**Example.** Maximize

$$z = 3x_1 + x_2$$

subject to:

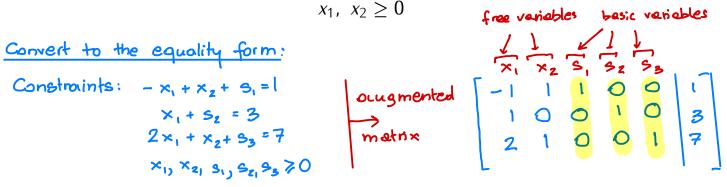
$$-x_1 + x_2 \le 1$$

$$x_1 \le 3$$

$$2x_1 + x_2 \le 7$$

$$x_1, x_2 > 0$$

×1, ×2, 3, 5, 5, 9, 7



We want to maximize

$$z = 3 \times_1 + \times_2 + \frac{0}{9} + \frac{0}{9} + \frac{0}{9} + \frac{0}{9}$$

Coefficients of basic variables are all 0.

Note: By selling the free variables to O we get a basic fearible solution:

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ y_1 = 1 \\ y_2 = 3 \\ y_2 = 7 \end{cases}$$
 this gives:  $z = 0$ 

Goal: Look for other basic feasible solutions that make the value of z larger.

Simplex tableau

## The current basic feasible solution:

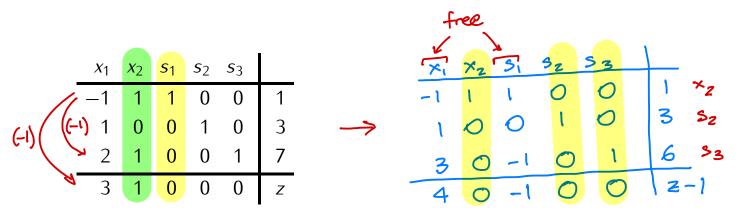
free 
$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ S_1 = 1 \\ S_2 = 3 \\ S_3 = 7 \end{cases} \mapsto Z = 0$$

## The pivot step:

- i) We can increase the value of z by increasing either  $x_1$  or  $x_2$ . Let's try to increase  $x_2$ , keep  $x_1 = 0$ .
- 2) Change of  $x_z$  will affect the values of the bosic variables  $s_1$ ,  $s_{z_1}$   $s_3$ . These variables must stay >0, which restricts possible values of  $x_z$ .

Since by assumption x1=0 we have:

All these conditions are satisfied if  $x_2 \le 1$ . For the biggest increase of z we set  $x_2 = 1$  which makes  $s_1 = 0$ . Since we mant free variables to have value 0, and basic variables to have non-zero values, we want to make  $x_2$  into a basic variable and make  $s_1$  free,



The new basic feasible solution

free 
$$\begin{cases} \begin{cases} x_1 = 0 \\ s_1 = 0 \end{cases} \\ \Rightarrow z = 1 \end{cases} \Rightarrow \begin{cases} z = 1 \\ z = 3 \end{cases}$$
 for  $z = 1$ 

The pivot step:

The free variables are x, and s.

Increasing s, will decrease 2,

but increasing x, will increase Z.

Thus we will increase x, while keeping  $x_2$ ,  $s_2$ ,  $s_3 > 0$ . (Note: we keep  $s_1 = 0$ )

$$- \times_{1} + \times_{2} = 1 \Rightarrow \times_{2} = 1 + \times_{1} \geqslant 0 - \text{no restriction on } \times_{1} \times_{1} + S_{2} = 3 \Rightarrow S_{2} = 3 - \times_{1} \geqslant 0 = 0$$
:  $\times_{1} + S_{3} = 6 \Rightarrow S_{3} = 6 - 3 \times_{1} \geqslant 0 = 0 = 3 \times_{1} \leqslant 6$ 

$$\times_{1} \leq 2$$

The biggest value of  $x_1$  satisfying all conditions is  $x_1 = 2$ Then  $s_3 = 0$ , so  $s_3$  becomes free and  $x_1$  basic.

<i>X</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>S</i> <sub>1</sub>	<b>s</b> <sub>2</sub>	<b>s</b> <sub>3</sub>	
<u>-1</u>	1	1	0	0	1
1	0	0	1	0	3
3	0	-1	0	1	1 3 6
4	0	-1	0	0	z — 1

<i>X</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>s</i> <sub>1</sub>	<b>s</b> <sub>2</sub>	<b>s</b> <sub>3</sub>	
0	1	<u>2</u> 3	0	<u>1</u>	3
0	0	$\frac{1}{3}$	1	$-\frac{1}{3}$	1
1	0	$-\frac{1}{3}$	0	1 3	2
0	0	<u>1</u>	0	$-\frac{4}{3}$	z – 9

## Geometric interpretation of the simplex method

Recall: Maximize

$$z = 3x_1 + x_2$$

subject to:

$$-x_1 + x_2 \le 1$$

$$x_1 \le 3$$

$$2x_1 + x_2 \le 7$$

$$x_1, x_2 \ge 0$$

