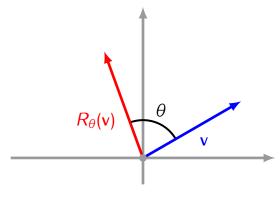
Problem: How to recognize if a function $f: \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation?

Example. Rotation by an angle θ :



 $R_{\theta} \colon \mathbb{R}^2 \to \mathbb{R}^2$

Definition

A function $T: \mathbb{R}^n \to \mathbb{R}^m$ is a *linear transformation* if it satisfies the following conditions:

- 1) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$
- 2) $T(c\mathbf{v}) = cT(\mathbf{v})$ for any $\mathbf{v} \in \mathbb{R}^n$ and any scalar c.

Proposition

Every matrix transformation is a linear transformation.

Theorem

Every linear transformation $T \colon \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation:

$$T = T_A$$

for some matrix A.

Corollary

If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation then $T = T_A$ where A is the matrix given by

$$A = [T(\mathbf{e}_1) \ T(\mathbf{e}_2) \ \dots \ T(\mathbf{e}_n)]$$

This matrix is called the *standard matrix* of T.

Example. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be the function given by

$$T\left(\left[\begin{array}{c} x_1 \\ x_2 \end{array}\right]\right) = \left[\begin{array}{c} x_1 + x_2 \\ 0 \\ 2x_1 \end{array}\right]$$

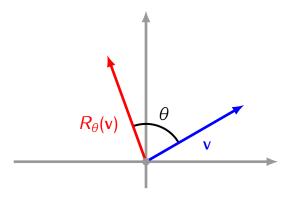
Check if T is a linear transformation. If it is, find its standard matrix.

Example. Let $S: \mathbb{R}^2 \to \mathbb{R}^3$ be the function given by

$$S\left(\left[\begin{array}{c} x_1 \\ x_2 \end{array}\right]\right) = \left[\begin{array}{c} 1 + x_2 \\ x_2 \\ 3x_1 \end{array}\right]$$

Check if S is a linear transformation. If it is, find its standard matrix.

Back to rotations:



 $R_{\theta} \colon \mathbb{R}^2 \to \mathbb{R}^2$

