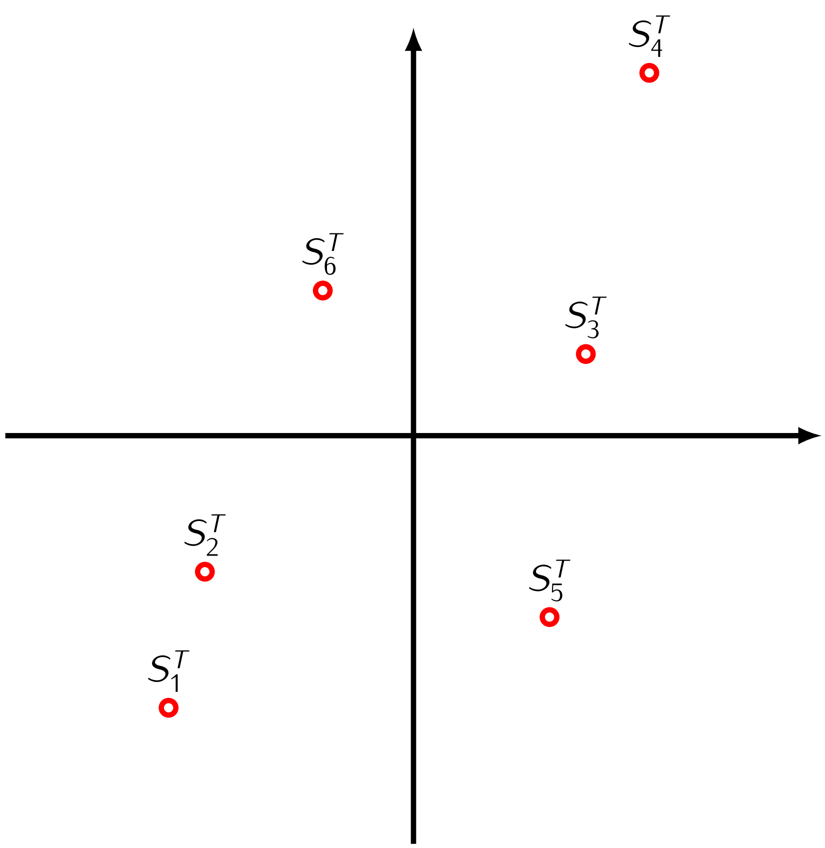
Example.

Demeaned data matrix:

$$A = \begin{array}{r} \\ \text{Aly} \\ \text{Bob} \\ \text{Chen} \\ \text{Deb} \\ \text{Emma} \\ \text{Finn} \end{array} \begin{array}{c} \begin{array}{cc} \text{Ex 1} & \text{Ex 2} \end{array} \\ \begin{bmatrix} -27 & -30 \\ -23 & -15 \\ 19 & 6 \\ 26 & 40 \\ 15 & -20 \\ -10 & 16 \end{bmatrix} \end{array} = \begin{bmatrix} \boxed{X_1} & \boxed{X_2} \end{bmatrix} = \begin{bmatrix} \boxed{S_1} \\ \boxed{S_2} \\ \boxed{S_3} \\ \boxed{S_4} \\ \boxed{S_5} \\ \boxed{S_6} \end{bmatrix}$$



Recall:

- The 1st principal axis of A is a vector $\mathbf{u}_1$ such that $\|\mathbf{u}_1\| = 1$ and $\mathrm{Var}(A\mathbf{u}_1)$ is the largest possible.

- The vector $Y_1 = A\mathbf{u}_1$ is called the 1st principal component of A.

Proposition

Let A an $N \times M$ demeaned data matrix

$$A = \begin{bmatrix} \boxed{\quad S_1 \quad} \\ \boxed{\quad S_2 \quad} \\ \vdots \\ \boxed{\quad S_N \quad} \end{bmatrix}$$

• The 1^{st} principal axis of A is the vector $\mathbf{u}_1 \in \mathbb{R}^M$ such that $||\mathbf{u}_1|| = 1$ and the number

$$\sum_{i=1}^{N} ||S_i^T - \text{proj}_{\mathbf{u}_1} S_i^T||^2$$

is the smallest possible.

• The 1^{st} principal component of A is the vector

$$Y_1 = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix}$$

such that $\text{proj}_{\mathbf{u}_1} S_i = c_i \mathbf{u}_1$ for $i = 1, \ldots, N$.