Example.
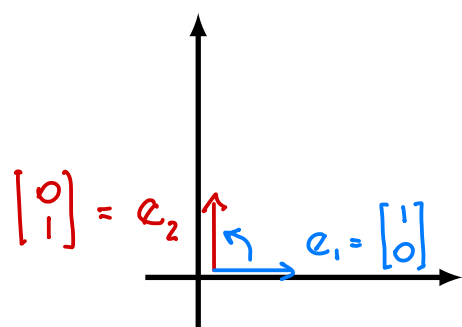
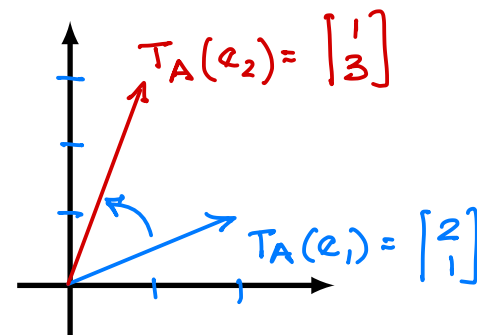$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ $\det A = 5 > 0$ $T_A : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$
                                                                    $v \longmapsto Av$

$T_A(e_2) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = e_2$ $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

T_A

$T_A(e_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

T_A preserves the direction of angles between vectors.
(We say that T_A preserves orientation)

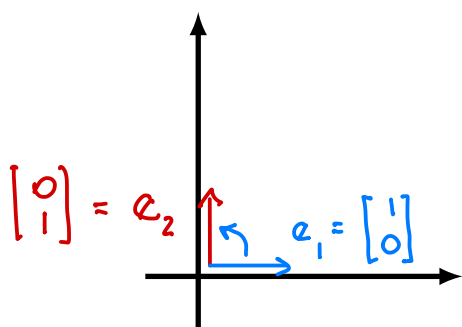**Example.**
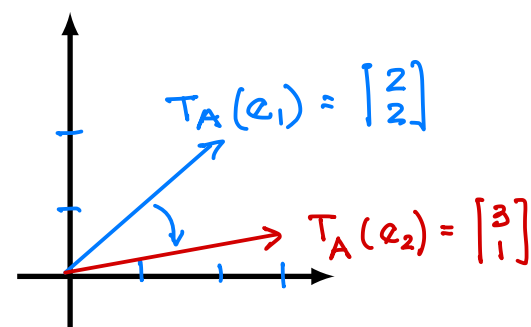
$A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$ $\det A = -4 < 0$ $T_A : \mathbb{R}^2 \to \mathbb{R}^2$
                                                                    $v \longmapsto Av$

$T_A(e_1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = e_2$ $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

T_A

$T_A(e_2) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

T_A reverses the direction of angles between vectors.
(We say that T_A reverses orientation)

Theorem

If A is a 2×2 matrix then the linear transformation $T_A \colon \mathbb{R}^2 \to \mathbb{R}^2$ preserves orientation if $\det A > 0$ and reverses orientation if $\det A < 0$.