Definition

Let A be an $m \times n$ matrix with columns $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ and let $\mathbf{w}$ be a vector in $\mathbb{R}^n$:

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_n \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

The product $A\mathbf{w}$ is a vector in $\mathbb{R}^m$ given by

$$A\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \ldots + c_n\mathbf{v}_n$$

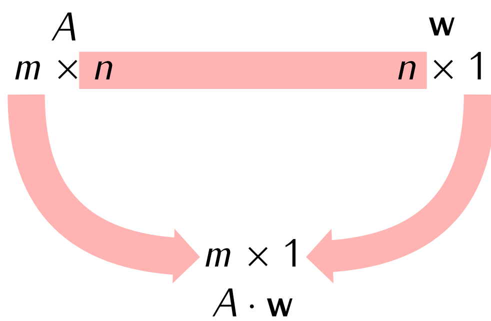
Example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad \mathbf{w} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

Properties of matrix-vector multiplication

1) The product $A\mathbf{w}$ is defined only if

$$(\text{number of columns of } A) = (\text{number of entries of } \mathbf{w})$$



2) $A(\mathbf{v} + \mathbf{w}) = A\mathbf{v} + A\mathbf{w}$

3) If c is a scalar then $A(c\mathbf{w}) = c(A\mathbf{w})$.

Example. Solve the matrix equation

$$\begin{bmatrix} 1 & 1 & -4 \\ 1 & -2 & 3 \\ 3 & -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

How to solve a matrix equation

$$A\mathbf{x} = \mathbf{b}$$

matrix equation

$$\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$$

augmented matrix

row reduction

$$\left[\text{ reduced matrix } \right]$$

read off solutions

$$\mathbf{x} = \ldots$$

solutions

Recall: A vector equation

$$x_1\mathbf{v}_1 + \ldots + x_n\mathbf{v}_n = \mathbf{b}$$

has a solution if and only if $\mathbf{b} \in \mathrm{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_n)$.

Definition

If A is a matrix with columns $\mathbf{v}_1, \ldots, \mathbf{v}_n$:

$$A = \begin{bmatrix} \mathbf{v}_1 & \ldots & \mathbf{v}_n \end{bmatrix}$$

then the set $\mathrm{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ is called the *column space* of A and it is denoted $\mathrm{Col}(A)$.