Definition

If

$$\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

are vectors in $\mathbb{R}^n$ then the *inner product* (or *dot product*) of $\mathbf{u}$ and $\mathbf{v}$ is the number

$$\mathbf{u} \cdot \mathbf{v} = a_1 b_1 + \ldots + a_n b_n$$

Properties of the dot product:

1) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

2) $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$

3) $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$

4) $\mathbf{u} \cdot \mathbf{u} \geq 0$ and $\mathbf{u} \cdot \mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$.

Definition

If $\mathbf{u} \in \mathbb{R}^n$ then the *length* (or the *norm*) of $\mathbf{u}$ is the number

$$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$

Note. If $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ then $||\mathbf{u}|| = \sqrt{a_1^2 + \ldots + a_n^2}$.

Properties of the norm:

1) $||u|| \geq 0$ and $||u|| = 0$ if and only if $\mathbf{u} = \mathbf{0}$.

2) $||cu|| = |c| \cdot ||u||$

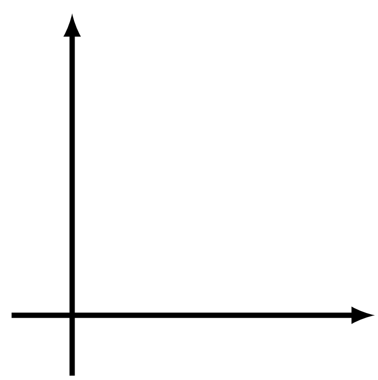
Definition

A vector $\mathbf{u} \in \mathbb{R}^n$ is an *unit vector* if $||\mathbf{u}|| = 1$.

Definition

If $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ then the *distance* between $\mathbf{u}$ and $\mathbf{v}$ is the number

$$\text{dist}(\mathbf{u}, \mathbf{v}) = ||\mathbf{u} - \mathbf{v}||$$



Note. If $\mathbf{u} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$ then

$$\text{dist}(\mathbf{u}, \mathbf{v}) = \sqrt{(a_1 - b_1)^2 + \ldots + (a_n - b_n)^2}$$

Definition

Vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ are *orthogonal* if $\mathbf{u} \cdot \mathbf{v} = 0$.

Pythagorean Theorem

Vectors $\mathbf{u}, \mathbf{v}$ are orthogonal if and only if

$$||\mathbf{u}||^2 + ||\mathbf{v}||^2 = ||\mathbf{u} + \mathbf{v}||^2$$