The *simplex method* is one of the main methods of solving linear programs.

Special assumptions (for now):

1) The program is in the equality form: we want to maximize

$$z = c_1 x_1 + \ldots + c_n x_n$$

subject to the constraints:

$$\begin{array}{c} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{array}$$

$$x_1,\ x_2,\ \ldots, x_n \geq 0$$

2) The coefficient matrix

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$$

is in the basic form.

3) $b_i \geq 0$ for $i = 1, \ldots, m$.

Example. Maximize

$$z = 3x_1 + x_2$$

subject to:

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1 &\leq 3 \\ 2x_1 + x_2 &\leq 7 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Geometric interpretation of the simplex method

Recall: Maximize

$$z = 3x_1 + x_2$$

subject to:

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1 &\leq 3 \\ 2x_1 + x_2 &\leq 7 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$