

**C289043 이소연**

**인공지능 Problem Set #3**

**Problem #1:**

다음은 클로즈의 집합(set of clauses) 이다.

- (1)  $\sim P \vee \sim Q \vee R$
- (2)  $\sim P \vee Q \vee \sim R$
- (3)  $\sim Q \vee \sim R \vee \sim W$
- (4)  $Q \vee R$
- (5)  $Q \vee W$
- (6)  $P \vee \sim Q$
- (7)  $\sim Q \vee W$

효율적인 DPLL 알고리즘을 적용하여, 위 논리표현 전부를 참으로 만드는 모델(진리 값: truth assignment)이 있는지, 혹은 없는지를 판단해 보라.

Initial set of clauses S0:

- (1)  $\sim P \vee \sim Q \vee R$
- (2)  $\sim P \vee Q \vee \sim R$
- (3)  $\sim Q \vee \sim R \vee \sim W$
- (4)  $Q \vee R$
- (5)  $Q \vee W$
- (6)  $P \vee \sim Q$
- (7)  $\sim Q \vee W$

Initial valuation V0: All atoms unbound. Sequence of calls.

1. Call  $dp1(ATOMS, S0, V0)$

P, Q, R, W 모두 긍정, 부정이 혼합되어 등장  $\Rightarrow$  순수 리터럴 없음.

No pure literals, no Singleton clauses.

Try  $V[P] := \text{TRUE}$ ;  $V1$  is the valuation  $V1[P] = \text{True}$ .

Delete clauses 6, delete  $\sim P$  from 1, 2

S1:

(1)  $\sim Q \vee R$

(2)  $Q \vee \sim R$

(3)  $\sim Q \vee \sim R \vee \sim W$

(4)  $Q \vee R$

(5)  $Q \vee W$

(7)  $\sim Q \vee W$

2. Call  $dp1(ATOMS, S1, V1)$

No pure literals, no Singleton clauses.

Try  $V[Q] := \text{TRUE}$ ;  $V2$  is the valuation  $V2[P] = \text{True}$ ,  $V2[Q] = \text{TRUE}$ .

Delete clauses 2, 4, 5, delete  $\sim Q$  from 1, 3, 7

S2:

(1) R

(3)  $\sim R \vee \sim W$

(7) W

3. Call  $dp1(ATOMS, S2, V2)$

1 is a singleton clause with literal R;  $V[R] = \text{TRUE}$ ;

$V3$  is the valuation  $V3[P]=\text{TRUE}$ ,  $V3[Q]=\text{TRUE}$ ,  $V3[R]=\text{TRUE}$

Delete clause 1, delete  $\sim R$  from clause 3.

New set of clauses  $S3$ :

(3)  $\sim W$

(4)  $W$

⇒ 모순 발생

4. Call  $dp1(ATOMS, S3, V3)$

Try  $V[Q]=\text{FALSE}$ ;

$V4[P]=\text{TRUE}$ ,  $V4[Q]=\text{FALSE}$

Delete clauses 1, 3, 7, delete Q from 2, 4, 5

$S4$ :

(2)  $\sim R$

(4)  $R$

(5)  $W$

⇒ Singleton clause 끼리 모순 발생 (2,4)

5. Call  $\text{dp1}(\text{ATOMS}, \text{S4}, \text{V4})$

Try  $\text{V}[\text{P}] = \text{FALSE}$ ;

$\text{V5}[\text{P}] = \text{FALSE}$ ;

Delete clauses 1, 2, delete P from 6

S5:

(3)  $\sim Q \vee \sim R \vee \sim W$

(4)  $Q \vee R$

(5)  $Q \vee W$

(6)  $\sim Q$

(7)  $\sim Q \vee W$

6. Call  $\text{dp1}(\text{ATOMS}, \text{S5}, \text{V5})$

6 is a singleton clause with literal  $\sim Q$

$\text{V}[\text{Q}] = \text{FALSE}$

$\text{V6}[\text{P}] = \text{FALSE}$ ,  $\text{V6}[\text{Q}] = \text{FALSE}$ ;

Delete clauses 3, 6, 7, delete Q from 4, 5

S6:

(4) R

(5) W

7. Call  $\text{dp1}(\text{ATOMS}, \text{S6}, \text{V6})$

4 is a singleton clause with literal R

$\text{V}[\text{R}] = \text{TRUE}$

$\text{V7}[\text{P}] = \text{FALSE}$ ,  $\text{V7}[\text{Q}] = \text{FALSE}$ ,  $\text{V7}[\text{R}] = \text{TRUE}$ ;

Delete clauses 4

S7:

(5) W

8.

7. Call  $\text{dp1}(\text{ATOMS}, \text{S7}, \text{V7})$

5 is a singleton clause with literal W

$V[W]=\text{TRUE}$

$V8[P]=\text{FALSE}, V8[Q]=\text{FALSE}, V8[R]=\text{TRUE}, V8[W]=\text{TRUE}$

Delete clauses 5

S8: empty set of clauses.

→  $V[P]=\text{FALSE}, V[Q]=\text{FALSE}, V[R]=\text{TRUE}, V[W]=\text{TRUE}$  일 때 SAT 하다.

**Problem #2:**

**A.** 다음의 논리식을 CNF 형태로 변형하라. (여기서, “~” 은 negation 의미)

$$P \Leftrightarrow (Q \wedge \sim R).$$

$$W \Rightarrow P.$$

$$R \Leftrightarrow S.$$

$$S \Rightarrow P.$$

$$P \Rightarrow (\sim(Q \vee W) \vee S).$$

1.  $\sim P \vee Q$
2.  $\sim P \vee \sim R$
3.  $\sim Q \vee R \vee P$
4.  $\sim W \vee P$
5.  $\sim R \vee S$
6.  $\sim S \vee R$
7.  $\sim S \vee P$
8.  $\sim P \vee \sim Q \vee S$
9.  $\sim P \vee \sim W \vee S$

**B.** 효율적인 DPLL 알고리즘을 적용하여 satisfiable 한지 아닌지를 판단하라.

S0 :

- (1)  $\sim P \vee Q$
- (2)  $\sim P \vee \sim R$
- (3)  $\sim Q \vee R \vee P$
- (4)  $\sim W \vee P$
- (5)  $\sim R \vee S$
- (6)  $\sim S \vee R$
- (7)  $\sim S \vee P$
- (8)  $\sim P \vee \sim Q \vee S$
- (9)  $\sim P \vee \sim W \vee S$

Step 1. Call  $dp1(ATOMS, S0, V0)$

$\sim W$  is a pure literal. ( $W$  never appears)  $V1[W] = \text{FALSE}$ .

Delete clauses 4, 9

$S1$ :

- (1)  $\sim P \vee Q$
- (2)  $\sim P \vee \sim R$
- (3)  $\sim \sim Q \vee R \vee P$
- (5)  $\sim R \vee S$
- (6)  $\sim S \vee R$
- (7)  $\sim S \vee P$
- (8)  $\sim P \vee \sim Q \vee S$

No pure literals, no singleton clauses.

Try  $V[P] := \text{TRUE}$ ;  $V2$  is the valuation  $V2[P] = \text{TRUE}$ ,  $V2[W] = \text{FALSE}$ . Call  $propagate(P, S1, V2)$ :

Delete clauses 3, 7, delete  $\sim P$  from 1, 2, 8

New set of clauses  $S2$ :

- (1)  $Q$
- (2)  $\sim R$
- (5)  $\sim R \vee S$
- (6)  $\sim S \vee R$
- (8)  $\sim Q \vee S$

Step 2. Call  $dp1(ATOMS, S2, V2)$ .

1 is a singleton clause with literal  $Q$

$V[Q] = \text{TRUE}$

$V3[P] = \text{TRUE}$ ,  $V3[Q] = \text{TRUE}$ ,  $V3[W] = \text{FALSE}$  ;

Delete clause 1, delete  $\sim Q$  from 8

New set of clauses  $S3$ :

- (2)  $\sim R$
- (5)  $\sim R \vee S$
- (6)  $\sim S \vee R$
- (8)  $S$



3. Call  $\text{dp1}(\text{ATOMS}, \text{S3}, \text{V3})$ .

2 is a singleton clause with literal  $\sim R$

$\text{V}[\text{R}] = \text{FALSE}$ ;

$\sim \text{V4}[\text{P}] = \text{TRUE}$ ,  $\text{V4}[\text{Q}] = \text{TRUE}$ ,  $\text{V4}[\text{R}] = \text{FALSE}$ ,  $\text{V4}[\text{W}] = \text{FALSE}$  ;

Delete clause 2,5 , delete R from 5

New set of clauses  $\text{S4}$ :

(6)  $\sim \text{S}$

(8)  $\text{S}$

$\Rightarrow$  singleton clauses 간 모순 발생  $\Rightarrow \text{UNSAT}$

4. Call  $\text{dp1}(\text{ATOMS}, \text{S4}, \text{V4})$

TRY  $\text{V}[\text{P}] = \text{FALSE}$ ;

$\text{V5}[\text{P}] = \text{FALSE}$ ,  $\text{V5}[\text{W}] = \text{FALSE}$ .

Delete clause 1,2,8 , delete P from 3,7

$\text{S5}$ :

(3)  $\sim \text{Q} \vee \text{R}$

(5)  $\sim \text{R} \vee \text{S}$

(6)  $\sim \text{S} \vee \text{R}$

(7)  $\sim \text{S}$

5. Call  $\text{dp1}(\text{ATOMS}, \text{S5}, \text{V5})$

7 is a singleton clause with literal  $\sim \text{S}$

$\text{V}[\text{S}] = \text{FALSE}$ ;

$\text{V3}$  is the valuation  $\text{V6}[\text{P}] = \text{FALSE}$ ,  $\text{V6}[\text{S}] = \text{FALSE}$ ,  $\text{V6}[\text{W}] = \text{FALSE}$ .

Delete clause 6,7 , delete S from clause 5

$\text{S6}$ :

(3)  $\sim \text{Q} \vee \text{R}$

(5)  $\sim \text{R}$

6. Call  $\text{dp1}(\text{ATOMS}, \text{S6}, \text{V6})$

5 is a singleton clause with literal  $\sim \text{R}$

$\text{V}[\text{R}] = \text{FALSE}$ ;

$\text{V3}$  is the valuation  $\text{V7}[\text{P}] = \text{FALSE}$ ,  $\text{V7}[\text{R}] = \text{FALSE}$ ,  $\text{V7}[\text{S}] = \text{FALSE}$ ,  $\text{V7}[\text{W}] = \text{FALSE}$ .

Delete clause 5 , delete R from clause 3  
S7:

(3)  $\sim Q$

7. Call  $dp1(ATOMS, S7, V7)$

3 is a singleton clause with literal  $\sim Q$

$V[Q] = FALSE$ ;

V3 is the valuation  $V8[P] = FALSE, V8[Q]=FALSE, V8[R]=FALSE, V8[S]=FALSE, V8[W] = FALSE$

Delete clause 3

S8 : empty set of clauses.

→  $V[P] = FALSE, V[Q]=FALSE, V[R]=FALSE, V[S]=FALSE, V[W] = FALSE$  할 때 SAT 하다.

**Problem #3:**

KB 에 있는 아래 룰들을 사용하여  $\sim P_{1,2}$  가 TRUE 임을 증명해 보자.

(여기서,  $\sim$  은 negation;  $\Leftrightarrow$  는 iff 의미하며 두개의 룰로 나누어도 됨.)

**Rule 1:**  $\sim P_{1,1}$

**Rule 2:**  $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

**Rule 3:**  $B_{2,1} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{3,1}$

**Rule 4:**  $\sim B_{1,1}$

**Rule 5:**  $B_{2,1}$

**A. Forward chaining** 방법으로 “ $\sim P_{1,2}$  가 TRUE” 임을 추론해 보라.

1. R4 에서  $B_{1,1}$  은 거짓
2. R2 에서  
 $B_{1,1} \Rightarrow P_{1,2} \vee P_{2,1}$   
 $\sim B_{1,1} \Rightarrow \sim P_{1,2} \wedge \sim P_{2,1}$   
 $B_{1,1}$  이 거짓이므로  $P_{1,2}$  와  $P_{2,1}$  둘 다 거짓  
 $\rightarrow \sim P_{1,2}$  는 참

**B. Backward chaining** 방법으로 “ $\sim P_{1,2}$  가 TRUE“ 임을 추론해 보라.

목표 =  $\sim P_{1,2}$  가 TRUE (=  $P_{1,2}$  가 거짓)

1. R2 에서  $B_{1,1}$  이 거짓이면  $P_{1,2}$  와  $P_{2,1}$  모두 거짓  
     $\rightarrow P_{1,2}$  가 거짓이어야 하므로  $B_{1,1}$  과  $P_{2,1}$  모두 거짓이어야 함.
2. R4 에서  $B_{1,1}$  이 거짓인 것이 확인됨  
 $\rightarrow P_{1,2}$  가 거짓인 것을 확정할 수 있음  
 $\rightarrow \sim P_{1,2}$  는 참

**C. Resolution** 방법으로 “ $\sim P_{1,2}$  가 TRUE“ 임을 추론해 보라.

- 2, 4 결합  $\rightarrow \sim P_{1,2} \vee \sim P_{2,1}$   
 $P_{1,2}$  가 참이라면 Rule 2,4 에 모순이 발생한다.  
따라서  $P_{1,2}$  는 거짓이다.  
 $\rightarrow \sim P_{1,2}$  는 참