Systems of linear equations

$$\begin{cases} 2x_1 + 6x_2 - 6x_3 - 2x_4 = -4 \\ 4x_2 - 8x_3 = \ \ 4 \\ 2x_1 + 7x_2 - 8x_3 = -1 \end{cases}$$

Note.

- A consistent system of equations with free variables has infinitely many solutions
- Once we fix values of the free variables, the values of the basic variables are uniquely determined.

Pivoting

Pivoting is an operation that lets us modify which variables are basic and which are free.

$$\begin{cases} x_1 = & -4 & - 3x_3 \\ x_2 = & 1 & + 2x_3 \\ x_4 = & 1 & \\ x_3 = & \text{free} & \end{cases}$$

Note.

- The columns of the matrix corresponding to basic variables are linearly independent.

- The number basic (and free variables) does not depend on which variables are basic and which are free.

Definition

We will say that an $m \times n$ matrix is in a *basic form* if it contains m columns that correspond to the columns of the $m \times m$ identity matrix.

Note. If a matrix A is in the basic form then in a matrix equation $A\mathbf{x} = \mathbf{b}$ the columns of A corresponding the columns of the identity matrix give basic variables, and the other columns correspond to free variables.