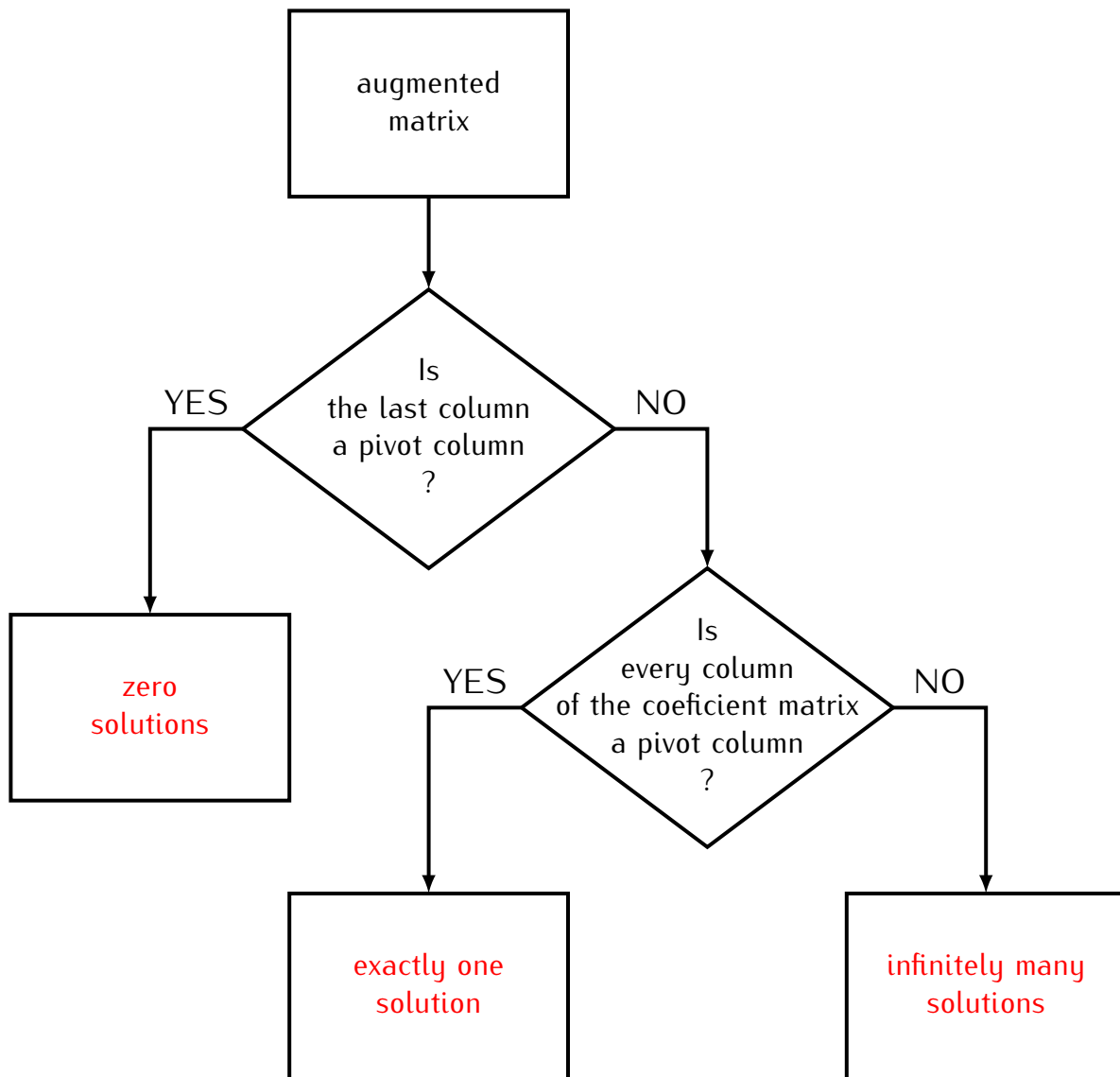> **Theorem**
>
> A system of linear equations can have either 0, 1, or infinitely many solutions.

Proof.

Recall:

<div style="border:1px solid black">

How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

make a matrix →

$$\begin{bmatrix} & & \\ & & \end{bmatrix}$$ augmented matrix

Gauss–Jordan elimination ↓

solutions

$$\begin{cases} x_1 = \ldots \\ x_2 = \ldots \\ x_3 = \ldots \end{cases}$$

← *read off solutions*

$$\begin{bmatrix} & & \\ & & \end{bmatrix}$$ matrix in reduced row echelon form

</div>

<u>**Next:**</u> Some applications of systems of linear equations:

- Computations of traffic flow.
- Balancing chemical equations.
- Google PageRank.
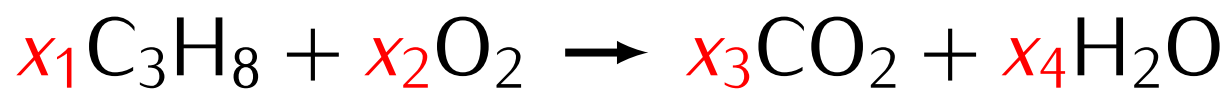
Computations of traffic flow
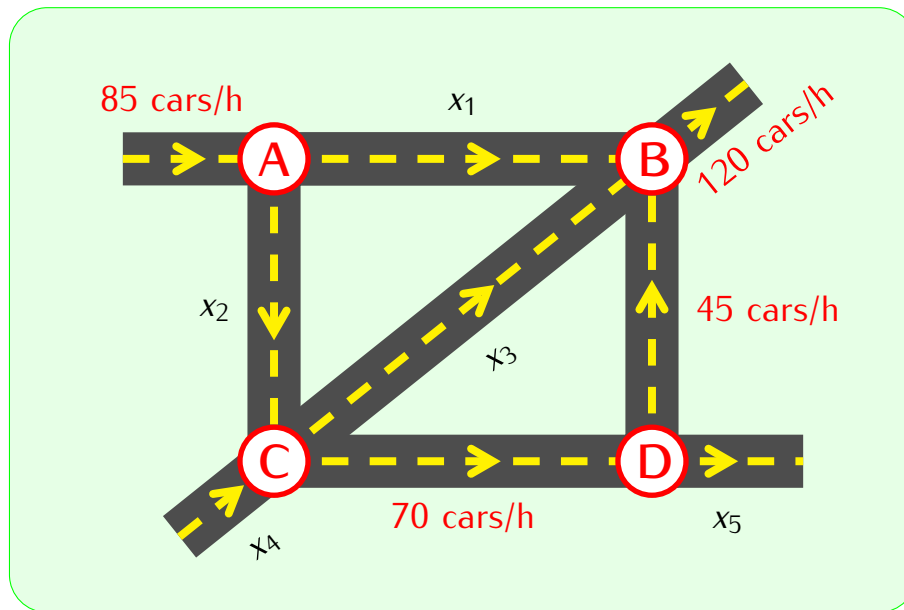
Problem. Find the flow rate of cars on each segment of streets.

Note:

- flow into an intersection = flow out of that intersection
- total flow in = total flow out

Balancing chemical equations

Burning propane:

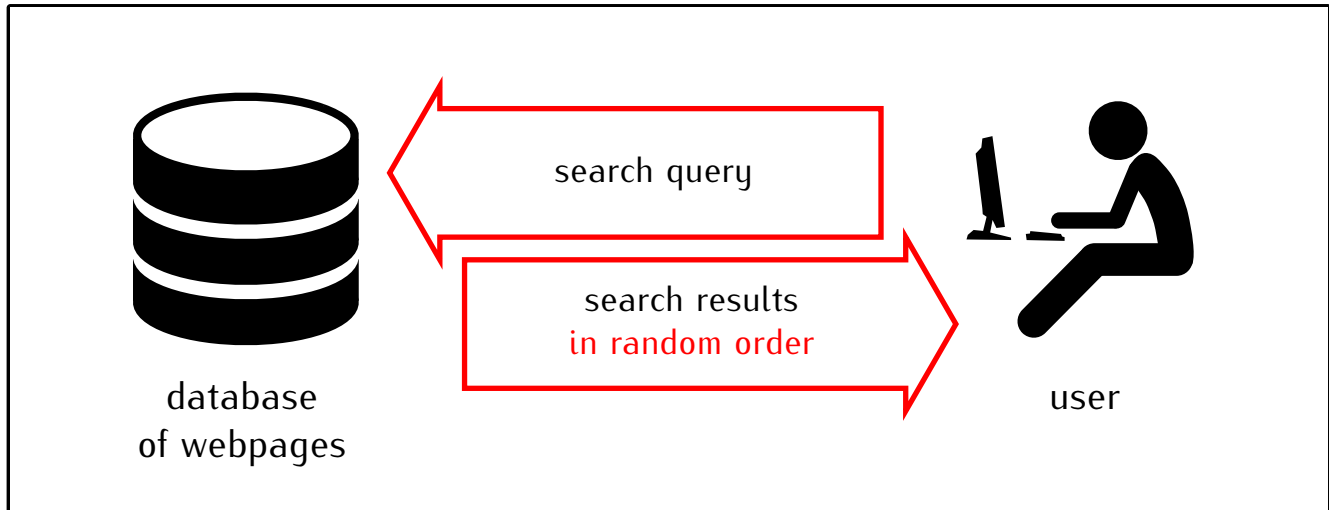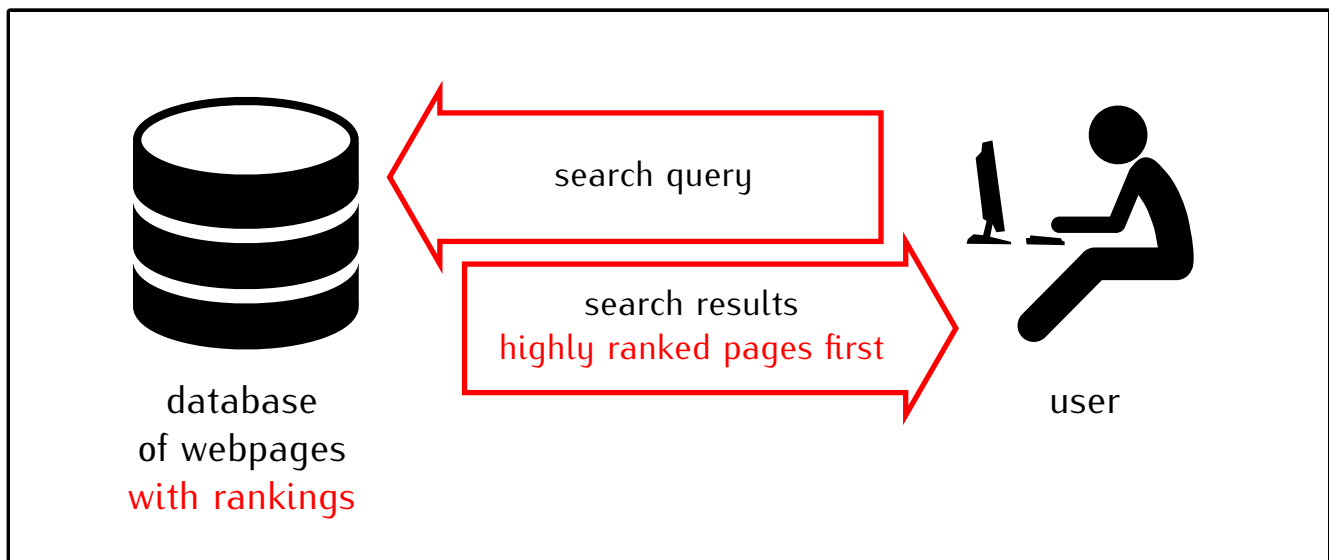$$x_1 C_3H_8 + x_2 O_2 \longrightarrow x_3 CO_2 + x_4 H_2O$$

Note:

- The numbers x_1, x_2, x_3, x_4 are integers.

- The number of atoms of each element on the left side is the same as the number of atoms of that element on the right side.
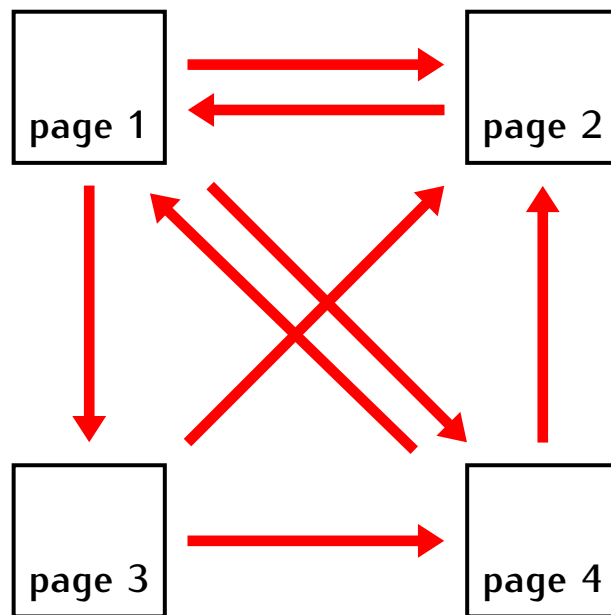
Google PageRank

Early search engines:

database
of webpages
search query
search results
in random order
user

Google search engine:

database
of webpages
with rankings
search query
search results
highly ranked pages first
user

How to rank webpages?

Very simple ranking:

$$\text{ranking of a page} = \begin{pmatrix} \text{number of links} \\ \text{pointing to that page} \end{pmatrix}$$

Network of web pages.

Problem. This is very easy to manipulate.