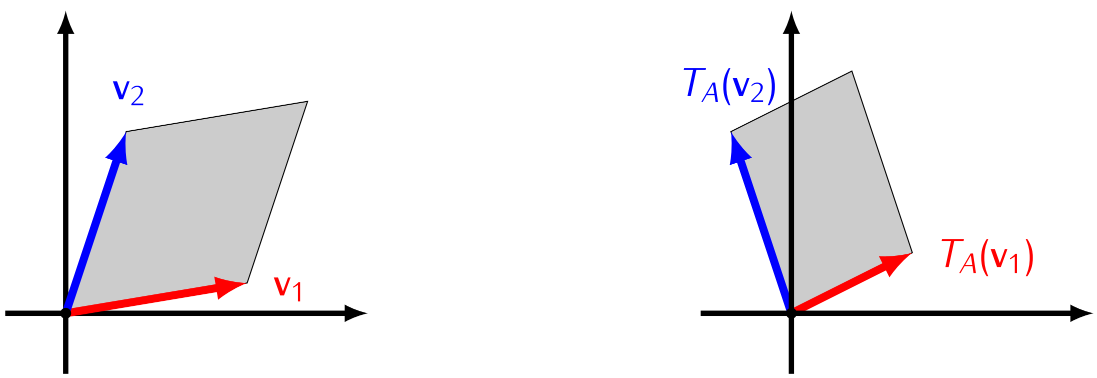
Recall: If A is a 2×2 matrix then it defines a linear transformation

$$T_A \colon \mathbb{R}^2 \to \mathbb{R}^2 \qquad T_A(\mathbf{v}) = A\mathbf{v}$$

Note. T_A maps parallelograms to parallelograms:



Theorem

If A is a 2×2 matrix and $\mathbf{v}_1, \mathbf{v}_1 \in \mathbb{R}^2$ then

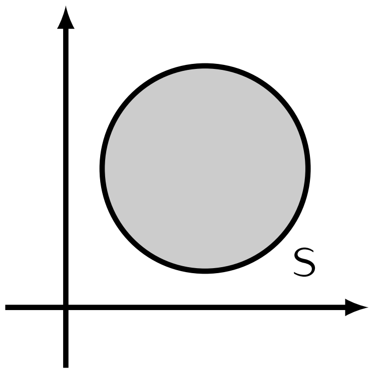
$$\text{area}(T_A(\mathbf{v}_1), T_A(\mathbf{v}_2)) = |\det A| \cdot \text{area}(\mathbf{v}_1, \mathbf{v}_2)$$

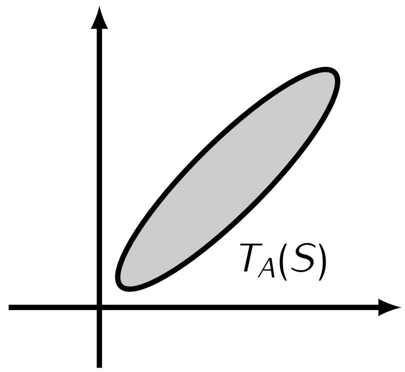
Generalization:

Theorem

If A is a 2×2 matrix then for any region S of $\mathbb{R}^2$ we have:

$$\text{area}(T_A(S)) = |\det A| \cdot \text{area}(S)$$





Idea of the proof.

The area of S can be approximated by the sum of small squares covering S.