

Other operations on matrices

## 1) Addition.

If  $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$ ,  $B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$  are  $m \times n$  matrices then

$$A + B = \begin{bmatrix} a_{11} + b_{11} & \dots & a_{1n} + b_{1n} \\ \vdots & & \vdots \\ a_{m1} + b_{m1} & \dots & a_{mn} + b_{mn} \end{bmatrix}$$

**Note.** The sum  $A + B$  is defined only if  $A$  and  $B$  have the same dimensions.

## 2) Scalar multiplication.

If  $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$ , and  $c$  is a scalar then

$$cA = \begin{bmatrix} ca_{11} & \dots & ca_{1n} \\ \vdots & & \vdots \\ ca_{m1} & \dots & ca_{mn} \end{bmatrix}$$

## Properties of matrix algebra

1)  $(AB)C = A(BC)$

2)  $(A + B)C = AC + BC$   
 $A(B + C) = AB + AC$

3)  $I_n$  = the  $n \times n$  identity matrix:

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

If  $A$  is an  $m \times n$  matrix then

$$A \cdot I_n = A$$

$$I_m \cdot A = A$$

## Non-commutativity of matrix multiplication

1) If  $AB$  is defined then  $BA$  need not be defined.

2) Even if both  $AB$  and  $BA$  are both defined then usually

$$AB \neq BA$$

## One more operation on matrices: matrix transpose

### Definition

The transpose of a matrix  $A$  is the matrix  $A^T$  such that

$$(\text{rows of } A^T) = (\text{columns of } A)$$

### Properties of transpose

- 1)  $(A^T)^T = A$
- 2)  $(A + B)^T = (A^T + B^T)$
- 3)  $(AB)^T = B^T A^T$