Definition

If $\mathbf{u} \in \mathbb{R}^n$ then the *length* (or the *norm*) of $\mathbf{u}$ is the number

$$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$

Note. If $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ then $||\mathbf{u}|| = \sqrt{a_1^2 + \ldots + a_n^2}$.

Properties of the norm:

1) $||u|| \geq 0$ and $||u|| = 0$ if and only if $\mathbf{u} = \mathbf{0}$.

2) $||cu|| = |c| \cdot ||u||$

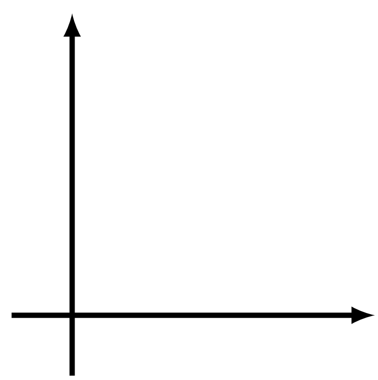
Definition

A vector $\mathbf{u} \in \mathbb{R}^n$ is an *unit vector* if $||\mathbf{u}|| = 1$.

Definition

If $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ then the *distance* between $\mathbf{u}$ and $\mathbf{v}$ is the number

$$\mathrm{dist}(\mathbf{u}, \mathbf{v}) = ||\mathbf{u} - \mathbf{v}||$$



Note. If $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ then

$$\mathrm{dist}(\mathbf{u}, \mathbf{v}) = \sqrt{(a_1 - b_1)^2 + \ldots + (a_n - b_n)^2}$$

Definition

Vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ are *orthogonal* if $\mathbf{u} \cdot \mathbf{v} = 0$.

Pythagorean Theorem

Vectors $\mathbf{u}, \mathbf{v}$ are orthogonal if and only if

$$||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$$

Definition

A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ in $\mathbb{R}^n$ is an *orthogonal set* if each pair each pair of vectors in this set is orthogonal, i.e.

$$\mathbf{v}_i \cdot \mathbf{v}_j = 0$$

for all $i \neq j$.

Example.

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is an orthogonal set in $\mathbb{R}^3$.

Example.

$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \right\}$ is another orthogonal set in $\mathbb{R}^3$.