<u>Next:</u> From systems of linear equations to vector equations.

$$\begin{cases} x_1 + 2x_2 = 4 \\ 2x_1 + 7x_2 = 9 \\ 4x_1 + x_2 = 0 \end{cases} \quad \Longrightarrow \quad x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 0 \end{bmatrix}$$

Why vectors and vector equations are useful:

- They show up in many applications (velocity vectors, force vectors etc.)
- They give a better geometric picture of systems of linear equations.

Definition

A *column vector* is a matrix with one column.

Note. Columns of a matrix are column vectors.

Notation

$\mathbb{R}^n$ is the set of all column vectors with n entries.

Operations on vectors in $\mathbb{R}^n$

1) Addition of vectors:

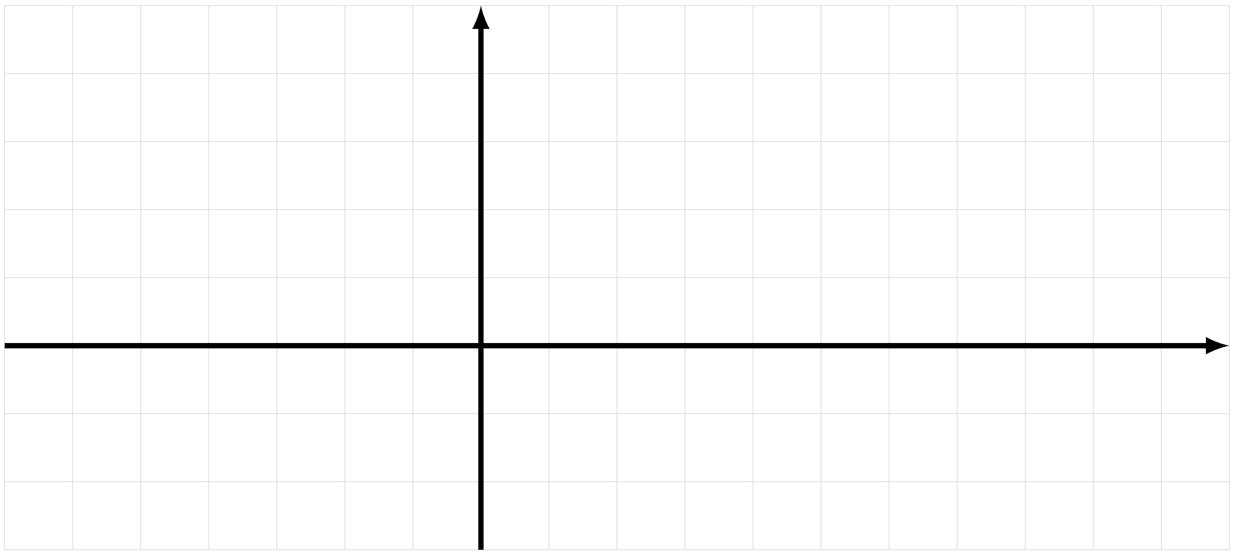
$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

2) Multiplication by scalars:

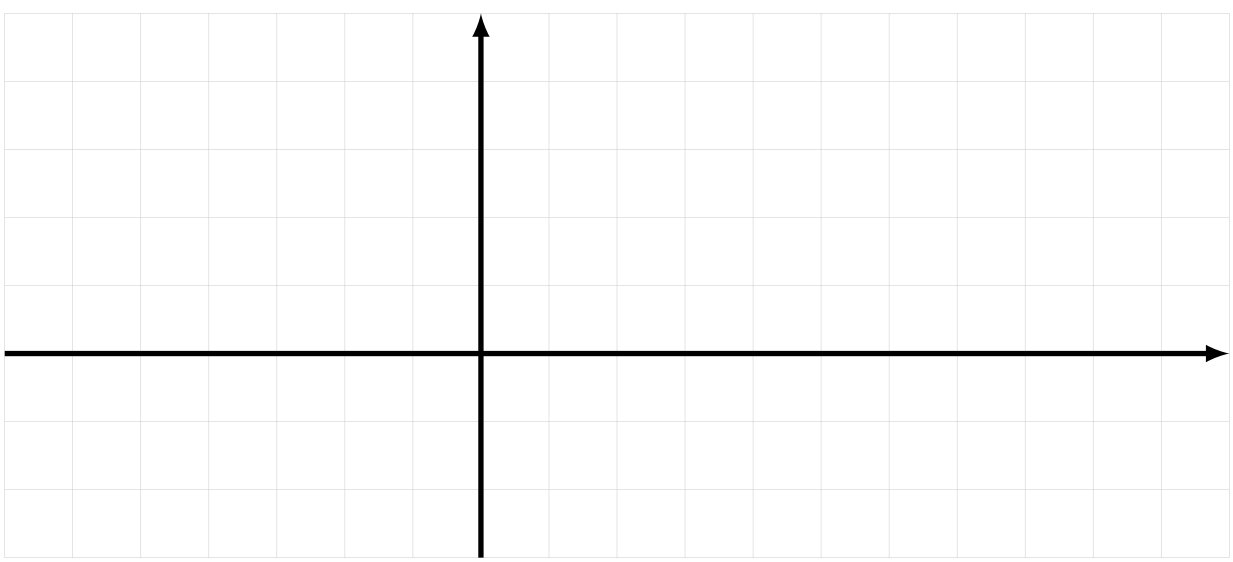
$$c \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} ca_1 \\ \vdots \\ ca_n \end{bmatrix}$$

Geometric interpretation of vectors in $\mathbb{R}^2$

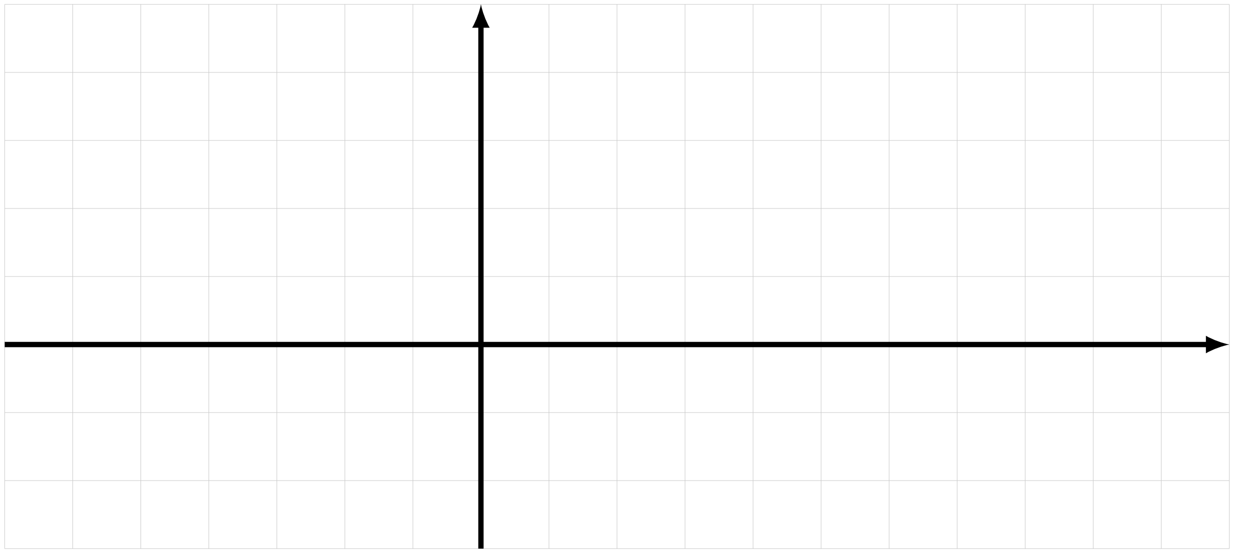
Vector coordinates:



Vector addition:



Scalar multiplication:



Vector equations

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \ldots + x_p\mathbf{v}_p = \mathbf{w}$$

Example. Solve the following vector equation:

$$x_1\begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

How to solve a vector equation

$$x_1\mathbf{v}_1 + \ldots + x_p\mathbf{v}_p = \mathbf{w}$$

vector of equation

make a matrix

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \ldots & \mathbf{v}_p & \mathbf{w} \end{array}\right]$$

augmented matrix

row reduction

$$\left[\text{ reduced matrix }\right]$$

read off solutions

$$\begin{cases} x_1 & = \ldots \\ \ldots & \ldots \\ x_p & = \ldots \end{cases}$$

solutions

Example: Target shooting.

At time $t = 0$ a target is observed at the position (x_0, y_0) moving in the direction of the vector v_t. The target is moving at such speed, that it travels the length of v_t in one second. A missile is positioned at the point $(0, 0)$. When fired, it will move vertically with such speed, that it will travel the length of the vector v_m in one second. After how many seconds should the missile be fired in order to intercept the target?