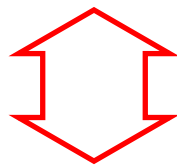


So far:

$$\begin{cases} 2x_1 + 4x_2 + 6x_3 + 3x_4 = 7 \\ 3x_1 + 2x_2 + 2x_3 + 9x_4 = 3 \\ 5x_1 + 8x_2 + 3x_3 + 3x_4 = 9 \end{cases}$$

system of  
linear equations



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 2 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 9 \end{bmatrix}$$

vector equation

Next:

$$\begin{bmatrix} 2 & 4 & 6 & 3 \\ 3 & 2 & 2 & 9 \\ 5 & 8 & 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 9 \end{bmatrix}$$

matrix equation

### Definition

Let  $A$  be an  $m \times n$  matrix with columns  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$  and let  $\mathbf{w}$  be a vector in  $\mathbb{R}^n$ :

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \dots & \mathbf{v}_n \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

The product  $A\mathbf{w}$  is a vector in  $\mathbb{R}^m$  given by

$$A\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_n\mathbf{v}_n$$

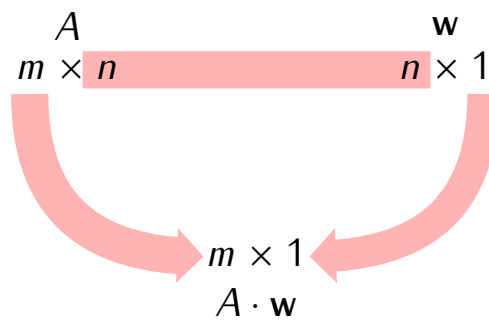
**Example.**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

## Properties of matrix-vector multiplication

1) The product  $Aw$  is defined only if

(number of columns of  $A$ ) = (number of entries of  $w$ )



2)  $A(v + w) = Av + Aw$

3) If  $c$  is a scalar then  $A(cw) = c(Aw)$ .

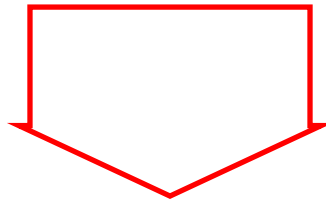
**Example.** Solve the matrix equation

$$\begin{bmatrix} 1 & 1 & -4 \\ 1 & -2 & 3 \\ 3 & -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

## How to solve a matrix equation

$$A\mathbf{x} = \mathbf{b}$$

matrix equation



$$\left[ \begin{array}{c|c} A & \mathbf{b} \end{array} \right]$$

augmented matrix



$$\left[ \text{reduced matrix} \right]$$



$$\mathbf{x} = \dots$$

solutions