Recall:

Definition

Let A be an $n \times n$ matrix. If $\mathbf{v} \in \mathbb{R}^n$ is a non-zero vector and λ is a scalar such that

$$A\mathbf{v} = \lambda\mathbf{v}$$

then we say that

- λ is an *eigenvalue* of A
- $\mathbf{v}$ is an *eigenvector* of A corresponding to λ.

Example.

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Note. Eigenvectors corresponding to a given eigenvalue λ form a subspace of $\mathbb{R}^n$ which is called the *eigenspace* corresponding to the eigenvalue λ.

Computation of eigenvalues

Notation. $I_n :=$ the $n \times n$ identity matrix.

Definition

If A is an $n \times n$ matrix then

$$P(\lambda) = \det(A - \lambda I_n)$$

is a polynomial of degree n. $P(\lambda)$ is called the *characteristic polynomial* of the matrix A.

Proposition

If A is a square matrix then

$$\text{eigenvalues of } A = \text{roots of } P(\lambda)$$

Example.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Computation of eigenvectors

Proposition

If λ is an eigenvalue of an $n \times n$ matrix A then

$$\left\{\begin{matrix}\text{eigenvectors of } A\\ \text{corresponding to } \lambda\end{matrix}\right\} = \left\{\begin{matrix}\text{vectors in}\\ \mathrm{Nul}(A - \lambda I_n)\end{matrix}\right\}$$

Example.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$