

Recall:

Let A be a matrix with a singular value decomposition

$$A = U\Sigma V^T$$

If

$$U = [\mathbf{u}_1 \ \dots \ \mathbf{u}_m] \quad V = [\mathbf{v}_1 \ \dots \ \mathbf{v}_n]$$

and $\sigma_1, \dots, \sigma_r$ are singular values of A then then

$$A = \sigma_1(\mathbf{u}_1\mathbf{v}_1^T) + \sigma_2(\mathbf{u}_2\mathbf{v}_2^T) + \dots + \sigma_r(\mathbf{u}_r\mathbf{v}_r^T)$$

Example: Movie ratings:

	Matrix	Amelie	Alien	Casablanca	Interstellar
user_1	5	0	5	0	4
user_2	5	0	3	0	5
user_3	0	5	0	5	1
user_4	1	5	0	4	0
user_5	4	0	4	0	3
user_6	0	5	0	4	0
user_7	3	0	3	0	2

Singular value decomposition of the matrix of movie ratings:

$$U = \begin{bmatrix} 0.6 & -0.1 & 0.3 & 0.2 & -0.2 & 0.7 & 0.2 \\ 0.5 & -0.1 & -0.8 & -0.2 & -0.1 & -0.1 & -0.1 \\ 0.1 & 0.6 & -0.2 & 0.7 & 0.4 & 0.0 & 0.0 \\ 0.1 & 0.5 & 0.1 & -0.7 & 0.4 & 0.1 & 0.2 \\ 0.5 & -0.1 & 0.3 & 0.1 & 0.1 & -0.7 & 0.4 \\ 0.1 & 0.6 & 0.1 & 0.0 & -0.8 & -0.1 & -0.2 \\ 0.3 & -0.1 & 0.3 & 0.0 & 0.3 & -0.1 & -0.8 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 13.6 & 0 & 0 & 0 & 0 \\ 0 & 11.4 & 0 & 0 & 0 \\ 0 & 0 & 1.9 & 0 & 0 \\ 0 & 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 0.6 & -0.1 & 0.0 & -0.7 & 0.4 \\ 0.1 & 0.7 & 0.1 & -0.3 & -0.6 \\ 0.5 & -0.1 & 0.7 & 0.4 & -0.2 \\ 0.1 & 0.6 & 0.0 & 0.4 & 0.7 \\ 0.5 & -0.1 & -0.7 & 0.4 & -0.3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 5 & 0 & 4 \\ 5 & 0 & 3 & 0 & 5 \\ 0 & 5 & 0 & 5 & 1 \\ 1 & 5 & 0 & 4 & 0 \\ 4 & 0 & 4 & 0 & 3 \\ 0 & 5 & 0 & 4 & 0 \\ 3 & 0 & 3 & 0 & 2 \end{bmatrix} \approx \begin{bmatrix} 0.6 & -0.1 \\ 0.5 & -0.1 \\ 0.1 & 0.6 \\ 0.1 & 0.5 \\ 0.5 & -0.1 \\ 0.1 & 0.6 \\ 0.3 & -0.1 \end{bmatrix} \cdot \begin{bmatrix} 13.6 & 0 \\ 0 & 11.4 \end{bmatrix} \cdot \begin{bmatrix} 0.6 & 0.1 & 0.5 & 0.1 & 0.5 \\ -0.1 & 0.7 & -0.1 & 0.6 & -0.1 \end{bmatrix}$$