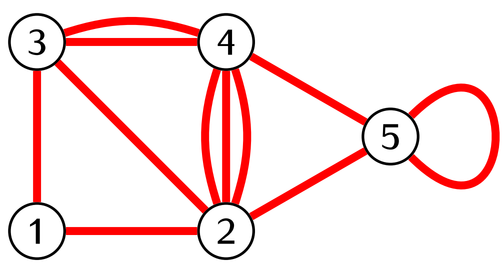
Recall:

Definition

A *graph* (or a *network*) is a pair $G = (V, E)$ where:

- V is the set of *vertices* (or *nodes*);
- E is the set of *edges*;
- each edge connects two vertices.

Note. We will usually denote vertices of a graph by positive integers: 1, 2, 3, etc.

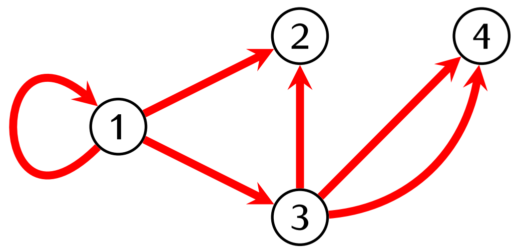


Examples:

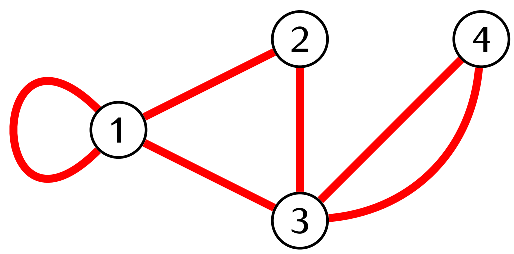
- computer networks
- social networks
- transportation networks
- citation networks
- ecological networks

Some types of graphs

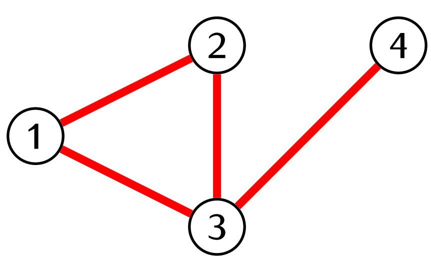
Directed graphs. Every edge has a direction pointing from one vertex to another.



Undirected graphs. Edges do not have a direction.



Simple graphs. There is at most one edge between any two vertices and there are no self-edges (i.e. edges that start and end in the same vertex).

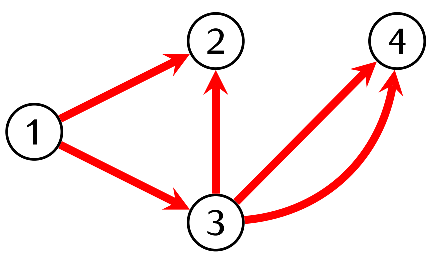


Definition

For a graph with vertices $1, 2, 3, \ldots, N$ the *adjacency matrix* of the graph is an $N \times N$ matrix $A = (a_{ij})$ such that

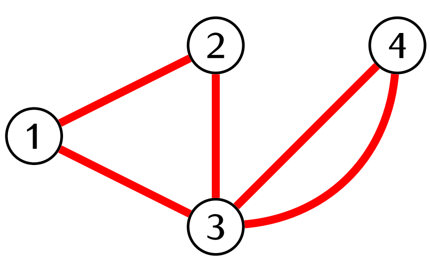
$$a_{ij} = (\text{the number of edges from } j \text{ to } i)$$

Example. Directed graph:



$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

Example. Undirected graph:



$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

Note: The adjacency matrix of an undirected graph is symmetric : $A = A^T$.