# MTH 309T Practice Exam 1

**1.** Let $A$ be a matrix and $\mathbf{v}$ be a vector given as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

**a)** Determine if $\mathbf{v}$ is in $\mathrm{Col}(A)$, where $\mathrm{Col}(A)$ is the column space of $A$.

**b)** Determine if $\mathbf{v}$ is in $\mathrm{Nul}(A)$, where $\mathrm{Nul}(A)$ is the null space of $A$.

**c)** Find an explicit description of $\mathrm{Nul}(A)$ by listing vectors that span the null space.

**2.** Consider the following matrices:

$$A = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

For each algebraic expression given below decide if it is defined. If it is, compute it. If it is not, give a reason why.

**a)** $C^T + 3B$ **b)** $CB + B$ **c)** $A^T BA$ **d)** $A^T C^{-1} A$ **e)** $CBC$

**3.** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first reflects points through the line $x_1 = x_2$ and then reflects points through the $x_1$-axis.

**a)** Find the standard $A$ matrix of $T$.

**b)** Find all vectors $\mathbf{v} \in \mathbb{R}^2$ such that $\mathbf{v} \in \mathrm{Nul}(A)$.

**4.** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix} \qquad\qquad T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \begin{bmatrix}3\\6\end{bmatrix}$$

**a)** Compute $T\left(\begin{bmatrix}3\\6\end{bmatrix}\right)$.

**b)** Find a vector $\mathbf{v}$ such that $\mathbf{v} \neq \mathbf{0}$ and $T(\mathbf{v}) = \mathbf{0}$.

**5.** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$, $B$ are matrices such that $AB$ is defined and is a square matrix (i.e. it has the same number of rows and columns) then $BA$ is also defined.

**b)** If $A$ is an $2 \times 2$ matrix such that $A\mathbf{v} = \mathbf{0}$ for some non-zero vector $\mathbf{v} \in \mathbb{R}^2$ then $A$ cannot be invertible.

**c)** If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set of vectors in $\mathbb{R}^2$ and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation then the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2)\}$ must be also linearly independent.

**d)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^2$ such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $\mathbf{v}$ must be in $\mathrm{Span}(\mathbf{u}, \mathbf{w})$.