

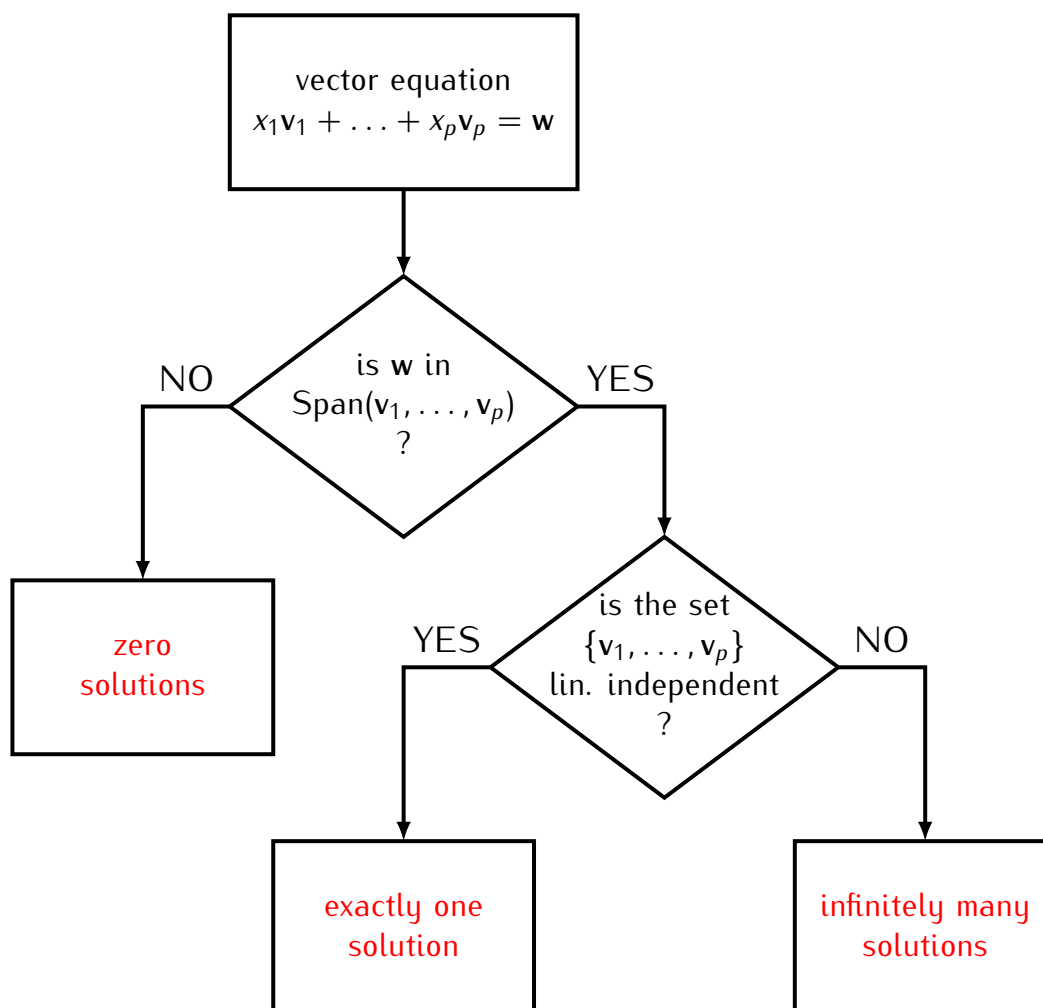
Recall:

1) $\text{Span}(\mathbf{v}_1, \dots, \mathbf{v}_p) = \left\{ \begin{array}{l} \text{the set of all} \\ \text{linear combinations} \\ c_1\mathbf{v}_1 + \dots + c_p\mathbf{v}_p \end{array} \right\}$

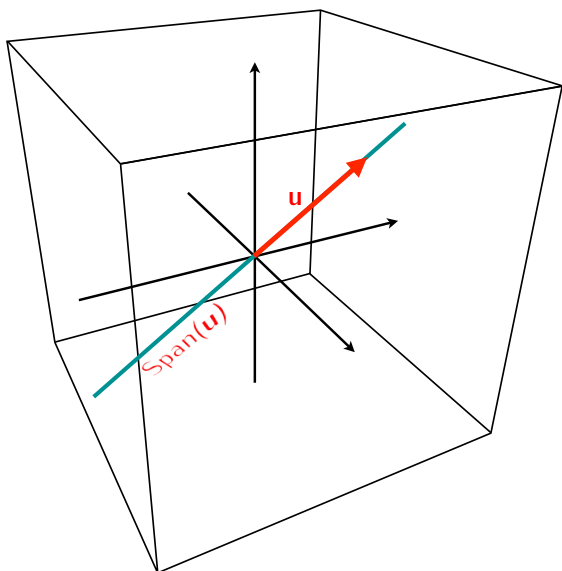
2) A set of vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is linearly independent if the equation

$$x_1\mathbf{v}_1 + \dots + x_p\mathbf{v}_p = \mathbf{0}$$

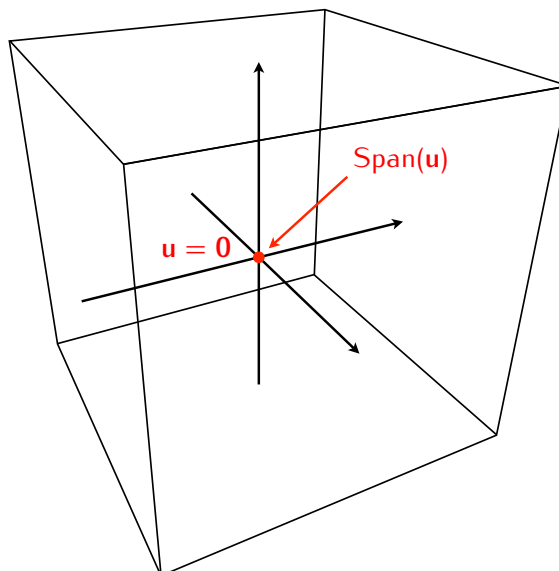
has only one, trivial solution $x_1 = 0, \dots, x_p = 0$.



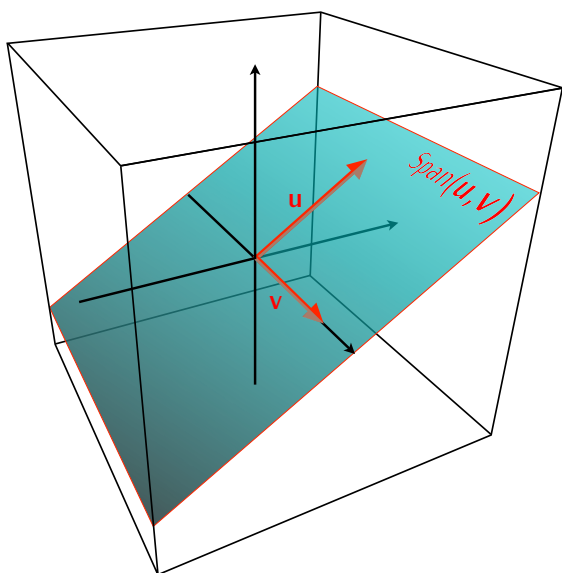
Linear independence vs. Span



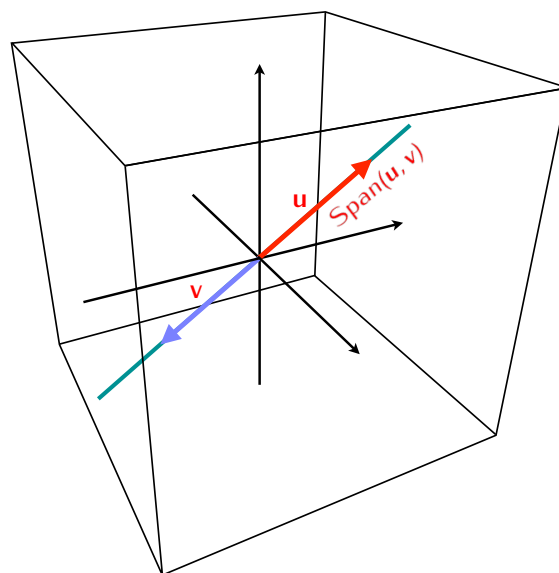
$\{u\}$ linearly independent



$\{u\}$ linearly dependent



$\{u, v\}$ linearly independent



$\{u, v\}$ linearly dependent

Theorem

If $\{v_1, \dots, v_p\}$ is a linearly dependent set of vectors in then:

- 1) for some v_i we have $v_i \in \text{Span}(v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_p)$.
- 2) for some v_i we have

$$\text{Span}(v_1, \dots, v_p) = \text{Span}(v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_p)$$

Example.

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$