

Next: From systems of linear equations to vector equations.

$$\begin{cases} x_1 + 2x_2 = 4 \\ 2x_1 + 7x_2 = 9 \\ 4x_1 + x_2 = 0 \end{cases} \quad \Rightarrow \quad x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 0 \end{bmatrix}$$

**Why vectors and vector equations are useful:**

- They show up in many applications (velocity vectors, force vectors etc.)
- They give a better geometric picture of systems of linear equations.

### Definition

A *column vector* is a matrix with one column.

**Note.** Columns of a matrix are column vectors.

### Notation

$\mathbb{R}^n$  is the set of all column vectors with  $n$  entries.

## Operations on vectors in $\mathbb{R}^n$

### 1) Addition of vectors:

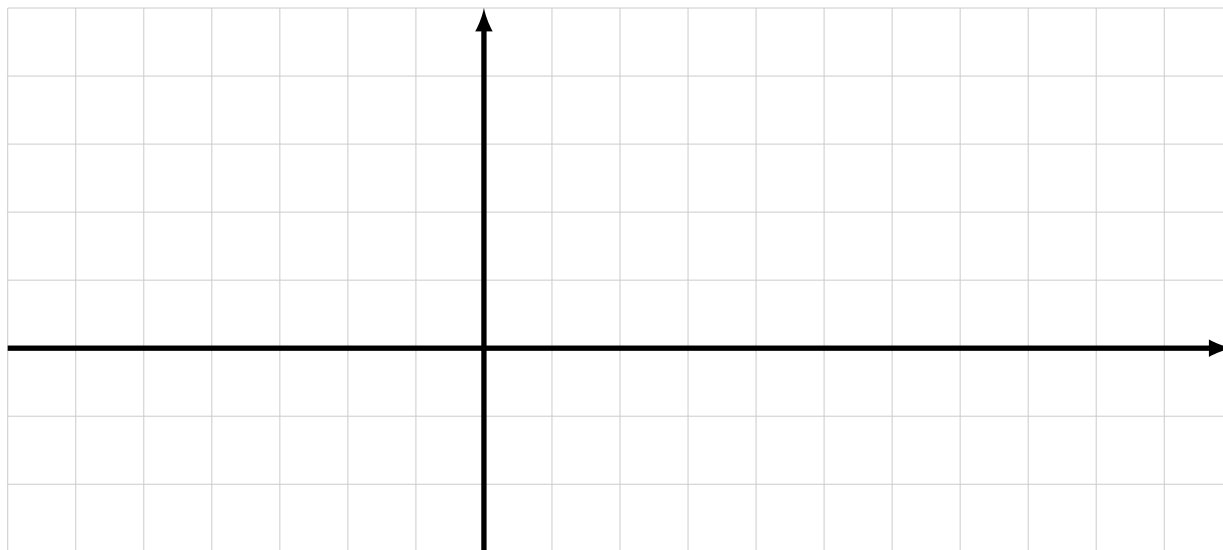
$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

### 2) Multiplication by scalars:

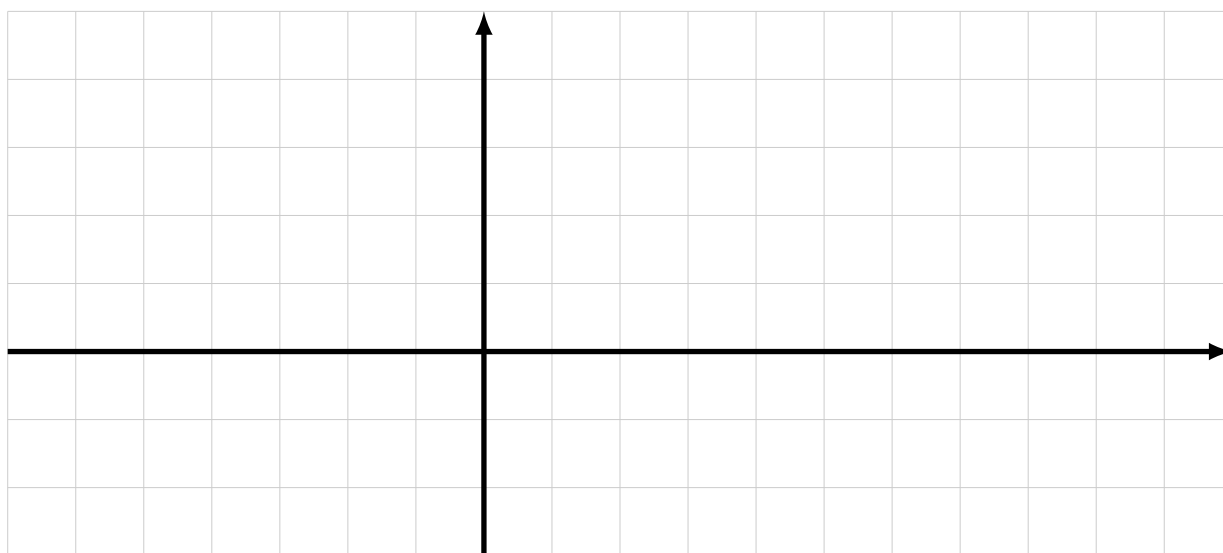
$$c \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} ca_1 \\ \vdots \\ ca_n \end{bmatrix}$$

## Geometric interpretation of vectors in $\mathbb{R}^2$

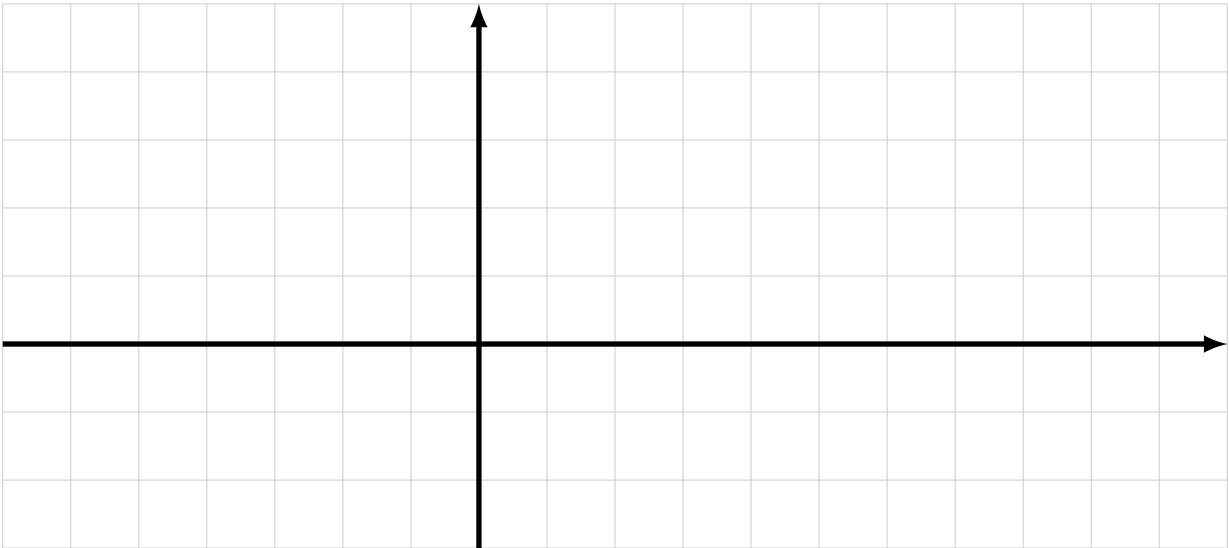
Vector coordinates:



Vector addition:



Scalar multiplication:



## Vector equations

$$x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + \dots + x_p \mathbf{v}_p = \mathbf{w}$$

**Example.** Solve the following vector equation:

$$x_1 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

## How to solve a vector equation

$$x_1 \mathbf{v}_1 + \dots + x_p \mathbf{v}_p = \mathbf{w}$$

vector of equation

*make  
a matrix*

$$\left[ \mathbf{v}_1 \quad \dots \quad \mathbf{v}_p \mid \mathbf{w} \right]$$

augmented matrix

*row  
reduction*

$$\left[ \text{reduced matrix} \right]$$

*read off  
solutions*

$$\begin{cases} x_1 = \dots \\ \dots \dots \\ x_p = \dots \end{cases}$$

solutions

**Example:** Target shooting.

At time  $t = 0$  a target is observed at the position  $(x_0, y_0)$  moving in the direction of the vector  $v_t$ . The target is moving at such speed, that it travels the length of  $v_t$  in one second. A missile is positioned at the point  $(0, 0)$ . When fired, it will move vertically with such speed, that it will travel the length of the vector  $v_m$  in one second. After how many seconds should the missile be fired in order to intercept the target?

