Recall:

Definition

Let A be an $n \times n$ matrix. If $\mathbf{v} \in \mathbb{R}^n$ is a non-zero vector and λ is a scalar such that

$$A\mathbf{v} = \lambda \mathbf{v}$$

then we say that

- ullet λ is an eigenvalue of A
- v is an *eigenvector* of A corresponding to λ .

Example.

$$A = \left[\begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right]$$

Note. Eigenvectors corresponding to a given eigenvalue λ form a subspace of \mathbb{R}^n which is called the *eigenspace* corresponding to the eigenvalue λ .

Computation of eigenvalues

Notation. $I_n := \text{the } n \times n \text{ identity matrix.}$

Definition

If A is an $n \times n$ matrix then

$$P(\lambda) = \det(A - \lambda I_n)$$

is a polynomial of degree n. $P(\lambda)$ is called the *characteristic polynomial* of the matrix A.

Proposition

If A is a square matrix then

eigenvalues of
$$A = \text{roots of } P(\lambda)$$

Example.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Computation of eigenvectors

Proposition

If λ is an eigenvalue of an $n \times n$ matrix A then

$$\begin{cases} \text{eigenvectors of } A \\ \text{corresponding to } \lambda \end{cases} = \begin{cases} \text{vectors in} \\ \text{Nul}(A - \lambda I_n) \end{cases}$$

Example.

$$A = \left[\begin{array}{ccc} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{array} \right]$$