Recall: If A is an $m \times n$ matrix then

$$A \cdot \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix}$$

Definition

If A is an $m \times n$ matrix then the function

$$T_A \colon \mathbb{R}^n \to \mathbb{R}^m$$

given by $T_A(\mathbf{v}) = A\mathbf{v}$ is called the *matrix transformation associated to* A.

Example.

Let $T_A \colon \mathbb{R}^3 \to \mathbb{R}^2$ be the matrix transformation defined by the matrix

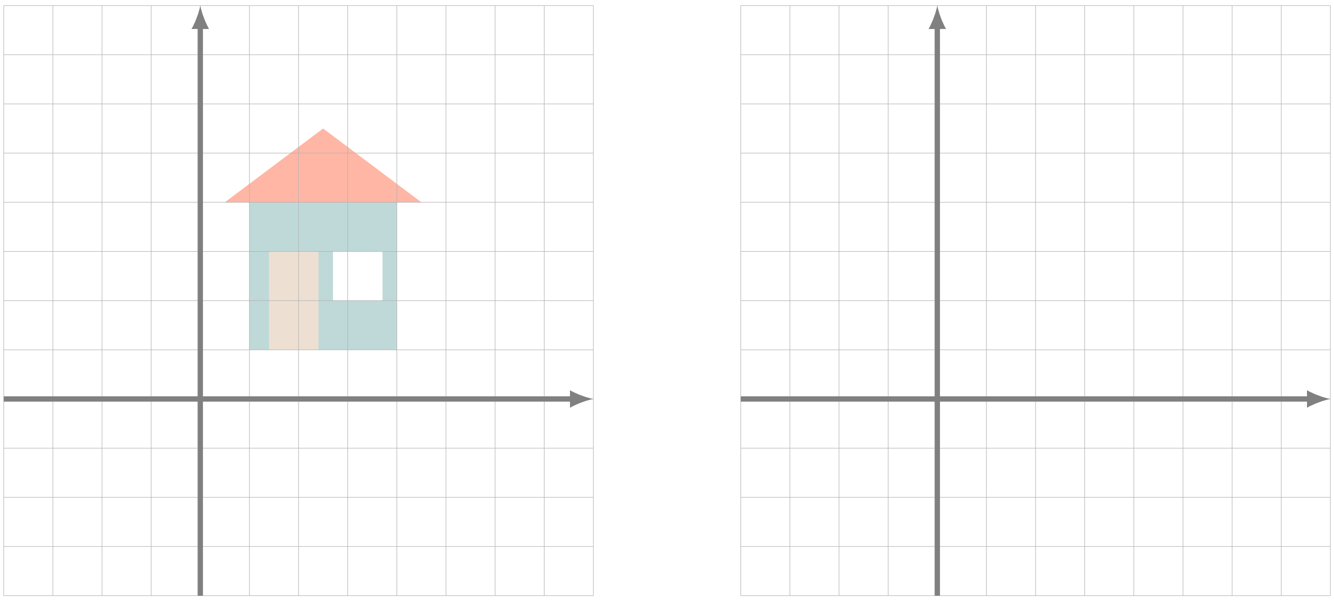
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{bmatrix}$$

1) Compute $T_A(\mathbf{v})$ where $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

2) Find a vector $\mathbf{v}$ such that $T_A(\mathbf{v}) = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$.

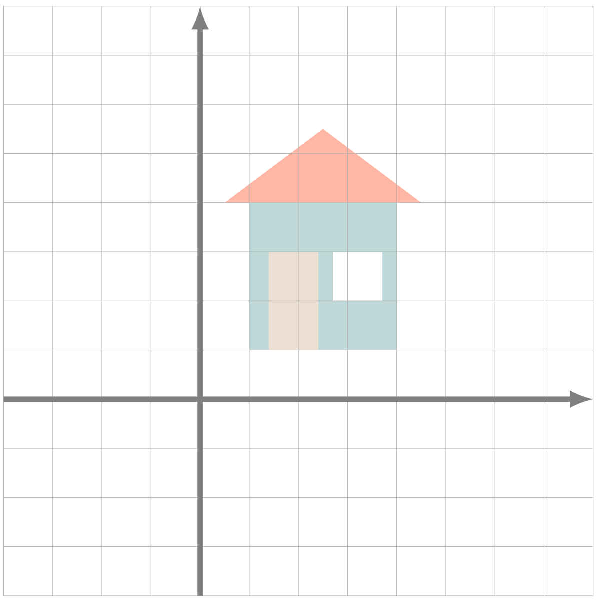
Geometric interpretation of matrix transformations $\mathbb{R}^2 \to \mathbb{R}^2$

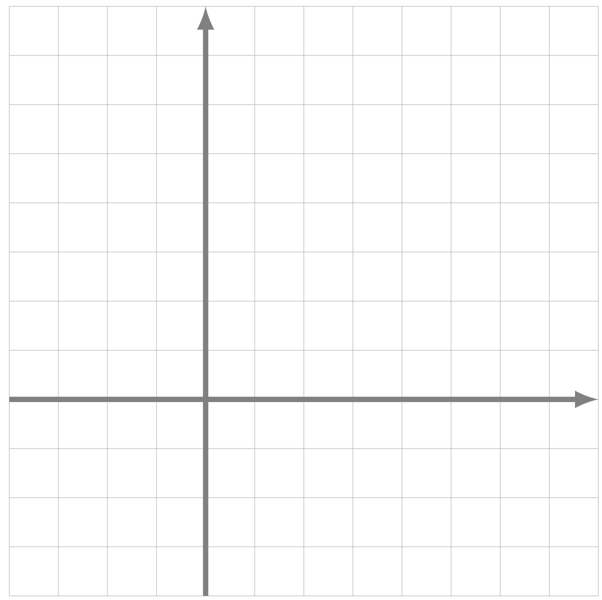
$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

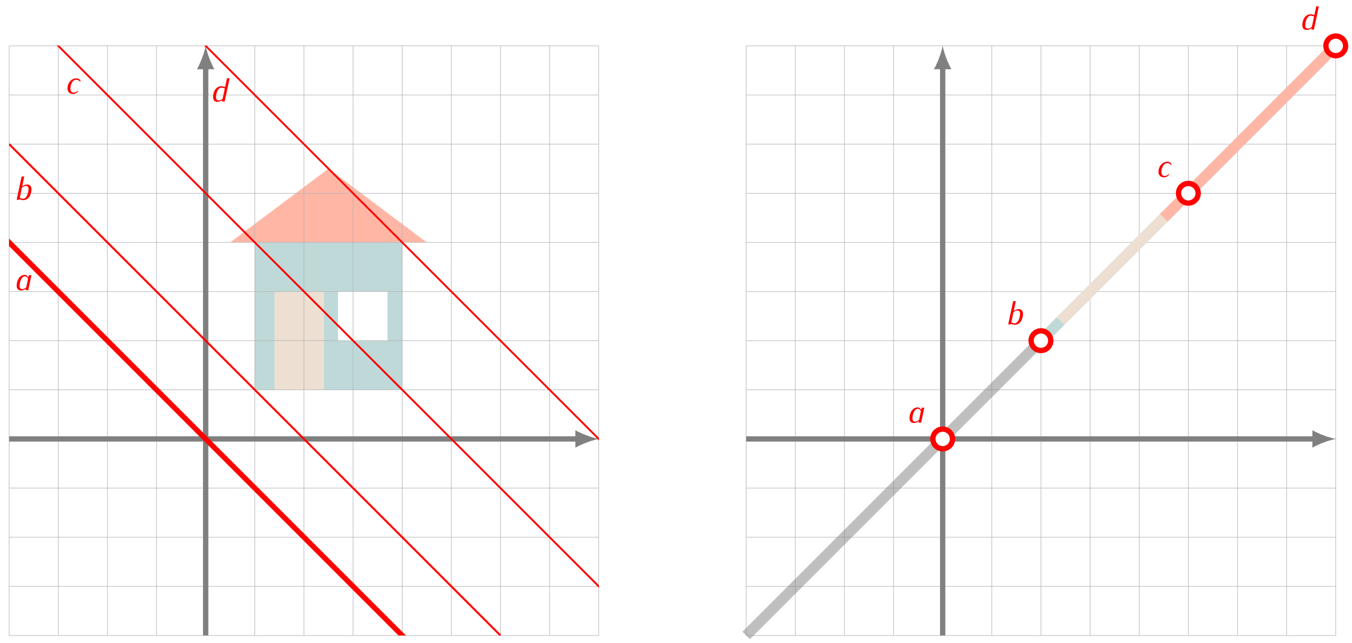


Example.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$






a
b
c
d
a
b
c
d

Note

If $T_A\colon \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation associated to a matrix A then:

- $\mathrm{Col}(A) =$ the set of values of T_A.
- $\mathrm{Nul}(A) =$ the set of vectors $\mathbf{v}$ such that $T_A(\mathbf{v}) = \mathbf{0}$.
- $T_A(\mathbf{v}) = T_A(\mathbf{w})$ if and only if $\mathbf{w} = \mathbf{v} + \mathbf{n}$ for some $\mathbf{n} \in \mathrm{Nul}(A)$.