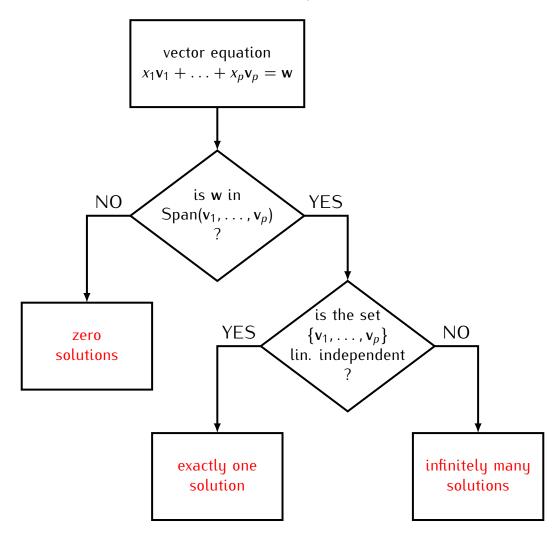
Recall:

1) Span(
$$v_1, ..., v_p$$
) =
$$\begin{cases} \text{the set of all} \\ \text{linear combinations} \\ c_1 v_1 + ... + c_p v_p \end{cases}$$

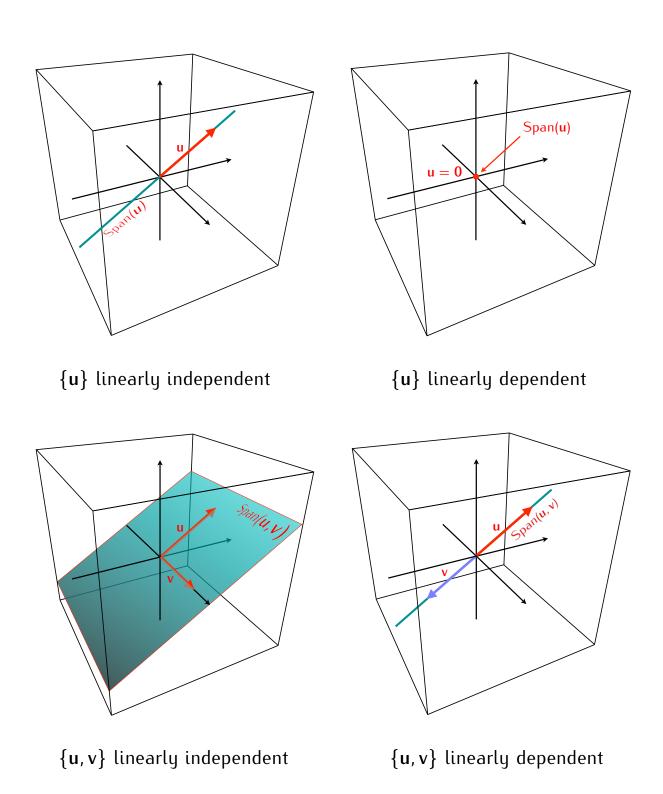
2) A set of vectors $\{v_1, \ldots, v_p\}$ is linearly independent if the equation

$$x_1\mathbf{v}_1+\ldots+x_p\mathbf{v}_p=\mathbf{0}$$

has only one, trivial solution $x_1 = 0, \ldots, x_p = 0$.



Linear independence vs. Span



Theorem

If $\{v_1, \ldots, v_p\}$ is a linearly dependent set of vectors in then:

- 1) for some v_i we have $v_i \in \text{Span}(v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_p)$.
- 2) for some v_i we have

$$\mathsf{Span}(\mathsf{v}_1,\ldots,\mathsf{v}_p)=\mathsf{Span}(\mathsf{v}_1,\ldots,\mathsf{v}_{i-1},\mathsf{v}_{i+1},\ldots,\mathsf{v}_p)$$

Example.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$