Instructions.

In this problem you will be given a few statements. For each statement you need to decide if it is true or not, and justify your answer.

How to justify your answer.

• In order to show that a statement is false, it suffices to give a counterexample. For example, consider the the statement:

The last digit of every even number is either 2, 4, or 8.

To show that this statement is false, it is enough to point out that, for example, 10 is an even number, but its last digit is 0.

• In order to show that a statement is true, you need to provide a reasoning explaining why it is true in all instances. Giving one example when it is true will not suffice, since the statement may not work in some other cases. For example, consider the the statement:

If n is an even number then $n+2$ is also an even number.

You can justify that this is true as follows. Even numbers are integers which are multiples of 2. If n is even then $n = 2m$ for some integer m. Then $n+2 = 2m+2 = 2(m+1)$, which shows that $n+2$ is even.

Note

This problem will not be collected or graded. However, problems of this type will appear on exams in this course. Sample solutions are provided at the end of this assignment.

For each of the statements given below decide if it is true or false. If you decide that it is true, justify your answer. If you think it is false give a counterexample.

a) If a system of linear equations has more variables than equations then it must have infinitely many solutions.

b) If a system of linear equations has more equations than variables, then it has no solutions.

c) If A is a 3×6 matrix then A can have at most 3 pivot columns.

d) If A is an augmented matrix of a system of linear equations such that all entries of the column of constants are equal to 0, then the system of equations represented by A is consistent.

Here are solutions to the questions from the previous page. You should try to answer all questions by yourself before reading these solutions.

a) FALSE. A system which has more variables than equations may have zero solutions. For example: the system

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 + x_2 + x_3 = 2 \end{cases}$$

has no solutions.

b) FALSE. For example, the system

$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \\ 3x_1 + 3x_2 = 3 \end{cases}$$

has infinitely many solutions.

c) TRUE. Pivot columns are columns which after row reduction contain leading ones. Since there can be only one leading one in every row of a matrix, and the matrix A has 3 rows, thus after row reduction we can have at most 3 leading ones, and so at most 3 pivot columns.

d) TRUE. If the column of constants consists of zeros, then after row reduction is will still consist of zeros. This means that this column will not contain a leading one, and so the system of equations represented by the matrix A will be consistent.