1. A company manufacturing widgets has the following demand for widgets for the first four months of the year:

	January	**February**	**March**	**April**
widgets ordered	500	1000	1600	1400

The company has to decide how many widgets it should produce each month.

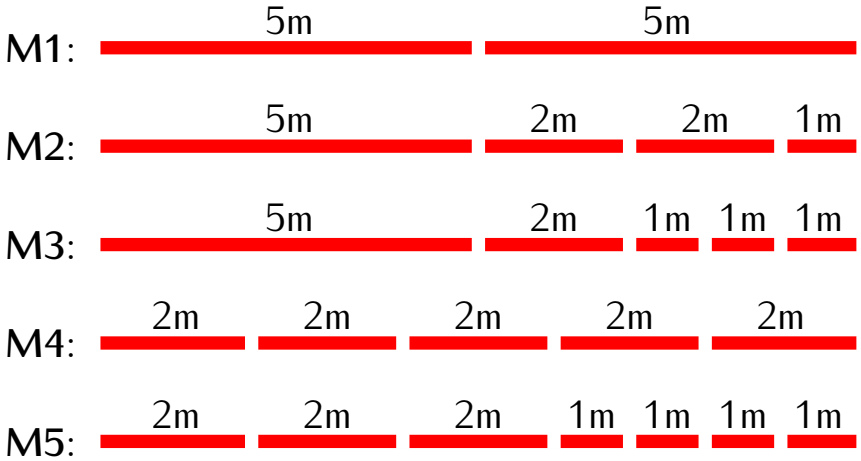
It costs $5 to produce each widget. The maximum production per month is 1200 widgets. Widgets produced one month can be delivered either that same month or stored in a warehouse and delivered some other month. It costs the company $3 to store one widget from one month to the next.

a) Formulate as a linear program the problem of determining how many widgets should be produced each month to meet the demand and minimize the total production and storage costs.

b) Assume that by working overtime the company can manufacture up to 400 additional widgets per month but it costs $10 to produce each of these additional widgets. Formulate a linear program that minimized the total production and storage costs in this case.

In each case explain what are the decision variables, the objective function, and the constraints.

2. A steel company manufactures 10 meter long metal rods. The rods are then cut into smaller pieces that are sold to customers. The company has five machines **M1** – **M5** that cut the rods as follows.



The company received an order for

- 175 rods of length 5m
- 100 rods of length 2m
- 225 rods of length 1m.

The company needs to plan how many 10 meter rods should be cut using each machine to fulfill this order.

Formulate a linear program that will minimize the total number of 10 meter rods that need to be cut. Explain what are the decision variables, the objective function, and the constraints.

3. Convert the following linear programming problem to the equality form:

Minimize

$$z = x_1 - x_2$$

subject to constraints

$$\begin{aligned} 2x_1 + x_2 &\geq 3 \\ 3x_1 - x_2 &\leq 7 \\ x_1 &\geq 0 \\ x_2 &\in \mathbb{R} \end{aligned}$$

4. For the following linear programming problem, plot the feasible region and use it to solve the problem.

Find the maximum of the function

$$z = 5x_1 + 7x_2$$

subject to constraints

$$\begin{aligned} 2x_1 + 3x_2 &\geq 6 \\ 3x_1 - x_2 &\leq 15 \\ -x_1 + x_2 &\leq 4 \\ 2x_1 + 5x_2 &\leq 27 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

5. Solve problem 4 using Python and `scipy.optimize.linprog` function. Submit a printout or a screenshot showing your code and the code output.