

**Recall:** The simplex method works under the following assumptions:

- 1) The program is in the equality form: we want to maximize

$$Z = c_1x_1 + \dots + c_nx_n$$

subject to the constraints

$$\begin{aligned} a_{11}x_1 + \dots + a_{1n}x_n &= b_1 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{m1}x_1 + \dots + a_{mn}x_n &= b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

- 2) The coefficient matrix

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

is in the basic form.

- 3)  $b_i \geq 0$  for  $i = 1, \dots, m$ .

- These assumptions assure that the program has a basic feasible solution.
- **Phase I of the simplex method** finds some basic feasible solution or verifies that no feasible solutions exists.

**Example.** Maximize

$$z = x_1 + 2x_2$$

subject to:

$$x_1 + 3x_2 + x_3 = 2$$

$$-x_2 - x_3 = -1$$

$$x_1, x_2, x_3 \geq 0$$

## Phase I of the simplex method

**Assumption:** The linear program is in the equality form with constraints

$$a_{i1}x_1 + \dots + a_{in}x_n = b_i$$

for  $i = 1, \dots, m$ .

1) Modify the constraints, if needed, so that  $b_i \geq 0$  for  $i = 1, \dots, m$ .

2) Add an additional variable  $s_i$  to the  $i$ -th constraint for  $i = 1, \dots, m$ .

**Note.** The augmented matrix of the new constraints is in the basic form.

3) Use the simplex method to minimize the function

$$Z = s_1 + \dots + s_m$$

with the new constraints. If the minimum is non-zero, then the original linear program has not feasible solutions.

If the minimum is  $z = 0$ , then the solution that gives the minimum has  $s_i = 0$  for  $i = 1, \dots, m$ . In such case, values of the variables  $x_1, \dots, x_n$  give a basic feasible solution of the original linear program.