Recall: Nul(A) can be always described as a span of some vectors.

Example. Find the null space of the matrix

$$A = \left[\begin{array}{rrrr} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{array} \right]$$

Example. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

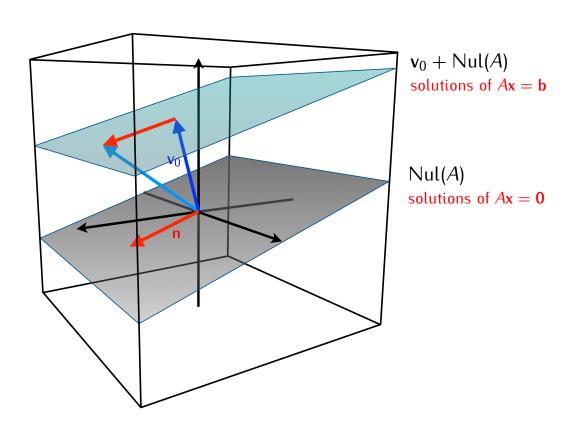
$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Proposition

Let \mathbf{v}_0 be some chosen solution of a matrix equation $A\mathbf{x}=\mathbf{b}$. Then any other solution \mathbf{v} of this equation is of the form

$$\mathbf{v}=\mathbf{v}_0+\mathbf{n}$$

where $n \in Nul(A)$.



Recall: If A is an $m \times n$ matrix then

$$A \cdot \left[\begin{array}{c} b_1 \\ \vdots \\ b_n \end{array} \right] = \left[\begin{array}{c} c_1 \\ \vdots \\ c_m \end{array} \right]$$

Definition

If A is an $m \times n$ matrix then the function

$$T_A \colon \mathbb{R}^n \to \mathbb{R}^m$$

given by $T_A(\mathbf{v}) = A\mathbf{v}$ is called the matrix transformation associated to A.