Definition

A complex number is a number of the form

$$z = a + bi$$

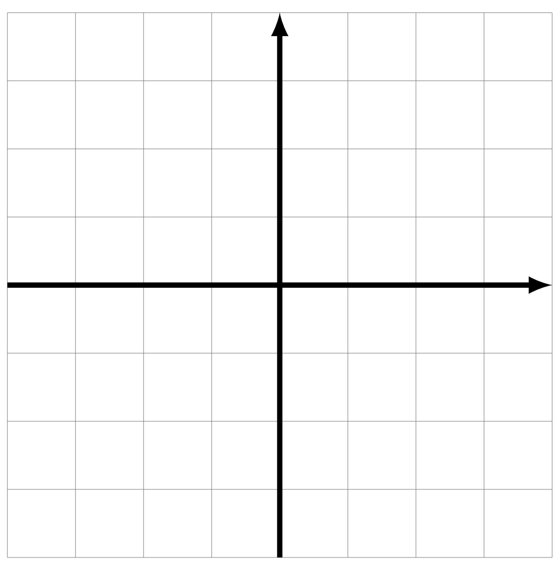
where

- $a, b \in \mathbb{R}$
- i is an *imaginary unit* which satisfies $i^2 = -1$.

Notation. If $z = a + bi$ is a complex number then

- $\mathrm{Re}(z) = a$ is called the *real part* of z
- $\mathrm{Im}(z) = b$ is called the *imaginary part* of z
- $\mathbb{C}$ denotes the set of all complex numbers.

Geometric interpretation of complex numbers



Note. Every real number is a complex number.

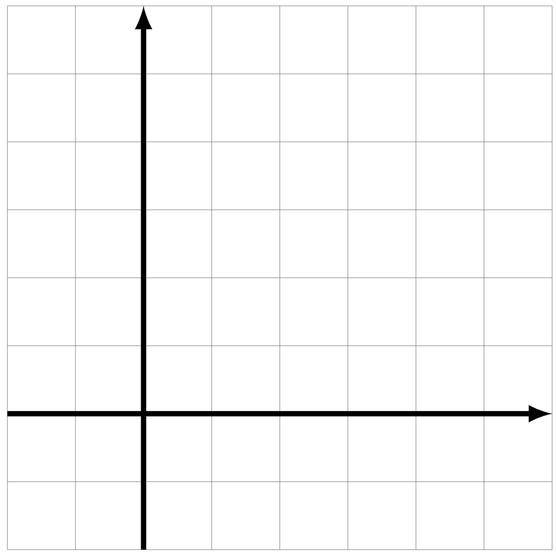
Addition, subtraction, multiplication

Cojugate of complex number and division

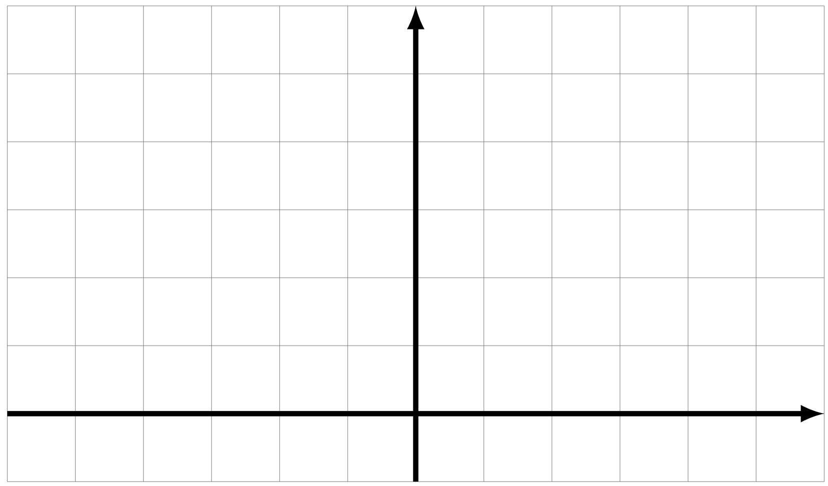
Definition

The *conjugate* of a complex number $z = a + b\mathrm{i}$ is the number $\bar{z} = a - b\mathrm{i}$.

Polar form of a complex number



Geometric interpretation of multiplication



Powers and roots of complex numbers

The Fundamental Theorem of Algebra

Let $P(x)$ be a polynomial of degree $n \geq 1$ with complex coefficient:

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

$a_n, \ldots, a_0 \in \mathbb{C}$. Then there exist complex numbers $z_1, \ldots, z_n$ such that

$$P(x) = a_n (x - z_1) \cdot \ldots \cdot (x - z_n)$$

As a consequence $P(z_i) = 0$ for $i = 1, \ldots, n$.

Example.

$P(x) = x^2 - 4x + 13$