The *simplex method* is one of the main methods of solving linear programs.

**Special assumptions (for now):**

**1)** The program is in the equality form: we want to maximize

$$z = c_1 x_1 + \ldots + c_n x_n$$

subject to the constraints:

$$a_{11} x_1 + \ldots + a_{1n} x_n = b_1$$
$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots$$
$$a_{m1} x_1 + \ldots + a_{mn} x_n = b_m$$

$$x_1, \ x_2, \ \ldots, x_n \geq 0$$

**2)** The coefficient matrix

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$$

is in the basic form.

**3)** $b_i \geq 0$ for $i = 1, \ldots, m$.

**Example.**

$$\begin{bmatrix} -1 & 0 & 1 & 1 & 0 & | & 1 \\ 1 & 1 & 0 & 0 & 0 & | & 3 \\ 2 & 0 & 0 & 1 & 1 & | & 7 \end{bmatrix}$$

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## Basic feasible solutions

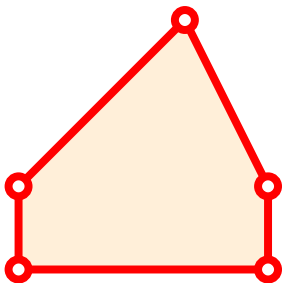*Basic feasible solutions* are the solutions obtained by setting all free variables to 0.

**Example.**

$$\begin{bmatrix} -1 & 0 & 1 & 1 & 0 & | & 1 \\ 1 & 1 & 0 & 0 & 0 & | & 3 \\ 2 & 0 & 0 & 1 & 1 & | & 7 \end{bmatrix}$$

$$\begin{cases} x_3 = 1 + x_1 - x_4 \\ x_2 = 3 - x_1 \\ x_5 = 7 - 2x_1 - x_4 \\ x_1 = \text{free} \\ x_5 = \text{free} \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

## Geometric interpretation



## The pivot step

$$\begin{bmatrix} -1 & 0 & 1 & 1 & 0 & | & 1 \\ 1 & 1 & 0 & 0 & 0 & | & 3 \\ 2 & 0 & 0 & 1 & 1 & | & 7 \end{bmatrix}$$

## One more assumption

**4)** In the objective function $z = c_1 x_1 + \ldots + c_n x_n$ the coefficients $c_i$ corresponding to basic variables are equal to 0.

**Example.** The objective function:

$$4x_1 + 2x_2 + x_4 = z$$

Constraints:

$$-x_1 + x_3 + x_4 = 1$$
$$x_1 + x_2 = 3$$
$$2x_1 + x_4 + x_5 = 7$$

$$x_1, \ x_2, \ x_3, \ x_4, \ x_5 \geq 0$$