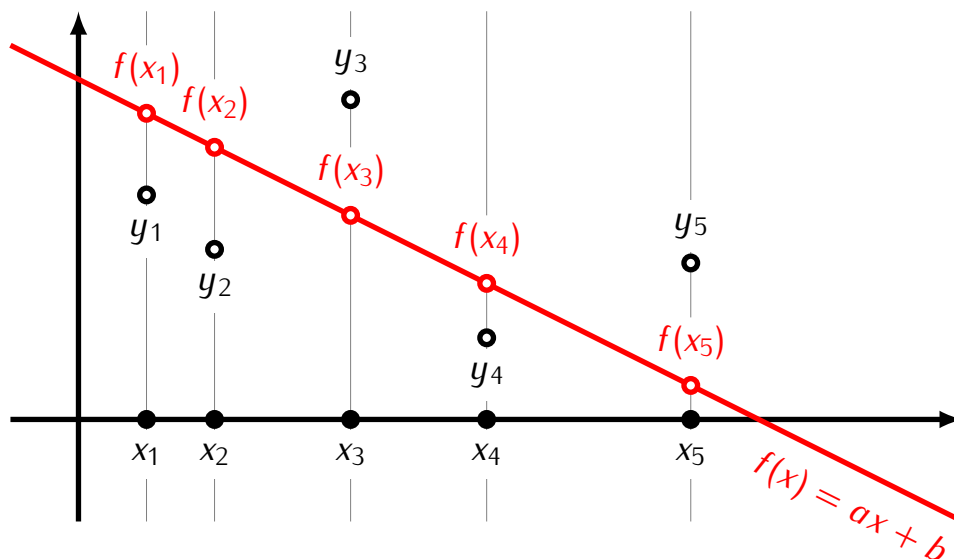


Application: Least square lines**Definition**

If  $(x_1, y_1), \dots, (x_p, y_p)$  are points on the plane then the *least square line* for these points is the line given by an equation  $f(x) = ax + b$  such that the number

$$\text{dist} \left( \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}, \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_p) \end{bmatrix} \right) = \sqrt{(y_1 - f(x_1))^2 + \dots + (y_p - f(x_p))^2}$$

is the smallest possible.

### Proposition

The line  $f(x) = ax + b$  is the least square line for points  $(x_1, y_1), \dots, (x_p, y_p)$  if the vector  $\begin{bmatrix} a \\ b \end{bmatrix}$  is the least square solution of the equation

$$\begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_p & 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

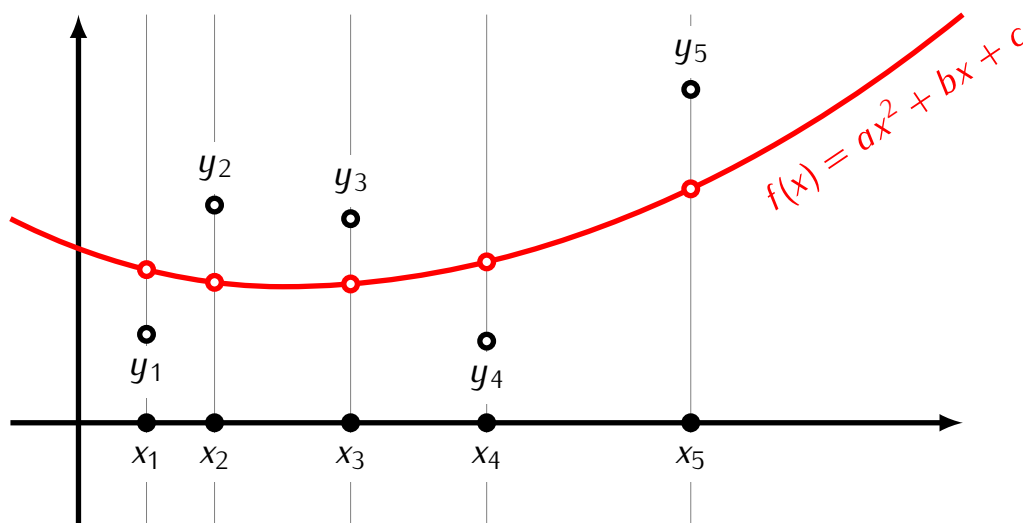
**Example.** Find the equation of the least square line for the points  $(0, 0)$ ,  $(1, 1)$ ,  $(3, 1)$ ,  $(5, 3)$ .



### Application: Least square curves

The above procedure can be used to determine curves other than lines that fit a set of points in the least square sense.

#### Example: Least square parabolas



#### Definition

If  $(x_1, y_1), \dots, (x_p, y_p)$  are points on the plane then the *least square parabola* for these points is the parabola given by an equation  $f(x) = ax^2 + bx + c$  such that the number

$$\text{dist} \left( \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}, \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_p) \end{bmatrix} \right) = \sqrt{(y_1 - f(x_1))^2 + \dots + (y_p - f(x_p))^2}$$

is the smallest possible.

### Proposition

The parabola  $f(x) = ax^2 + bx + c$  is the least square parabola for points  $(x_1, y_1), \dots, (x_p, y_p)$  if the vector  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$  is the least square solution of the equation

$$\begin{bmatrix} x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots \\ x_p^2 & x_p & 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix}$$

**Example.** Find the equation of the least square parabola for the points  $(-2, 2)$ ,  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 3)$ .

