

Sets

In general sets will be denoted by capital letters: A, B, C, \dots

Frequently used sets:

\emptyset = the empty set (i.e. the set that contains no elements)

$\mathbb{N} = \{0, 1, 2, \dots\}$ the set of natural numbers

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ the set of integers

$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ the set of positive integers

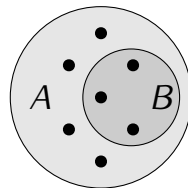
\mathbb{Q} = the set of rational numbers

\mathbb{R} = the set of real numbers

We will write $x \in A$ to denote that x is an element of the set A and $y \notin A$ to indicate that y is not an element of A . For example, $5 \in \mathbb{Z}$, $\frac{1}{3} \notin \mathbb{Z}$.

Subsets

A set B is a *subset* of a set A if every element of B is in A . In such case we write $B \subseteq A$.



A set B is a *proper subset* of A if $B \subseteq A$ and $B \neq A$.

Example. Some subsets of \mathbb{Z} :

- $A = \{n \in \mathbb{Z} \mid n > 10\}$ - the set of integers greater than 10.
- $B = \{n \in \mathbb{Z} \mid n = 2k \text{ for some } k \in \mathbb{Z}\}$ - the set of even integers.

Operations on sets

- The *union* of sets A and B is the set $A \cup B$ that consists of all elements that belong to either A or B (or both):

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

- The *intersection* of sets A and B is the set $A \cap B$ that consists of all elements that belong to both A and B :

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- The *difference* of sets A and B is the set $A \setminus B$ that consists of all elements that belong to A but not to B :

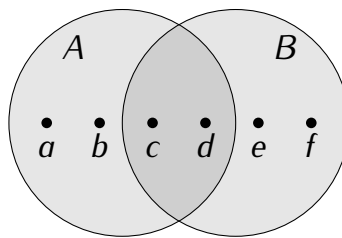
$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

Example. If $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ then:

$$A \cup B = \{a, b, c, d, e, f\}$$

$$A \cap B = \{c, d\}$$

$$A \setminus B = \{a, b\}$$



Note. We say that sets A and B are *disjoint sets* if $A \cap B = \emptyset$.

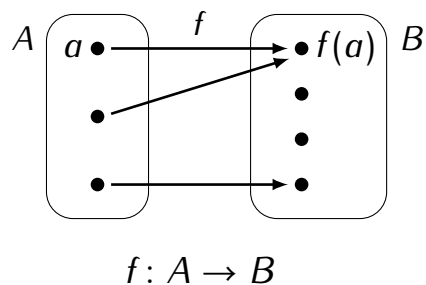
- The *Cartesian product* of sets A , B is the set consisting of all ordered pairs of elements of A and B :

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

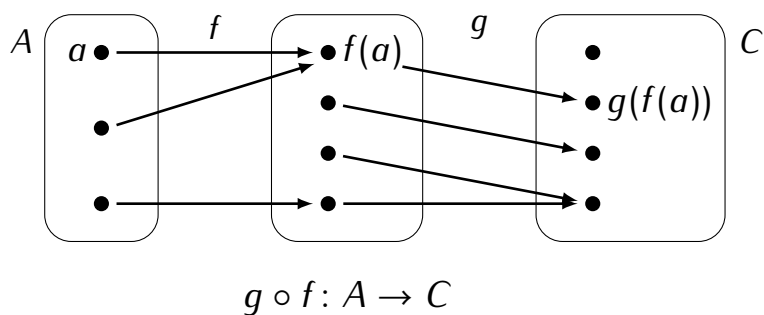
Example. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then

$$A \times B = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$$

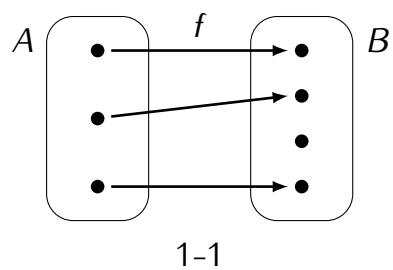
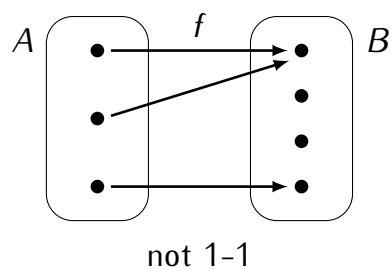
Functions



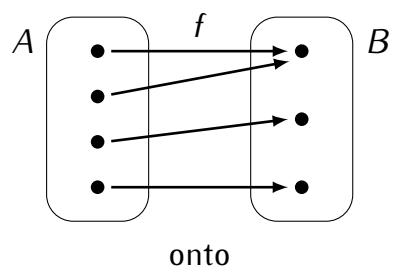
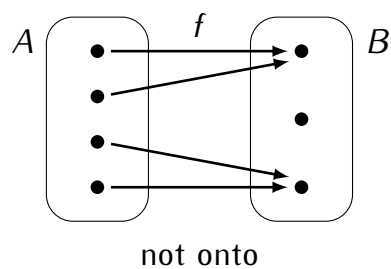
Composition of functions:



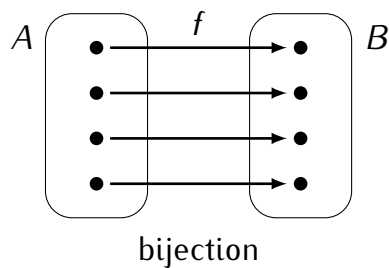
- A function $f: A \rightarrow B$ is *1-1* if $f(a) = f(a')$ only if $a = a'$.



- A function $f: A \rightarrow B$ is *onto* if for every $b \in B$ there is $a \in A$ such that $f(a) = b$.



- A function $f: A \rightarrow B$ is a *bijection* if f is both 1-1 and onto.



Note. If $f: A \rightarrow B$ is a bijection then the inverse function $f^{-1}: B \rightarrow A$ exists and it is also a bijection. Then for every $a \in A$ and $b \in B$ we have:

$$f^{-1}(f(a)) = a \quad \text{and} \quad f(f^{-1}(b)) = b$$

Cardinality

We will denote by $|A|$ the cardinality of the set A . For a finite set, this is the number of elements of A .

Note. If A, B are sets, then $|A| = |B|$ if and only if there exists a bijection $f: A \rightarrow B$.