering the parameters from data 1 of Table 4, we obtain $R_0 = 1$ if the infection rate $\alpha = 0.35$. Under this condition, system (2) occurs a transcritical bifurcation, and spreaders S change from unstable to stable, as shown in Fig. 6.

Table 4: Parameter for transcritical bifurcation.

Parameter	α	β	γ	ε	η	λ	δ	μ
Data 1	[0.0003-1]	[0.0001-1]	0.05	0.1	0.15	0.1	0.2	0.05

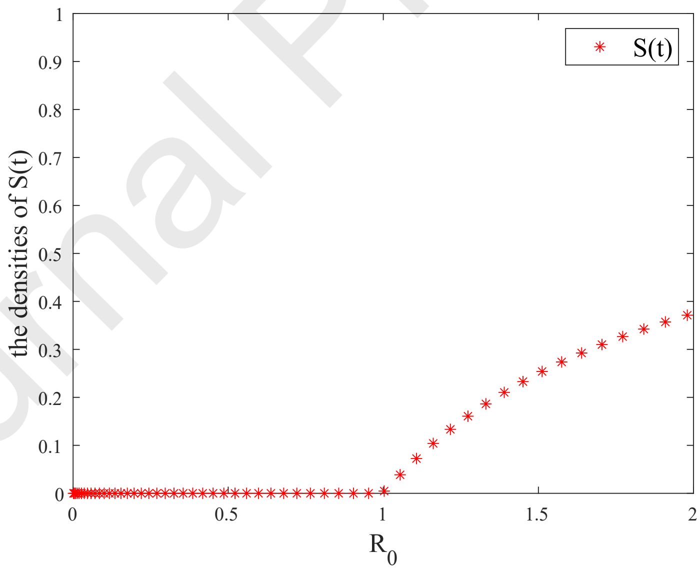


Fig. 6. Transcritical bifurcation at $R_0 = 1$.

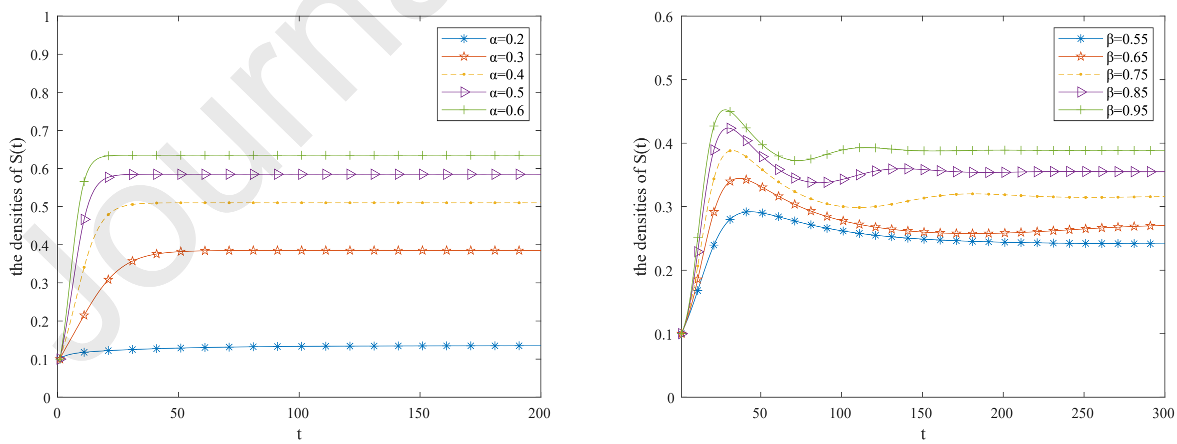
Therefore, spreaders S can change from an inactive state to an active one if $R_0 = 1$. If $R_0 > 1$, the densities of spreaders at the stable state increase along with the increase of R_0, but the increasing rate reduces gradually. The result indicates that rumors will not always exist, nor will they continue to increase rapidly. The basic reproduction number can control the propagating speed of rumors and the densities of spreaders. If the basic reproduction number becomes smaller in actual situations, the scope of the spreader influence will become smaller, the densities of the other three states in the system will increase, and the spread of rumors will be hindered.

5.3 Influence of Important Parameters for the Spread Node

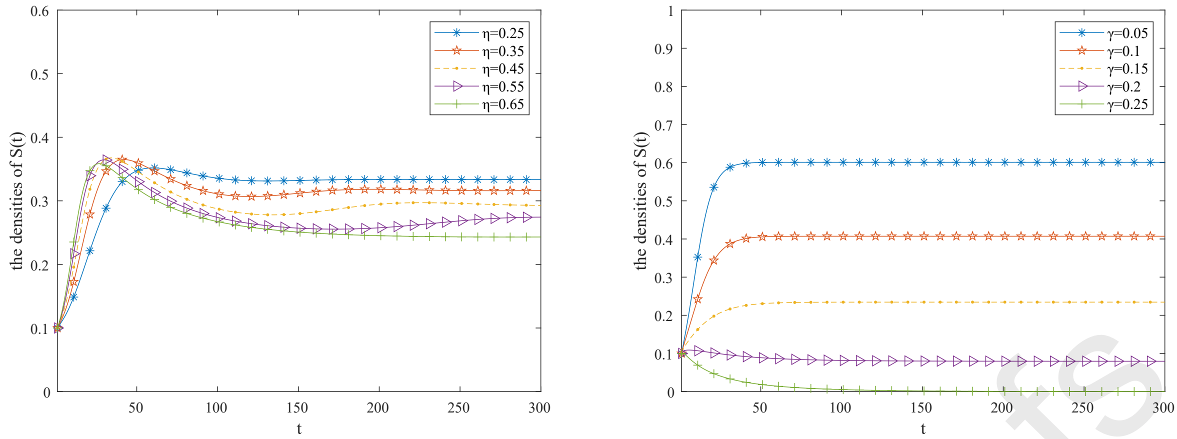
According to system (2), the densities of spreaders are closely related to the infection rates α and η, the recovery rate γ, and the discussion rate β. This section will present the densities evolution of spreaders with different α, β, η, and γ. Fig. 7 represents the density curves of spreaders for different α, β, η, and γ. Data 1 to data 4 in Table 5 correspond to Figs. 7(a) to (d), respectively.

Table 5: Parameter for spread nodes.

Parameter	α	β	γ	ε	η	λ	δ	μ
Data 1	[0.2-0.6]	0.2	0.1	0.72	0.5	0.3	0.1	0.05
Data 2	0.1	[0.55-0.95]	0.1	0.72	0.45	0.2	0.2	0.05
Data 3	0.1	0.7	0.01	0.72	[0.25-0.65]	0.2	0.2	0.05
Data 4	0.35	0.3	[0.05-0.25]	0.72	0.5	0.2	0.2	0.05



(a) (b)



(c) (d)

Fig. 7. The densities evolution of spreaders with different α (a), β (b), η (c), and γ (d).

From Figs. 7(a) and 7(b), it can also be seen that the peaks of spreaders S become bigger as the infection rate α and discussion rate β increase. In contrast, the peak values of spread nodes $S(t)$ become smaller as the infection rate η and recovery rate γ increase in Figs. 7(c) and 7(d). Therefore, the larger discussion rate β and infection rate α are, the smaller recovery rate γ and infection rate η are, and the more widespread rumors in social networks are. In other words, if the α and β are appropriately reduced, or the η and γ are increased, the transmission of rumors in the system can be reduced, which conforms to the rumor spreading characteristics in social networks.

5.4 Sensitivity Analysis

The sensitivity analysis of the basic reproduction number related to model parameters are performed. This paper focus on the simulation of the basic reproduction number with different the infection rate α, the recruitment and exit rate μ, the forget rate ε, and the recovery rate γ.

Table 6: Parameter for R_0.

Parameter	α	γ	ε	δ	μ
Data 1	[0-0.8]	0.1	0.1	0.1	0.05
Data 2	0.5	0.1	0.1	0.1	[0.1-0.5]
Data 3	0.2	0.1	[0.1-1]	0.1	0.05
Data 4	0.6	[0.1-0.6]	0.15	0.1	0.05

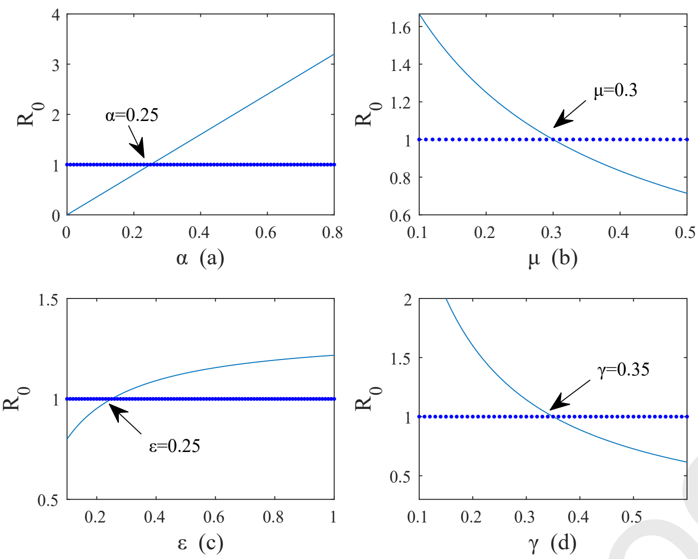


Fig. 8. Parameter for α (a), μ (b), ε (c), and γ (d) for R_0.

Choosing parameters in Table 6 from data 1 to data 4, it can obtain that $\alpha = 0.25$, $\mu = 0.3$, $\varepsilon = 0.25$, and $\gamma = 0.35$ if $R_0 = 1$. As seen from Figs. 8(a) and (c), it is obvious that R_0 become larger with the rise of the infection rate α and the forget rate ε, which means that the greater these parameters are, the greater R_0 will be. In other words, the larger the α and ε are, the more ignorances and discussants may believe rumors and become spreaders, and the more removers can forget rumors and become ignorances again, which increases the risk of re-infection.

Conversely, it is apparent that R_0 decrease with the increases of the recruitment and exit rate μ and the recovery rate γ in Fig. 8, which means that increasing these parameter values can reduce the values of R_0. In other words, the larger the μ and γ are, the smaller the R_0 will become until they are less than one, which can effectively control the rumor propagation. In summary, rumors will eventually be controlled and not be spread if $R_0 < 1$.

5.5 Model Validation

5.5.1 Validation by comparing ISRI model

This paper proposes the IDSRI model by considering the impact of discussants based on the

ISRI rumor propagation model. By comparing with the ISRI model, it is intuitively to reveal the influence of discussants of the IDSRI model on the rumor propagation process. Except for the discussion rate β, the infection rate η and the recovery rate λ, the other parameters of the IDSRI model are the same with all parameters of the ISRI model. Table 7 lists the specific parameter values.

Table 7: Parameter for IDSRI and ISRI models.

Model	**IDSRI Model**							
	ISRI Model							
Parameter	α	γ	ε	δ	μ	β	η	λ
Data 1	0.2	0.05	0.4	0.1	0.08	0.85	0.4	0.3
Data 2	0.2	0.05	0.4	0.1	0.08	0.85	0.4	0.03
Data 3	0.2	0.05	0.4	0.1	0.08	0.85	0.04	0.3
Data 4	0.2	0.05	0.4	0.1	0.08	0.6	0.4	0.3

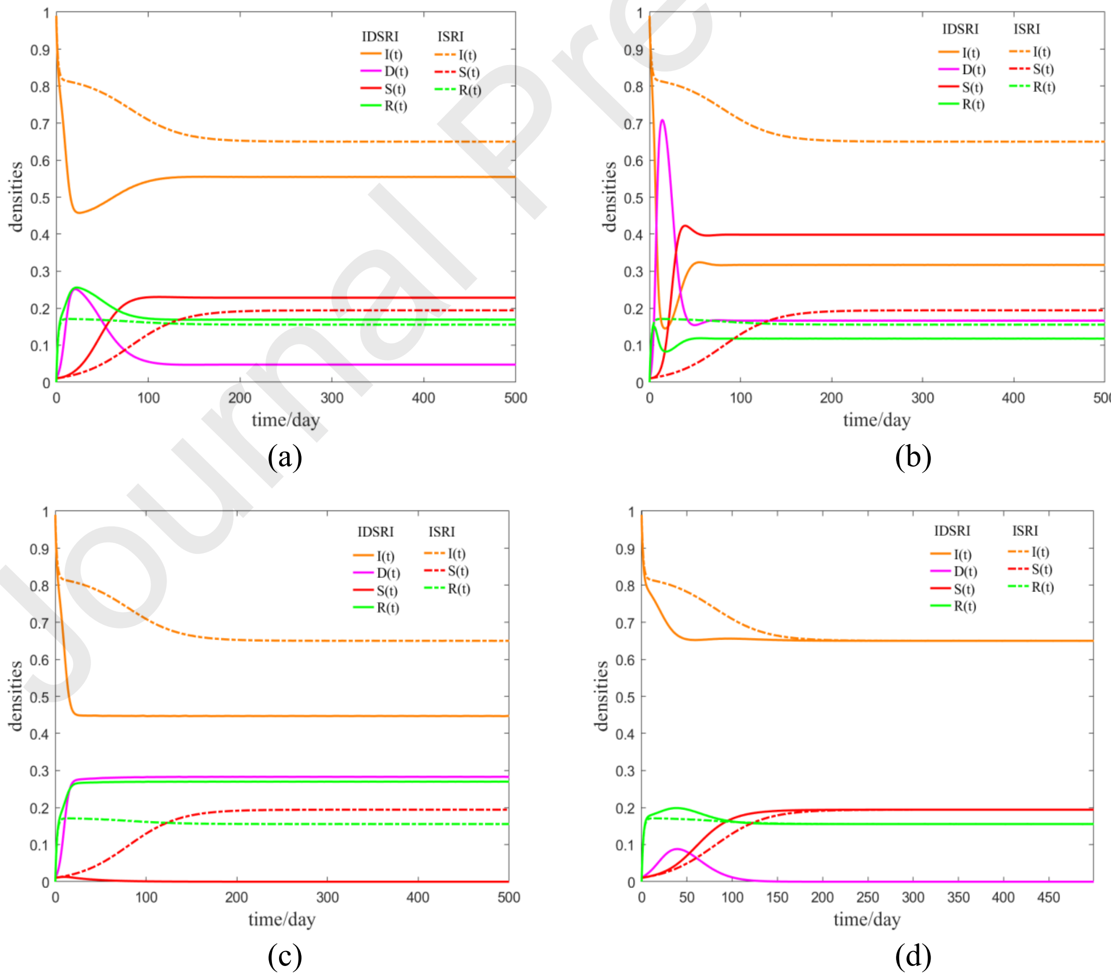


Fig. 10. Comparison of the IDSRI model and the ISRI model

By changing the values of the unique parameters β, η and λ in the IDSRI model, four groups of experiments can be obtained. Data 1 to data 4 in Table 7 correspond to Figs. 10(a) to (d), respectively. From Fig. 10(a), when there are discussants in the system, the densities of ignorances, spreaders, and removers are different from the ISRI model. As can be seen from Fig. 10(b), the densities of removers decrease and the densities of spreaders increase when the value of λ is reduced. The reason for this is that the probability of discussants becoming removers has decreased, resulting in lower densities of removers and higher densities of discussants. Although the infection rate η has not changed, the densities of spreaders increase as the densities of discussants increase.

The densities of removers increase and the densities of spreaders decrease as the infection rate η decreases in Fig. 10(c). Obviously, lowering the value of η will lead to a decrease in the densities of spreaders. As the densities of discussants increase, the densities of removers increase. From Fig. 10(d), as the densities of discussants drop to zero, the densities of ignorances, spreaders, and removers of the two models gradually become the same.

Compared with the ISRI model, the presence of discussants affects the densities of the other three groups. The values of the infection rate η and the recovery rate λ affect the discussant's choice between becoming a spreader or a remover. As the λ declines, increasing amounts of discussants become spreaders. Similarly, as the η decreases, an increasing number of discussants become removers. The value of the β affects the densities of discussants. If the value of the β is too small, the discussant in the system will eventually disappear.

5.5.2 Validation by actual dataset

This section will use an actual dataset to validate the IDSRI model. The Newly Emerged Rumors in Twitter dataset [1] contains 12 different rumor events. Each row of the dataset represents a tweet associated with the rumor, and each column of the dataset explains the information about that tweet. By comparing these 12 datasets, it is found that the data scale of the Dataset_R12 is larger than those of the other 11 datasets. Therefore, the Dataset_R12 in Newly Emerged Rumors in Twitter is chosen to verify the rumor spreading. This dataset collects tweets about the rumor "A screenshot from MyLife.com confirms that mail bomb suspect Cesar Sayoc was registered as a Democrat" from 14:00 on October 26th to 20:00 on October 29th, 2018. According to the attributes of the dataset, the tweets with the state “r” in this dataset is set as rumor tweets, corresponding to the *S*-state in our model. The cumulative number of *S*-state per ten minutes is counted, and the data of the first 34 hours is used to estimate the all parameters of the model. The data of the later hours is employed to validate the model.

Table 8

Parameter for estimate value.

Parameter	α	β	γ	ε	η	λ	δ	μ
Estimate Value	0.1	0.9798	0.0228	0.045	0.062	0.932	0.177	0.001

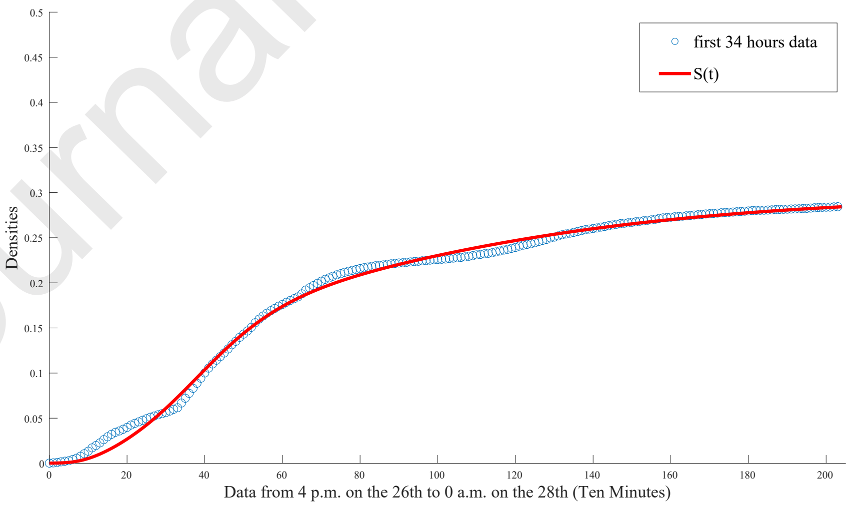


Fig. 11. The *S*-state curve of the model and the real data curve.

In the existing models, the values of model parameters are usually set directly by researchers or based on experience, without fully considering the actual rumor spreading. In order to effectively predict the rumor spreading trend, the parameters in the IDSRI model need to estimate. The least-square function is used to continuously adjust all parameter values of the model, iterating from 0.001 to 0.999, and obtain the estimated values of all parameters, as shown in Table 8. Fig. 11 demonstrates that the *S*-state curve of the model and the real data curve in the optimal fitting state. The scattering curve describes the first 34 hours of actual data, and the solid curve represents the densities of the *S*-state in the model.

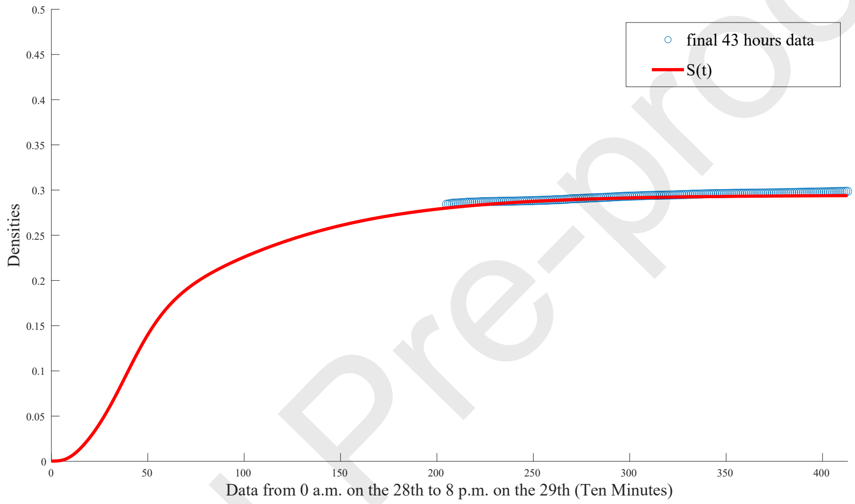


Fig. 12. The *S*-state curve of the model and the real data curve.

Next, the parameter values in Table 8 and the final 43 hours of data are employed to verify our model. First, the parameter values in Table 8 are substituted into system (2) to obtain the densities curve of the *S*-state. Second, the scattering curve for the final 43 hours of data is plotted, as shown in Fig. 12. Last, the R-squared is 0.9544, and the results show that the IDSRI model can well predict the rumor spreading trend of the next 43 hours. Therefore, the IDSRI model can effectively study the law of rumor spreading in social networks.

6. Conclusion

This paper proposes a new rumor spreading model–IDSRI. First, the total population is divided into four categories: ignorance, discussant, spreader, and remover, and its differential dynamics equations are proposed. Second, the local asymptotic stability of the four equilibria are analyzed by the Routh-Hurwitz stability criterion and the Lyapunov stability theorem. Furthermore, the global stability of the model is proved by establishing a Lyapunov function. In addition, the transcritical bifurcation at the rumor-free equilibrium are studied by applying Castillo-Chavez and Song bifurcation theorem. Next, numerical simulations are performed to verify the validity of the above theory, including local asymptotic stability analysis of the four equilibria, transcritical bifurcation, the effect of important parameters for the densities of spreaders, and the sensitivity analysis of the basic reproduction number. Finally, we verify the effectiveness of the model in real social networks by comparing the ISRI model and using a real dataset to conduct parameter estimation and prediction.

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