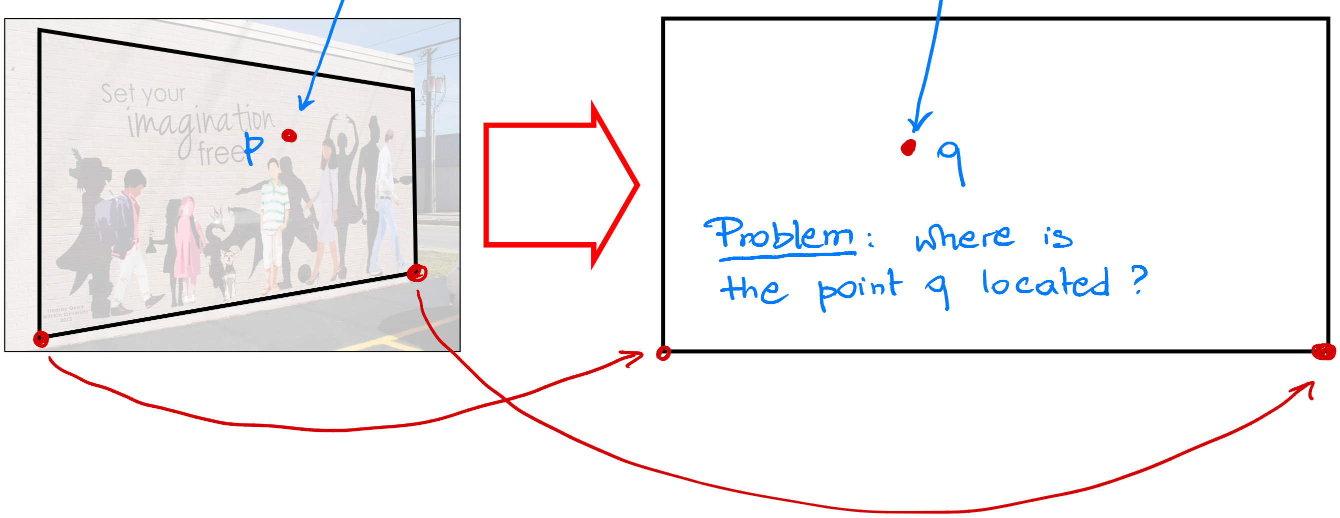
**What we want:**


photo taken at an angle straightened image

**What we have:**

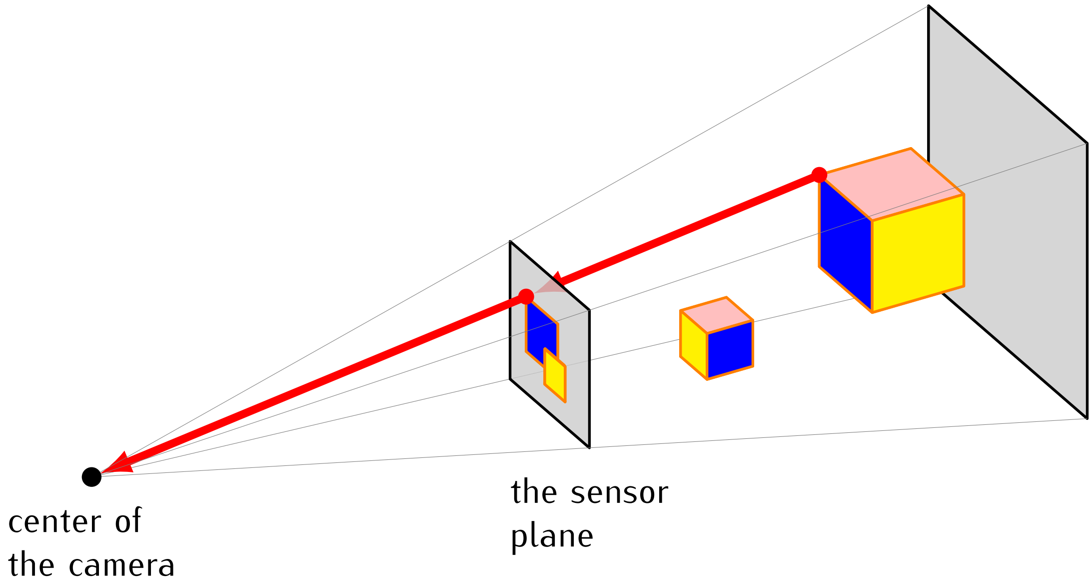
point of the original image

the corresponding point of the straightened image




Problem: where is the point q located ?

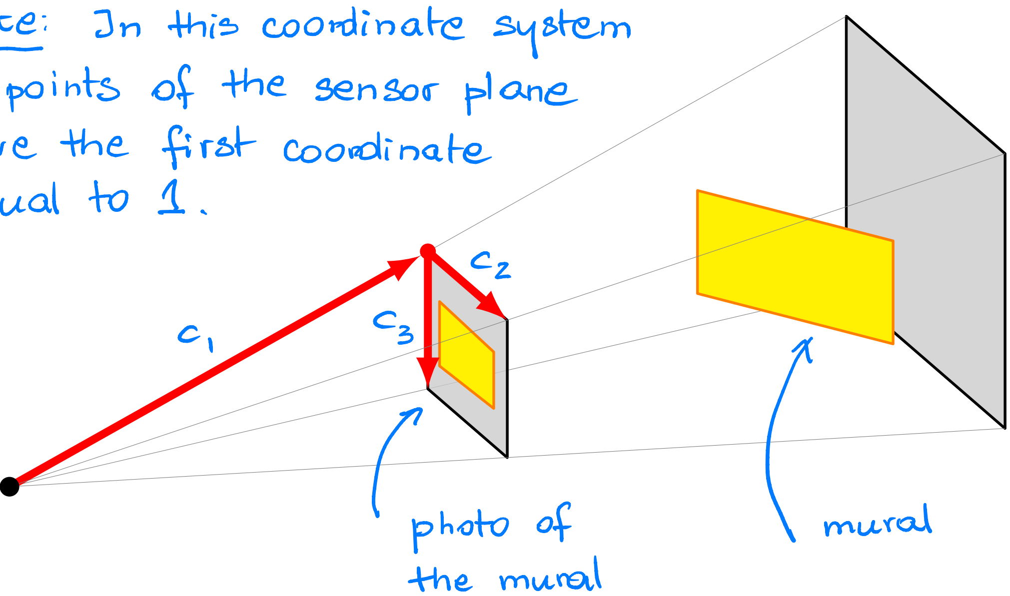
## Image formation in a camera

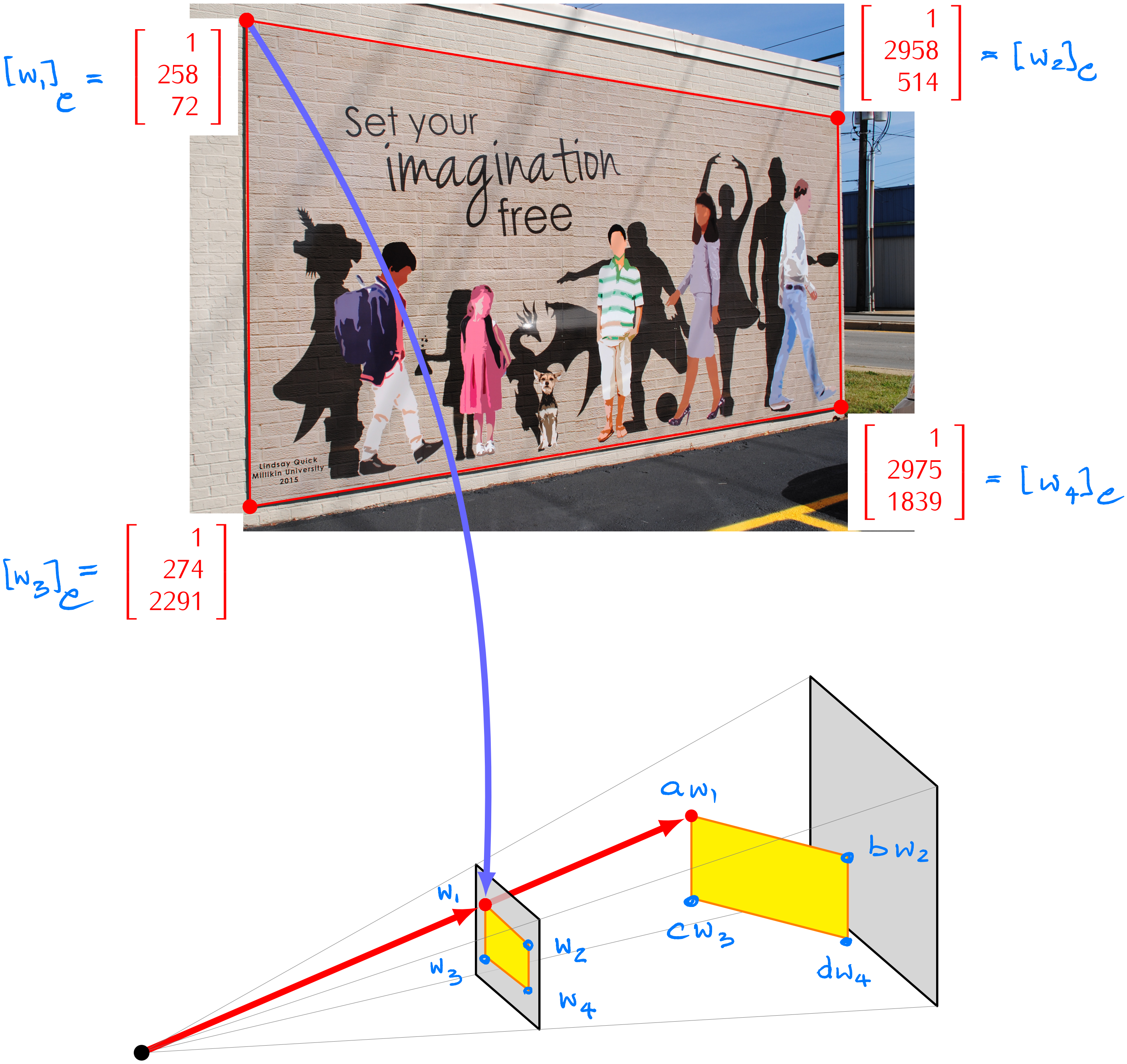



## The camera coordinate system $\mathcal{C}$

$\mathcal{C} = \{c_1, c_2, c_3\}$

Note: In this coordinate system all points of the sensor plane have the first coordinate equal to 1.






$$[aw_1]_e = a \cdot [w_1]_e = a \cdot \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix}$$

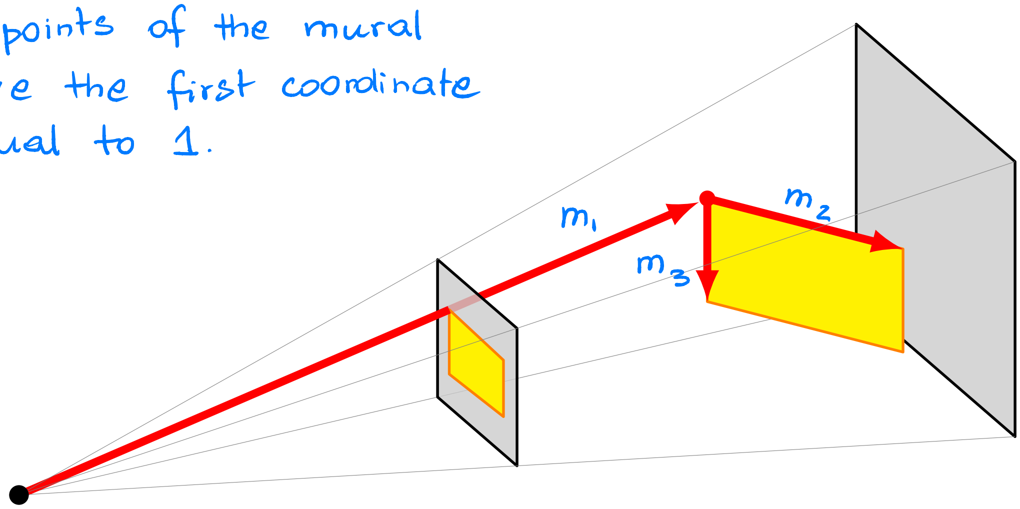
$$[bw_2]_e = b\,[w_2]_e = \dots$$

$$[cw_3]_e = c\,[w_3]_e = \dots$$

$$[dw_4]_e = d\,[w_4]_e = \dots$$

$\mathcal{M} = \{m_1, m_2, m_3\}$

Note: In this coordinate system all points of the mural have the first coordinate equal to 1.




Note: If the mural coordinates of a point $p$ are $[p]_\mu = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then the mural coordinates of the point $q = tp$ are

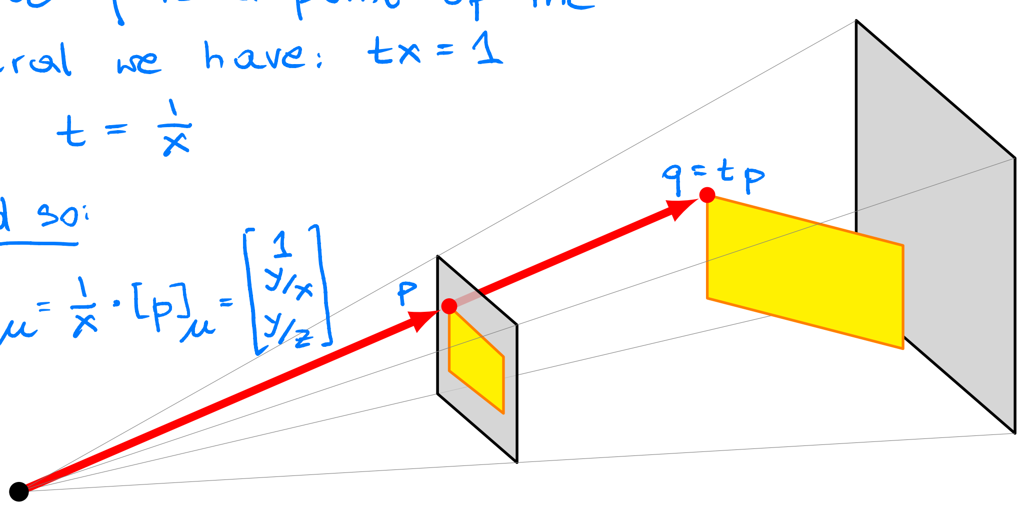
$$[q]_\mu = t[p]_\mu = t \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix}$$

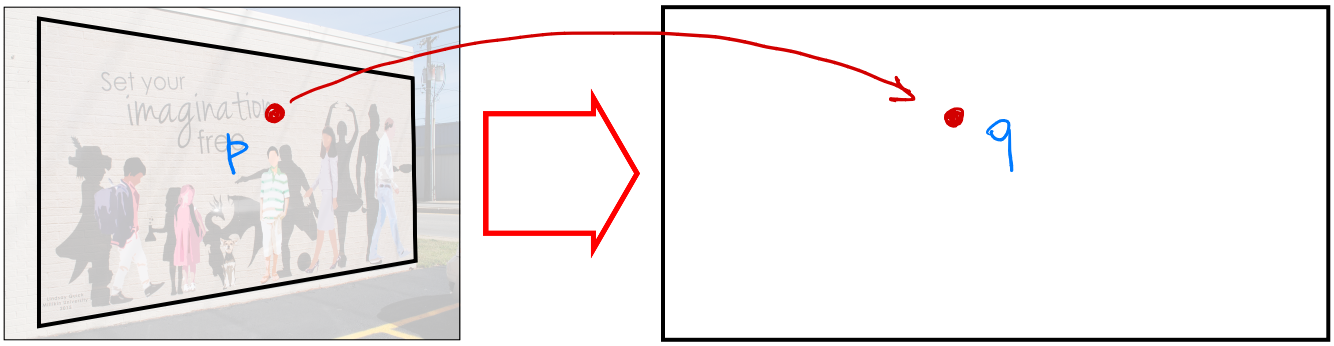
Since $q$ is a point of the mural we have: $tx = 1$

so: $t = \frac{1}{x}$

and so:

$$[q]_\mu = \frac{1}{x} \cdot [p]_\mu = \begin{bmatrix} 1 \\ y/x \\ y/z \end{bmatrix}$$






**Upshot:**

**We know:** $[p]_{\mathcal{C}}$ = the camera coordinates of $p$.

**We want:** $[q]_{\mathcal{M}}$ = the mural coordinates of $q$

**Strategy:** Compute $[p]_{\mathcal{M}}$ = the mural coordinates of $p$.

Then, if $[p]_{\mathcal{M}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then $[q]_{\mathcal{M}} = \begin{bmatrix} 1 \\ y/x \\ z/x \end{bmatrix}$

**Note:**

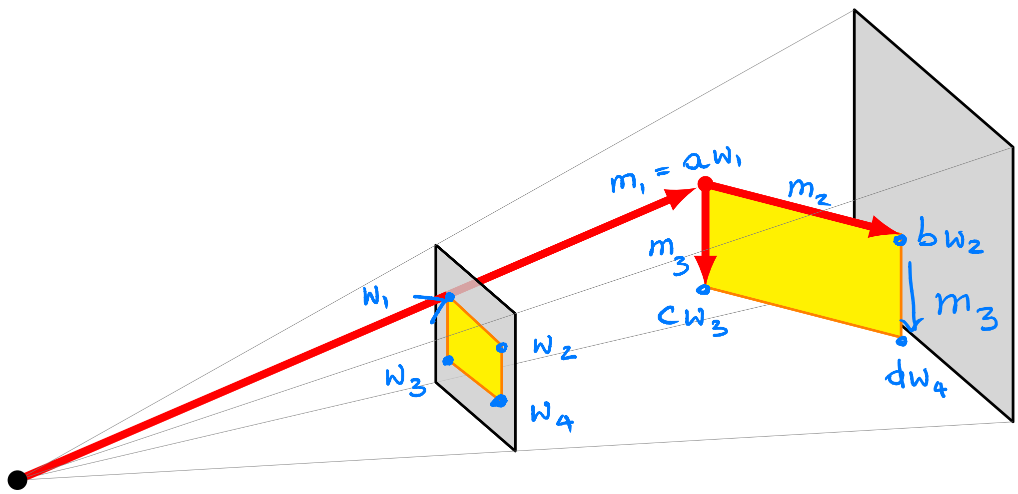
1) $[p]_{\mathcal{M}} = \underbrace{P_{\mathcal{M} \leftarrow \mathcal{C}}} \cdot [p]_{\mathcal{C}}$

the change of coordinates matrix from $\mathcal{C}$ to $\mathcal{M}$.

2) It will suffice to compute $P_{\mathcal{C} \leftarrow \mathcal{M}}$ since $P_{\mathcal{M} \leftarrow \mathcal{C}} = (P_{\mathcal{C} \leftarrow \mathcal{M}})^{-1}$.

## From mural coordinates to camera coordinates

$$P_{\mathcal{C}\leftarrow\mathcal{M}} = \left[\ [\mathbf{m}_1]_{\mathcal{C}}\ \ [\mathbf{m}_2]_{\mathcal{C}}\ \ [\mathbf{m}_3]_{\mathcal{C}}\ \right]$$




We have:

$$\boxed{m_1 = a w_1}$$

$m_1 + m_2 = b w_2$ so: $m_2 = b w_2 - m_1$

$$\boxed{m_2 = b w_2 - a w_1}$$

$m_1 + m_3 = c w_3$ so: $\boxed{m_3 = c w_3 - a w_1}$

This gives:

$$[m_1]_{\mathcal{C}} = a \cdot [w_1]_{\mathcal{C}} = a \cdot \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix}$$

$$[m_2]_{\mathcal{C}} = b \cdot [w_2]_{\mathcal{C}} - a[w_1]_{\mathcal{C}} = b \cdot \begin{bmatrix} 1 \\ 2958 \\ 514 \end{bmatrix} - a \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix}$$

$$[m_3]_{\mathcal{C}} = c \cdot [w_3]_{\mathcal{C}} - a[w_1]_{\mathcal{C}} = c \cdot \begin{bmatrix} 1 \\ 274 \\ 2291 \end{bmatrix} - a \cdot \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix}$$

**Problem:** What are the numbers $a$, $b$, $c$?

Note: $m_1 + m_2 + m_3 = d \cdot w_4$

This gives:

$$(\cancel{aw_1}) + (bw_2 - \cancel{aw_1}) + (cw_3 - aw_1) = dw_4$$

$$\boxed{bw_2 + cw_3 - aw_1 = dw_4}$$

So: $b[w_2]_\mu + c[w_3]_\mu - a[w_1]_\mu = d[w_4]_\mu$

$$\boxed{b \cdot \begin{bmatrix} 1 \\ 2958 \\ 514 \end{bmatrix} + c \begin{bmatrix} 1 \\ 274 \\ 2291 \end{bmatrix} - a \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix} = d \cdot \begin{bmatrix} 1 \\ 2975 \\ 1839 \end{bmatrix}}$$

Problem: 3 equations, 4 unknowns, so we can't have a single solution for $a, b, c, d$.

Good news:

1) For our computations the value of $d$ does not matter. We can set it to any non-zero number (e.g. $d = 1$)

2) Once the value of $d$ is fixed, the values of $a, b, c$ are uniquely determined. This lets us compute $[m_1]_e$, $[m_2]_e$, $[m_3]_e$, and so we obtain the matrix $P_{e \leftarrow \mu}$.