2) Scalar multiplication.

If $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$, and c is a scalar then

$$cA = \begin{bmatrix} ca_{11} & \dots & ca_{1n} \\ \vdots & & \vdots \\ ca_{m1} & \dots & ca_{mn} \end{bmatrix}$$

Properties of matrix algebra

1) $(AB)C = A(BC)$

2) $(A + B)C = AC + BC$
$A(B + C) = AB + AC$

3) $I_n =$ the $n \times n$ identity matrix:

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

If A is an $m \times n$ matrix then

$$A \cdot I_n = A$$

$$I_m \cdot A = A$$

Non-commutativity of matrix multiplication

1) If AB is defined then BA need not be defined.

2) Even if both AB and BA are both defined then usually

$$AB \neq BA$$

One more operation on matrices: matrix transpose

Definition

The transpose of a matrix A is the matrix A^T such that

$$(\text{rows of } A^T) = (\text{columns of } A)$$

Properties of transpose

1) $(A^T)^T = A$
2) $(A+B)^T = (A^T + B^T)$
3) $(AB)^T = B^T A^T$

Operations on matrices so far:

- addition/subtraction $A \pm B$
- scalar multiplication $c \cdot A$
- matrix multiplication $A \cdot B$
- matrix transpose A^T

Next: How to divide matrices?

Definition

A matrix A is *invertible* if there exists a matrix B such that

$$A \cdot B = B \cdot A = I$$

(where $I =$ the identity matrix). In such case we say that B is the *inverse* of A and we write $B = A^{-1}$.