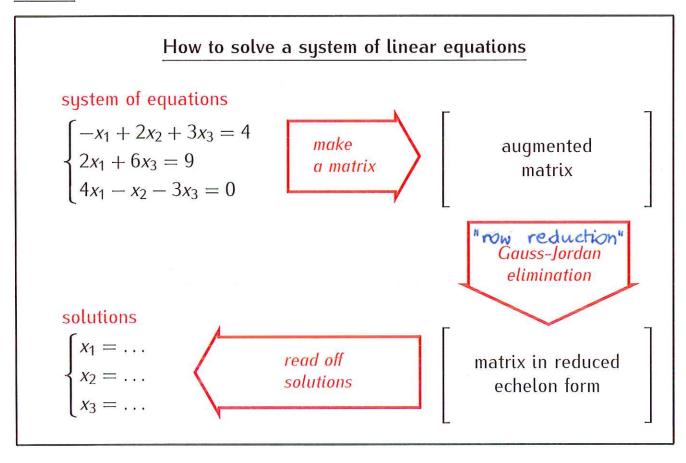
Recall:

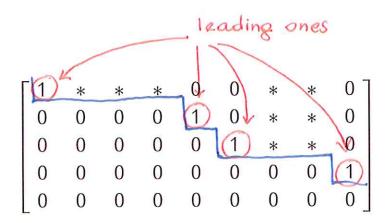


- Every system of linear equations can be represented by a matrix
- Elementary row operations:
 - interchange of two rows
 - multiplication of a row by a non-zero number
 - addition of a multiple of one row to another row.
- Elementary row operations do not change solutions of systems of linear equations.

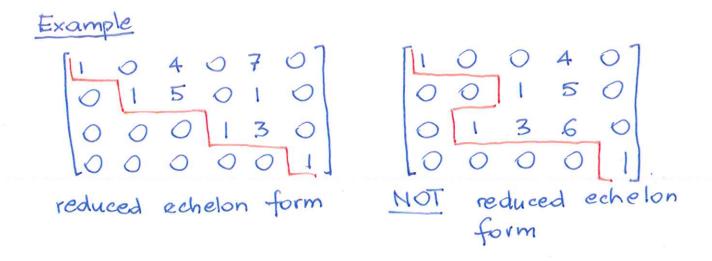
Definition

A matrix is in the reduced echelon form if:

- the non-zero entry of each row is a 1 ("a leading one");
- the leading one in each row is to the right of the leading one in the row above it;
- all entries above each leading one are 0.



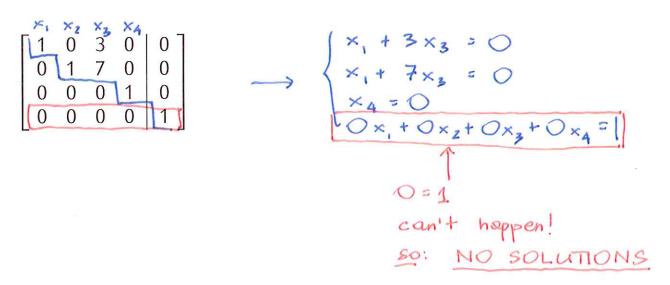
(* = any number)



Fact

If a system of linear equations is represented by a matrix in the reduced echelon form then it is easy to solve the system.

Example

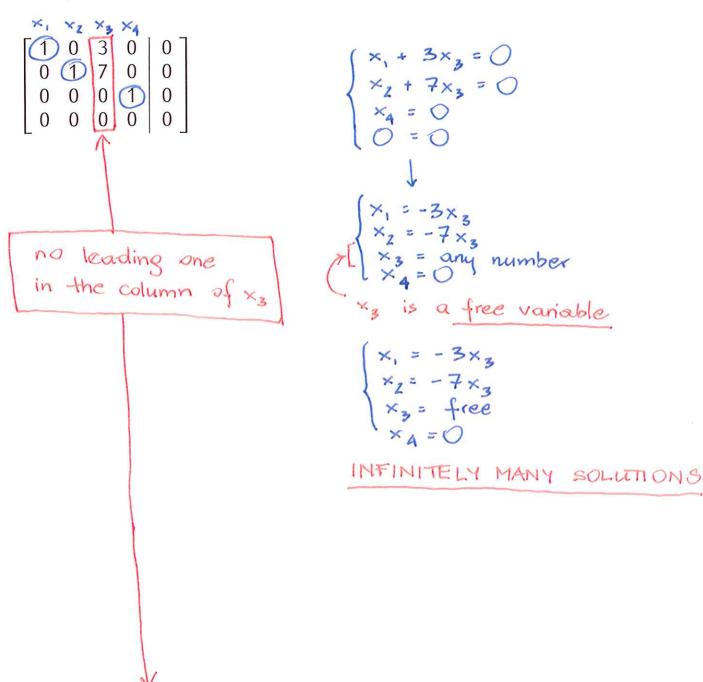


Proposition

A matrix in the reduced echelon form represents an inconsistent system if and only if it contains a row of the form

i.e. with the leading one in the last column.

Example



Note

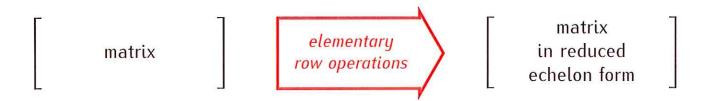
In an augmented matrix in the reduced echelon form free variables correspond to columns of the coefficient matrix that do not contain leading ones.

Example

Note

A matrix in the reduced echelon represents a system of equations with exactly one solution if and only if it has a leading one in every column except for the last one.

Gauss-Jordan elimination process (= row reduction)



- 1 Interchange rows, if necessary, to bring a non-zero element to the top of the first non-zero column of the matrix.
- (2) Multiply the first row so that its first non-zero entry becomes 1.
- 3 Add multiples of the first row to eliminate non-zero entries below the leading one.
- \bigcirc Ignore the first row; apply steps 1-3 to the rest of the matrix.
- (5) Eliminate non-zero entries above all leading ones.

Example.

$$\begin{bmatrix} 0 & 4 & -8 & 0 & 4 \\ 2 & 6 & -6 & -2 & -4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix}$$

$$\downarrow$$

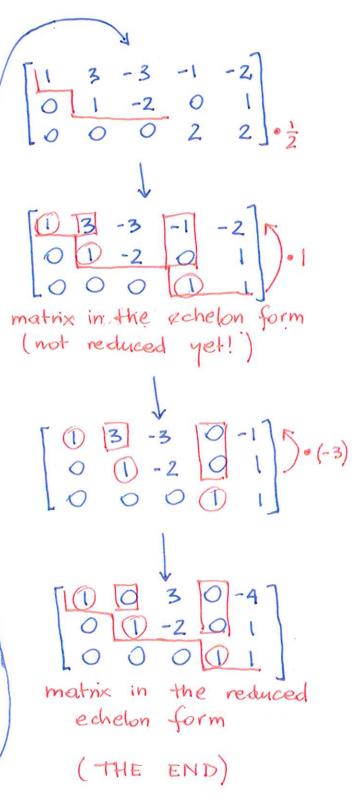
$$\begin{bmatrix} 2 & 6 & -6 & -2 & -4 \\ 0 & 4 & -8 & 0 & 4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 4 & -8 & 0 & 4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix} \cdot (-2)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 4 & -8 & 0 & 4 \\ 0 & 1 & -2 & 2 & 3 \end{bmatrix} \cdot (-1)$$



How to solve systems of linear equations: example

$$\begin{cases} 4x_2 - 8x_3 = 4 \\ 2x_1 + 6x_2 - 6x_3 - 2x_4 = -4 \\ 2x_1 + 7x_2 - 8x_3 = -1 \end{cases}$$

$$\begin{cases} x_1 \times x_2 \times x_3 \times x_4 \\ 2 \times x_1 + 6 \times x_2 - 6 \times x_3 - 2 \times x_4 = -4 \\ 2 \times x_1 + 7 \times x_2 - 8 \times x_3 = -1 \end{cases}$$

$$\begin{cases} x_1 \times x_2 \times x_3 \times x_4 \\ x_2 \times x_3 \times x_4 + x_4 = -1 \end{cases}$$

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$$\begin{cases} x_1 \times x_2 \times x_3 \times x_4 \\ x_3 \times x_4 + x_4 = -1 \end{cases}$$

$$\begin{cases} x_1 \times x_2 \times x_3 \times x_4 \\ x_4 \times x_4 = -1 \end{cases}$$

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