Linear programs with Python:

`scipy.optimize.linprog` (looks for the minimum of the objective function).

Cases when a solution of a linear program may not exist:
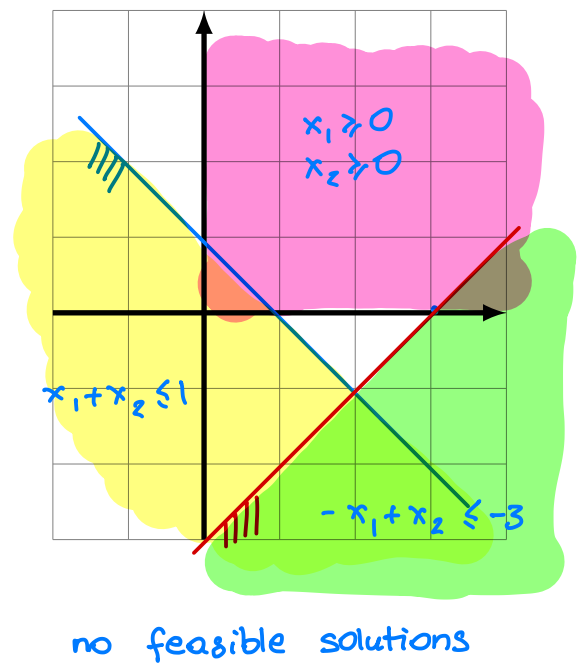
Infeasibilty: There are no feasible solutions.

Example. Maximize $z = 2x_1 + x_2$ subject to

$$x_1 + x_2 \leq 1$$
$$-x_1 + x_2 \leq -3$$
$$x_1,\ x_2 \geq 0$$

$x_1 \geq 0$
$x_2 \geq 0$

$x_1 + x_2 \leq 1$

$-x_1 + x_2 \leq -3$

no feasible solutions

Unboundedness: The objective function has no minimum (or maximum) in the feasible region.
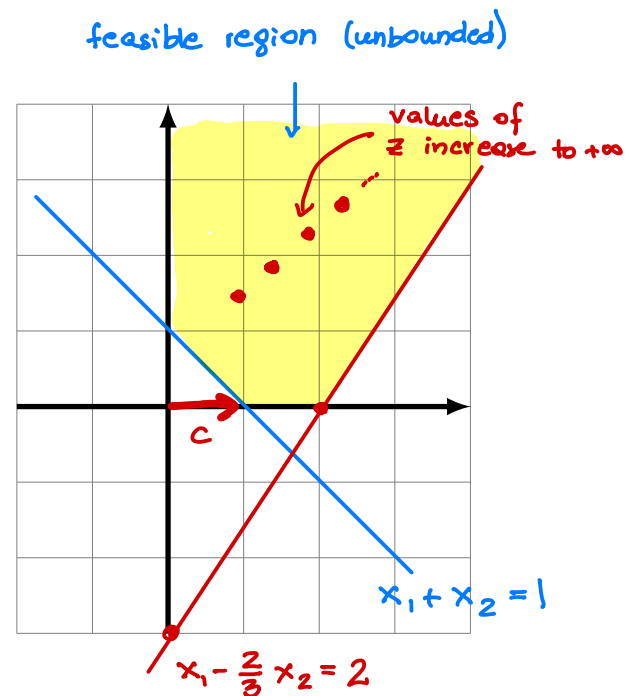
$$c \cdot x$$
$$z = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Example. Maximize $z = x_1 + 0x_2$ subject to

$$x_1 - \tfrac{2}{3}x_2 \leq 2$$
$$x_1 + x_2 \geq 1$$
$$x_1, \ x_2 \geq 0$$

feasible region (unbounded)

values of z increase to $+\infty$

c

$x_1 + x_2 = 1$

$x_1 - \tfrac{2}{3}x_2 = 2$

Note. Even when the feasible region is unbounded the objective function may have a maximum or a minimum in this region.