Theorem

Let A and B be $n \times n$ matrices. If B is obtained from A by multiplying one row (or one column) of A by a scalar k then

$$\det B = k \cdot \det A$$

Example.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

Corollary

If a square matrix A contains a row or column consisting of zeros, then $\det A = 0$.

Theorem

Let A and B be $n \times n$ matrices. If B is obtained from A by adding a multiple of one row of A to another row (or adding a multiple of one column to another column) then

$$\det B = \det A$$

Example.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

Theorem

Let A and B be $n \times n$ matrices. If B is obtained from A by interchanging two rows (or two columns) then

$$\det B = -\det A$$

Example.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

Definition

An square matrix is *upper triangular* is all its entries below the main diagonal are 0.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix}$$

Proposition

If A is an upper triangular matrix as above then

$$\det A = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$$