

**Problem:** What are the numbers  $a$ ,  $b$ ,  $c$ ?

**Definition**

If

$$\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

are vectors in  $\mathbb{R}^n$  then the *inner product* (or *dot product*) of  $\mathbf{u}$  and  $\mathbf{v}$  is the number

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + \dots + a_n b_n$$

**Properties of the dot product:**

- 1)  $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
- 2)  $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$
- 3)  $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$
- 4)  $\mathbf{u} \cdot \mathbf{u} \geq 0$  and  $\mathbf{u} \cdot \mathbf{u} = 0$  if and only if  $\mathbf{u} = \mathbf{0}$ .

### Definition

If  $\mathbf{u} \in \mathbb{R}^n$  then the *length* (or the *norm*) of  $\mathbf{u}$  is the number

$$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$

**Note.** If  $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$  then  $\|\mathbf{u}\| = \sqrt{a_1^2 + \dots + a_n^2}$ .

### Properties of the norm:

- 1)  $\|\mathbf{u}\| \geq 0$  and  $\|\mathbf{u}\| = 0$  if and only if  $\mathbf{u} = \mathbf{0}$ .
- 2)  $\|c\mathbf{u}\| = |c| \cdot \|\mathbf{u}\|$

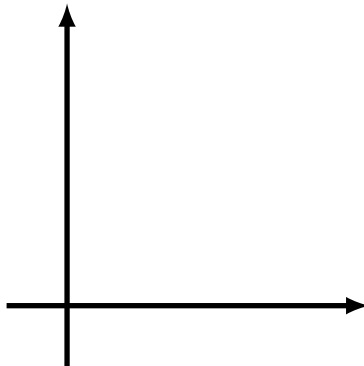
### Definition

A vector  $\mathbf{u} \in \mathbb{R}^n$  is an *unit vector* if  $\|\mathbf{u}\| = 1$ .

### Definition

If  $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$  then the *distance* between  $\mathbf{u}$  and  $\mathbf{v}$  is the number

$$\text{dist}(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|$$



**Note.** If  $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ ,  $\mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$  then

$$\text{dist}(\mathbf{u}, \mathbf{v}) = \sqrt{(a_1 - b_1)^2 + \dots + (a_n - b_n)^2}$$