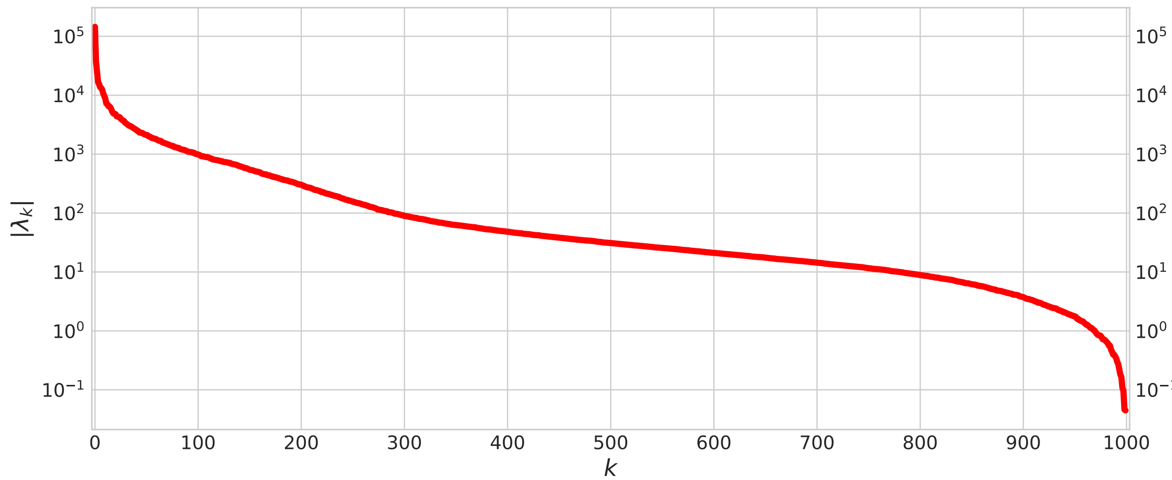
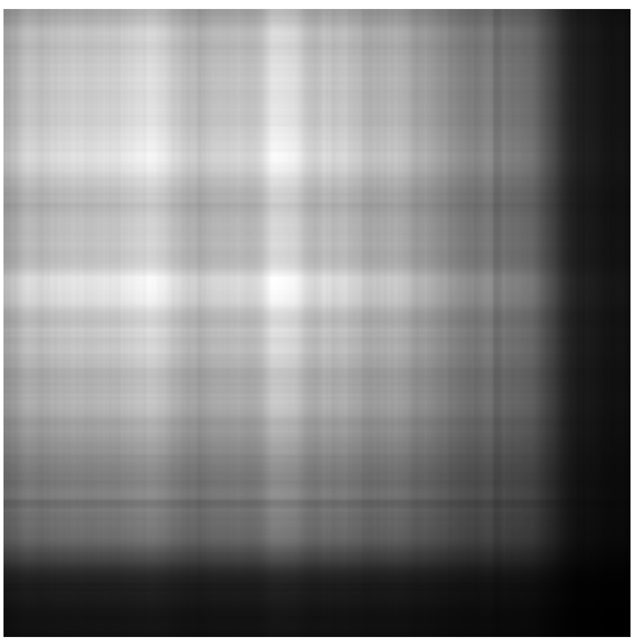
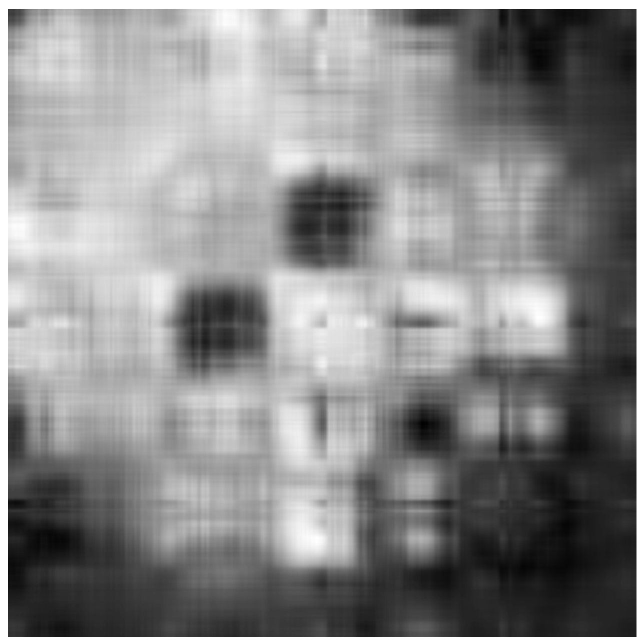
Eigenvalues of the matrix A

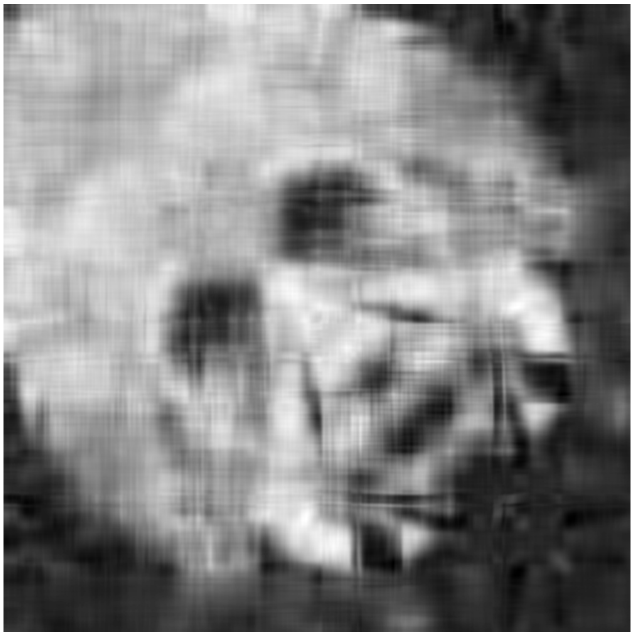




matrix B_1
1001 bytes
compression 1000:1

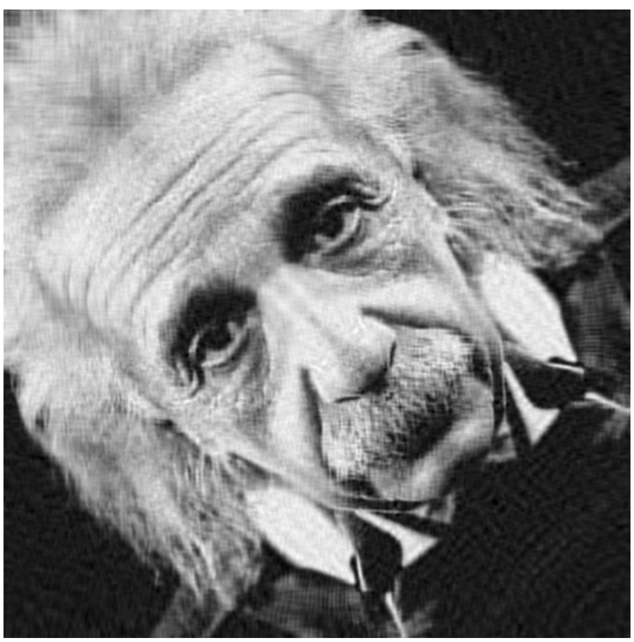


matrix B_5
5005 bytes
compression 200:1

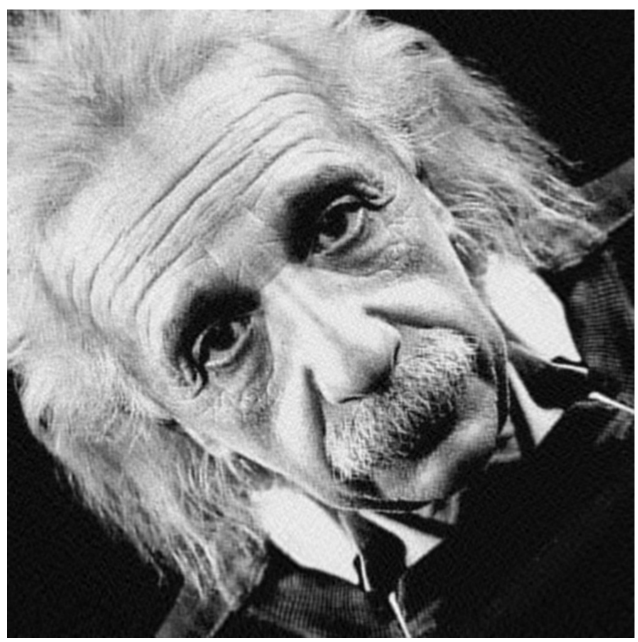


matrix $\mathbf{B}_{10}$
10,010 bytes
compression 100:1

matrix $\mathbf{B}_{20}$
20,020 bytes
compression 50:1



matrix $\mathbf{B}_{50}$
50,050 bytes
compression 20:1



matrix $\mathbf{B}_{100}$
100,100 bytes
compression 10:1

Theorem

Any A an $m \times n$ matrix can be written as a product

$$A = U\Sigma V^T$$

where:

- $U = \begin{bmatrix} \mathbf{u}_1 & \dots & \mathbf{u}_m \end{bmatrix}$ is an $m \times m$ orthogonal matrix.
- $V = \begin{bmatrix} \mathbf{v}_1 & \dots & \mathbf{v}_n \end{bmatrix}$ is an $n \times n$ orthogonal matrix.
- Σ is an $m \times n$ matrix of the following form:

$$\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \sigma_n \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \sigma_m & 0 & \cdots & 0 \end{bmatrix}$$

(if $n \leq m$) (if $n \geq m$)

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq 0$.

Note.

- The numbers $\sigma_1, \sigma_2, \ldots$ are called *singular values* of A.
- The vectors $\mathbf{u}_1, \ldots, \mathbf{u}_m$ are called *left singular vectors* of A.
- Then the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are called *right singular vectors* of A.
- The formula $A = U\Sigma V^T$ is called a *singular value decomposition (SVD)* of A.
- The matrix Σ is uniquely determined, but U and V depend on some choices.