lf

$$\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

are vectors in \mathbb{R}^n then the *inner product* (or *dot product*) of \mathbf{u} and \mathbf{v} is the number

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + \ldots + a_n b_n$$

Properties of the dot product:

1)
$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$

2)
$$(u + v) \cdot w = u \cdot w + v \cdot w$$

3)
$$(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$$

4)
$$\mathbf{u} \cdot \mathbf{u} \geq 0$$
 and $\mathbf{u} \cdot \mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$.

If $\mathbf{u} \in \mathbb{R}^n$ then the *length* (or the *norm*) of \mathbf{u} is the number

$$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$

Note. If
$$\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$
 then $||\mathbf{u}|| = \sqrt{a_1^2 + \ldots + a_n^2}$.

Properties of the norm:

1)
$$||u|| \ge 0$$
 and $||u|| = 0$ if and only if $u = 0$.

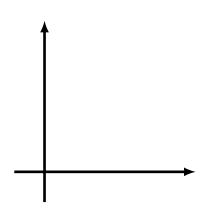
2)
$$||cu|| = |c| \cdot ||u||$$

A vector $\mathbf{u} \in \mathbb{R}^n$ is an *unit vector* if $||\mathbf{u}|| = 1$.

Definition

If $\mathbf{u},\mathbf{v} \in \mathbb{R}^n$ then the *distance* between \mathbf{u} and \mathbf{v} is the number

$$dist(\mathbf{u}, \mathbf{v}) = ||\mathbf{u} - \mathbf{v}||$$



Note. If
$$\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$$
, $\mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ then

dist(u, v) =
$$\sqrt{(a_1 - b_1)^2 + \ldots + (a_n - b_n)^2}$$

Vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.

Pythagorean Theorem

Vectors \mathbf{u}, \mathbf{v} are orthogonal if and only if

$$||u||^2 + ||v||^2 = ||u + v||^2$$