

$$\begin{array}{c}
 \text{pivot positions} \\
 \left[\begin{array}{cccc|c}
 \textcircled{0} & 4 & -8 & 0 & 4 \\
 2 & \textcircled{6} & -6 & -2 & -4 \\
 2 & 7 & -8 & \textcircled{0} & -1
 \end{array} \right]
 \end{array}
 \xrightarrow{\text{row reduction}}
 \begin{array}{c}
 \text{leading ones} \\
 \left[\begin{array}{cccc|c}
 \textcircled{1} & 0 & 3 & 0 & -4 \\
 0 & \textcircled{1} & -2 & 0 & 1 \\
 0 & 0 & 0 & \textcircled{1} & 1
 \end{array} \right]
 \end{array}$$

pivot columns

Definition

A *pivot position* in a matrix is a position that after the row reduction contains a leading one.

A *pivot column* of a matrix is a column that contains a pivot position.

Theorem

- 1) A system of linear equations is inconsistent if and only if the last column of its augmented matrix is a pivot column.
- 2) Free variables of the system correspond to non-pivot columns of the coefficient matrix.
- 3) The system has only one solution if and only if every column of its augmented matrix is a pivot column, except for the last column.

Theorem

A system of linear equations can have either 0, 1, or infinitely many solutions.

Proof.

