

Next:

How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

*make
a matrix*

augmented
matrix

*Gauss-Jordan
elimination*

solutions

$$\begin{cases} x_1 = \dots \\ x_2 = \dots \\ x_3 = \dots \end{cases}$$

*read off
solutions*

matrix in reduced
echelon form

Matrices

matrix = rectangular array of numbers

$$\begin{matrix} \text{3 columns} \\ \left[\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array} \right] \end{matrix} \left. \vphantom{\begin{matrix} \text{3 columns} \\ \left[\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array} \right] \end{matrix}} \right\} \text{2 rows}$$

2x3 matrix

$$\begin{matrix} \text{3 columns} \\ \left[\begin{array}{ccc} 1 & 2 & 0 \\ 7 & -5 & 1 \\ 8 & 10 & 7 \\ 6 & 4 & 3 \end{array} \right] \end{matrix} \left. \vphantom{\begin{matrix} \text{3 columns} \\ \left[\begin{array}{ccc} 1 & 2 & 0 \\ 7 & -5 & 1 \\ 8 & 10 & 7 \\ 6 & 4 & 3 \end{array} \right] \end{matrix}} \right\} \text{4 rows}$$

4x3 matrix

Note

Every system of linear equations can be represented by a matrix.

Example.

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

$$\begin{matrix} x_1 & x_2 & x_3 \\ \left[\begin{array}{ccc|c} -1 & 2 & 3 & 4 \\ 2 & 0 & 6 & 9 \\ 4 & -1 & 3 & 0 \end{array} \right] \end{matrix}$$

the coefficient matrix the column of constants

the augmented matrix of the system of equations

Elementary row operations:

1) Interchange of two rows.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \xrightarrow{\text{Interchange Row 1 and Row 3}} \begin{bmatrix} 4 & 3 & 0 & 7 \\ 0 & 1 & 5 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

2) Multiplication of a row by a non-zero number.

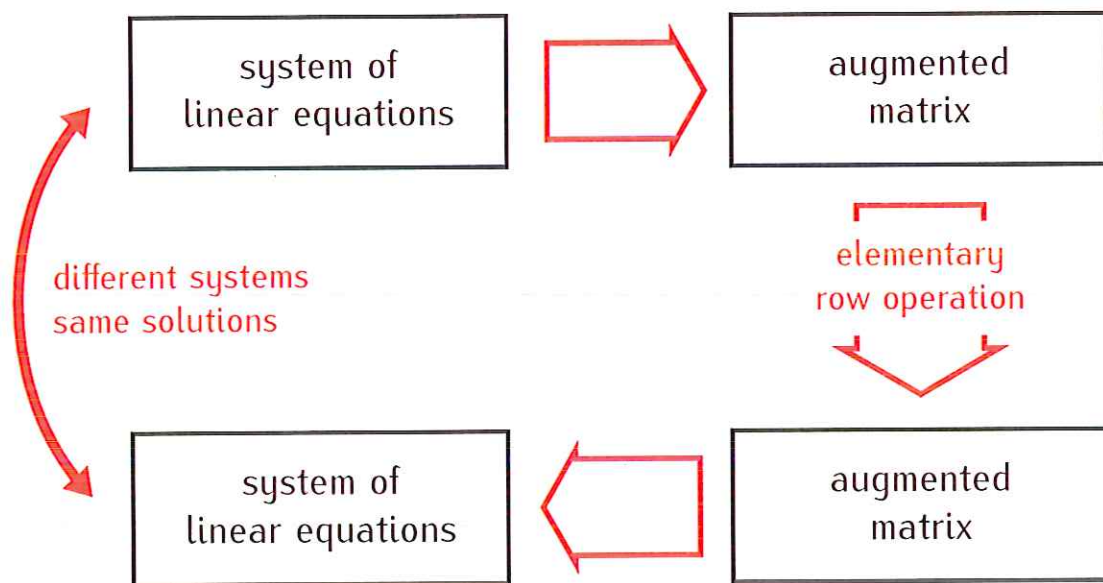
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \xrightarrow{\text{Row 3} \times 2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 8 & 6 & 0 & 14 \end{bmatrix}$$

3) Addition of a multiple of one row to another row.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix} \xrightarrow{\text{Row 3} + 3 \times \text{Row 2}} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 7 & 14 & 13 \\ 4 & 3 & 0 & 7 \end{bmatrix}$$

Proposition

Elementary row operations do not change solutions of the system of equations represented by a matrix.



$$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 - x_2 + 5x_3 = 0 \\ 3x_1 + 6x_2 - x_3 = 1 \end{cases} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & -1 & 5 & 0 \\ 3 & 6 & -1 & 1 \end{array} \right]$$

$$\begin{cases} 3x_1 + 6x_2 - x_3 = 1 \\ 2x_1 - x_2 + 5x_3 = 0 \\ x_1 + 2x_2 + 3x_3 = 4 \end{cases} \leftarrow \left[\begin{array}{ccc|c} 3 & 6 & -1 & 1 \\ 2 & -1 & 5 & 0 \\ 1 & 2 & 3 & 4 \end{array} \right]$$

(interchange of rows)

(multipl. of a row)

(addition of a multiple
of one row to another)

= (interchange of equations)

= (multipl. of an equation)

(addition of a multiple
of one equation to another)