A BAYESIAN ANALYSIS ON THE ACCURACY OF BEAM MODELS

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ABSTRACT

We use mathematical models to predict the behaviour of physical systems, whether it be structural beams, bridges, cells or circuits. This short paper aims to use how a Bayesian approach to assess quantitatively how well two different beam models, namely Euler-Bernolli beam theory and Finite Element Analysis (FEA), predict the deflection of a thin shelled structure.

1 Introduction

Introduction

2 Bayesian Analysis

3 Application to the Mechanics of Thin Shell Structures

In this section we demonstrate the application of the proposed methodology to the deflection of thin shell structures. Thin shells are curved solids with one dimension significantly smaller than the other two. They are prevalent in nature, e.g., as insect wings or biological membranes and in engineering, most prominently in aerospace and automotive. Carefully designed curved thin shells have a load carrying capacity which is usually significantly higher than comparable flat structures.

3.1 Problem Description

We consider the composite beam shown in Figure 1 consisting of a gyroid core and two face plates. The gyroid is a triply periodic minimal surface with zero mean curvature and has recently been extensively explored in additive manufacturing applications, see e.g. Hussein et al. (2013); Abueidda et al. (2017). As known, cellular solids like the gyroid core can have mechanical properties that are orders of magnitude different from their constituent materials (Fleck et al., 2010). The length of the beam is 0.8, its height, i.e. distance between the top and bottom plates, is 0.1 and its width is 0.1. The gyroid core is described by the algebraic function

$$sin(\lambda x)cos(\lambda y) - sin(\lambda y)cos(\lambda z) - sin(\lambda y)cos(\lambda x) = 0 \tag{1}$$

with $\lambda = 20$. The core and the two plates are modelled as thin shells and have a thickness of $t = 0.003$. The beam is clamped at its left end, and at its right end the bottom plate is simply supported at a distance 0.025 away from the boundary. The Young's modulus and the Poisson's ratio are $E = 2.30GPa$ and $\nu = 0.3$. The top plate is subjected to a uniform pressure $f(x) = 3$ acting in the negative z direction.

The gyroid beam is a complex geometry that can only be manufactured via additive methods, such as 3D printing. Thin shells are curved solids with one dimension significantly smaller than the other two. They are prevalent in nature, e.g., as insect wings or biological membranes and in engineering, most prominently in aerospace and automotive. Carefully designed curved thin shells have a load carrying capacity which is usually significantly higher than comparable flat structures.

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3.2 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j|y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \tag{2}$$

3.2.1 Headings: third level

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4 Examples of citations, figures, tables, references

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The documentation for `natbib` may be found at

`http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf`

Of note is the command `\citet`, which produces citations appropriate for use in inline text. For example,

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> Hasselmo, et al. (1995) investigated…

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4.1 Figures

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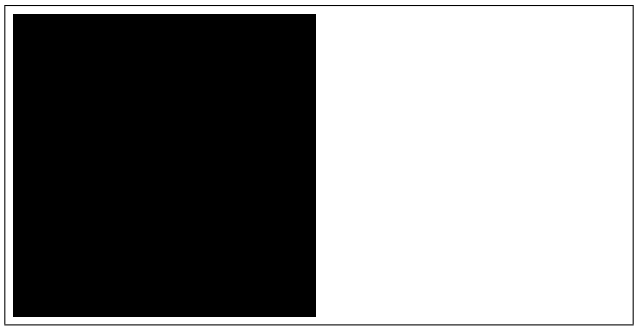

Figure 1: Sample figure caption.

Table 1: Sample table title

	Part	
Name	Description	Size (μm)
Dendrite	Input terminal	$\sim$100
Axon	Output terminal	$\sim$10
Soma	Cell body	up to 10^6

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4.2 Tables

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4.3 Lists

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References

[1] George Kour and Raid Saabne. Real-time segmentation of on-line handwritten arabic script. In *Frontiers in Handwriting Recognition (ICFHR), 2014 14th International Conference on*, pages 417–422. IEEE, 2014.

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[3] Guy Hadash, Einat Kermany, Boaz Carmeli, Ofer Lavi, George Kour, and Alon Jacovi. Estimate and replace: A novel approach to integrating deep neural networks with existing applications. *arXiv preprint arXiv:1804.09028*, 2018.

[1] Sample of the first footnote.