1. Solve the following linear program using the simplex method. At each pivot step write the basic feasible solution obtained and indicate which variable becomes basic and which becomes free.

a) Maximize the function

$$z = 4x_1 + 6x_2$$

subject to constraints:

$$\begin{aligned} -x_1 + x_2 &\leq 11 \\ x_1 + x_2 &\leq 27 \\ 2x_1 + 5x_2 &\leq 90 \\ x_1, x_2 &\geq 0 \end{aligned}$$

b) Sketch the feasible region for above linear program and number the extreme points in the order they appear in your simplex method calculations.

2. Assume that in the process of solving a linear program using the simplex method, you obtain the following tableau:

$$\begin{array}{cccc|c} x_1 & x_2 & \dots\ \dots\ \dots & & \\ \hline a_1 & 1 & \dots\ \dots\ \dots & & b_1 \\ a_2 & 0 & \dots\ \dots\ \dots & & b_2 \\ \dots & \dots & \dots\ \dots\ \dots & & \dots \\ a_n & 0 & \dots\ \dots\ \dots & & b_n \\ \hline c_1 & 0 & \dots\ \dots\ \dots & & z - d \end{array}$$

Here $c_1 \neq 0$. The variable x_1 is free and x_2 is basic. Assume also that in the next pivot step x_1 becomes basic and x_2 free. Explain why x_2 cannot become basic again in the pivot step that occurs immediately after that.

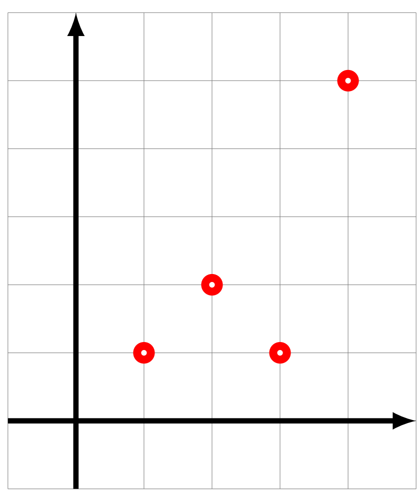
Hint. What can you say about c_1 and a_1 in this tableau? Why?

3. Assume that the simplex tableau of a linear program has n variables and m constraints (with $n \geq m$). What is the largest number of basic feasible solutions this linear program can have? Why?

Note. In the worst case, the simplex method may travel through most of these basic feasible solutions before it reaches the maximum, but typically it gets to the maximum much faster.

4. a) Write a linear program to find an equation of the linear function $f(x) = ax + b$ that best fits the following points in the L_1 sense:

$$(1, 1),\ (2, 2),\ (3, 1),\ (4, 5)$$

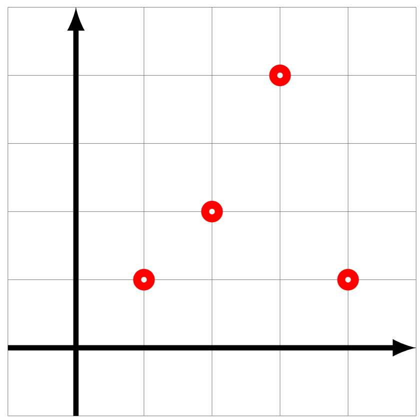


b) Solve this linear problem using `scipy.optimmize.linprog`. Include a printout or a screenshot showing your code and the results of computations.

c) Write the equation of the function $f(x)$ computed in part b) and plot this function together with the points given in part a).

5. a) Write a linear program to find an equation of the second degree polynomial $g(x) = ax^2 + bx + c$ that best fits the following points in the L_1 sense:

$$(1,1),\ (2,2),\ (3,4),\ (4,1)$$



b) Solve this linear problem using `scipy.optimmize.linprog`. Include a printout or a screenshot showing your code and the results of computations.

c) Write the equation of the function $g(x)$ computed in part b) and plot this function together with the points given in part a).