

Matrices

matrix = rectangular array of numbers

Example.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 7 & -5 & 1 \\ 8 & 10 & 7 \\ 6 & 4 & 3 \end{bmatrix}$$

Note

Every system of linear equations can be represented by a matrix.

Example.

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

Elementary row operations:

1) Interchange of two rows.

Example.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix}$$

2) Multiplication of a row by a non-zero number.

Example.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix}$$

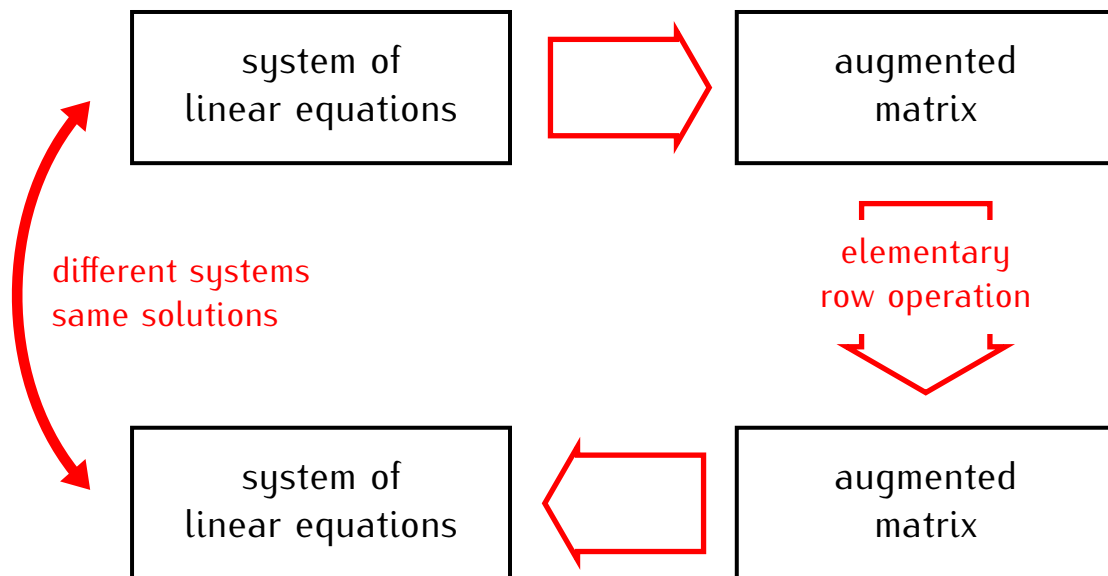
3) Addition of a multiple of one row to another row.

Example.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 1 \\ 4 & 3 & 0 & 7 \end{bmatrix}$$

Proposition


Elementary row operations do not change solutions of the system of equations represented by a matrix.




Recall:How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$



*make
a matrix*



augmented
matrix

*Gauss-Jordan
elimination*

solutions

$$\begin{cases} x_1 = \dots \\ x_2 = \dots \\ x_3 = \dots \end{cases}$$



*read off
solutions*

matrix in reduced
row echelon form

- Every system of linear equations can be represented by a matrix
- Elementary row operations:
 - interchange of two rows
 - multiplication of a row by a non-zero number
 - addition of a multiple of one row to another row.
- Elementary row operations do not change solutions of systems of linear equations.