Recall:

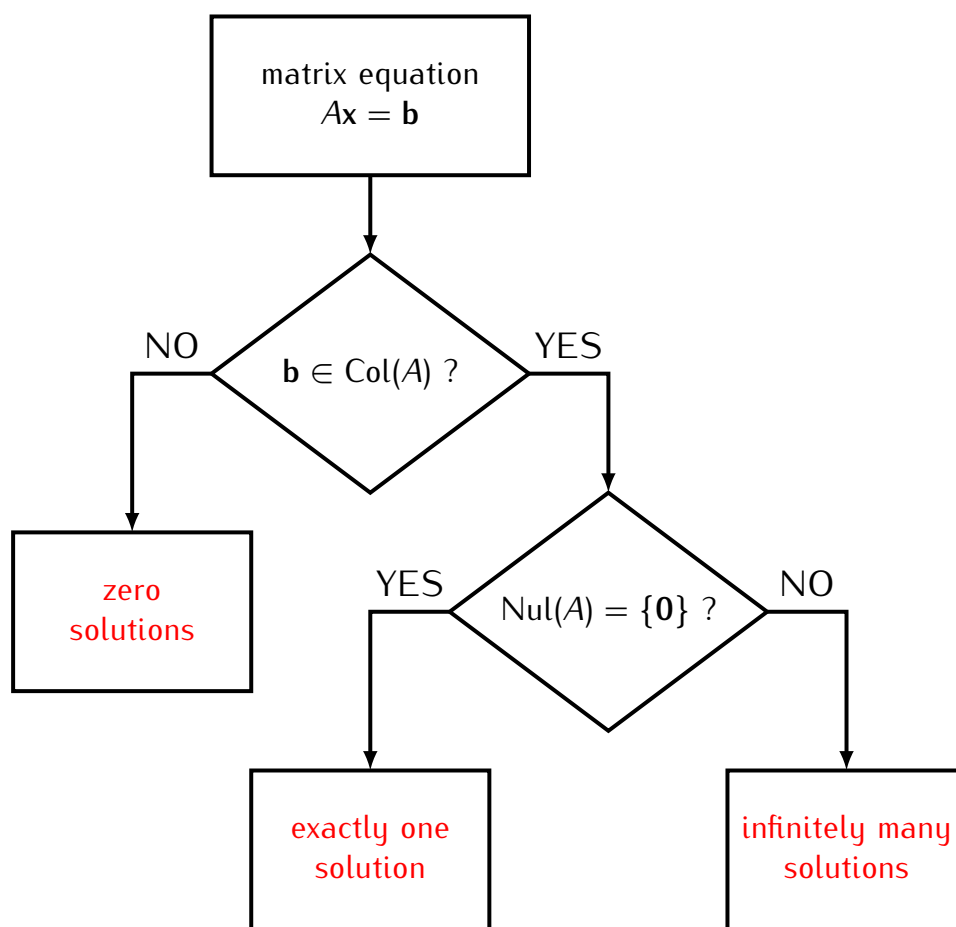**1)** We can multiply vectors by matrices.

2) Matrix equation: $A\mathbf{x} = \mathbf{b}$

$\mathrm{Col}(A) = (\text{span of column vectors of } A)$

$\mathrm{Nul}(A) = (\text{set of solutions of } A\mathbf{x} = \mathbf{0})$

Recall: Nul(A) can be always described as a span of some vectors.

Example. Find the null space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix}$$

Example. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Proposition

Let v_0 be some chosen solution of a matrix equation $A\mathbf{x} = \mathbf{b}$. Then any other solution $\mathbf{v}$ of this equation is of the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{n}$$

where $\mathbf{n} \in \mathrm{Nul}(A)$.

$\mathbf{v}_0 + \mathrm{Nul}(A)$
solutions of $A\mathbf{x} = \mathbf{b}$

$\mathrm{Nul}(A)$
solutions of $A\mathbf{x} = \mathbf{0}$