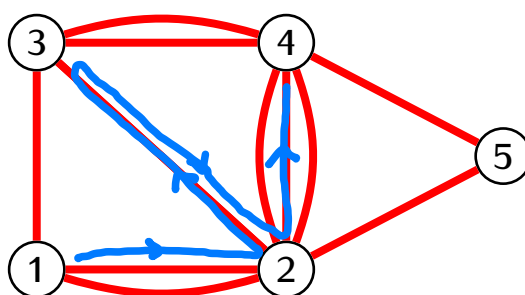


**Definition**

A *path* in a graph is a sequence of edges such that each edge ends at the vertex when the next edge begins.



**Example.** In the graph pictured above, how many paths of length 2 are there that start at the vertex 2 and end at the vertex 4?

Adjacency matrix:

$$\begin{array}{c}
 \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 3 & 1 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 3 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}
 \end{array}$$

← edges to vertex 4

↑ edges from vertex 2

$$\begin{array}{c}
 \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ 4 & \begin{bmatrix} 0 & 3 & 2 & 0 & 1 \end{bmatrix} \end{matrix} \cdot \begin{matrix} \begin{matrix} 2 \\ 2 \\ 1 \\ 3 \\ 1 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \end{matrix}
 \end{array}
 = \underbrace{0 \cdot 2}_{4 \leftarrow 1 \leftarrow 4} + \underbrace{3 \cdot 0}_{4 \leftarrow 2 \leftarrow 2} + \underbrace{2 \cdot 1}_{4 \leftarrow 3 \leftarrow 2} + \underbrace{0 \cdot 3}_{4 \leftarrow 4 \leftarrow 2} + \underbrace{1 \cdot 1}_{4 \leftarrow 5 \leftarrow 2}$$

↑ the entry in the 4<sup>th</sup> row, 2<sup>nd</sup> column of the matrix  $A \cdot A$

### Proposition

Let  $A$  be the the adjacency matrix of a graph.

The entry  $b_{ij}$  of the matrix  $A^2 = (b_{ij})$  gives the number of paths of length 2 that start at the vertex  $j$  and terminate at the vertex  $i$ .

In general, for any  $n \geq 1$  the entry  $c_{ij}$  of the matrix  $A^n = (c_{ij})$  gives the number of paths of length  $n$  that start at the vertex  $j$  and terminate at the vertex  $i$ .