Integer programs are the same as as a linear program except we want solutions that are integers.

The general form of an integer program

Find the minimum (or the maximum) of the objective function

$$z = c_1x_1 + \ldots + c_nx_n$$

subject to constraints:

$$a_{i1}x_1 + \ldots + a_{in}x_n \begin{matrix} \leq \\ = \\ \geq \end{matrix} b_i$$

for $i = 1, \ldots, m$, where $x_j \in \mathbb{Z}$ for $j = 1, \ldots, n$.

Note. Integer programs are much more difficult to solve that linear programs. In general, there is no efficient method for solving them.

LP Relaxation: Given an integer program solve it using the simplex method. Possibilities:

- The solution consists of integers and so gives a solution of the integer program.
- The solution does not consist of integers but gives a useful estimate for the integer program.
- The solution does not consist of integers and is not a useful for the integer program – other methods are needed (usually involving solving more linear programs).

Example. Why rounding solutions of a linear program to integers is not enough:

Maximize

$$z = 5x_1 + 8x_2$$

subject to:

$$\begin{aligned} x_1 + x_1 &\leq 6 \\ 5x_1 + 9x_2 &\leq 45 \\ x_1,\ x_2 &\geq 0 \\ x_1, x_2 &\in \mathbb{Z} \end{aligned}$$