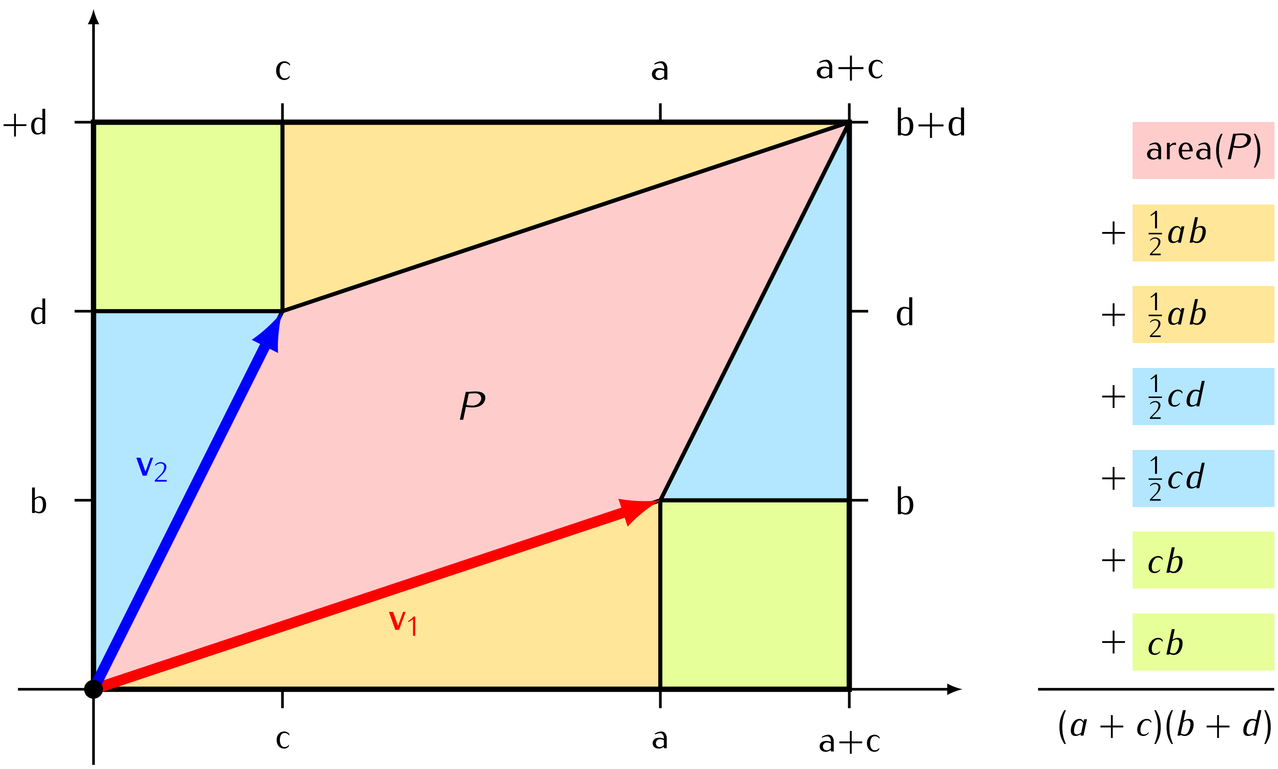
Theorem

If $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^2$ then

$$\operatorname{area}(\mathbf{v}_1, \mathbf{v}_2) = \left|\det\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}\right|$$

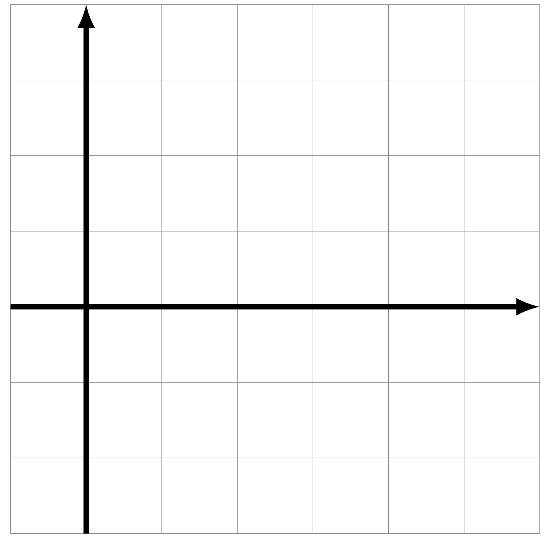
Idea of the proof.

$$\mathbf{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} c \\ d \end{bmatrix}$$

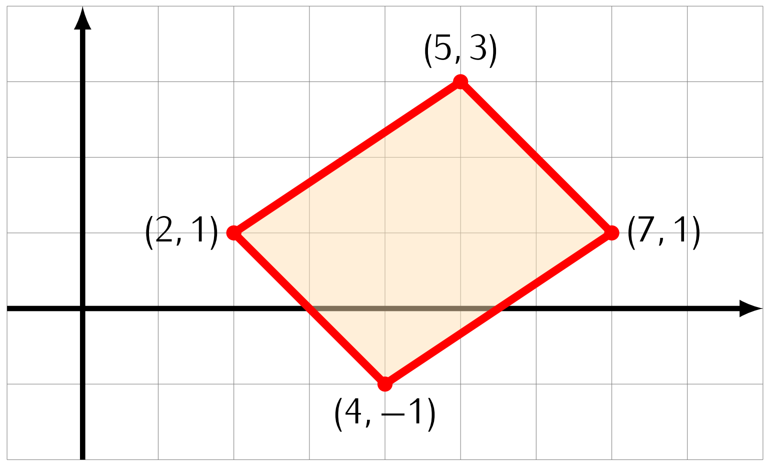


Example.

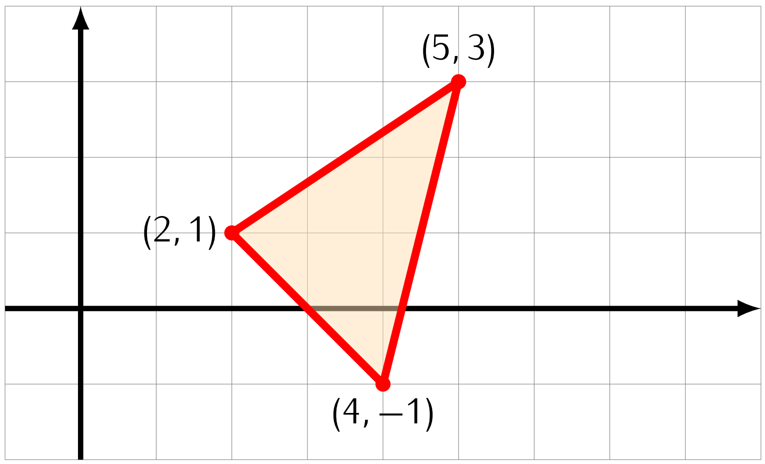
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



Example. Calculate the area of the parallelogram with vertices at the points $(2, 1)$, $(5, 3)$, $(7, 1)$, $(4, -1)$.



Example. Calculate the area of the triangle with vertices at the points $(2, 1)$, $(5, 3)$, $(4, -1)$.



Note. In order to compute areas of other polygons, subdivide them into triangles.