## Sets

In general sets will be denoted by capital letters: $A, B, C, \ldots$

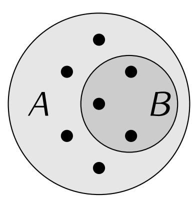
Frequently used sets:

$\varnothing =$ the empty set (i.e. the set that contains no elements)
$\mathbb{N} = \{0, 1, 2, \ldots\}$ the set of natural numbers
$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ the set of integers
$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ the set of positive integers
$\mathbb{Q} =$ the set of rational numbers
$\mathbb{R} =$ the set of real numbers

We will write $x \in A$ to denote that $x$ is an element of the set $A$ and $y \notin A$ to indicate that $y$ is not an element of $A$. For example, $5 \in \mathbb{Z}$, $\frac{1}{3} \notin \mathbb{Z}$.

## Subsets

A set $B$ is a *subset* of a set $A$ if every element of $B$ is in $A$. In such case we write $B \subseteq A$.




A set $B$ is a *proper subset* of $A$ if $B \subseteq A$ and $B \neq A$.

**Example.** Some subsets of $\mathbb{Z}$:

- $A = \{n \in \mathbb{Z} \mid n > 10\}$ – the set of integers greater than 10.
- $B = \{n \in \mathbb{Z} \mid n = 2k \text{ for some } k \in \mathbb{Z}\}$ – the set of even integers.

## Operations on sets

• The *union* of sets $A$ and $B$ is the set $A \cup B$ that consists of all elements that belong to either $A$ or $B$ (or both):

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

• The *intersection* of sets $A$ and $B$ is the set $A \cap B$ that consists of all elements that belong to both $A$ and $B$:

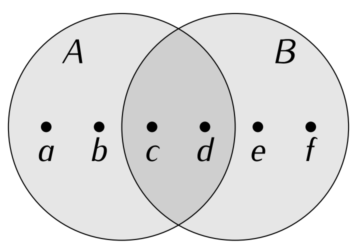
$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

• The *difference* of sets $A$ and $B$ is the set $A \setminus B$ that consists of all elements that belong to $A$ but not to $B$:

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

**Example.** If $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ then:

$$\begin{aligned} A \cup B &= \{a, b, c, d, e, f\} \\ A \cap B &= \{c, d\} \\ A \setminus B &= \{a, b\} \end{aligned}$$




**Note.** We say that sets $A$ and $B$ are *disjoint sets* if $A \cap B = \varnothing$.

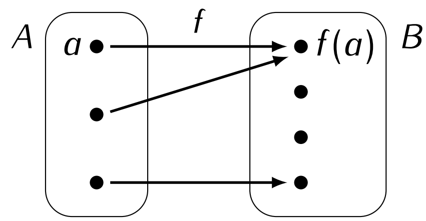
• The *Cartesian product* of sets $A$, $B$ is the set consisting of all ordered pairs of elements of $A$ and $B$:

$$A \times B = \{(a, b) \mid a \in A, \quad b \in B\}$$

**Example.** If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then

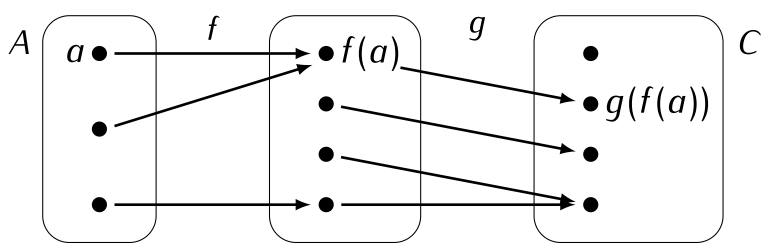
$$A \times B = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$$

## Functions



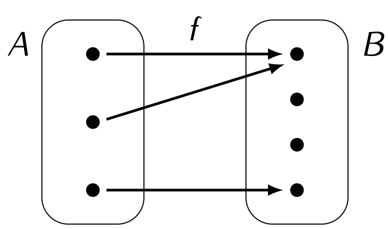

$f\colon A \to B$

Composition of functions:

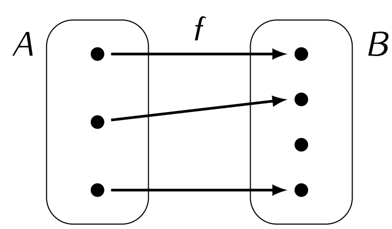



$g \circ f\colon A \to C$

• A function $f\colon A \to B$ is *1-1* if $f(a) = f(a')$ only if $a = a'$.

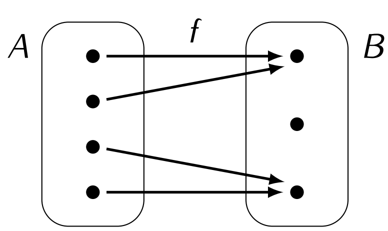



not 1-1

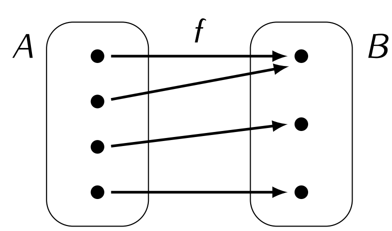



1-1

• A function $f\colon A \to B$ is *onto* if for every $b \in B$ there is $a \in A$ such that $f(a) = b$

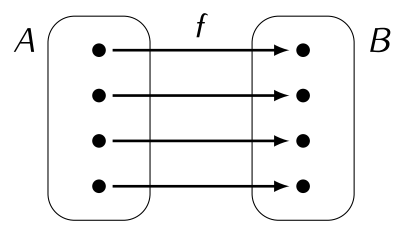



not onto




onto

- A function $f\colon A \to B$ is a *bijection* if $f$ is both 1-1 and onto.



bijection

**Note.** If $f\colon A \to B$ is a bijection then the inverse function $f^{-1}\colon B \to A$ exists and it is also a bijection. Then for every $a \in A$ and $b \in B$ we have:

$$f^{-1}(f(a)) = a \quad \text{and} \quad f(f^{-1}(b)) = b$$

## Cardinality

We will denote by $|A|$ the cardinality of the set $A$. For a finite set, this is the number of elements of $A$.

**Note.** If $A$, $B$ are sets, then $|A| = |B|$ if and only if there exists a bijection $f\colon A \to B$.