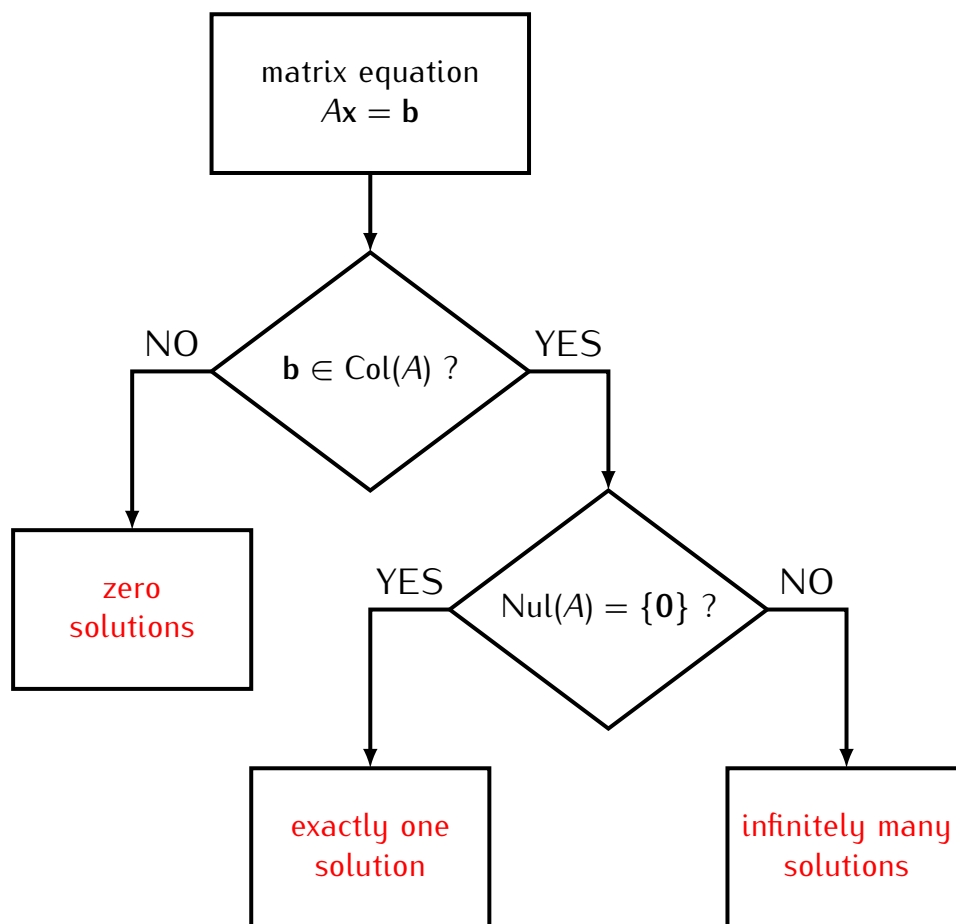


Recall:

- 1) We can multiply vectors by matrices.
- 2) Matrix equation: $A\mathbf{x} = \mathbf{b}$



$\text{Col}(A)$ = (span of column vectors of A)

$\text{Nul}(A)$ = (set of solutions of $A\mathbf{x} = \mathbf{0}$)

Recall: $\text{Nul}(A)$ can be always described as a span of some vectors.

Example. Find the null space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix}$$

Example. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Proposition

Let \mathbf{v}_0 be some chosen solution of a matrix equation $A\mathbf{x} = \mathbf{b}$. Then any other solution \mathbf{v} of this equation is of the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{n}$$

where $\mathbf{n} \in \text{Nul}(A)$.

