Example. A company needs to hire people for 5 different positions $P_1, \ldots, P_5$. There are 7 candidates $C_1, \ldots, C_7$ who interviewed for these positions. The table below shows the interview score (higher is better) how each person is qualified for each position. Blank entries indicate the score of 0 (i.e. a candidate is either not suitable or not interested in the corresponding position).

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_1	70	90		75	55		60
P_2	40	95	85			80	
P_3	50		75		70		65
P_4			60	80		35	
P_5		75		70		35	20

Which candidate should be offered which position so that the sum of scores of the assignment is the largest possible?

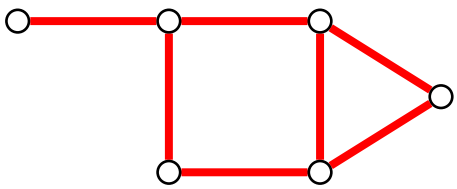
Goal: All basic feasible solutions of the assignment problem consist of integers.

Definition

A *graph* (or a *network*) is a pair $G = (V, E)$ where:

- V is the set of *vertices* (or *nodes*);
- E is the set of *edges*;
- each edge connects two vertices.

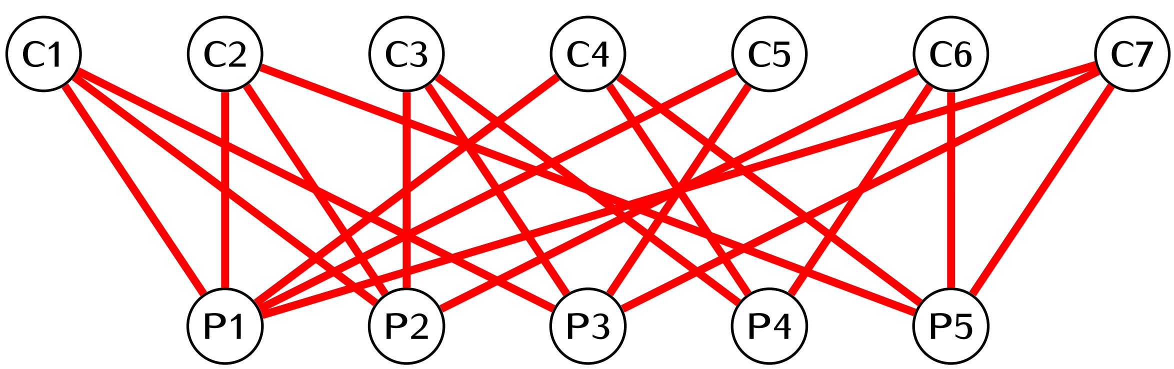
Example.



Definition

A *bipartite graph* is a graph $G = (V, E)$ such the set of nodes is a union of two disjoint subsets $V = V_1 \cup V_2$ and that every edge connects some node in V_1 with some node in V_2.

Example. Bipartite graph for the assignment problem:

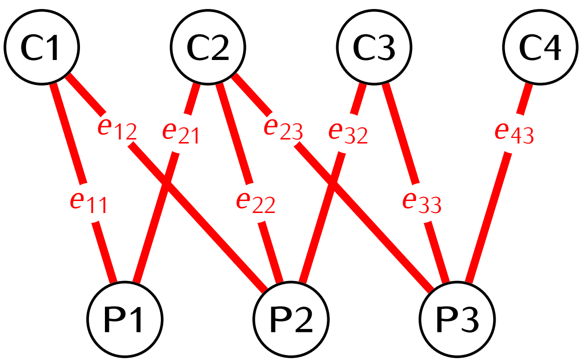


Definition

The *edge incidence matrix* of a graph $G = (V, E)$ is a matrix A such that:

- rows of A are labeled by vertices of G
- columns of A are labeled by edges of G
- the entry in the row of a vertex $\mathbf{v}$ and the column of an edge $\mathbf{e}$ is 1 if the edge $\mathbf{e}$ is attached to $\mathbf{v}$; otherwise it is 0.

Example.



$$
\begin{array}{c@{}c}
 & \begin{array}{cccccccc} e_{11} & e_{12} & e_{21} & e_{22} & e_{23} & e_{32} & e_{33} & e_{43} \end{array} \\
\begin{array}{c} \mathrm{C1} \\ \mathrm{C2} \\ \mathrm{C3} \\ \mathrm{C4} \\ \mathrm{P1} \\ \mathrm{P1} \\ \mathrm{P3} \end{array} &
\left[\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right]
\end{array}
$$