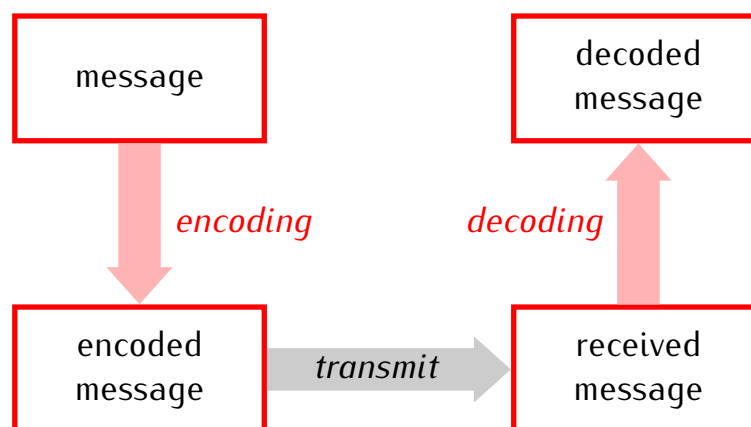


### Basic scheme of error correction



**Working assumption for this lecture:** We expect at most one transmission error in any message up to 20 bits long.

**A simple error correcting code:** triple repeat.

message: 1011

**Problem:** The encoded message is 3 times longer than the original message.

**Better error correction:** Hamming (7,4) code.

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

encoding matrix

$$D = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

decoding matrix

message: 10111101

**Encoding.**

1) Split the message into vectors with 4 entries, and multiply each vector by the encoding matrix  $E$ .

2) Reduce all numbers obtained in step 1 modulo 2. That is, write 0 for each even number and 1 for each odd number.

Encoded message: 1 0 1 1 0 1 0 1 1 0 1 0 0 1

Received message: 1 0 1 1 0 1 0 1 1 1 1 1 0 0 1

**Decoding.** Split the received message into vectors with 7 entries, multiply each vector by the decoding matrix  $D$ , and reduce modulo 2.

Decoded message:

**How the Hamming code works:**