A simple error correcting code: triple repeat.

message: 1011

Problem: The encoded message is 3 times longer than the original message.

Better error correction: Hamming (7,4) code.

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

encoding matrix

decoding matrix

message: 10111101

Encoding.

1) Split the message into vectors with 4 entries, and multiply each vector by the encoding matrix E.

2) Reduce all numbers obtained in step 1 modulo 2. That is, add or subtract from each number a multiple of 2 to get either 0 or 1.

Encoded message:

Received message:

Decoding. Split the received message into vectors with 7 entries, multiply each vector by the decoding matrix D, and reduce modulo 2.

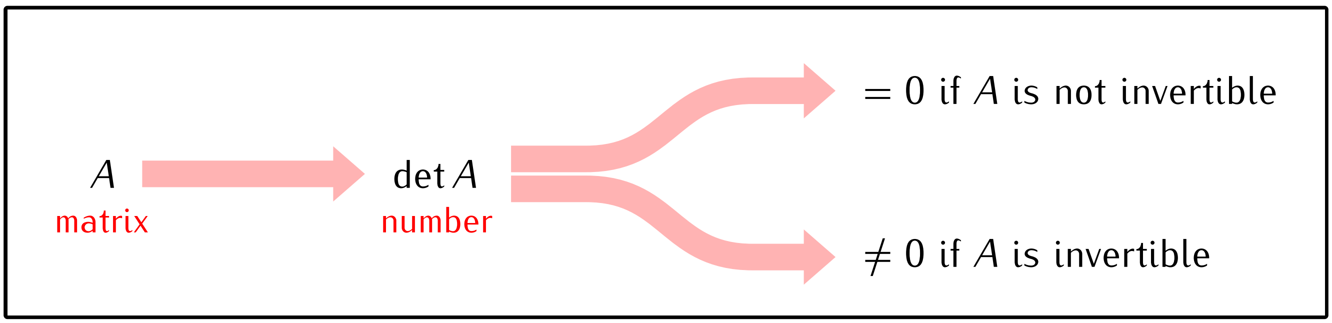
Decoded message:

How the Hamming code works:

<u>**Recall:**</u> If an $n \times n$ matrix A is invertible then:

- the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b} \in \mathbb{R}^n$
- the linear transformation $T_A \colon \mathbb{R}^n \to \mathbb{R}^n$, $T_A(\mathbf{v}) = A\mathbf{v}$ has an inverse function.

Determinants recognize which matrices are invertible:



Example: Determinant for a 1×1 matrix.

$$A = \begin{bmatrix} a \end{bmatrix}$$