

- The size of this image is 1000×1000 pixels.
- The color of each pixel is represented by an integer between 0 (black) and 255 (white).
- The whole image is described by a (symmetric) matrix A consisting of $1000 \times 1000 = 1,000,000$ numbers
- Each number is stored in 1 byte, so the image file size is 1,000,000 bytes (\approx 1 MB).

How to make the image file smaller:

1) Find the spectral decomposition of the matrix A:

$$A = \lambda_1(\mathbf{u}_1\mathbf{u}_1^T) + \lambda_2(\mathbf{u}_2\mathbf{u}_2^T) + \ldots + \lambda_{1000}(\mathbf{u}_{1000}\mathbf{u}_{1000}^T)$$

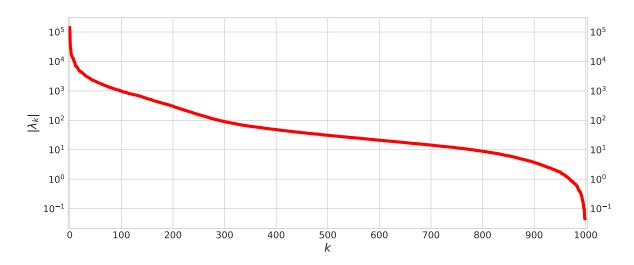
where $|\lambda_1| \ge |\lambda_2| \ge \ldots \ge |\lambda_{1000}|$.

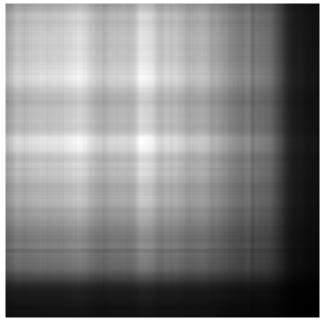
2) For k = 1, ..., 1000 define:

$$B_k = \lambda_1(\mathbf{u}_1\mathbf{u}_1^T) + \lambda_2(\mathbf{u}_2\mathbf{u}_2^T) + \ldots + \lambda_k(\mathbf{u}_k\mathbf{u}_k^T)$$

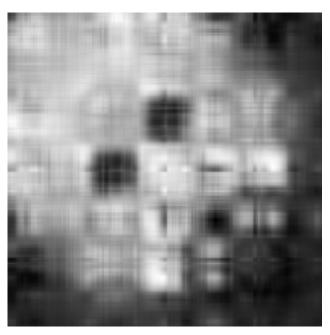
This matrix approximates the matrix A and can be stored using $k \cdot (1000 + 1)$ numbers (i.e. $k \cdot (1000 + 1)$ bytes).

Eigenvalues of the matrix A

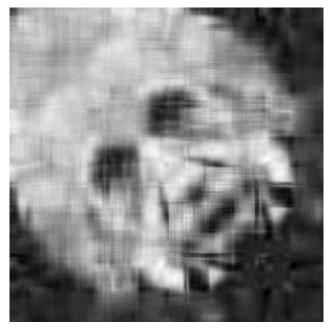




matrix B₁ 1001 bytes compression 1000:1



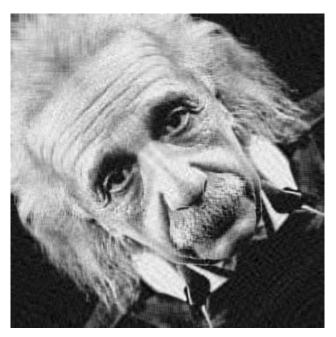
matrix B₅ 5005 bytes compression 200:1



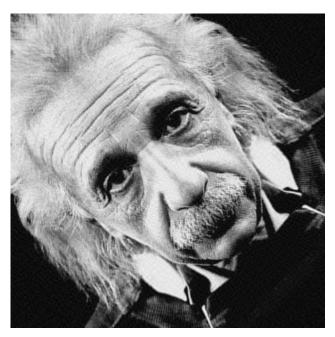
matrix B₁₀ 10,010 bytes compression 100:1



 $\begin{array}{l} \textbf{matrix} \ B_{20} \\ 20,020 \ bytes \\ \textbf{compression} \ 50:1 \end{array}$



 $\begin{array}{l} \textbf{matrix} \ B_{50} \\ 50,\!050 \ bytes \\ \textbf{compression} \ 20:1 \end{array}$



 $\begin{array}{l} \textbf{matrix} \ B_{100} \\ 100,\!100 \ bytes \\ \textbf{compression} \ 10:\!1 \end{array}$