

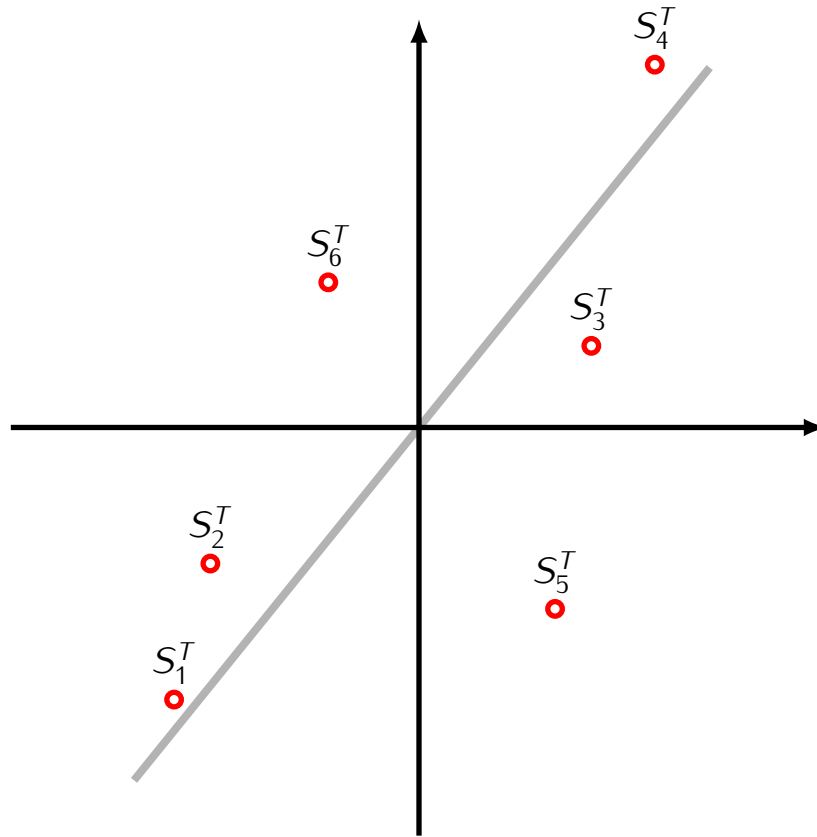
Example.

Demeaned data matrix:

$$A = \begin{array}{c} \text{Ex 1} \quad \text{Ex 2} \\ \text{Aly} \begin{bmatrix} -27 & -30 \end{bmatrix} \\ \text{Bob} \begin{bmatrix} -23 & -15 \end{bmatrix} \\ \text{Chen} \begin{bmatrix} 19 & 6 \end{bmatrix} \\ \text{Deb} \begin{bmatrix} 26 & 40 \end{bmatrix} \\ \text{Emma} \begin{bmatrix} 15 & -20 \end{bmatrix} \\ \text{Finn} \begin{bmatrix} -10 & 16 \end{bmatrix} \end{array} = \begin{bmatrix} \boxed{} & \boxed{} \end{bmatrix} = \begin{bmatrix} \boxed{S_1} \\ \boxed{S_2} \\ \boxed{S_3} \\ \boxed{S_4} \\ \boxed{S_5} \\ \boxed{S_6} \end{bmatrix}$$

Let \mathbf{u}_1 be the 1st principal axis of A , and let Y_1 be the 1st principal component of A :

$$Y_1 = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix} = \begin{bmatrix} S_1 \mathbf{u}_1 \\ S_2 \mathbf{u}_1 \\ \vdots \\ S_N \mathbf{u}_1 \end{bmatrix}$$



The projection matrix

The difference matrix

Definition

Let A an $N \times M$ demeaned data matrix

$$A = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{bmatrix}$$

Let \mathbf{u}_1 be the 1st principal axis of A , and let Y_1 be the 1st principal component of A .

The 2^{nd} *principal axis* of A is the 1st principal axis of the difference matrix

$$D_1 = A - Y_1 \mathbf{u}_1^T$$

The 2^{nd} *principal component* Y_2 of A is the 1st principal component of the matrix D_1 .

Computation of the 2nd principal axis of A

Proposition

Given a demeaned data matrix A , let

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$$

be eigenvalues of the covariance matrix C_A and let $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N$ be orthonormal vectors such that \mathbf{u}_i is an eigenvector of C_A corresponding to the eigenvalue λ_i .

- The 2^{nd} *principal axis* of A is the vector \mathbf{u}_2 .
- The 2^{nd} *principal component* of A is the vector $Y_2 = A\mathbf{u}_2$.
- We have $\text{Var}(Y_2) = \lambda_2$.
- In addition, $\text{Cov}(Y_1, Y_2) = 0$.

The i^{th} principal component

Proposition/Definition

Given a demeaned data matrix A , let

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$$

be eigenvalues of the covariance matrix C_A and let $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N$ be orthonormal vectors such that \mathbf{u}_i is an eigenvector of C_A corresponding to the eigenvalue λ_i .

- The i^{th} *principal axis* of A is the vector \mathbf{u}_i .
- The i^{th} *principal component* of A is the vector $Y_i = A\mathbf{u}_i$.
- We have $\text{Var}(Y_i) = \lambda_i$.
- In addition, $\text{Cov}(Y_i, Y_j) = 0$ if $i \neq j$.