MTH 309T Practice Exam 3

1. Consider the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace V.

b) Compute the vector $\mathrm{proj}_V\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto V.

2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$ **b)** $\lambda = -1$ **c)** $\lambda = -2$

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ which satisfy $||\mathbf{u} + \mathbf{v}|| = ||\mathbf{u} - \mathbf{v}||$ then $\mathbf{u}$ must be orthogonal to $\mathbf{v}$.

b) If A is a 3×3 matrix which is both symmetric and orthogonal then $A^3 = A$.

c) If A is an $n \times n$ matrix and $\mathbf{v}$ is eigenvector of A, then $\mathbf{v}$ is also an eigenvector of A^2.

d) If A is an $n \times n$ matrix and λ is eigenvalue of A, then λ is also an eigenvalue of A^2.