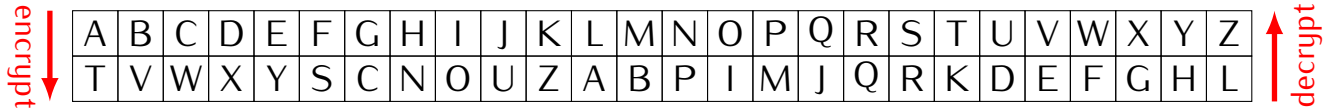
Substitution cipher: Replace each letter of the alphabet by some other letter.

Example.



encryption/decryption key

message: TOP SECRET

Hill cipher: Use matrix multiplication

Example.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 2 & 0 & -1 \end{bmatrix}$$

encryption key
invertible matrix

decryption key
matrix inverse

message: TOP SECRET

Encryption:

1) Replace letters by numbers:

_	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

2) Since the key is a 3×3 matrix split the number sequence numbers in vectors with 3 entries each.

3) Multiply each vector by the encryption matrix A.

4) Write the new vectors as a sequence of numbers.

We can do better, but the next part will not work with an arbitrary invertible matrix A. It will work though e.g. if all entries of A and A^{-1} are integers.

5) Reduce all numbers obtained in step 4 modulo 27. That is, add or subtract from each number a multiple of 27 to get a number between 0 and 26.

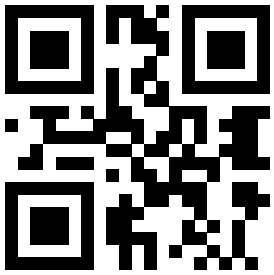
6) Replace numbers by letters.

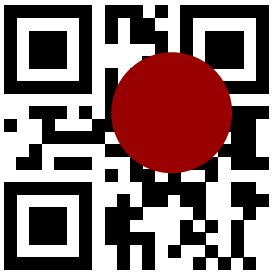
Decryption.

1) Replace letters by numbers, split into vectors, and multiply each vector by A^{-1}

2) Write the new vectors as a sequence of numbers, reduce each number modulo 27.

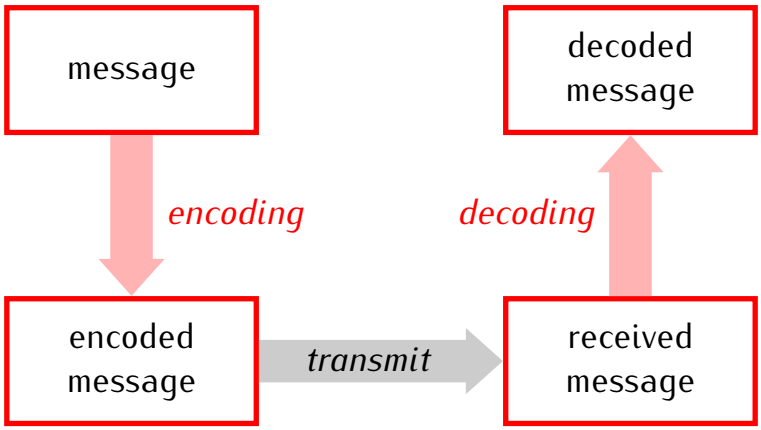
3) Replace numbers by letters





Problem: How to detect and correct transmission errors?

Basic scheme of error correction



Working assumption for this lecture: We expect at most one transmission error in any message up to 20 bits long.