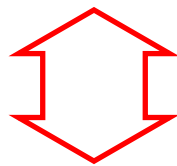


So far:

$$\begin{cases} 2x_1 + 4x_2 + 6x_3 + 3x_4 = 7 \\ 3x_1 + 2x_2 + 2x_3 + 9x_4 = 3 \\ 5x_1 + 8x_2 + 3x_3 + 3x_4 = 9 \end{cases}$$

system of
linear equations



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 2 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 9 \end{bmatrix}$$

vector equation

Next:

$$\begin{bmatrix} 2 & 4 & 6 & 3 \\ 3 & 2 & 2 & 9 \\ 5 & 8 & 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 9 \end{bmatrix}$$

matrix equation

Definition

Let A be an $m \times n$ matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ and let \mathbf{w} be a vector in \mathbb{R}^n :

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_n \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

The product $A\mathbf{w}$ is a vector in \mathbb{R}^m given by

$$A\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_n\mathbf{v}_n$$

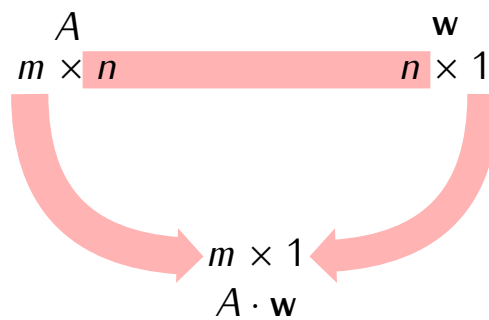
Example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

Properties of matrix-vector multiplication

1) The product Aw is defined only if

(number of columns of A) = (number of entries of w)



2) $A(v + w) = Av + Aw$

3) If c is a scalar then $A(cw) = c(Aw)$.

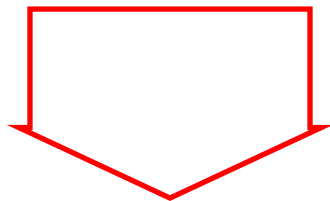
Example. Solve the matrix equation

$$\begin{bmatrix} 1 & 1 & -4 \\ 1 & -2 & 3 \\ 3 & -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

How to solve a matrix equation

$$A\mathbf{x} = \mathbf{b}$$

matrix equation



$$\left[\begin{array}{c|c} A & \mathbf{b} \end{array} \right]$$

augmented matrix



$$\left[\text{reduced matrix} \right]$$



$$\mathbf{x} = \dots$$

solutions