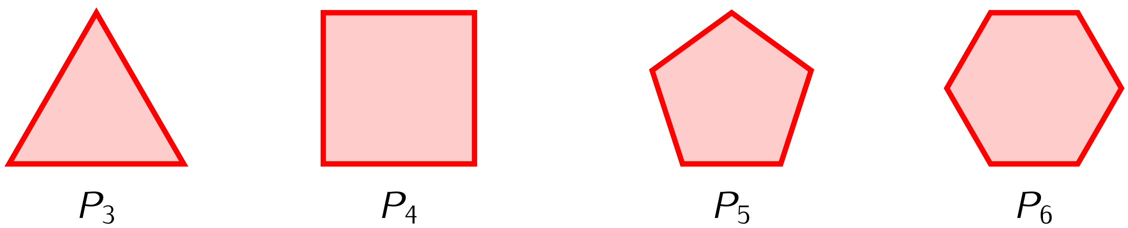
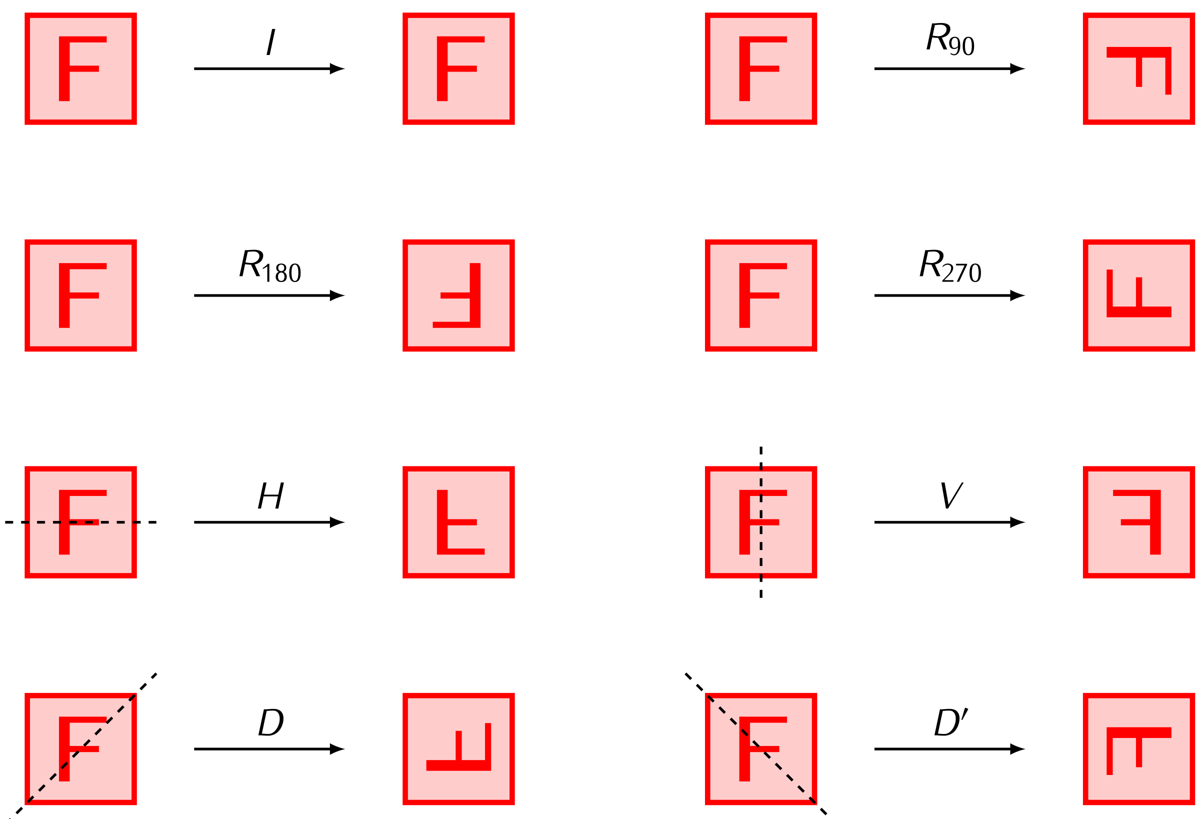
Regular polygons P_n with n sides:



Symmetries of P_4:



Composition of symmetries:

Composition table of symmetries of a square:

$\circ$	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
I	I	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_{90}	R_{90}	R_{180}	R_{270}	I	D'	D	H	V
R_{180}	R_{180}	R_{270}	I	R_{90}	V	H	D'	D
R_{270}	R_{270}	I	R_{90}	R_{180}	D	D'	V	H
H	H	D	V	D'	I	R_{180}	R_{90}	R_{270}
V	V	D'	H	D	R_{180}	I	R_{270}	R_{90}
D	D	H	D'	V	R_{270}	R_{90}	I	R_{180}
D'	D'	V	D	H	R_{90}	R_{270}	R_{180}	I

For $n \geq 3$ the dihedral group D_n is defined as follows:

- **Elements of D_n:** symmetries of the regular polygon with n sides.
- **Group operation:** Composition of symmetries (e.g. $V \circ D = R_{270}$).
- **The identity element:** The identity symmetry I.

Definition 3.1

The *order* of a group G is the number of elements of G. It is denoted by $|G|$.

If G has infinitely many elements, we write $|G| = \infty$.

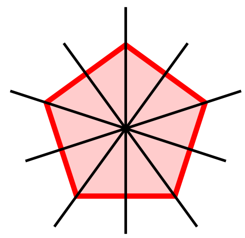
Theorem 3.2

For $n \geq 3$ we have $|D_n| = 2n$.

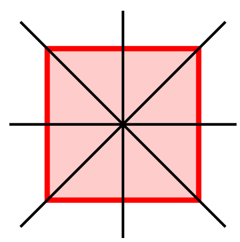
Note. The dihedral group D_n consists of the following elements:

1) n rotations by the angles of $k \cdot \frac{360}{n}$ degrees for $k = 0, \ldots, n-1$. For $k = 0$ this gives the rotation by 0 degrees, i.e. the identity symmetry.

2) n reflections with respect to different symmetry axes. If n is odd, there is one symmetry axis for each vertex of the polygon P_n:



If n is even there are $\frac{n}{2}$ symmetry axes passing through pairs of opposite vertices and $\frac{n}{2}$ symmetry axes crossing opposite sides of P_n:



Definition 3.3

Let G be a group, and let $S \subseteq G$ be a subset of G. We say that the set S *generates* G if every element of G can be obtained as a product of some elements of S and inverses of elements of S.