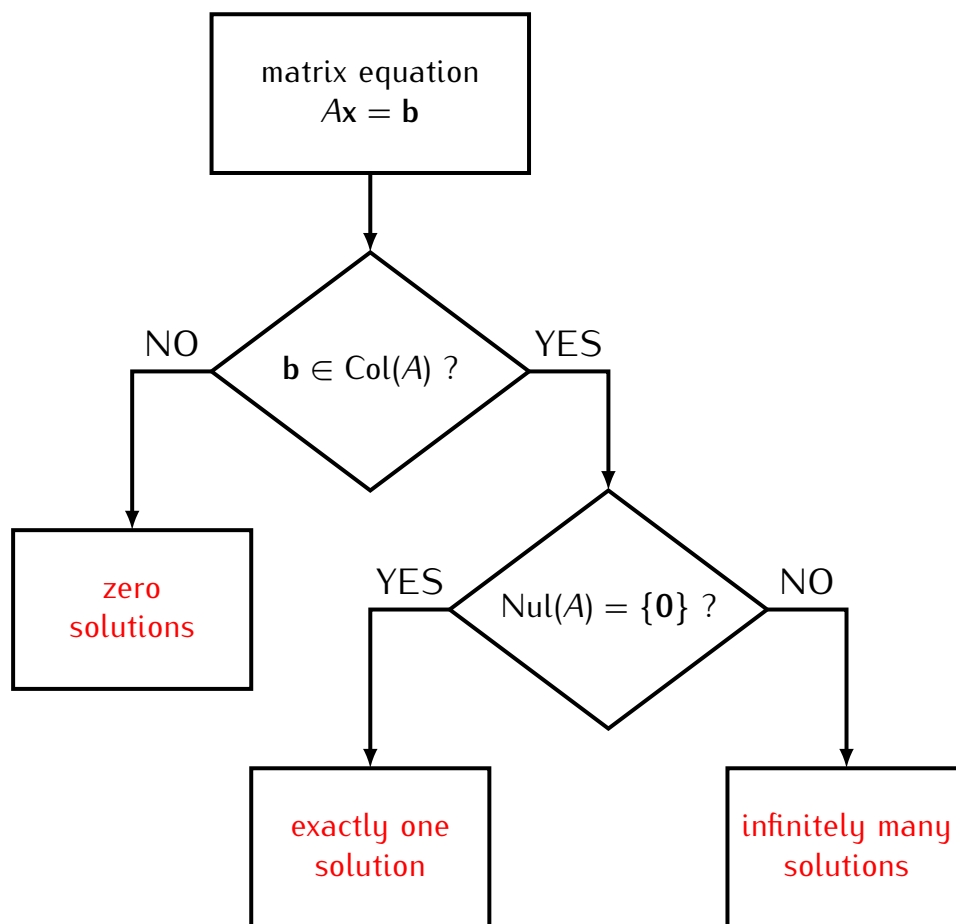


Recall:

- 1) We can multiply vectors by matrices.
- 2) Matrix equation:  $A\mathbf{x} = \mathbf{b}$



$\text{Col}(A)$  = (span of column vectors of  $A$ )

$\text{Nul}(A)$  = (set of solutions of  $A\mathbf{x} = \mathbf{0}$ )

**Recall:**  $\text{Nul}(A)$  can be always described as a span of some vectors.

**Example.** Find the null space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix}$$

**Example.** Solve the matrix equation  $A\mathbf{x} = \mathbf{b}$  where

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

## Proposition

Let  $\mathbf{v}_0$  be some chosen solution of a matrix equation  $A\mathbf{x} = \mathbf{b}$ . Then any other solution  $\mathbf{v}$  of this equation is of the form

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{n}$$

where  $\mathbf{n} \in \text{Nul}(A)$ .

