Example. Compute A^{-1} for

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Recall: If A is an invertible matrix then the equation $A\mathbf{x} = \mathbf{b}$ has only one solution: $\mathbf{x} = A^{-1}\mathbf{b}$.

Definition

If A is an $n \times n$ matrix and $\mathbf{b} \in \mathbb{R}^n$ then $A_i(\mathbf{b})$ is the matrix obtained by replacing the i^{th} column of A with $\mathbf{b}$.

Example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

Theorem (Cramer's Rule)

If A is an $n \times n$ invertible matrix and $\mathbf{b} \in \mathbb{R}^n$ then the unique solution of the equation

$$A\mathbf{x} = \mathbf{b}$$

is given by

$$\mathbf{x} = \frac{1}{\det A} \begin{bmatrix} \det A_1(\mathbf{b}) \\ \vdots \\ \det A_n(\mathbf{b}) \end{bmatrix}$$

Example. Solve the equation

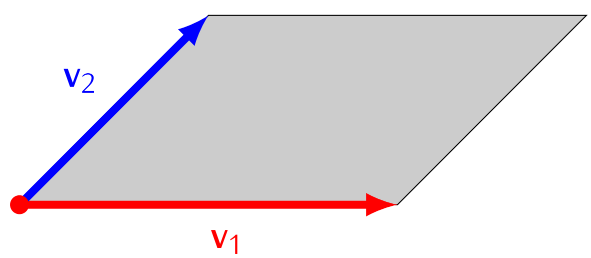
$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Recall:

A → $\det A$ → $= 0$ if A is not invertible

matrix number $\neq 0$ if A is invertible

Note. Any two vectors in $\mathbb{R}^2$ define a parallelogram:



Notation

$$\text{area}(\mathbf{v}_1, \mathbf{v}_2) = \begin{pmatrix} \text{area of the parallelogram} \\ \text{defined by } \mathbf{v}_1 \text{ and } \mathbf{v}_2 \end{pmatrix}$$