1. Consider an $n \times n$ stochastic matrix P and two vectors $\mathbf{u}$ and $\mathbf{v}$:

$$P = \begin{bmatrix} p_{11} & \dots & p_{1n} \\ \vdots & & \vdots \\ p_{n1} & \dots & p_{nn} \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

Show that if $\mathbf{v} = P\mathbf{u}$ then

$$x_1 + \cdots + x_n = y_1 + \cdots + y_n$$

2. A gambler repeatedly plays a game where he bets X dollars and then can either win this amount with probability 0.4, or loose it with probability 0.6. At the beginning the gambler has \$3 and aims to play until he either loses all his money or gets to \$7. He considers two strategies:

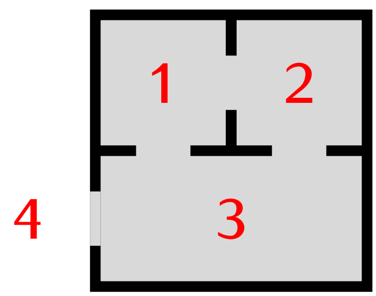
(I) Each time bet \$1.

(II) Each time bet as much as possible, but not more than needed to get to \$7.

a) Describe the strategy (I) as a Markov chain. Explain what are the states of this Markov chain and write its transition matrix. Compute the probability that the gambler will win \$7 using this strategy.

b) Repeat part a) for strategy (II).

3. An ogre is chasing a hobbit in a house. There are three rooms in the house, connected by doors as shown in the picture. There is also a door in room 3 that leads outside (4).



Every minute the ogre and the hobbit can decide to either stay in the room they are in, or to use one of the doors in that room to change location. However, the ogre never goes outside the house. At each step each of them chooses what to do randomly. For example, if the ogre is one minute in room 3, there are equal chances that the next minute he will be in the same room, move to room 1 or move to room 2. Similarly, if the hobbit is in room 3, then with equal chances he will stay in that room, move to room 1, move to room 2 or go outside.

The ogre and the hobbit choose their moves independently. This means that if $p_{i,j}$ if the probability that the hobbit will move from i to j and $q_{k,l}$ is the probability that the ogre will move from k to l, then the probability that the hobbit will move from i to j and the ogre will move from k to l at the same time is $p_{ij}q_{kl}$.

The chase stops when either the ogre and hobbit get to the same room (and then the ogre catches the hobbit), or the hobbit escapes outside.

a) Describe this chase as a Markov chain. Explain what are the states of this Markov chain. Identify states that are absorbing, and write the transition matrix.

b) Assume that at the beginning the hobbit is in room 3 and the ogre is in room 1. What are the chances that the hobbit will eventually escape? Explain you computations.

c) Repeat part b) assuming that at the beginning the hobbit is in room 2 and the ogre in room 3.