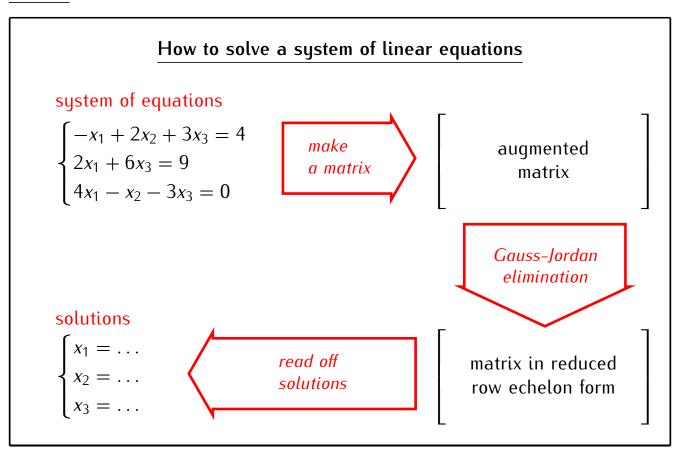
### Recall:



- Every system of linear equations can be represented by a matrix
- Elementary row operations:
  - interchange of two rows
  - multiplication of a row by a non-zero number
  - addition of a multiple of one row to another row.
- Elementary row operations do not change solutions of systems of linear equations.

#### **Definition**

A matrix is in the row echelon form if:

- 1) the first non-zero entry of each row is a 1 ("a leading one");
- 2) the leading one in each row is to the right of the leading one in the row above it.

A matrix is in the reduced row echelon form if in addition it satisfies:

3) all entries above each leading one are 0.

$$\begin{bmatrix} 1 & * & * & * & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(\* = any number)

## Example

$$\begin{bmatrix} 1 & 0 & 4 & 0 & 7 & 0 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 4 & 6 & 7 & 0 \\ 0 & 1 & 5 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 4 & 0 & 7 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

#### **Fact**

If a system of linear equations is represented by a matrix in the reduced row echelon form then it is easy to solve the system.

## Example

$$\left[\begin{array}{ccc|ccc|c}
1 & 0 & 3 & 0 & 0 \\
0 & 1 & 7 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{array}\right]$$

# Proposition

A matrix in the reduced row echelon form represents an inconsistent system if and only if it contains a row of the form

i.e. with the leading one in the last column.

# Example

$$\left[\begin{array}{ccc|ccc|c}
1 & 0 & 3 & 0 & 0 \\
0 & 1 & 7 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]$$

### Note

In an augmented matrix in the reduced row echelon form free variables correspond to columns of the coefficient matrix that do not contain leading ones.

# Example

$$\left[\begin{array}{ccc|ccc|c}
1 & 0 & 0 & 0 & 5 \\
0 & 1 & 0 & 0 & 6 \\
0 & 0 & 1 & 0 & 7 \\
0 & 0 & 0 & 1 & 8
\end{array}\right]$$

### Note

A matrix in the reduced row echelon form represents a system of equations with exactly one solution if and only if it has a leading one in every column except for the last one.