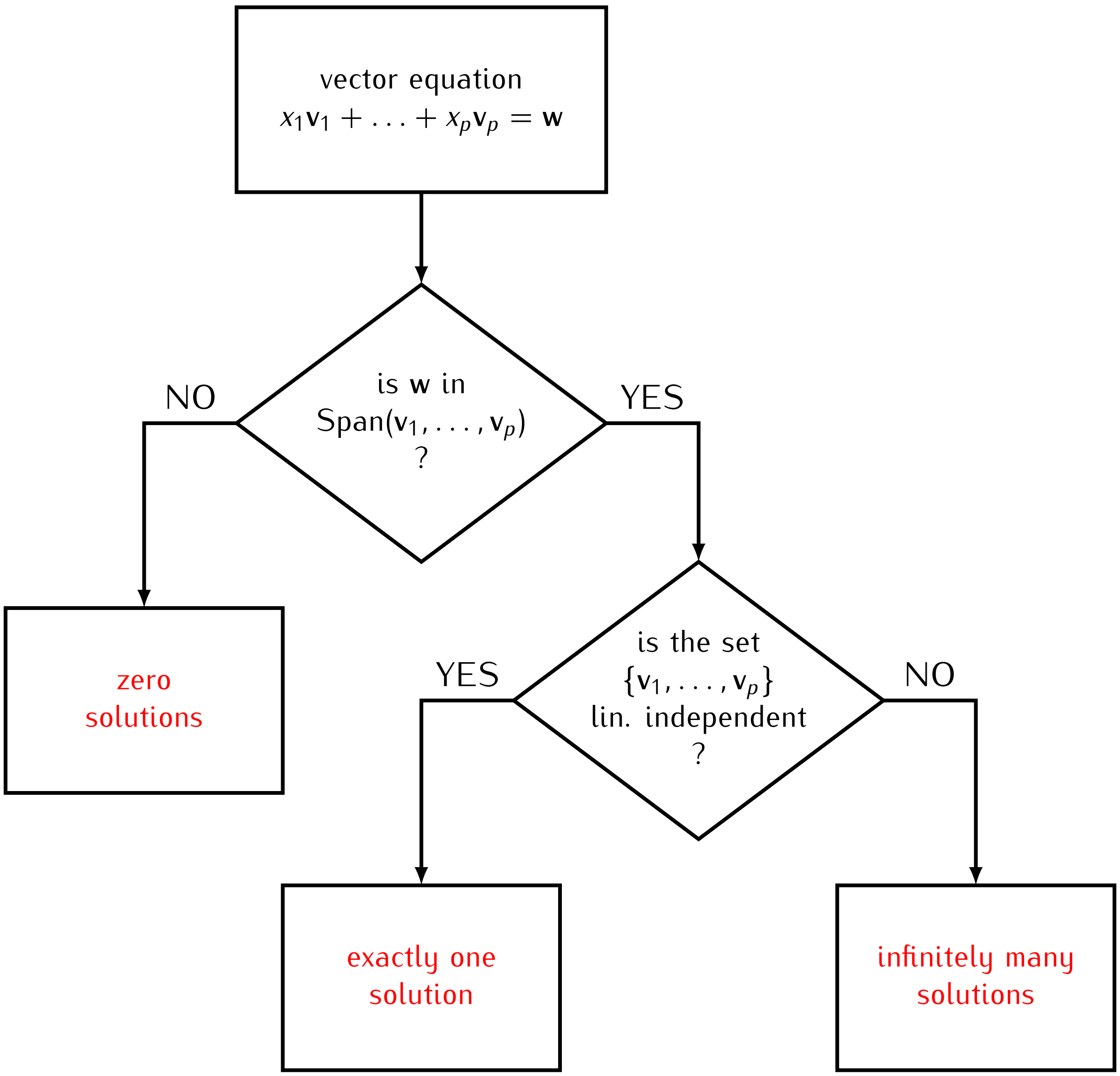
Upshot: how to find the number of solutions of a vector equation



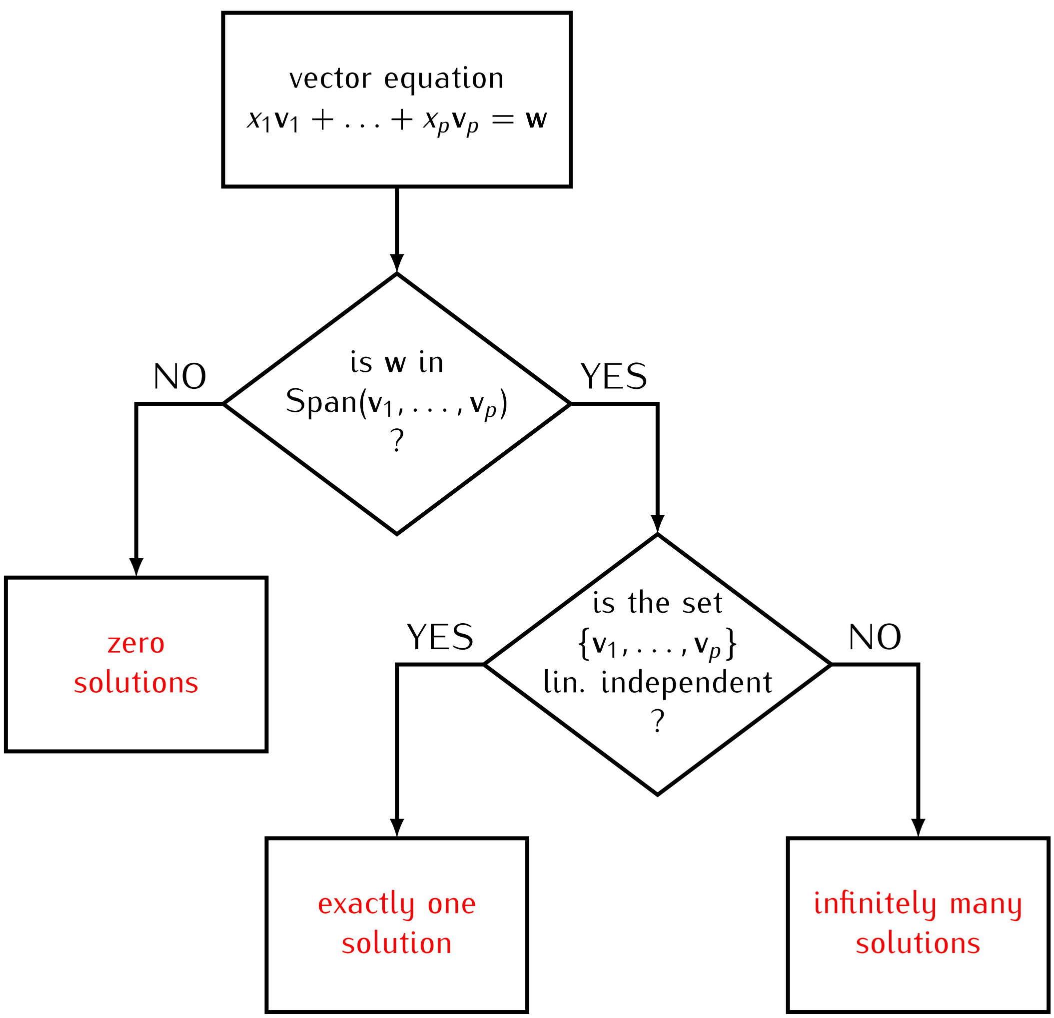
Recall:

1) $$\mathrm{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_p) = \left\{ \begin{array}{c} \text{the set of all} \\ \text{linear combinations} \\ c_1\mathbf{v}_1 + \ldots + c_p\mathbf{v}_p \end{array} \right\}$$

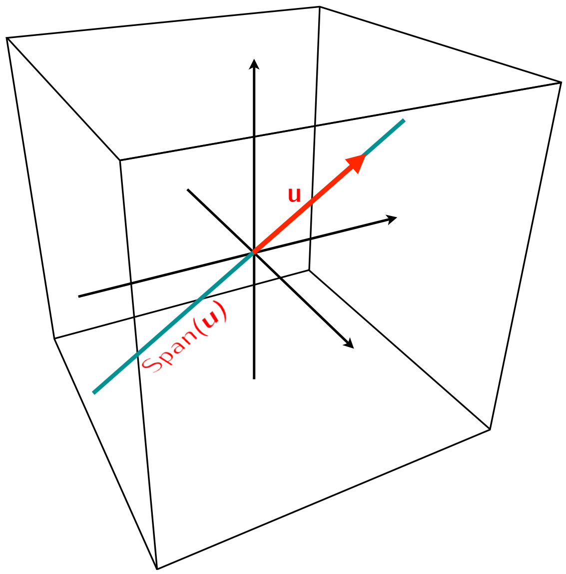
2) A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly independent if the equation

$$x_1\mathbf{v}_1 + \ldots + x_p\mathbf{v}_p = \mathbf{0}$$

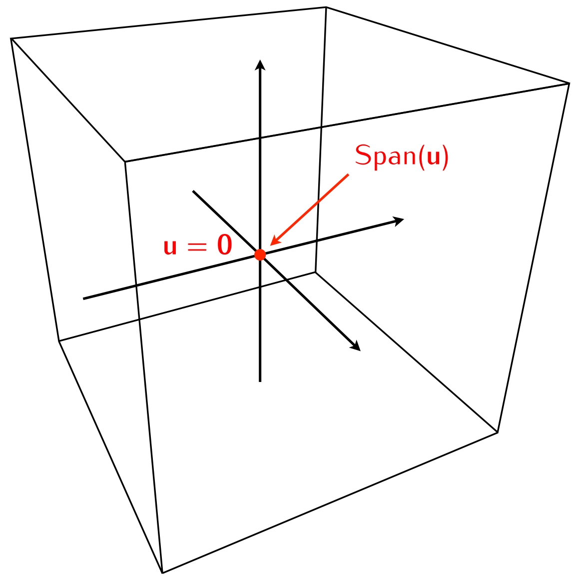
has only one, trivial solution $x_1 = 0, \ldots, x_p = 0$.



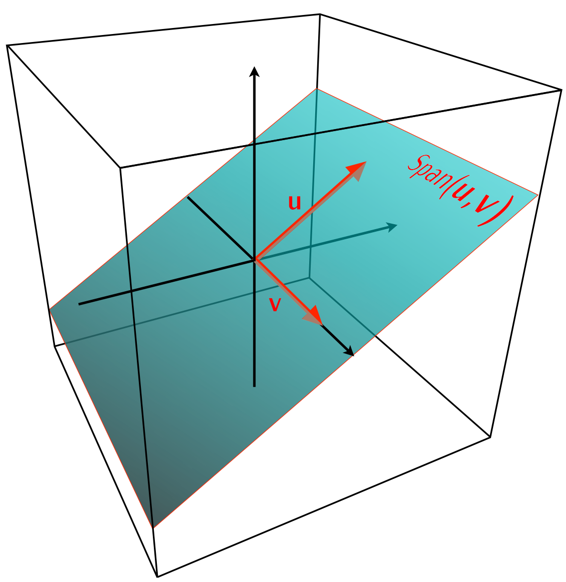
Linear independence vs. Span



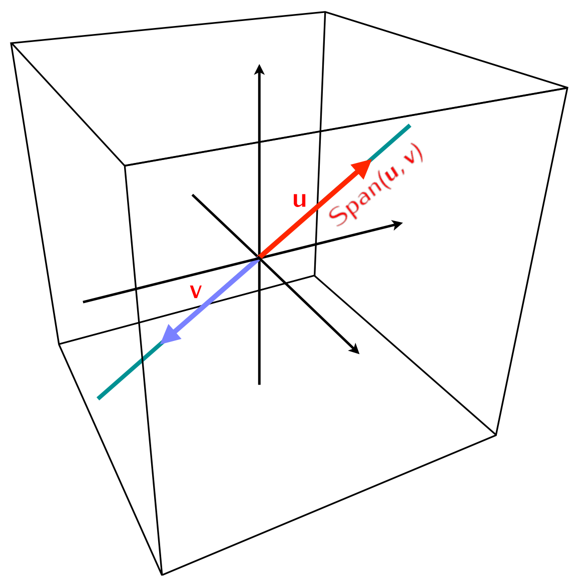
$\{\mathbf{u}\}$ linearly independent



$\{\mathbf{u}\}$ linearly dependent



$\{\mathbf{u}, \mathbf{v}\}$ linearly independent



$\{\mathbf{u}, \mathbf{v}\}$ linearly dependent