Recall:

1) If A is an $m \times n$ matrix then the function

$$T_A \colon \mathbb{R}^n \to \mathbb{R}^m$$

defined by $T_A(\mathbf{v}) = A\mathbf{v}$ is called the matrix transformation associated to A.

2) A function $T \colon \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation if

(ii) $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$
(ii) $T(c\mathbf{v}) = cT(\mathbf{v})$

3) Every matrix transformation is a linear transformation.

4) Every linear transformation $T \colon \mathbb{R}^n \to \mathbb{R}^m$ is a matrix transformation:

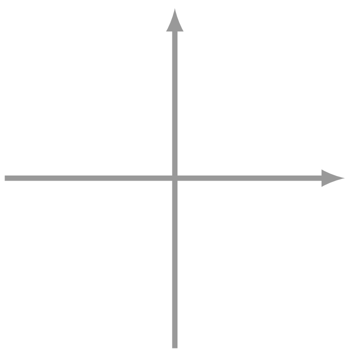
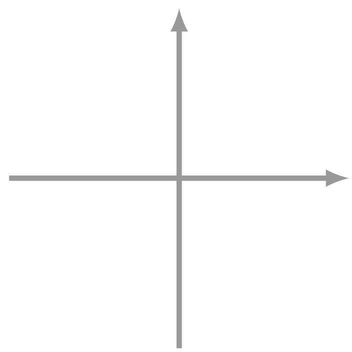
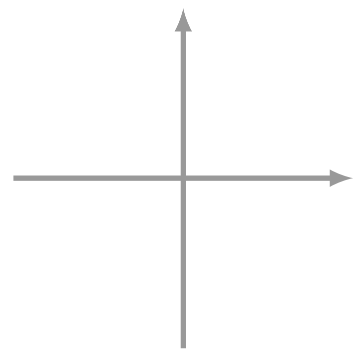
$$T(\mathbf{v}) = A\mathbf{v}$$

where

$$A = \begin{bmatrix} T(\mathbf{e}_1) & T(\mathbf{e}_2) & \dots & T(\mathbf{e}_n) \end{bmatrix}$$

The matrix A is called the standard matrix of T.

Composition of linear transformations

Theorem

If $S\colon \mathbb{R}^n \to \mathbb{R}^m$ and $T\colon \mathbb{R}^m \to \mathbb{R}^k$ are linear transformation then the composition

$$T \circ S\colon \mathbb{R}^n \to \mathbb{R}^k$$

is also a linear transformation.

Upshot. The function $T \circ S$ is represented by some matrix C:

$$T \circ S(\mathbf{v}) = C\mathbf{v}$$

Question. Let $S\colon \mathbb{R}^n \to \mathbb{R}^m$ and $T\colon \mathbb{R}^m \to \mathbb{R}^k$ be linear transformations, and let

- B is the standard matrix of S
- A is the standard matrix of T

What if the standard matrix of $T \circ S\colon \mathbb{R}^n \to \mathbb{R}^k$?

Definition

Let

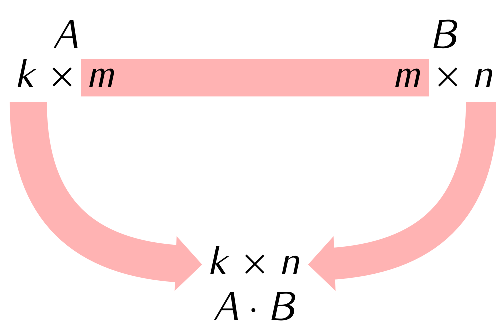
- A be an $k \times m$ matrix
- $B = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_n \end{bmatrix}$ be an $m \times n$ matrix

Then $A \cdot B$ is an $k \times n$ matrix given by

$$A \cdot B = \begin{bmatrix} A\mathbf{v}_1 & A\mathbf{v}_2 & \dots & A\mathbf{v}_n \end{bmatrix}$$

Note. The product $A \cdot B$ is defined only if

$$\text{(number of columns of } A) \;=\; \text{(number of rows of } B)$$



Example.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ 4 & 5 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$