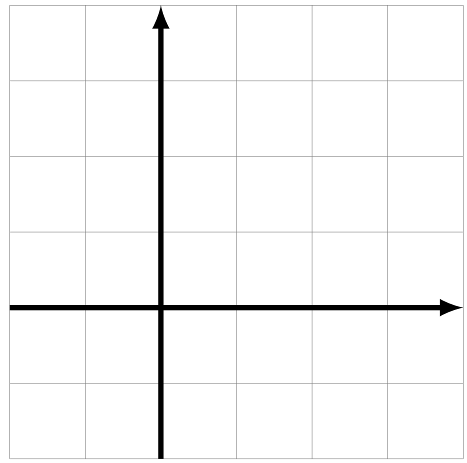
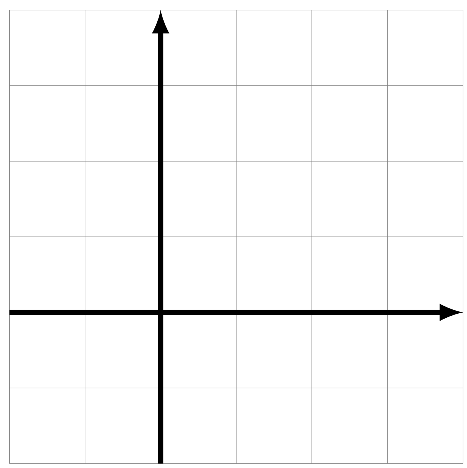
Question: How many solutions a system of linear equations can have?

Example: Systems of equations in 2 variables.

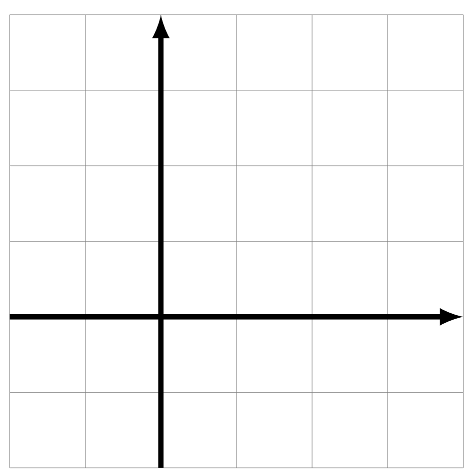
$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \end{cases}$$



$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + x_2 = 2 \end{cases}$$

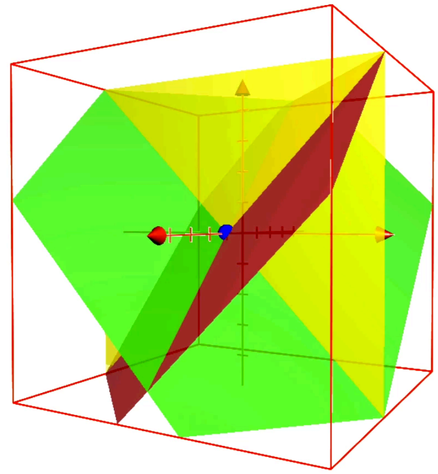


$$\begin{cases} x_1 + x_2 = 1 \\ 2x_1 + 2x_2 = 2 \end{cases}$$

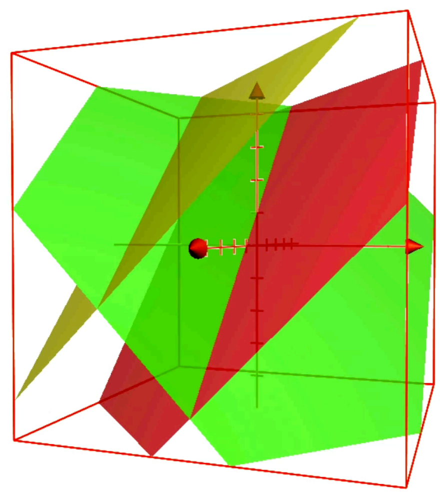


Example: Systems of equations in 3 variables.

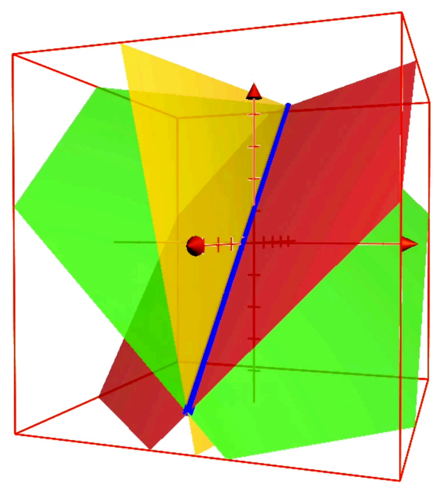
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 = 1 \end{cases}$$



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 6 \end{cases}$$



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 + 5x_2 + x_3 = 1 \end{cases}$$



In general:

A system of linear equations can have either

- no solutions
- exactly one solution
- infinitely many solutions

Definition

If as system of linear equations which has no solutions is called an *inconsistent system*. Otherwise the system is *consistent*.

Next:

How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$

make a matrix

[augmented matrix]

Gauss–Jordan elimination

[matrix in reduced row echelon form]

read off solutions

solutions

$$\begin{cases} x_1 = \dots \\ x_2 = \dots \\ x_3 = \dots \end{cases}$$