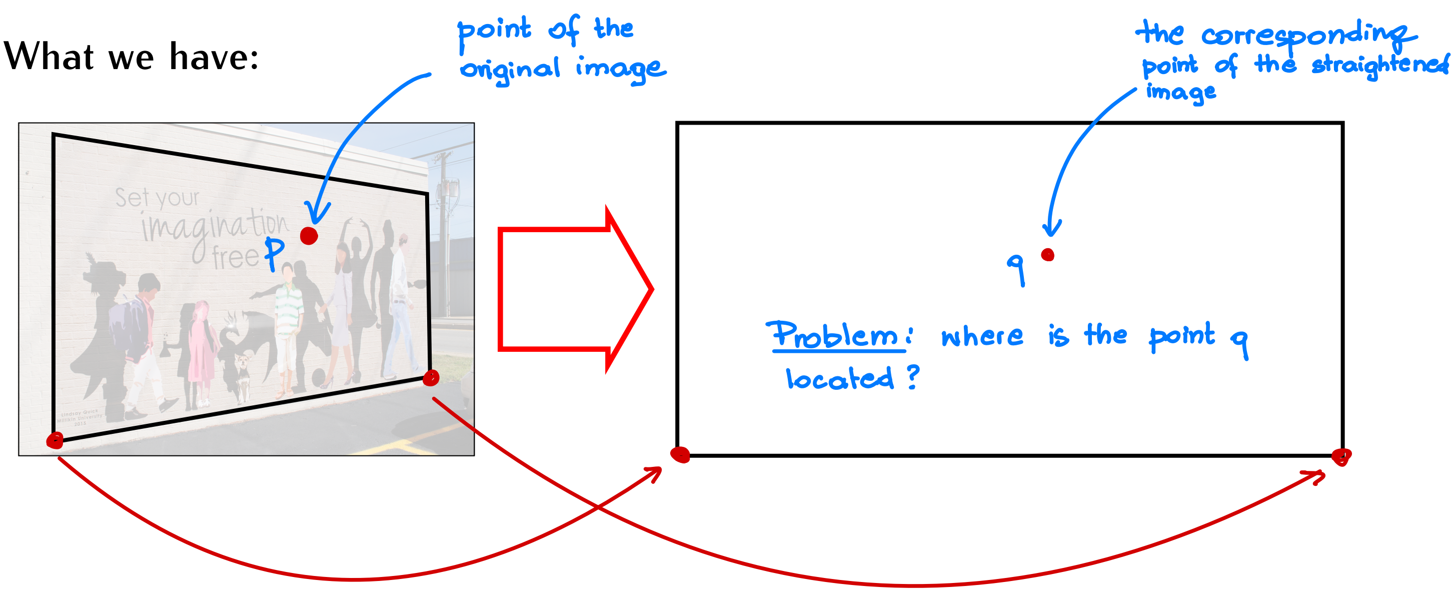
What we want:

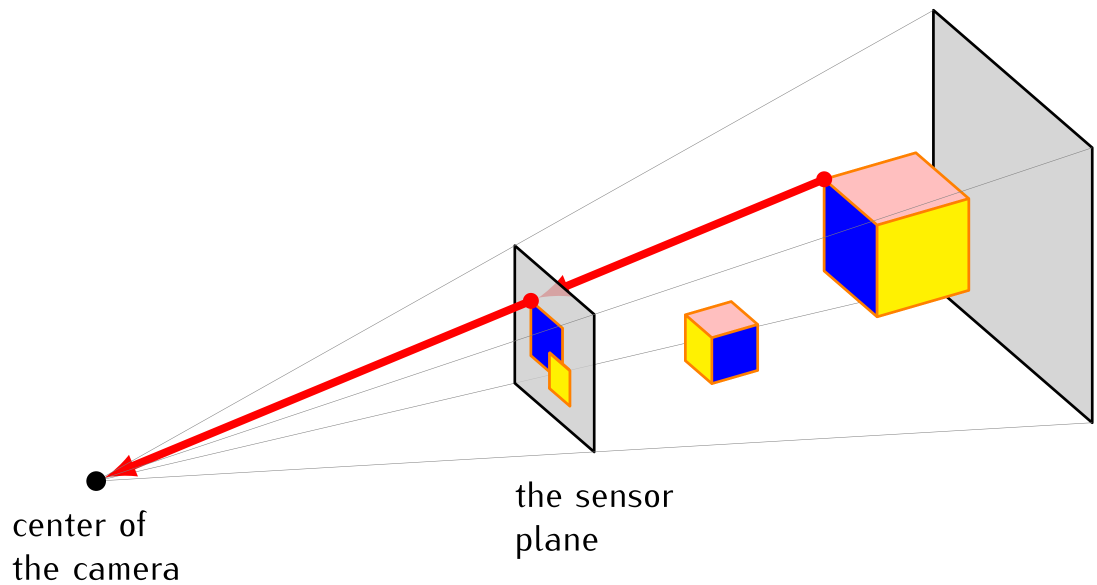
photo taken at an angle straightened image

What we have:



<u>Problem:</u> where is the point q located?

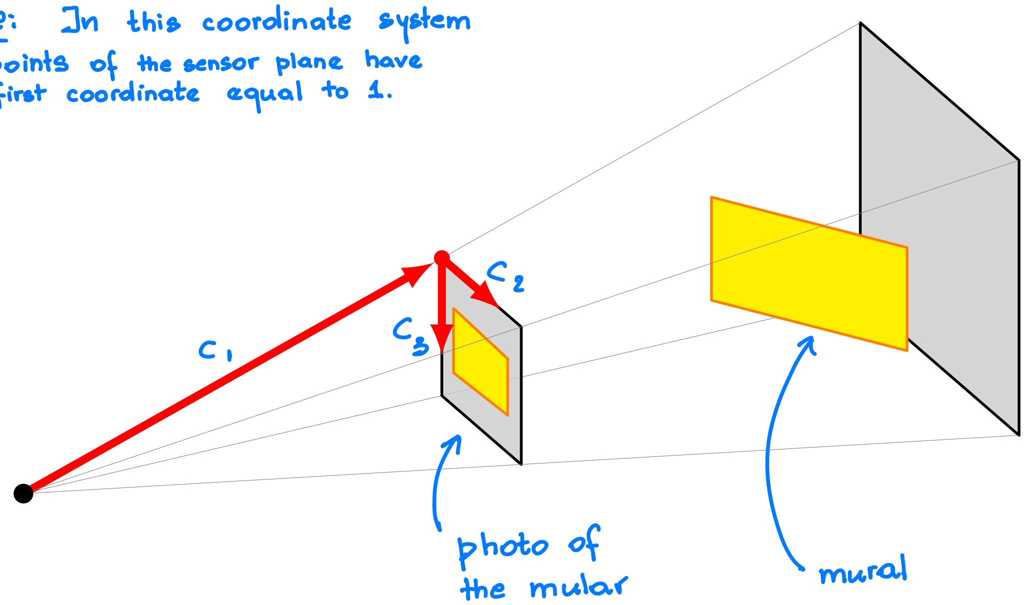
Image formation in a camera

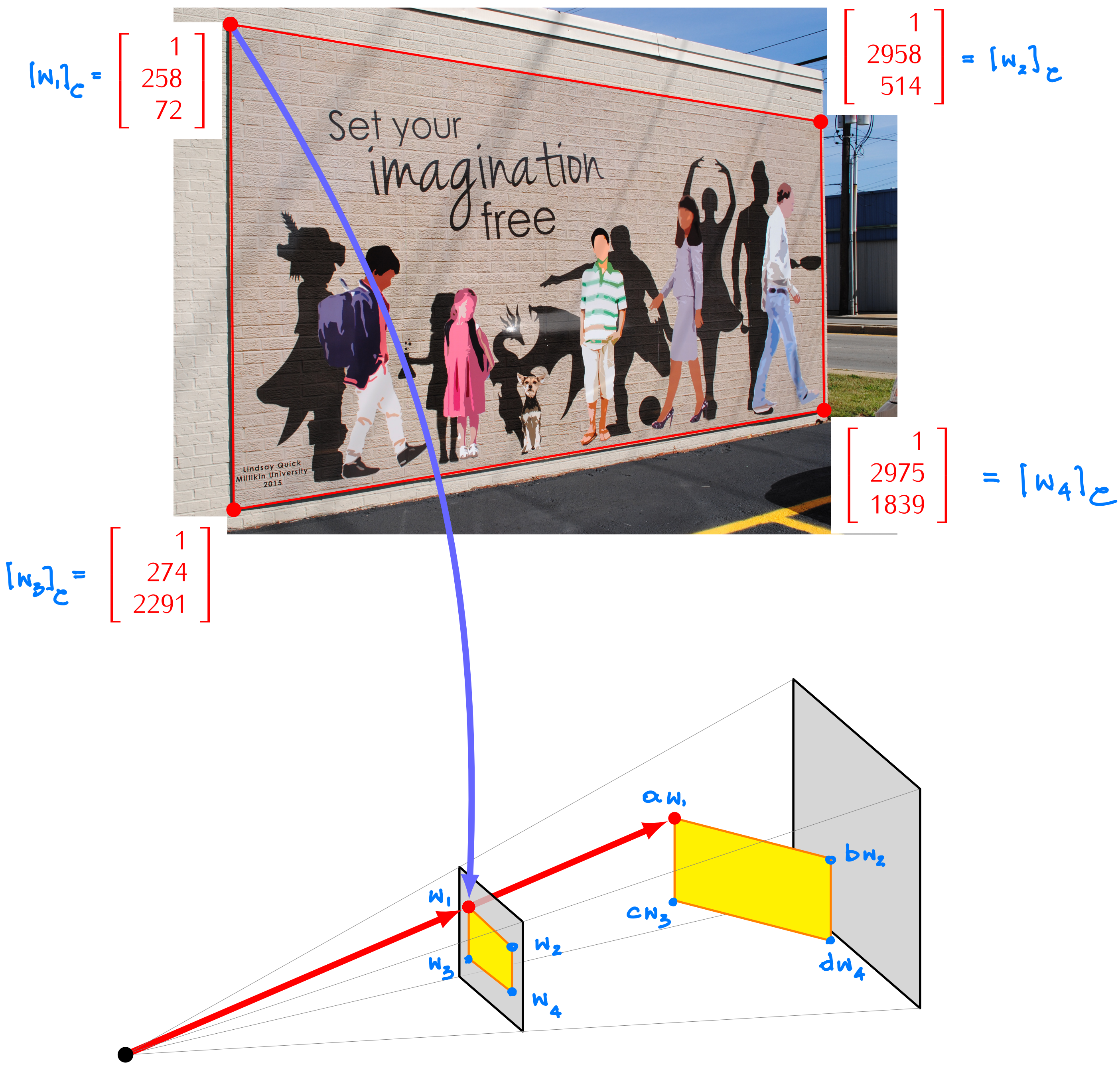


The camera coordinate system $\mathcal{C}$

$\mathcal{C} = \{c_1, c_2, c_3\}$

Note: In this coordinate system all points of the sensor plane have the first coordinate equal to 1.





$$[aw_1]_e = a \cdot [w_1]_e = a \cdot \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix}$$

$$[bw_2]_e = b \cdot [w_2]_e = \ldots$$

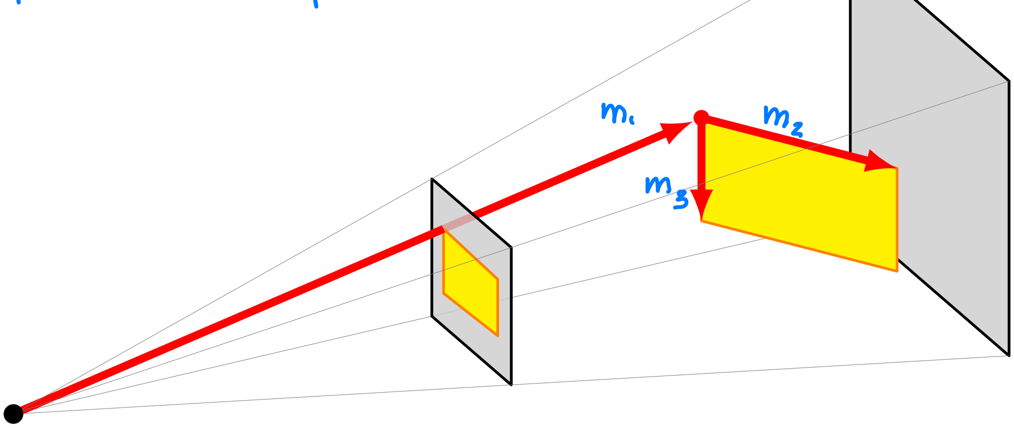
$$[cw_3]_e = c \cdot [w_3]_e = \ldots$$

$$[dw_4]_e = d \cdot [w_4]_e = \ldots$$

The mural coordinate system $\mathcal{M}$

$$\mathcal{M} = \{m_1, m_2, m_3\}$$

Note: In this coordinate system all points of the mural have the first coordinate equal to 1

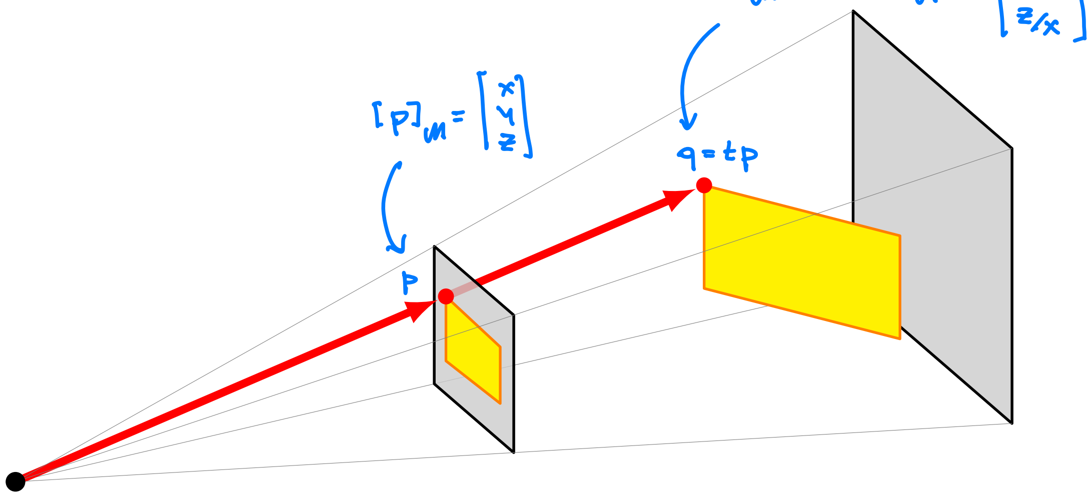


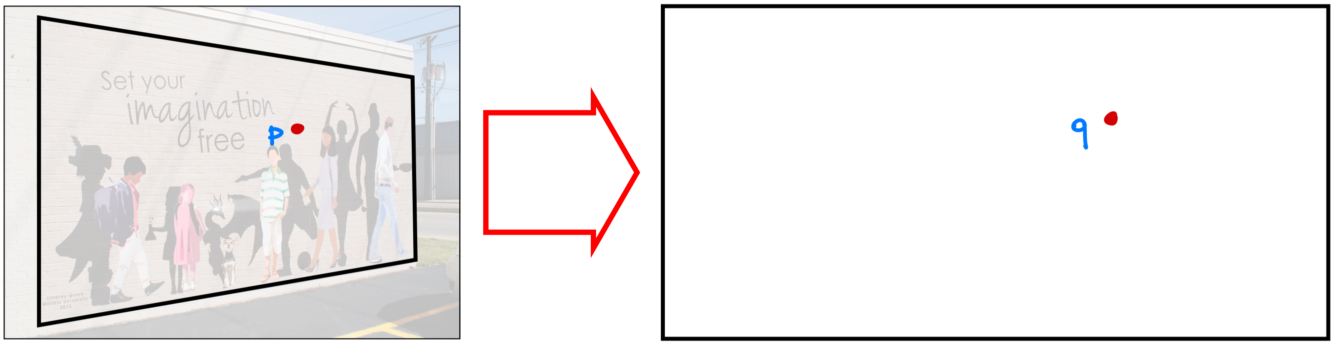
Note: If the mural coordinates of a point p are $[p]_{\mathcal{M}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then the mural coordinates of the point $q = tp$ are

$$[q]_{\mathcal{M}} = t \cdot [p]_{\mathcal{M}} = t \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix}$$

If q is a point of the mural, we have $tx = 1$, so $t = \frac{1}{x}$

$$[q]_{\mathcal{M}} = \frac{1}{x} \cdot [p]_{\mathcal{M}} = \begin{bmatrix} 1 \\ y/x \\ z/x \end{bmatrix}$$





Upshot:

We know: $[p]_{\mathcal{C}}$ = the camera coordinates of p

We want: $[q]_{\mathcal{M}}$ = the mural coordinates of q

Strategy: Compute $[p]_{\mathcal{M}}$ = the mural coordinates of p.

Then, if $[p]_{\mathcal{M}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ then $[q]_{\mathcal{M}} = \begin{bmatrix} 1 \\ y/x \\ z/x \end{bmatrix}$

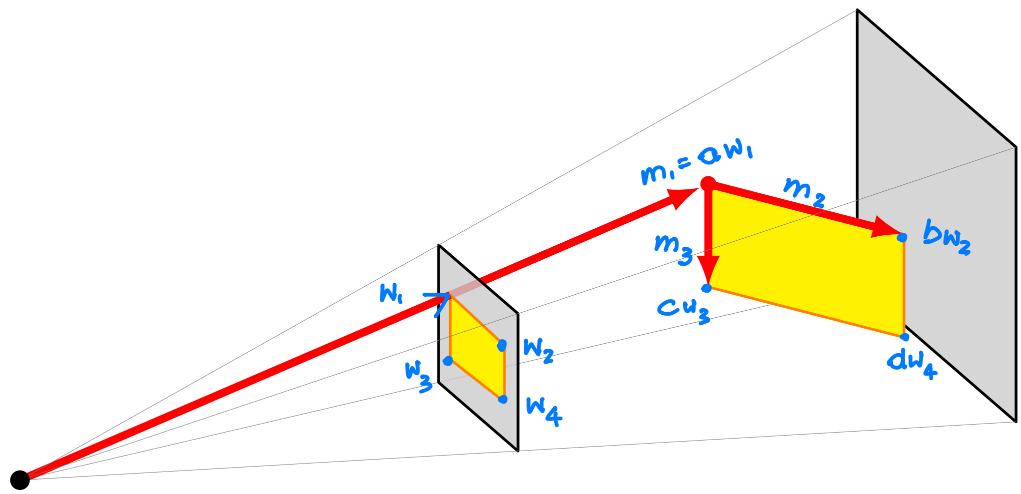
Note:

1) $[p]_{\mathcal{M}} = P_{\mathcal{M} \leftarrow \mathcal{C}} \cdot [p]_{\mathcal{C}}$

2) It will suffice to compute $P_{\mathcal{C} \leftarrow \mathcal{M}}$ since $P_{\mathcal{M} \leftarrow \mathcal{C}} = (P_{\mathcal{C} \leftarrow \mathcal{M}})^{-1}$.

From mural coordinates to camera coordinates

$$P_{\mathcal{C}\leftarrow\mathcal{M}} = \left[\ [\mathbf{m}_1]_{\mathcal{C}}\ \ [\mathbf{m}_2]_{\mathcal{C}}\ \ [\mathbf{m}_3]_{\mathcal{C}}\ \right]$$



We have:

$m_1 = aw_1$

$m_1 + m_2 = bw_2$ so: $m_2 = bw_2 - m_1$

$\boxed{m_2 = bw_2 - aw_1}$

$m_1 + m_3 = cw_3$ so: $\boxed{m_3 = cw_3 - aw_1}$

This gives:

$$[m_1]_{\mathcal{C}} = a\cdot[w_1]_{\mathcal{C}} = a\cdot\begin{bmatrix}1\\258\\72\end{bmatrix}$$

$$[m_2]_{\mathcal{C}} = b\cdot[w_2]_{\mathcal{C}} - a[w_1]_{\mathcal{C}} = b\cdot\begin{bmatrix}1\\2958\\514\end{bmatrix} - a\cdot\begin{bmatrix}1\\258\\72\end{bmatrix}$$

$$[m_3]_{\mathcal{C}} = c[w_3]_{\mathcal{C}} - a[w_1]_{\mathcal{C}} = c\cdot\begin{bmatrix}1\\274\\2291\end{bmatrix} - a\cdot\begin{bmatrix}1\\258\\72\end{bmatrix}$$

Problem: What are the numbers a, b, c?

Note: $m_1 + m_2 + m_3 = d w_4$

This gives: $\cancel{aw_1} + (bw_2 - \cancel{aw_1}) + (cw_3 - aw_1) = dw_4$

$bw_2 + cw_3 - aw_1 + dw_4$

So: $\boxed{b[w_2]_\mathcal{E} + c[w_3]_\mathcal{E} - a[w_1]_\mathcal{E} = d[w_4]_\mathcal{E}}$

$$b \cdot \begin{bmatrix} 1 \\ 2958 \\ 514 \end{bmatrix} + c \begin{bmatrix} 1 \\ 274 \\ 2291 \end{bmatrix} - a \begin{bmatrix} 1 \\ 258 \\ 72 \end{bmatrix} = d \begin{bmatrix} 1 \\ 2975 \\ 1839 \end{bmatrix}$$

Problem: 3 equations, 4 unknowns, so we can't have a single solution for a, b, c, d.

Good news:

1) For our computations the value of d does not matter, we can set it to any non-zero number

2) Once the value of d is fixed, the values of a, b, c are uniquely determined. This lets us compute $[m_1]_\mathcal{E}$, $[m_2]_\mathcal{E}$, $[m_3]_\mathcal{E}$, and so we obtain the matrix $P_{\mathcal{E} \leftarrow \mathcal{M}}$.