

Example.

- Acme Inc. makes two types of widgets: **WG1** and **WG2**.
- Each widget must go through two processes: **assembly** and **testing**.
- The number of hours required to complete each process is as follows:

	assembly	testing
WG1	3	1
WG2	7	3

- Acme Inc. has three plants in New York, Texas, and Minnesota.
- Hourly cost (in dollars) of each process in each plant is as follows:

	NY	TX	MN
assembly	10	15	12
testing	15	20	15

Problem. What is the cost of producing each type of widgets in each plant?

Other operations on matrices

1) Addition.

If $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$ are $m \times n$ matrices then

$$A + B = \begin{bmatrix} a_{11} + b_{11} & \dots & a_{1n} + b_{1n} \\ \vdots & & \vdots \\ a_{m1} + b_{m1} & \dots & a_{mn} + b_{mn} \end{bmatrix}$$

Note. The sum $A + B$ is defined only if A and B have the same dimensions.

2) Scalar multiplication.

If $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$, and c is a scalar then

$$cA = \begin{bmatrix} ca_{11} & \dots & ca_{1n} \\ \vdots & & \vdots \\ ca_{m1} & \dots & ca_{mn} \end{bmatrix}$$

Properties of matrix algebra

1) $(AB)C = A(BC)$

2) $(A + B)C = AC + BC$
 $A(B + C) = AB + AC$

3) I_n = the $n \times n$ identity matrix:

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

If A is an $m \times n$ matrix then

$$A \cdot I_n = A$$

$$I_m \cdot A = A$$

Non-commutativity of matrix multiplication

1) If AB is defined then BA need not be defined.

2) Even if both AB and BA are both defined then usually

$$AB \neq BA$$