What we want:

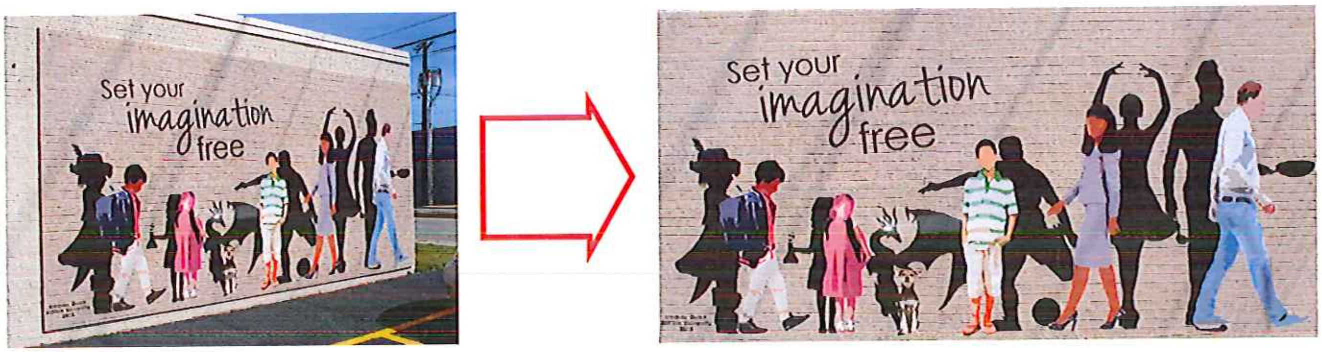


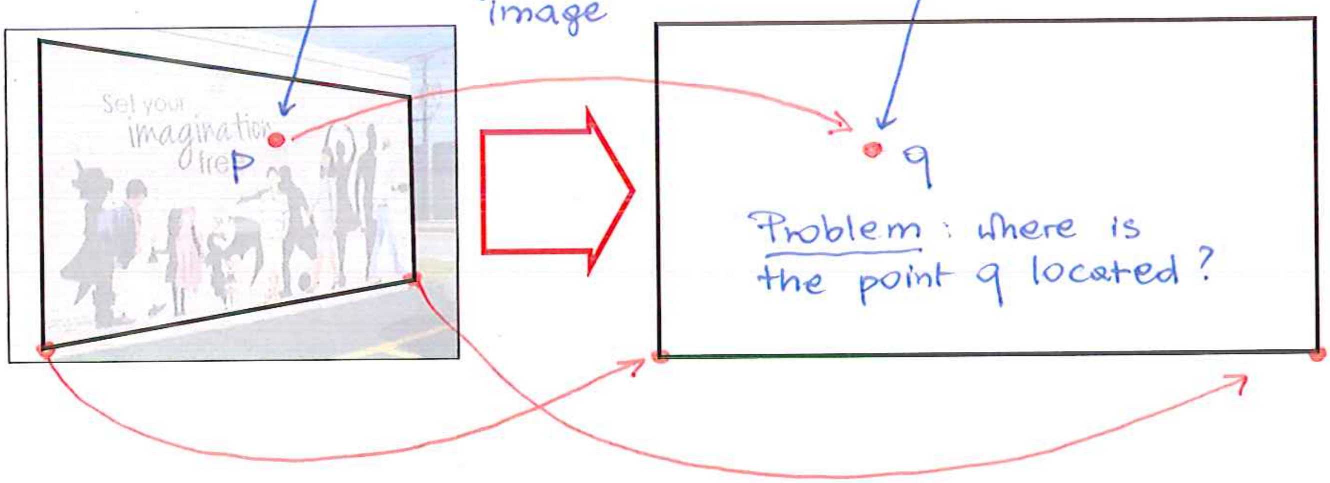
photo taken at an angle

straightened image

What we have:

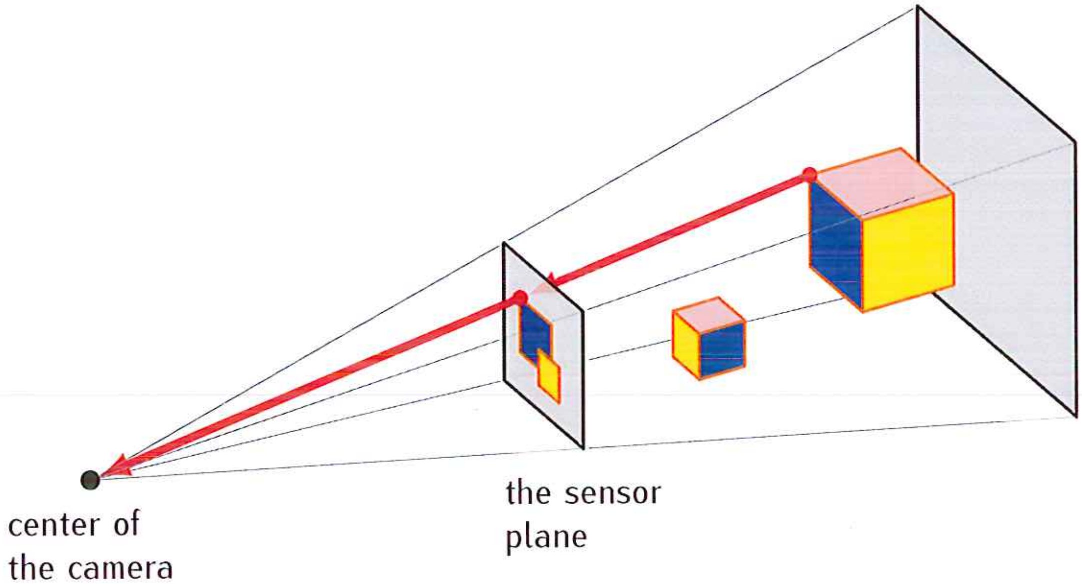
point of the original image

the corresponding point of the straightened image



Problem: where is the point q located?

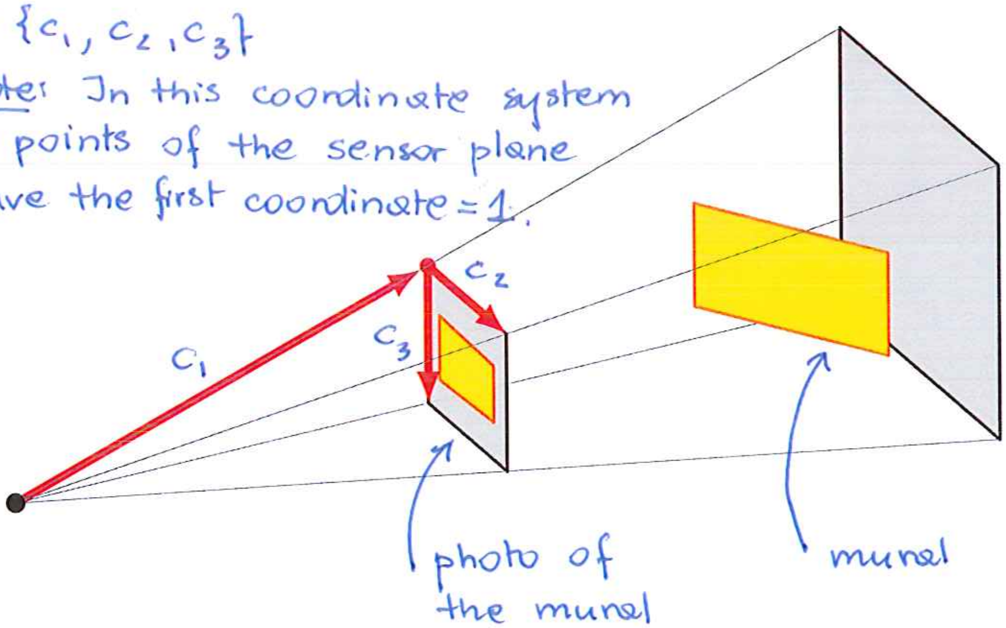
Image formation in a camera

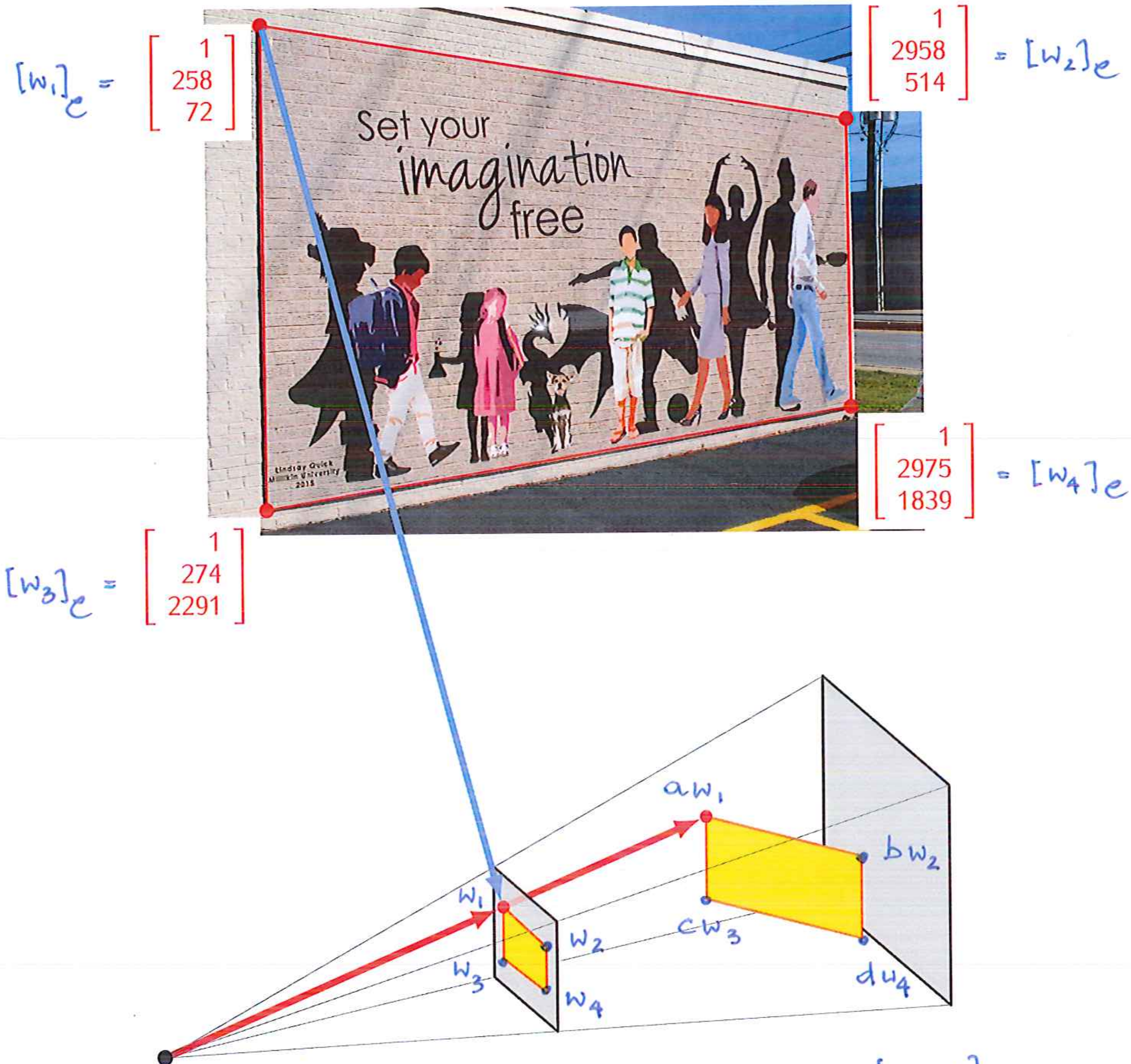


The camera coordinate system $\mathcal{C}$

$\mathcal{C} = \{c_1, c_2, c_3\}$

Note: In this coordinate system all points of the sensor plane have the first coordinate $= 1$.





$$[aw_1]_e = a\cdot[w_1]_e = a\cdot\begin{bmatrix}1\\258\\72\end{bmatrix}$$

$$[bw_2]_e = b[w_2]_e = \dots$$

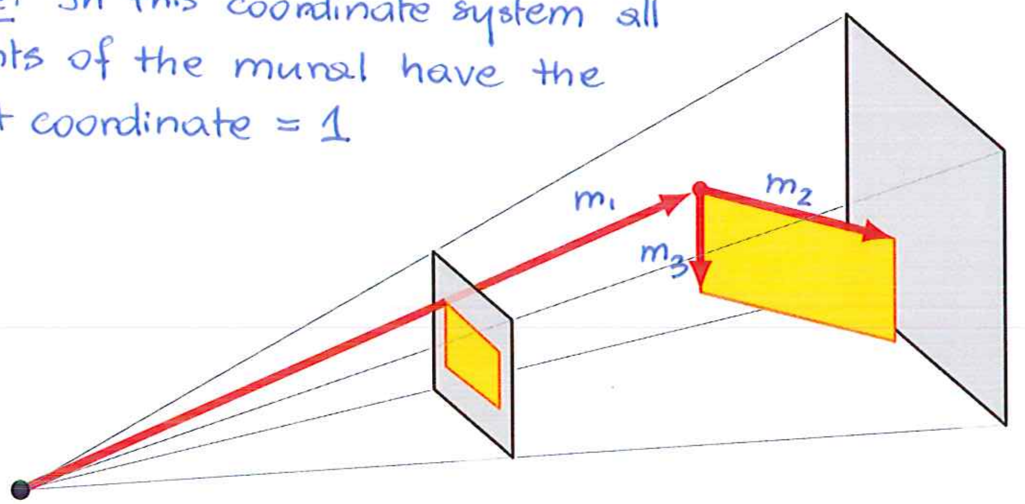
$$[cw_3]_e = c\cdot[w_3]_e = \dots$$

$$[dw_4]_e = d\cdot[w_4]_e = \dots$$

The mural coordinate system $\mathcal{M}$

$\mathcal{M} = \{m_1, m_2, m_3\}$

Note: In this coordinate system all points of the mural have the first coordinate $= 1$



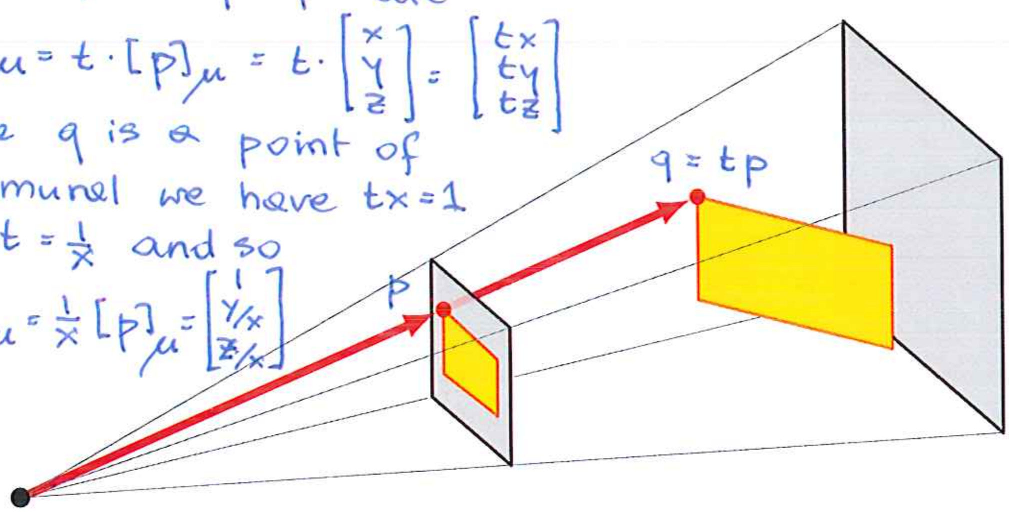
Note: If the mural coordinates of a point p are $[p]_\mathcal{M} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then the mural coordinates of the point $q = tp$ are

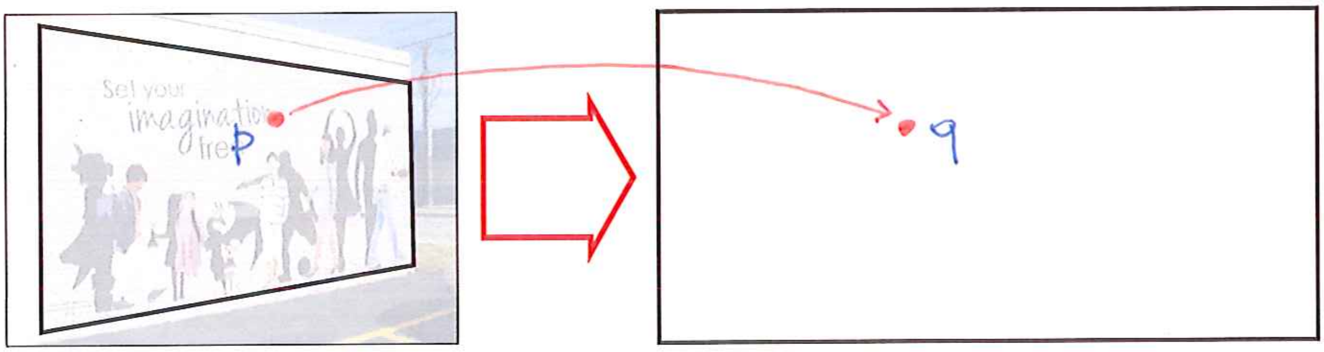
$$[q]_\mathcal{M} = t \cdot [p]_\mathcal{M} = t \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix}$$

Since q is a point of the mural we have $tx = 1$

So: $t = \frac{1}{x}$ and so

$$[q]_\mathcal{M} = \frac{1}{x}[p]_\mathcal{M} = \begin{bmatrix} 1 \\ y/x \\ z/x \end{bmatrix}$$





Upshot:

We know: $[p]_e$ = the camera coordinates of p

We want: $[q]_\mu$ = the mural coordinates of q

Strategy: Compute $[p]_\mu$ = the mural coordinates of p.

Then, if $[p]_\mu = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then $[q]_\mu = \begin{bmatrix} 1 \\ y/x \\ z/x \end{bmatrix}$

Note: 1) $[p]_\mu = \underbrace{(P_{\mu \leftarrow e})}_{\text{the change of coordinates matrix from } e \text{ to } \mu} \cdot [p]_e$

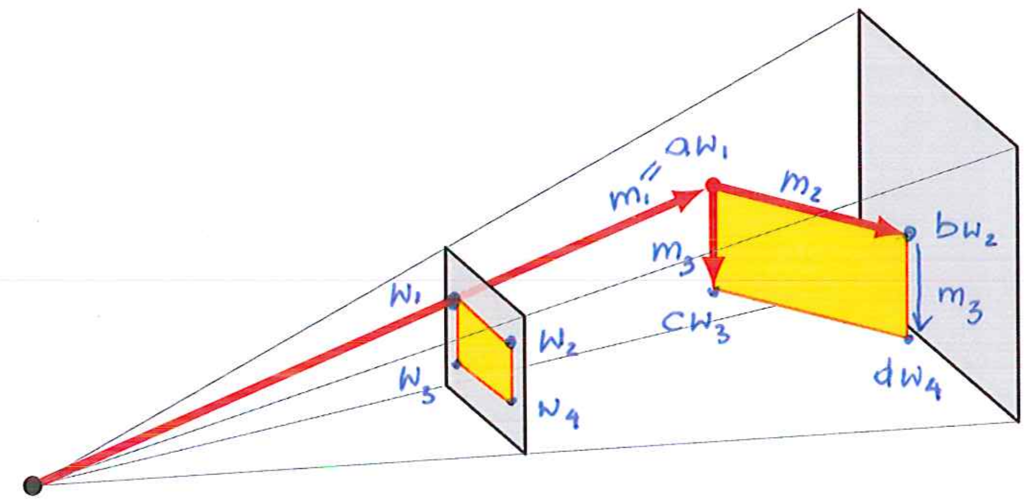
so we just need to compute $P_{\mu \leftarrow e}$

2) It will suffice to compute $P_{e \leftarrow \mu}$

since $P_{\mu \leftarrow e} = (P_{e \leftarrow \mu})^{-1}$

<u>From mural coordinates to camera coordinates</u>

$$P_{\mathcal{C}\leftarrow\mathcal{M}} = \left[\ [\mathbf{m}_1]_{\mathcal{C}}\ \ [\mathbf{m}_2]_{\mathcal{C}}\ \ [\mathbf{m}_3]_{\mathcal{C}}\ \right]$$



We have:

$$\boxed{m_1 = aw_1}$$

$m_1 + m_2 = bw_2$ so: $m_2 = bw_2 - m_1$

$$\boxed{m_2 = bw_2 - aw_1}$$

$m_1 + m_3 = cw_3$ so: $m_3 = cw_3 - m_1$

$$\boxed{m_3 = cw_3 - aw_1}$$

This gives

$$[m_1]_e = a\cdot[w_1]_e = a\cdot\begin{bmatrix}1\\258\\72\end{bmatrix}$$

$$[m_2]_e = b[w_2]_e - a[w_1]_e = b\cdot\begin{bmatrix}1\\2958\\514\end{bmatrix} - a\begin{bmatrix}1\\258\\72\end{bmatrix}$$

$$[m_3]_e = c[w_3] - a[w_1]_e = c\cdot\begin{bmatrix}1\\274\\2291\end{bmatrix} - a\begin{bmatrix}1\\258\\72\end{bmatrix}$$

Problem: What are the numbers a, b, c?

Note: $m_1 + m_2 + m_3 = dw_4$

This gives:

$$(\cancel{aw_1}) + (bw_2 - \cancel{aw_1}) + (cw_3 - aw_1) = dw_4$$

$$\boxed{bw_2 + cw_3 - aw_1 = dw_4}$$

So:

$$b[w_2]_\mu + c[w_3]_\mu - a[w_1]_\mu = d[w_4]_\mu$$

$$\boxed{b \cdot \begin{bmatrix} 1 \\ 2958 \\ 514 \end{bmatrix} + c \begin{bmatrix} 1 \\ 274 \\ 2291 \end{bmatrix} - a \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix} = d \begin{bmatrix} 1 \\ 2975 \\ 1839 \end{bmatrix}}$$

Problem: 3 equations, 4 unknowns, so we can't have a unique solution for a, b, c, d.

Good news:

1) For our computations the value of d does not matter. We can set it to any non-zero number (e.g. $d = 1$)

2) Once the value of d is fixed, the values of a, b, c are uniquely determined. This lets us compute $[m_1]_e, [m_2]_e, [m_3]_e$ and so we obtain the matrix $P_{e \leftarrow \mu}$.