> **Definition**
>
> A *column vector* is a matrix with one column.

Note. Columns of a matrix are column vectors.

> **Notation**
>
> $\mathbb{R}^n$ is the set of all column vectors with n entries.

Operations on vectors in $\mathbb{R}^n$
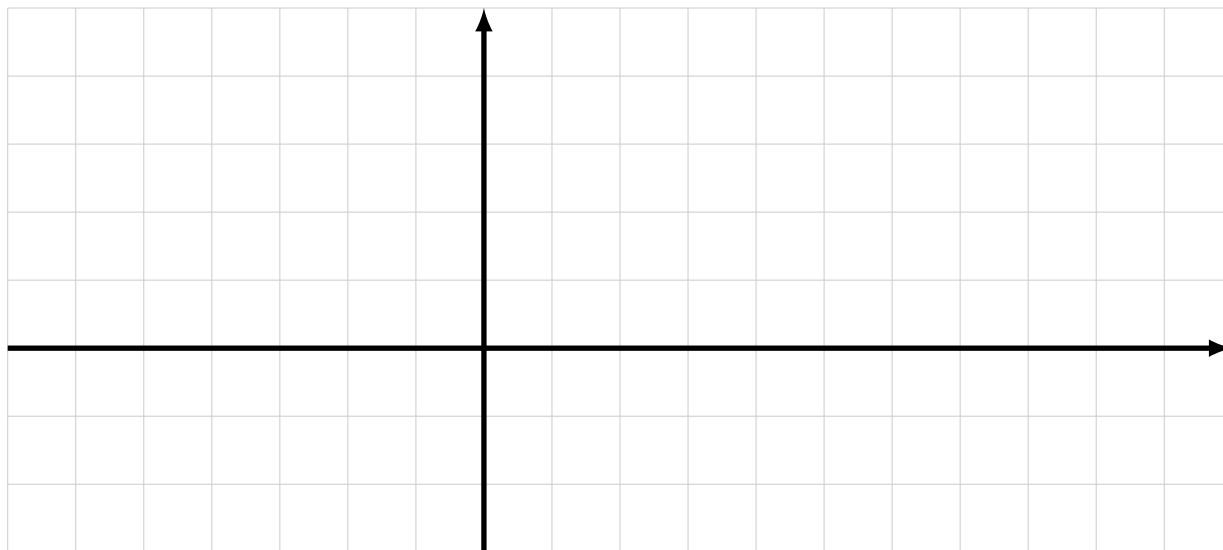
1) Addition of vectors:

$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{bmatrix}$$
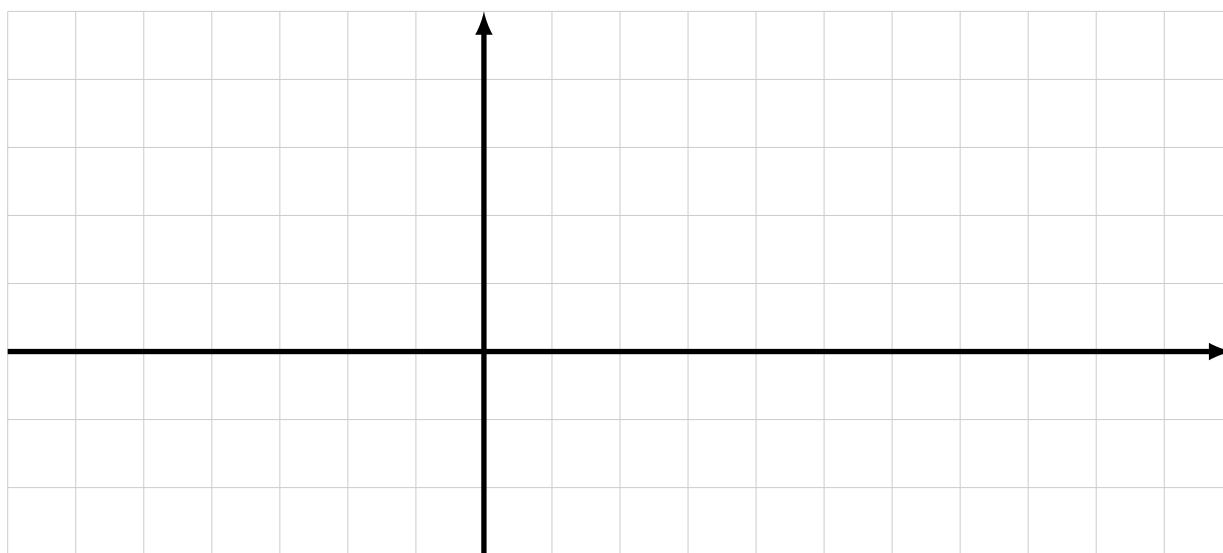
2) Multiplication by scalars:

$$c \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} ca_1 \\ \vdots \\ ca_n \end{bmatrix}$$

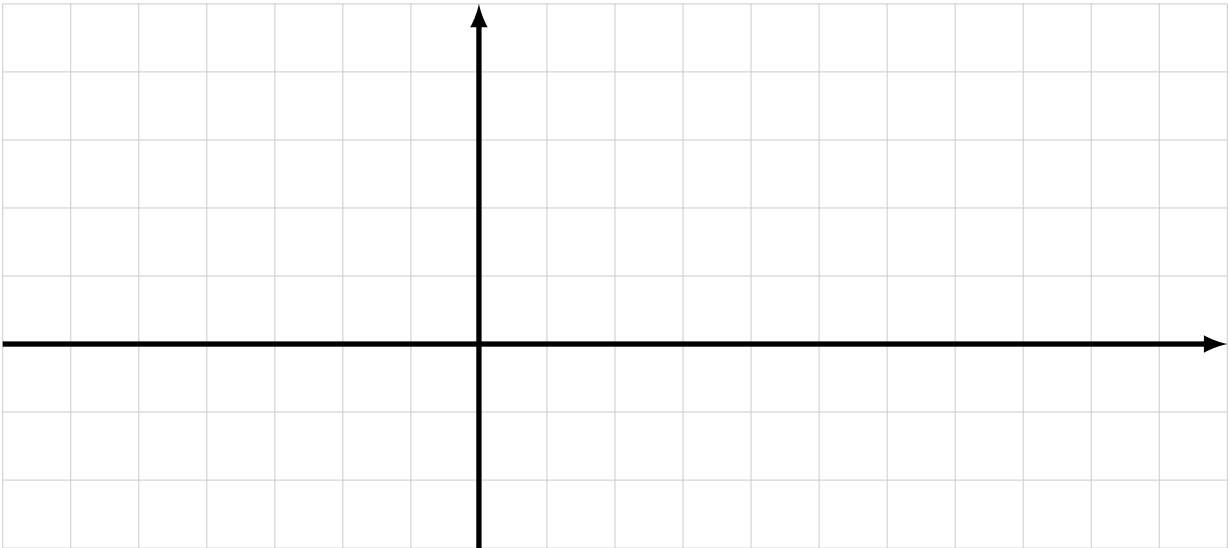# Geometric interpretation of vectors in $\mathbb{R}^2$

Vector coordinates:

Vector addition:

Scalar multiplication:

Vector equations

$$x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + \ldots + x_p \mathbf{v}_p = \mathbf{w}$$

Example. Solve the following vector equation:

$$x_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

How to solve a vector equation

$$x_1\mathbf{v}_1 + \ldots + x_p\mathbf{v}_p = \mathbf{w}$$

vector of equation

make
a matrix

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \ldots & \mathbf{v}_p & \mathbf{w} \end{array}\right]$$

augmented matrix

row
reduction

$$\left[\, \text{reduced matrix} \,\right]$$

read off
solutions

$$\begin{cases} x_1 & = \ldots \\ \ldots & \ldots \\ x_p & = \ldots \end{cases}$$

solutions

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Example: Target shooting.

At time $t = 0$ a target is observed at the position (x_0, y_0) moving in the direction of the vector v_t. The target is moving at such speed, that it travels the length of v_t in one second. A missile is positioned at the point $(0, 0)$. When fired, it will move vertically with such speed, that it will travel the length of the vector v_m in one second. After how many seconds should the missile be fired in order to intercept the target?
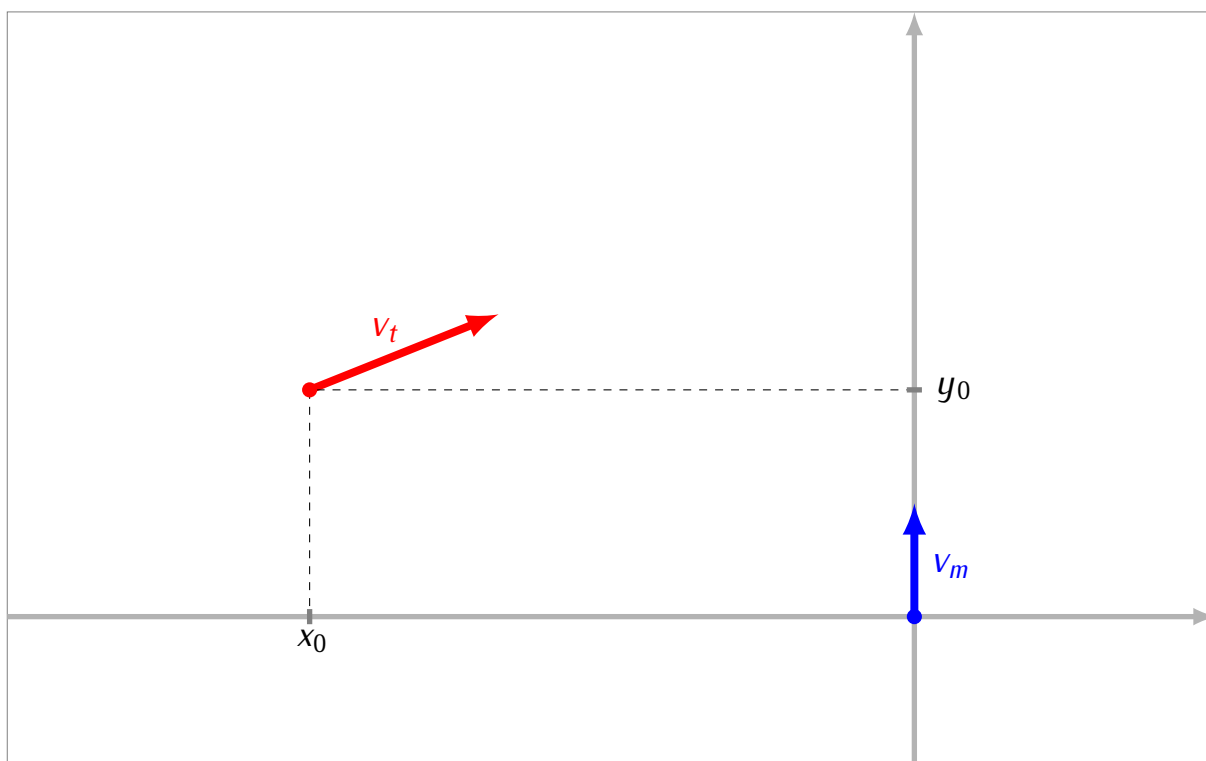
Recall:

Vector equations are equivalent to systems of linear equations:

$$x_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \qquad \Longleftrightarrow \qquad \begin{cases} 2x_1 + 4x_2 = 7 \\ 3x_1 + 2x_2 = 3 \end{cases}$$

<div align="center">

vector
equation   system of
linear equations

</div>

Upshot. A vector equation can have either:
- no solutions
- exactly one solution
- infinitely many solutions

Next:
- When does a vector equation have a solution?
- When does it have exactly one solution?