**Recall:**

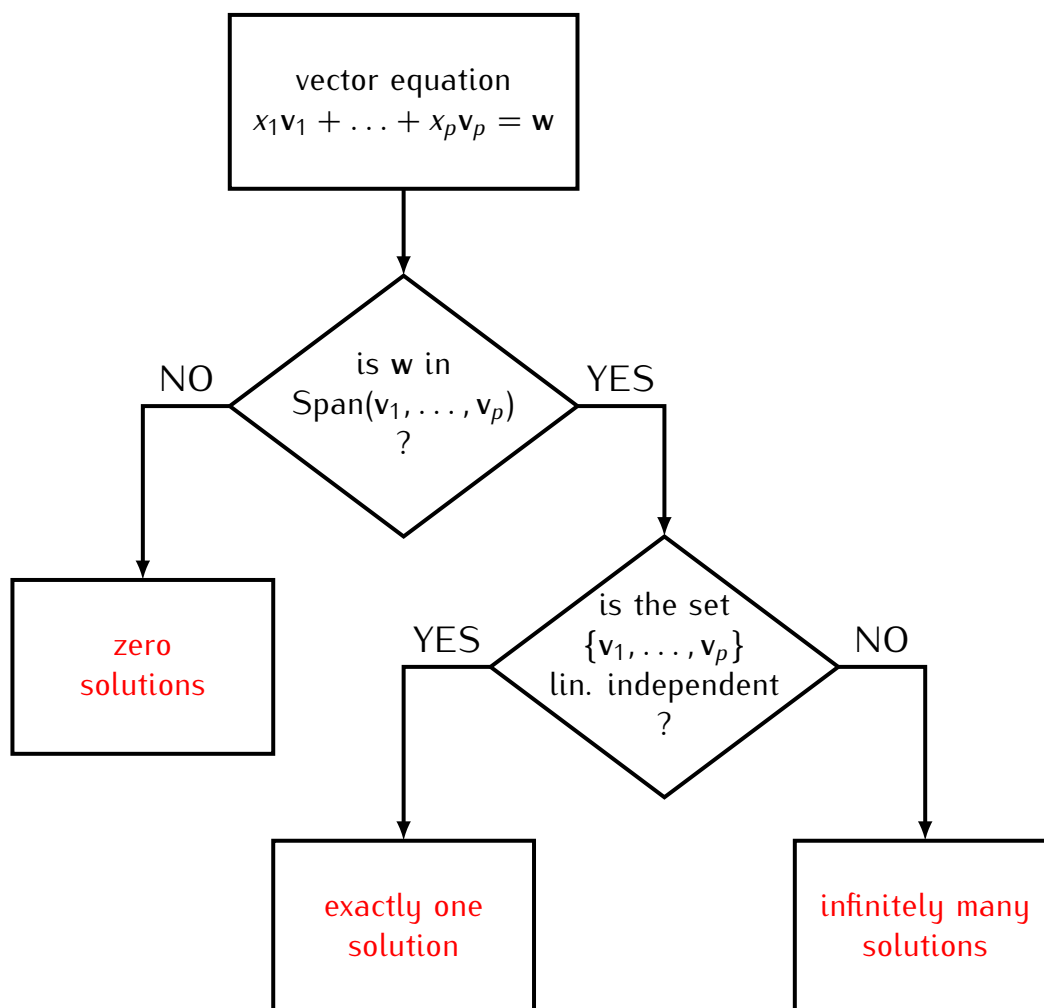**1)** $\text{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_p) = \left\{ \begin{array}{c} \text{the set of all} \\ \text{linear combinations} \\ c_1\mathbf{v}_1 + \ldots + c_p\mathbf{v}_p \end{array} \right\}$
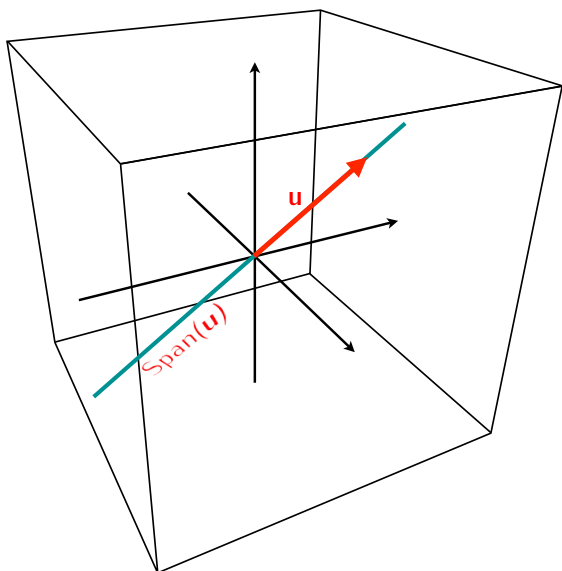
**2)** A set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly independent if the equation

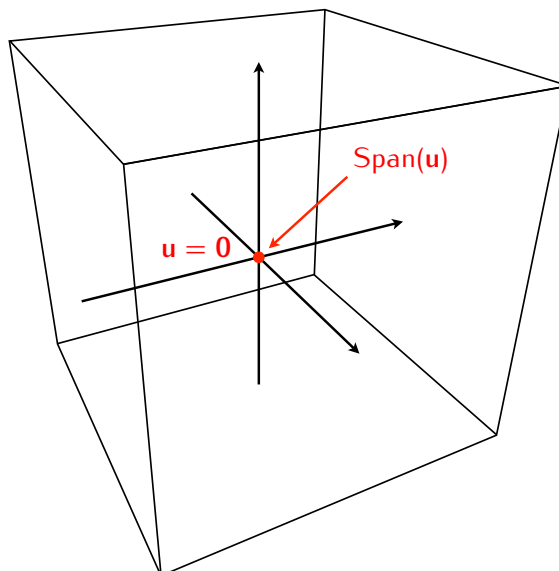$$x_1\mathbf{v}_1 + \ldots + x_p\mathbf{v}_p = \mathbf{0}$$

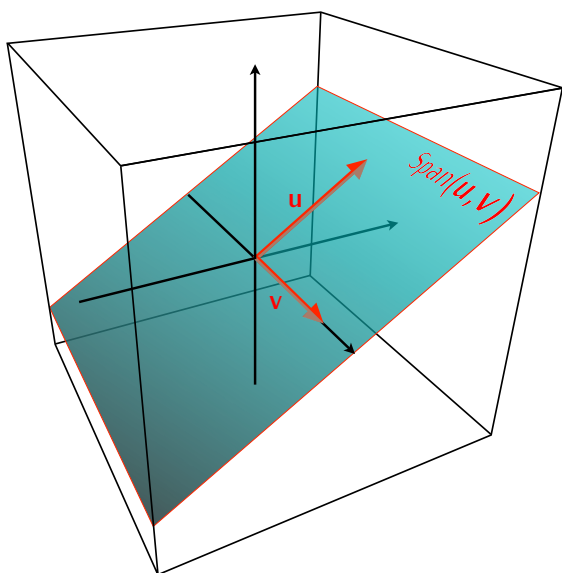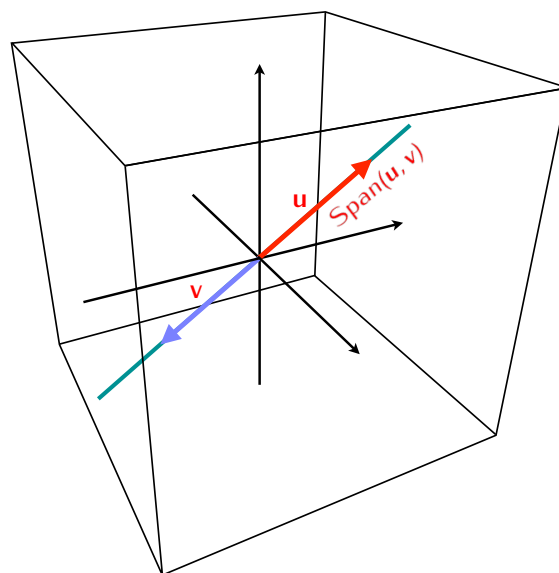has only one, trivial solution $x_1 = 0, \ldots, x_p = 0$.

```
                    ┌─────────────────────────┐
                    │    vector equation      │
                    │ x₁v₁ + ... + xₚvₚ = w   │
                    └─────────────────────────┘
                                │
                                ▼
                          ╱─────────────╲
              NO         ╱   is w in      ╲        YES
        ┌────────────── │ Span(v₁,...,vₚ)  │ ──────────────┐
        │                ╲       ?        ╱                │
        │                  ╲─────────────╱                 ▼
        ▼                                            ╱──────────────╲
  ┌──────────┐                           YES        ╱   is the set    ╲      NO
  │   zero   │                     ┌──────────────│ {v₁,...,vₚ}       │────────┐
  │solutions │                     │               ╲ lin. independent ╱        │
  └──────────┘                     │                 ╲      ?       ╱          │
                                   ▼                   ╲────────────╱           ▼
                            ┌──────────────┐                          ┌──────────────┐
                            │ exactly one  │                          │infinitely many│
                            │  solution    │                          │   solutions   │
                            └──────────────┘                          └──────────────┘
```

# Linear independence vs. Span



{**u**} linearly independent

{**u**} linearly dependent

{**u**, **v**} linearly independent

{**u**, **v**} linearly dependent

> **Theorem**
>
> If $\{v_1, \ldots, v_p\}$ is a linearly dependent set of vectors in then:
>
> **1)** for some $v_i$ we have $v_i \in \text{Span}(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_p)$.
>
> **2)** for some $v_i$ we have
>
> $$\text{Span}(v_1, \ldots, v_p) = \text{Span}(v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_p)$$

**Example.**

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$