- The size of this image is 1000× 1000 pixels.

- The color of each pixel is represented by an integer between 0 (black) and 255 (white).

- The whole image is described by a (symmetric) matrix $A$ consisting of $1000 \times 1000 = 1,000,000$ numbers

- Each number is stored in 1 byte, so the image file size is $1,000,000$ bytes ($\approx 1$ MB).

**How to make the image file smaller:**

**1)** Find the spectral decomposition of the matrix $A$:

$$A = \lambda_1(\mathbf{u}_1\mathbf{u}_1^T) + \lambda_2(\mathbf{u}_2\mathbf{u}_2^T) + \ldots + \lambda_{1000}(\mathbf{u}_{1000}\mathbf{u}_{1000}^T)$$
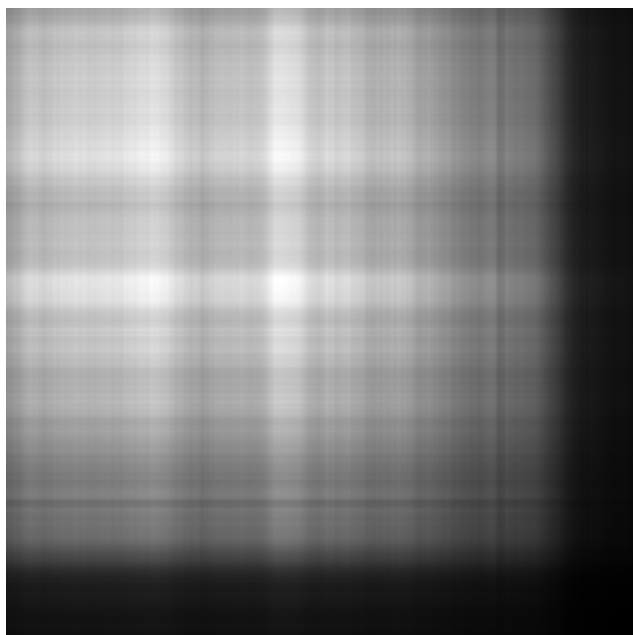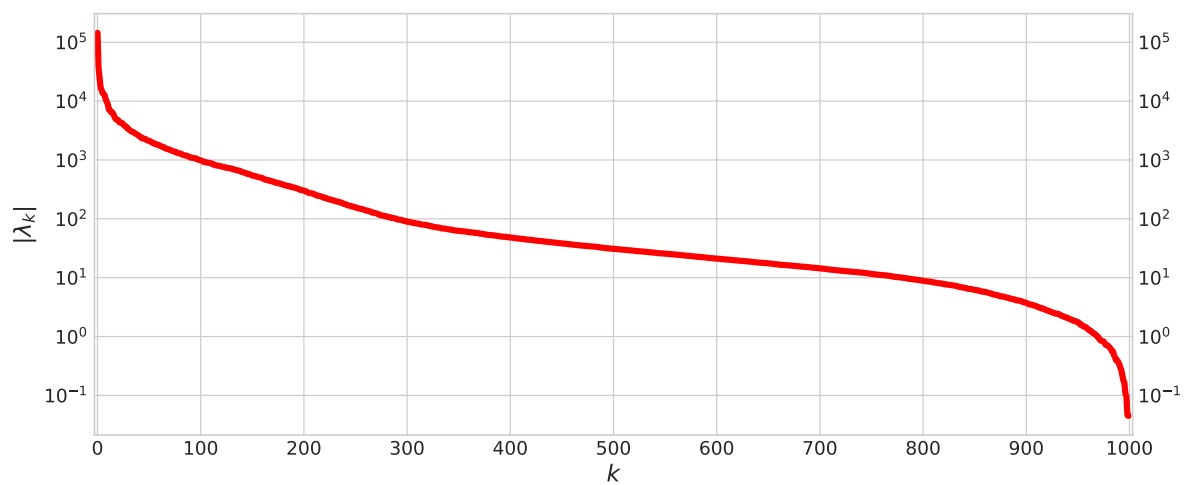
where $|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_{1000}|$.
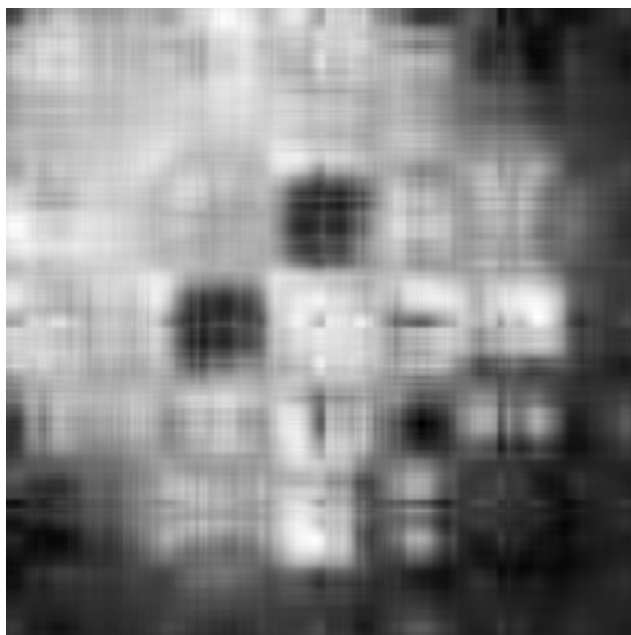
**2)** For $k = 1, \ldots, 1000$ define:

$$B_k = \lambda_1(\mathbf{u}_1\mathbf{u}_1^T) + \lambda_2(\mathbf{u}_2\mathbf{u}_2^T) + \ldots + \lambda_k(\mathbf{u}_k\mathbf{u}_k^T)$$

This matrix approximates the matrix $A$ and can be stored using $k \cdot (1000 + 1)$ numbers (i.e. $k \cdot (1000 + 1)$ bytes).
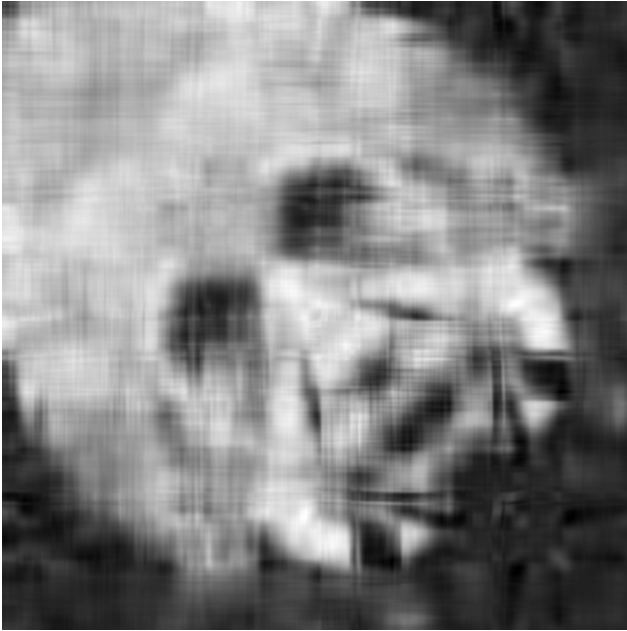
# Eigenvalues of the matrix A
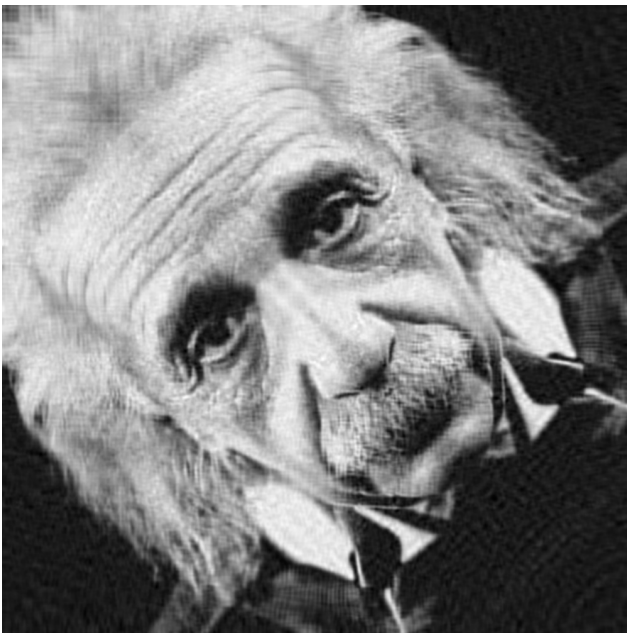




**matrix B₁**
1001 bytes
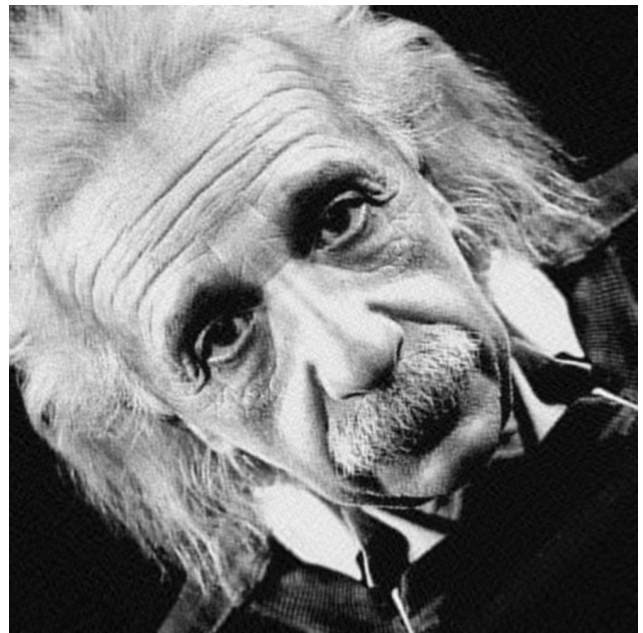compression 1000:1

**matrix B₅**
5005 bytes
compression 200:1

**matrix** $B_{10}$
10,010 bytes
compression 100:1



**matrix** $B_{20}$
20,020 bytes
compression 50:1



**matrix** $B_{50}$
50,050 bytes
compression 20:1



**matrix** $B_{100}$
100,100 bytes
compression 10:1