<u>Recall:</u> The general form of a linear program

For the objective variables $x_1, \ldots, x_n$ find the minimum (or the maximum) of the objective function

$$z = c_1 x_1 + \ldots + c_n x_n$$

subject to the constraints:

$$a_{i1} x_1 + \ldots + a_{in} x_n \begin{array}{c} \leq \\ = \\ \geq \end{array} b_i$$

for $i = 1, \ldots, m$, and possibly $x_j \geq 0$ for $j = 1, \ldots, n$.

The <u>*equality* (or *standard*) form of a linear program:</u>

- we are looking for the maximum;
- all constraints are of the form

$$a_{i1} x_1 + \ldots + a_{in} x_n = b_i$$

- we require that $x_j \geq 0$ for $j = 1, \ldots, n$.

Every linear program can be converted to the equality form.

- finding minimum of $z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

  $\|$

  finding maximum of $z = -(c_1 x_1 + c_2 x_2 + \ldots + c_n x_n)$

  $\qquad\qquad\qquad\quad = -c_1 x_1 - c_2 x_2 - \ldots - c_n x_n$

- if we have a constraint of the form

  $$a_{i1} x_1 + \ldots + a_{in} x_n \leq b_i$$

  then we can replace it by

  $$a_{i1} x_1 + \ldots + a_{in} x_n + s_i = b_i$$

  where $s_i \geq 0$ is a new <u>slack variable</u>.

- a constraint of the form

  $$a_{i1} x_1 + \ldots + a_{in} x_n \geq b_i$$

  is equivalent to

  $$a_{i1} x_1 + \ldots + a_{in} x_n - s_i = b_i$$

  where $s_i \geq 0$.

- if x_j is an unrestricted variable (i.e. x_j can be any real number) then we can replace it by $x_j = x_j^+ - x_j^-$ where x_j^+, x_j^- are new variables such that $x_j^+, x_j^- \geq 0$.

Example. Convert the following linear program to the equality form.

Minimize the function
$$z = 6x_1 - 10x_2$$

subject to the constraints:
$$5x_1 + 7x_2 \leq 8$$
$$4x_1 + 2x_2 \geq 10$$
$$x_1 \geq 0$$
$$x_2 \in \mathbb{R}$$

Solution:

Maximize

$$z = -6x_1 + 10x_2$$

Constraints:

$$5x_1 + 7x_2 + s_1 = 8$$
$$4x_1 + 2x_2 - s_2 = 10$$

$$x_1 \geq 0$$
$$s_1 \geq 0$$
$$s_2 \geq 0$$
$$x_2 \in \mathbb{R}$$

$\xrightarrow{}$

$x_2 = x_2^+ - x_2^-$

Maximize:

$$z = -6x_1 + 10x_2^+ - 10x_2^- + 0s_1 + 0s_2$$

$$5x_1 + 7x_2^+ - 7x_2^- + s_1 = 8$$
$$4x_1 + 2x_2^+ - 2x_2^- - s_2 = 10$$

$$x_1 \geq 0$$
$$x_2^+ \geq 0$$
$$x_2^- \geq 0$$
$$s_1 \geq 0$$
$$s_2 \geq 0$$

the equality form

The *inequality* form of a linear program:

- we are looking for the maximum;
- all constraints are of the form

$$a_{i1}x_1 + \ldots + a_{in}x_n \leq b_i$$

- we require that $x_j \geq 0$ for $j = 1, \ldots, n$.

Fact

Every linear program can be converted to the inequality form.

- if we have a constraint of the form

$$a_{i1}x_1 + \ldots + a_{in}x_n = b_i$$

  then we can replace it by two constraints

$$a_{i1}x_1 + \ldots + a_{in}x_n \leq b_i$$
$$a_{i1}x_1 + \ldots + a_{in}x_n \geq b_i$$

Linear programs with Python:

`scipy.optimize.linprog` (looks for the minimum of the objective function).