

$$A = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{km} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$$

$$AB = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \vdots & & \vdots \\ c_{k1} & \dots & c_{kn} \end{bmatrix}$$

$$c_{ij} = \underbrace{\begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{im} \end{bmatrix}}_{i^{\text{th}} \text{ row of } A} \cdot \underbrace{\begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{mj} \end{bmatrix}}_{j^{\text{th}} \text{ column of } B} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{im}b_{mj}$$

\uparrow
 the entry in
 i^{th} row
 j^{th} column

Example.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ 4 & 5 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

2×3 3×4
AB is defined and
it is a 2×4 matrix

$$AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}$$

$$c_{11} = [1^{\text{st}} \text{ row of } A] \cdot \begin{bmatrix} 1^{\text{st}} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [0 \ 1 \ 2] \cdot \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} = 0 \cdot 0 + 1 \cdot 4 + 2 \cdot 1 = \underline{\underline{6}}$$

$$c_{12} = [1^{\text{nd}} \text{ row of } A] \cdot \begin{bmatrix} 2^{\text{nd}} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [0 \ 1 \ 2] \cdot \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} = 0 \cdot (-1) + 1 \cdot 5 + 2 \cdot 2 = \underline{\underline{9}}$$

\vdots

$$c_{24} = [2^{\text{nd}} \text{ row of } A] \cdot \begin{bmatrix} 4^{\text{th}} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = [3 \ 4 \ 5] \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 3 \cdot 1 + 4 \cdot 0 + 5 \cdot 1 = \underline{\underline{8}}$$

$$AB = \begin{bmatrix} 6 & 9 & 7 & 2 \\ 21 & 27 & 25 & 8 \end{bmatrix}$$

Example.

- Acme Inc. makes two types of widgets: WG1 and WG2.
- Each widget must go through two processes: **assembly** and **testing**.
- The number of hours required to complete each process is as follows:

$$A = \begin{array}{c|cc} & \text{assembly} & \text{testing} \\ \hline \text{WG1} & 3 & 1 \\ \text{WG2} & 7 & 3 \end{array}$$

- Acme Inc. has three plants in New York, Texas, and Minnesota.
- Hourly cost (in dollars) of each process in each plant is as follows:

$$B = \begin{array}{c|ccc} & \text{NY} & \text{TX} & \text{MN} \\ \hline \text{assembly} & 10 & 15 & 12 \\ \text{testing} & 15 & 20 & 15 \end{array}$$

Problem. What is the cost of producing each type of widgets in each plant?

Cost of WG1 in TX: $3 \cdot 15 + 1 \cdot 20 = \65

$$[3 \ 1] \cdot \begin{bmatrix} 10 \\ 15 \end{bmatrix} = [\text{1st row } A] \cdot \begin{bmatrix} \text{2nd column} \\ B \end{bmatrix}$$

Cost of WG2 in NY = $7 \cdot 10 + 3 \cdot 15 = \115

$$[7 \ 3] \cdot \begin{bmatrix} 10 \\ 15 \end{bmatrix} = [\text{2nd row } A] \cdot \begin{bmatrix} \text{1st column} \\ B \end{bmatrix}$$

$$A \cdot B = \begin{array}{c|ccc} & \text{NY} & \text{TX} & \text{MN} \\ \hline \text{WG1} & 45 & 65 & 51 \\ \text{WG2} & 115 & 165 & 129 \end{array}$$