

How to solve systems of linear equations: example

$$\begin{cases} 4x_2 - 8x_3 = 4 \\ 2x_1 + 6x_2 - 6x_3 - 2x_4 = -4 \\ 2x_1 + 7x_2 - 8x_3 = -1 \end{cases}$$

$$\left[\begin{array}{cccc|c} 0 & 4 & -8 & 0 & 4 \\ 2 & 6 & -6 & -2 & -4 \\ 2 & 7 & -8 & 0 & -1 \end{array} \right] \xrightarrow{\text{row reduction}} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

Definition

A *pivot position* in a matrix is a position that after the row reduction contains a leading one.

A *pivot column* of a matrix is a column that contains a pivot position.

Theorem

- 1) A system of linear equations is inconsistent if and only if the last column of its augmented matrix is a pivot column.
- 2) Free variables of the system correspond to non-pivot columns of the coefficient matrix.
- 3) The system has only one solution if and only if every column of its augmented matrix is a pivot column, except for the last column.