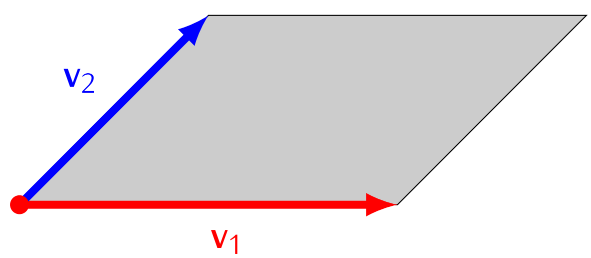
Recall:

A (matrix) $\longrightarrow$ $\det A$ (number)

- $= 0$ if A is not invertible
- $\neq 0$ if A is invertible

Note. Any two vectors in $\mathbb{R}^2$ define a parallelogram:



Notation

$$\text{area}(\mathbf{v}_1, \mathbf{v}_2) = \begin{pmatrix} \text{area of the parallelogram} \\ \text{defined by } \mathbf{v}_1 \text{ and } \mathbf{v}_2 \end{pmatrix}$$

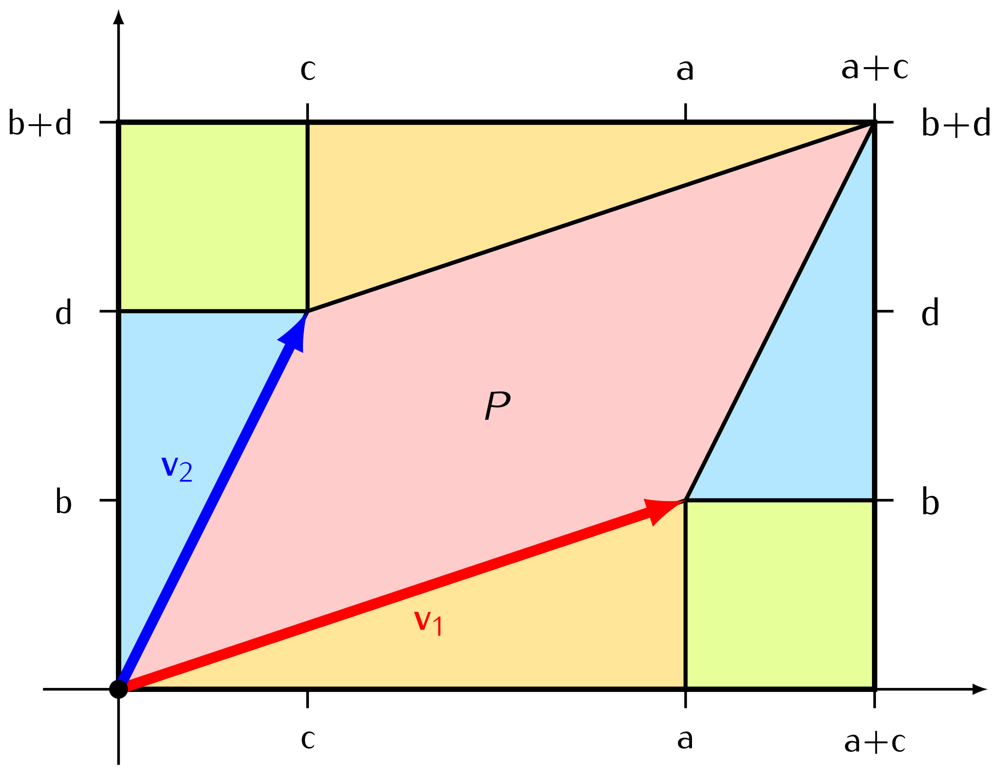
Theorem

If $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^2$ then

$$\text{area}(\mathbf{v}_1, \mathbf{v}_2) = \left| \det \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \right|$$

Idea of the proof.

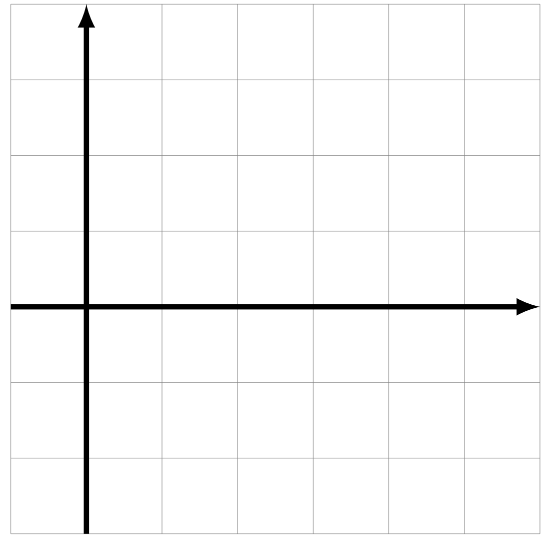
$$\mathbf{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} c \\ d \end{bmatrix}$$



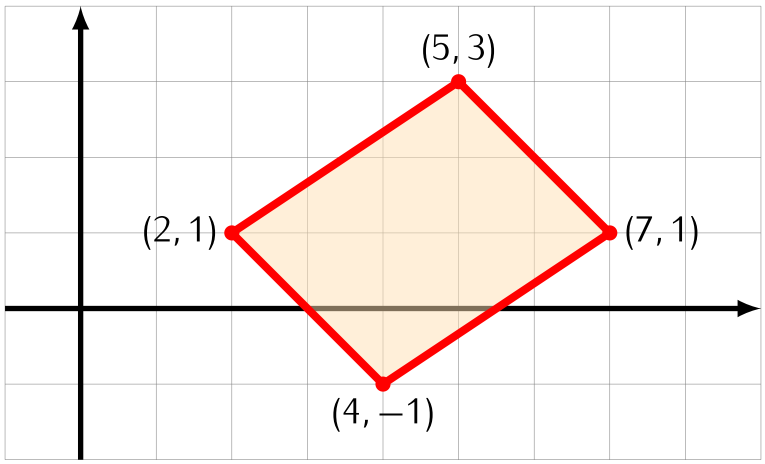
$$\begin{array}{r} \text{area}(P) \\ +\ \frac{1}{2}ab \\ +\ \frac{1}{2}ab \\ +\ \frac{1}{2}cd \\ +\ \frac{1}{2}cd \\ +\ cb \\ +\ cb \\ \hline (a+c)(b+d) \end{array}$$

Example.

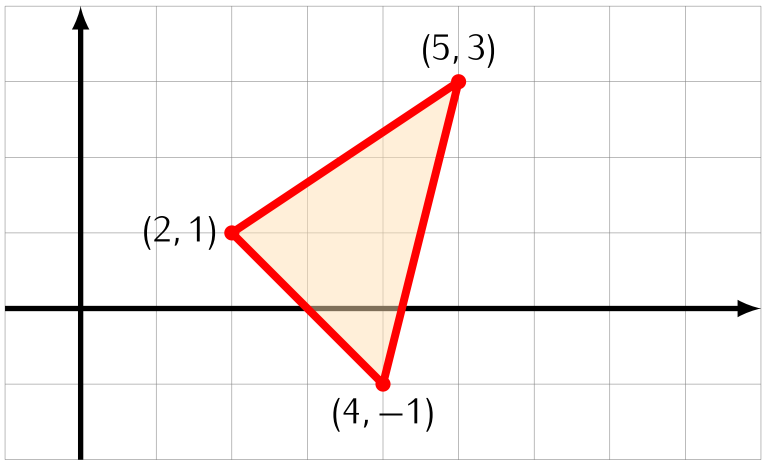
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



Example. Calculate the area of the parallelogram with vertices at the points $(2, 1)$, $(5, 3)$, $(7, 1)$, $(4, -1)$.



Example. Calculate the area of the triangle with vertices at the points $(2, 1)$, $(5, 3)$, $(4, -1)$.



Note. In order to compute areas of other polygons, subdivide them into triangles.