

1. We have the following table with 5 columns of numbers:

C_1	C_2	C_3	C_4	C_5
4	7	11	9	5
12	2	10	11	6
8	12	6	11	5
2	12	8	7	10
10	1	1	10	9
6	11	2	3	12
5	13	13	6	12

We want to permute the columns of this table so that the sum of entries in the highlighted spots is the largest possible:

C_{i_1}	C_{i_2}	C_{i_3}	C_{i_4}	C_{i_5}

Explain how to represent this problem as an assignment problem. In particular, write the score table for the assignment problem, explain what each row and each columns represents and how you computed the scores in the table. Then use Python (with the code provided for this homework) to solve this assignment problem. You don't need to include Python computations in your solution.

Write the solution of the assignment problem (i.e. which order of columns gives the maximum sum and what is this maximum sum).

2. TrueBlue Airlines operate flights between BUF airport in Buffalo and MIA airport in Miami, FL. The daily flight schedule is as follows:

Flights from BUF to MIA:

Flight	Depart BUF	Arrive MIA
101	5 AM	8 AM
102	8 AM	11 AM
103	9 AM	12 PM
104	1 PM	4 PM
105	4 PM	7 PM
106	12 AM	3 AM

Flights from MIA to BUF:

Flight	Depart MIA	Arrive BUF
201	3 AM	6 AM
202	10 AM	1 PM
203	11 AM	2 PM
204	2 PM	5 PM
205	7 PM	10 PM
206	11 PM	2 AM

The airline needs to schedule crews for these flights. Each crew will be based either in Buffalo or Miami. A single crew will operate a pair of flights, flying from their home city to the other airport and then flying back home.

The crews need to be scheduled so that the total time they spend waiting for their return flights is the smallest possible. However, each crew must have a layover between flights of at least 2 hours.

Set up an assignment problem that will determine:

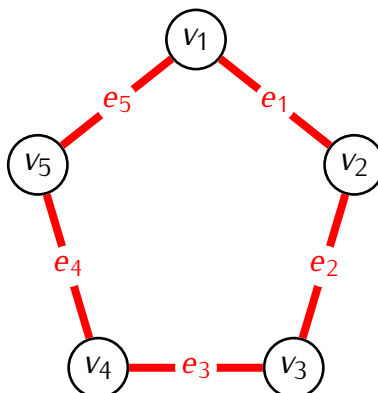
- which pairs of flights should be operated by the same crew
- in which city (Buffalo or Miami) the crew operating a given pair of flights will be based.

Write the score table for this assignment problem, explain what each row and each columns of the table represents and how you computed the scores in the table.

Then use Python (with the code provided for this homework) to solve this assignment problem. You don't need to include Python computations in your solution.

Write the solution of this problem that lists pairs of flight operated by a single crew, the home city of each crew, and the total time the crews will wait for the return flights.

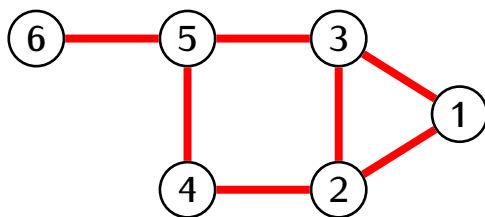
3. For $n \geq 3$, the n -cycle graph C_n is a graph with vertices v_1, v_2, \dots, v_n , such that there is a single edge e_i connecting v_i with v_{i+1} for $i = 1, \dots, n-1$ and an edge e_n connecting v_n with v_1 . For example, here is a picture of the graph C_5 :



Let A_n be the incidence matrix of the graph C_n . Compute $\det A_n$. Explain your computations.

Note. The value of $\det A_n$ will depend on n . Also, the sign of the determinant will depend on the ordering of vertices and edges used to write the incidence matrix.

4. For a graph G , an independent set of G is a set of vertices S such that no two vertices in S are connected by an edge. For example, in the graph below the sets $\{5, 2\}$ and $\{6, 4, 1\}$ are examples of independent sets.

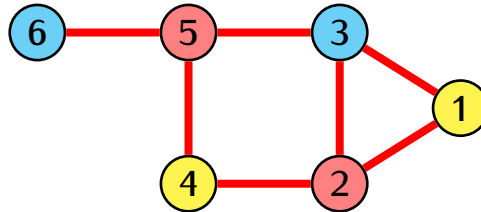


Write an integer program whose solution determines the biggest possible independent set of a given graph G (i.e. an independent set with the largest possible number of vertices).

Explain your reasoning: what is the meaning of the decision variables, of the objective function and the constraints.

5. For a graph G , a graph coloring is an assignment that associates to each vertex a label (color) in such way, that two vertices that are connected by an edge have different colors.

A graph with n vertices can be colored using n colors by assigning a different color to each vertex. However, usually fewer colors are needed. For example, the graph below has 6 vertices but can be colored using 3 colors:



Write an integer program that for a given graph G finds a coloring of the graph that uses the smallest possible number of colors. Explain your reasoning: what is the meaning of the decision variables, of the objective function, and the constraints.

Hint. Start with the assumption that the graph has n vertices and that there are n colors available: c_1, \dots, c_n . The integer program should minimize the number of colors selected from among c_1, \dots, c_n that are used to color the graph.