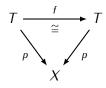
23 | Deck Transformations

23.1 Definition. Let $p: T \to X$ be a covering. A *deck transformation* of p is an isomorphism of coverings



23.2 Lemma. Let $F: \mathbb{C} \to \mathbb{D}$ be a functor such that for any $c, c' \in \mathbb{C}$ the map the map $\mathrm{Mor}_{\mathbb{C}}(c, c') \to \mathrm{Mor}_{\mathbb{D}}(F(c), F(c'))$ given by $f \mapsto F(f)$ is a bijection. A morphism $f: c \to c'$ i in \mathbb{C} is an isomorphism if and only if $F(f): F(c) \to F(c')$ is an isomorphism.
Proof. Exercise.
23.3 Corollary. Let X be a connected and locally path connected space, $x_0 \in X$, and let $p: T \to X$ be a path connected covering. The group of deck transformations $D(p)$ is isomorphic to the group of $\pi_1(X, x_0)$ -equivariant isomorphisms $p^{-1}(x_0) \to p^{-1}(x_0)$.
Proof. Exercise.

23.4 Definition. Let G be a group, and let $H \subseteq G$ be a subgroup. The *normalizer* of H in G is the subgroup $N_G(H) \subseteq G$ defined by

$$N_G(H) = \{ g \in G \mid gHg^{-1} = H \}$$

23.6 Proposition. Let G be a group, and let S is a transitive G-set. For any $s \in S$ there exists an isomorphism of groups

$$Iso_G(S) \cong N_G(G_s)/G_s$$

23.7 Proposition. Let X be a connected and locally path connected space, and let $x_0 \in X$. For a path connected covering $p \colon T \to X$ and $\tilde{x} \in p^{-1}(x_0)$ there exists an isomorphism of groups:

$$D(p) \cong N_{\pi_1(X,x_0)}(p_*(\pi_1(T,\tilde{x})))/p_*(\pi_1(T,\tilde{x}))$$