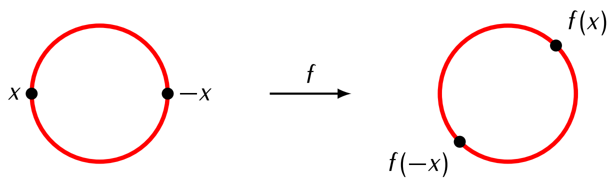
6 | Some Applications

6.1 Proposition. *The circle S^1 is not a retract of the disc D^2.*

6.2 Brouwer Fixed Point Theorem. *For each map $f\colon D^2 \to D^2$ there exists a point $x_0 \in D^2$ such that $f(x_0) = x_0$.*

6.3 Borsuk-Ulam Theorem. *For each map $f\colon S^2 \to \mathbb{R}^2$ there exists $x \in S^2$ such that $f(x) = f(-x)$.*

6.4 Lemma. *Let $f\colon S^1 \to S^1$ be a function such that $f(-x) = -f(x)$ for all $x \in S^1$:*



For any $x_0 \in X$ the homomorphism $f_*\colon \pi_1(S^1, x_0) \to \pi_1(S^1, f(x_0))$ is non-trivial.

Proof. Exercise. $\square$

6.5 Corollary. *There does not exist an embedding of S^2 into $\mathbb{R}^2$.*

6.6 Corollary. *If $A_1, A_2, A_3 \subseteq S^2$ are closed sets such that $A_1 \cup A_2 \cup A_3 = S^2$ then one of these sets contains a pair of antipodal points $\{x, -x\}$.*

6.7 The Fundamental Theorem of Algebra. *If $P(x)$ is a polynomial with coefficients in $\mathbb{C}$ and $\deg P(x) > 0$ then $P(z_0) = 0$ for some $z_0 \in \mathbb{C}$.*