Example.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \text{det } A = 5 > 0 \qquad T_A : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$T_A(a_2) : \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$T_A(a_1) : \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

TA preserves direction of angles between vectors (we say that TA preserves orientation)

Example.

$$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} \quad \det A = -4 < 0 \qquad T_A : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$

$$T_A(a_1) : \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$T_A(a_2) : \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

TA reverses direction of angles between vectors. (We say that TA reverses orientation).

Theorem

If A is a 2×2 matrix then the linear transformation $T_A \colon \mathbb{R}^2 \to \mathbb{R}^2$ preserves orientation if $\det A > 0$ and reverses orientation if $\det A < 0$.