$$A = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{km} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$$

$$AB = \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & & \vdots \\ c_{k1} & \dots & c_{km} \end{bmatrix}$$

$$\underbrace{c_{ij}}_{\substack{\text{the entry in} \\ i^{th} \text{ row} \\ j^{th} \text{ column}}} = \underbrace{\begin{bmatrix} a_{i1} & a_{i2} & a_{im} \end{bmatrix}}_{i^{th} \text{ row of } A} \cdot \underbrace{\begin{bmatrix} b_{1j} \\ b_{1j} \\ \vdots \\ b_{1j} \end{bmatrix}}_{j^{th} \text{ column of } B} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{im}b_{mj}$$

Example.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ 4 & 5 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

2×3 $\qquad$ 3×4

AB is defined and it is a 2×4 matrix

$$AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}$$

$$c_{11} = [1^{st} \text{ row of } A] \cdot \begin{bmatrix} 1^{st} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [0\ 1\ 2] \cdot \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} = 0 \cdot 0 + 1 \cdot 4 + 2 \cdot 1 = \underline{6}$$

$$c_{12} = [1^{nd} \text{ row of } A] \cdot \begin{bmatrix} 2^{nd} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [0\ 1\ 2] \cdot \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} = 0 \cdot (-1) + 1 \cdot 5 + 2 \cdot 2 = \underline{9}$$

$\vdots$

$$c_{24} = [2^{nd} \text{ row of } A] \cdot \begin{bmatrix} 4^{th} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [3\ 4\ 5] \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 3 \cdot 1 + 4 \cdot 0 + 5 \cdot 1 = \underline{8}$$

$$AB = \begin{bmatrix} 6 & 9 & 7 & 2 \\ 21 & 27 & 25 & 8 \end{bmatrix}$$

Example.

- Acme Inc. makes two types of widgets: **WG1** and **WG2**.
- Each widget must go though two processes: **assembly** and **testing**.
- The number of hour required to complete each process is as follows:

$$A = \begin{array}{c|cc} & \text{assembly} & \text{testing} \\ \hline \begin{array}{c}\text{WG1} \\ \text{WG}\cancel{1}\,2\end{array} & \multicolumn{2}{c}{\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}} \end{array}$$

- Acme Inc. has three plans in New York, Texas, and Minnesota.
- Hourly cost (in dollars) of each process in each plant is as follows:

$$B = \begin{array}{c|ccc} & \text{NY} & \text{TX} & \text{MN} \\ \hline \begin{array}{r}\text{assembly} \\ \text{testing}\end{array} & \multicolumn{3}{c}{\begin{bmatrix} 10 & 15 & 12 \\ 15 & 20 & 15 \end{bmatrix}} \end{array}$$

<u>**Problem.**</u> What is the cost of producing each type of widgets in each plant?

cost of WG1 in TX: $\underbrace{3\cdot 15 + 1\cdot 20}_{[4\ \ 1]\cdot\begin{bmatrix}10\\15\end{bmatrix}} = \$65$

$[4\ \ 1]\cdot\begin{bmatrix}10\\15\end{bmatrix} = [1^{st}\text{ row } A]\cdot\begin{bmatrix}2^{nd}\\\text{column}\\B\end{bmatrix}$

cost of WG2 in NY $= \underbrace{7\cdot 10 + 3\cdot 15}_{[7\ \ 3]\cdot\begin{bmatrix}10\\15\end{bmatrix}} = \$115$

$[7\ \ 3]\cdot\begin{bmatrix}10\\15\end{bmatrix} = [2^{nd}\text{ row } A]\cdot\begin{bmatrix}1^{st}\\\text{column}\\B\end{bmatrix}$

$$A\cdot B = \begin{array}{c|ccc} & \text{NY} & \text{TX} & \text{MN} \\ \begin{array}{c}\text{WG1} \\ \text{WG2}\end{array} & \multicolumn{3}{c}{\begin{bmatrix} 45 & 65 & 51 \\ 115 & 165 & 129 \end{bmatrix}} \end{array}$$