

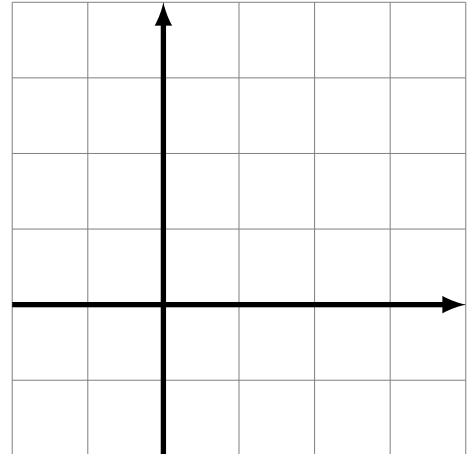
Systems of linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

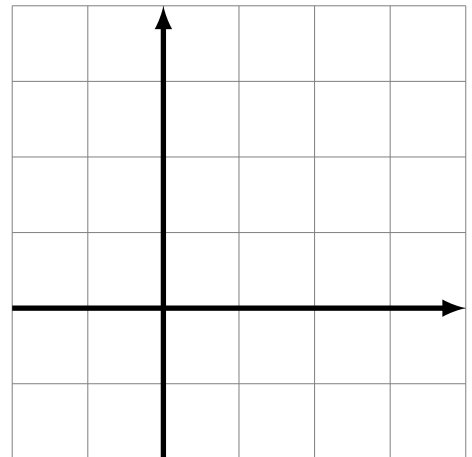
Question: How many solutions a system of linear equations can have?

Example: Systems of equations in 2 variables.

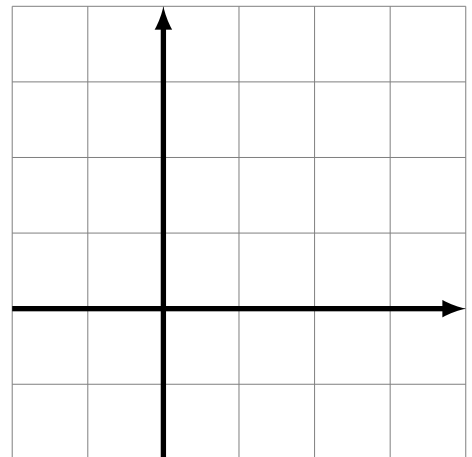
$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \end{cases}$$



$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + x_2 = 2 \end{cases}$$

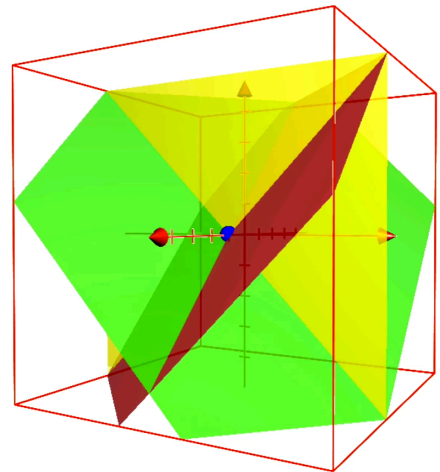


$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \end{cases}$$

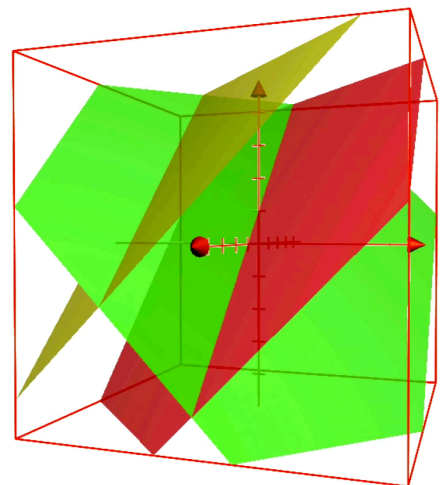


Example: Systems of equations in 3 variables.

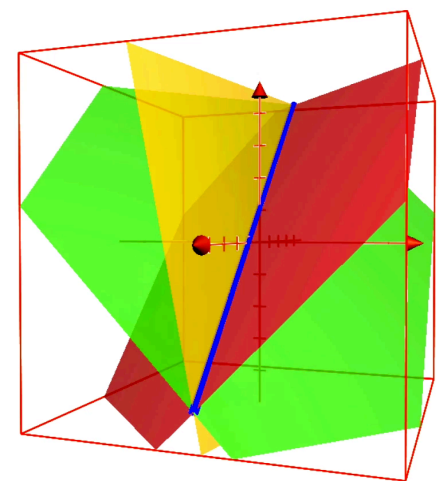
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 = 1 \end{cases}$$



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 6 \end{cases}$$



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 + 5x_2 + x_3 = 1 \end{cases}$$



In general:

A system of linear equations can have either

- no solutions
- exactly one solution
- infinitely many solutions

Definition

A system of linear equations which has no solutions is called an *inconsistent system*. Otherwise the system is *consistent*.