Next:

How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

make a matrix →

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$ augmented matrix

Gauss-Jordan elimination

solutions

$$\begin{cases} x_1 = \ldots \\ x_2 = \ldots \\ x_3 = \ldots \end{cases}$$

← *read off solutions*

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$ matrix in reduced echelon form

Matrices

matrix = rectangular array of numbers

3 columns

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Big\} \; 2 \text{ rows}$$

2×3 matrix

3 columns

$$\begin{bmatrix} 1 & 2 & 0 \\ 7 & -5 & 1 \\ 8 & 10 & 7 \\ 6 & 4 & 3 \end{bmatrix} \Big\} \; 4 \text{ rows}$$

4×3 matrix

Note

Every system of linear equations can be represented by a matrix.

Example.

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & 3 & 4 \\ 2 & 0 & 6 & 9 \\ 4 & -1 & 3 & 0 \end{array} \right]$$

the coefficient matrix   the column of constants

the augmented matrix of the system of equations

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Elementary row operations:

1) Interchange of two rows.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 4 & 3 & 0 & 7 \\ 0 & 1 & 5 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

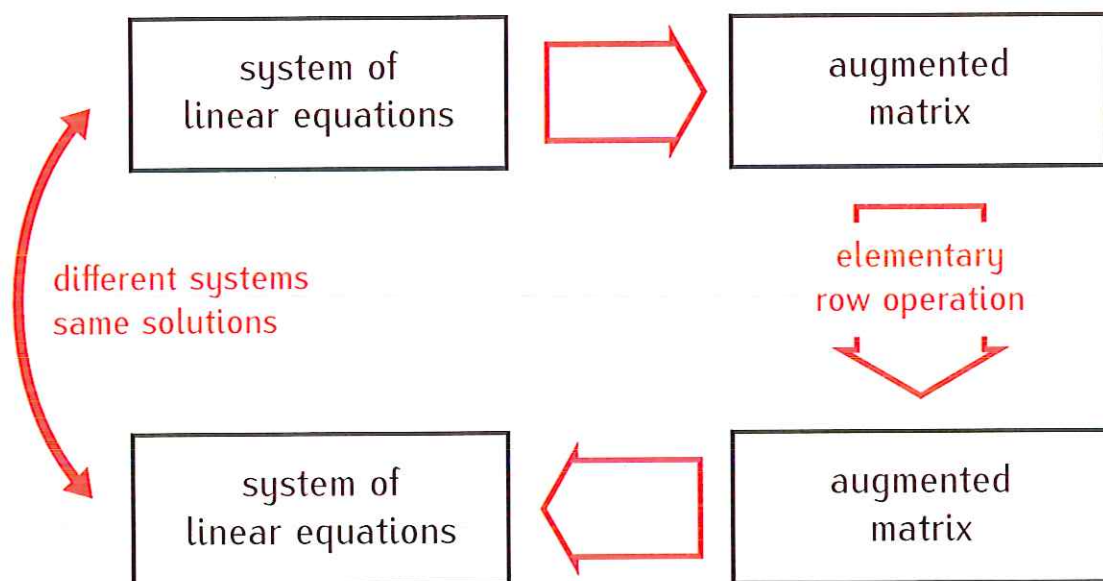## 2) Multiplication of a row by a non-zero number.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \cdot 2 \longrightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 8 & 6 & 0 & 14 \end{bmatrix}$$

3) Addition of a multiple of one row to another row.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \cdot 3 \longrightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 7 & 14 & 13 \\ 4 & 3 & 0 & 7 \end{bmatrix}$$

Elementary row operations do not change solutions of the system of equations represented by a matrix.

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┌──────────────┐          ┌──────────────┐
│  system of   │   ═══▶   │  augmented   │
│linear equations│        │   matrix     │
└──────────────┘          └──────────────┘
                                 │
        different systems    elementary
        same solutions       row operation
                                 ▼
┌──────────────┐          ┌──────────────┐
│  system of   │   ◀═══   │  augmented   │
│linear equations│        │   matrix     │
└──────────────┘          └──────────────┘
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$$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 - x_2 + 5x_3 = 0 \\ 3x_1 + 6x_2 - x_3 = 1 \end{cases} \longrightarrow \begin{bmatrix} 1 & 2 & 3 & | & 4 \\ 2 & -1 & 5 & | & 0 \\ 3 & 6 & -1 & | & 1 \end{bmatrix}$$

$$\begin{cases} 3x_1 + 6x_2 - x_3 = 1 \\ 2x_1 - x_2 + 5x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 4 \end{cases} \longleftarrow \begin{bmatrix} 3 & 6 & -1 & | & 1 \\ 2 & -1 & 5 & | & 0 \\ 1 & 2 & 3 & | & 4 \end{bmatrix}$$

(interchange of rows) = (interchange of equations)

(multipl. of a row) = (multipl. of an equation)

$\begin{pmatrix} \text{addition of a multiple} \\ \text{of one row to another} \end{pmatrix}$ = $\begin{pmatrix} \text{addition of a multiple} \\ \text{of one equation to another} \end{pmatrix}$