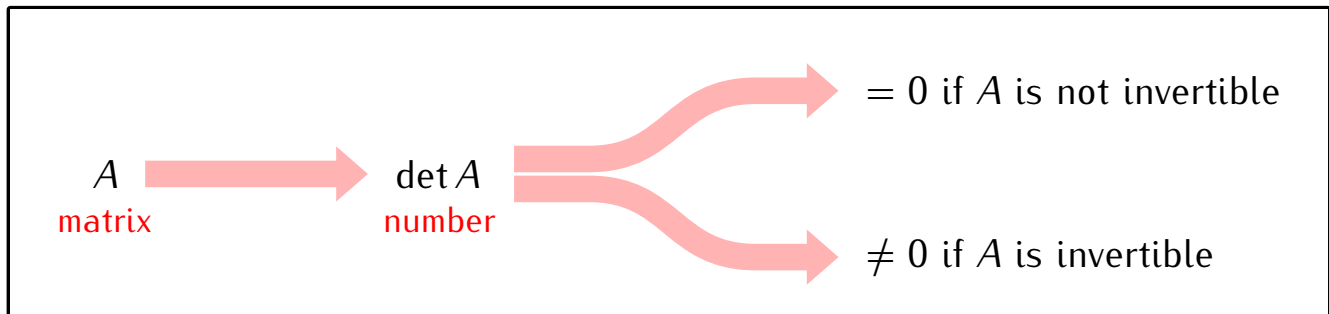
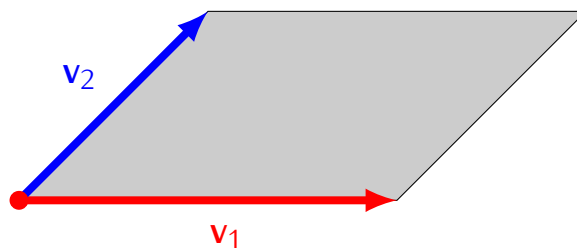


Recall:

Note. Any two vectors in \mathbb{R}^2 define a parallelogram:

**Notation**

$$\text{area}(v_1, v_2) = \left(\begin{array}{l} \text{area of the parallelogram} \\ \text{defined by } v_1 \text{ and } v_2 \end{array} \right)$$

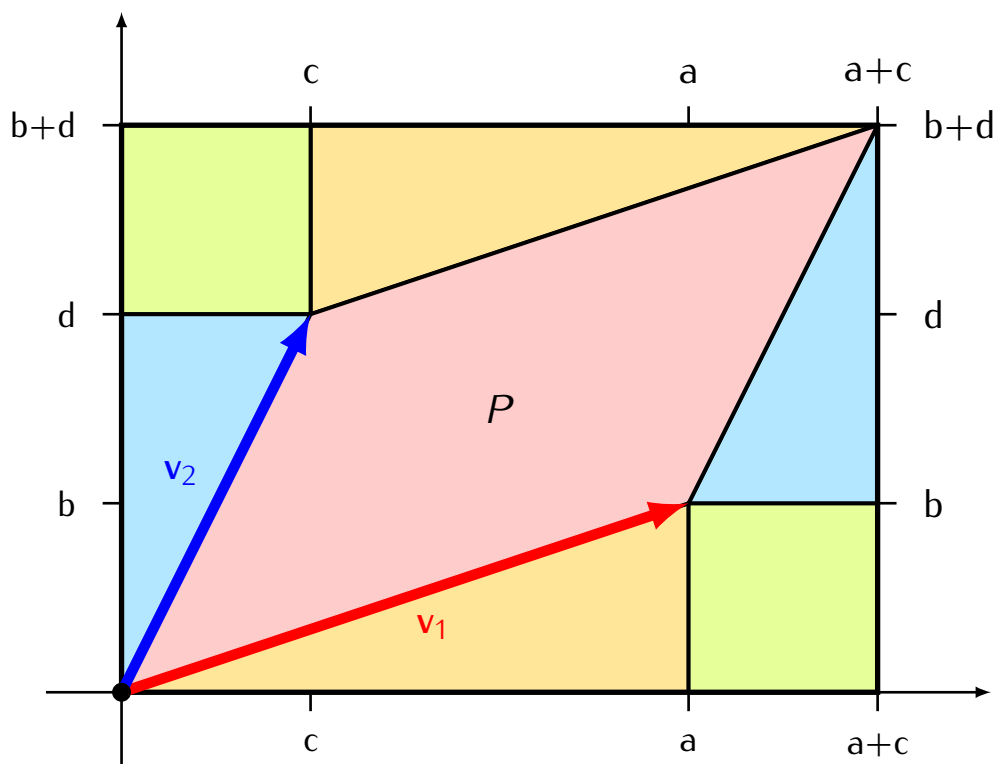
Theorem

If $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^2$ then

$$\text{area}(\mathbf{v}_1, \mathbf{v}_2) = \left| \det \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \right|$$

Idea of the proof.

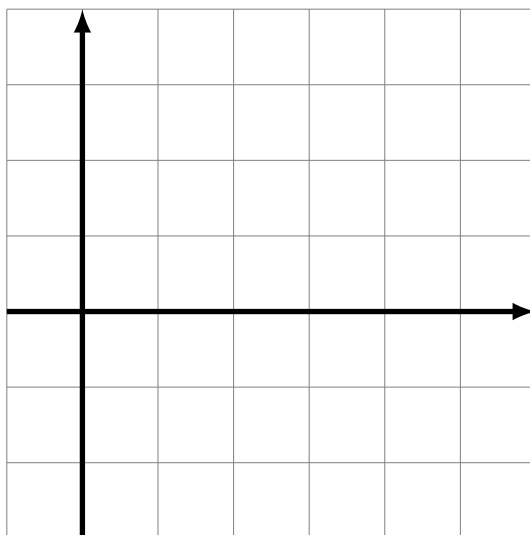
$$\mathbf{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} c \\ d \end{bmatrix}$$



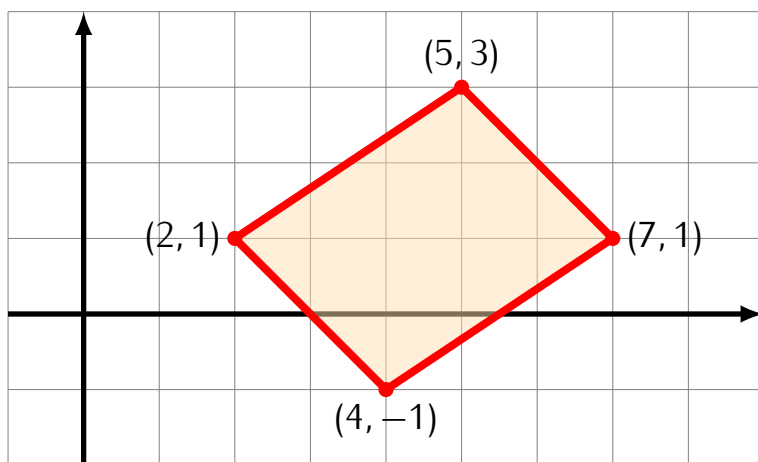
$$\begin{aligned}
 & \text{area}(P) \\
 & + \frac{1}{2}ab \\
 & + \frac{1}{2}ab \\
 & + \frac{1}{2}cd \\
 & + \frac{1}{2}cd \\
 & + cb \\
 & + cb \\
 & \hline
 & (a+c)(b+d)
 \end{aligned}$$

Example.

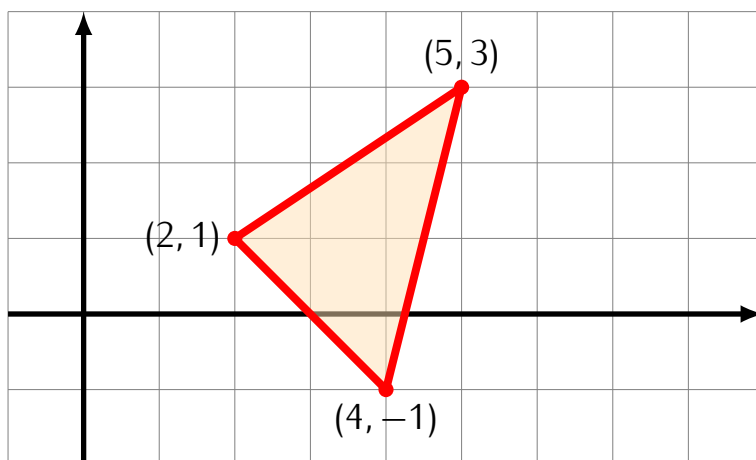
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



Example. Calculate the area of the parallelogram with vertices at the points $(2, 1)$, $(5, 3)$, $(7, 1)$, $(4, -1)$.



Example. Calculate the area of the triangle with vertices at the points $(2, 1)$, $(5, 3)$, $(4, -1)$.



Note. In order to compute areas of other polygons, subdivide them into triangles.

