Recall:

1) If A is an upper triangular matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix}$$

then $\det A = a_{11} \cdot a_{22} \cdot \ldots \cdot a_{nn}$.

Note. If A is a square matrix then the row echelon form of A is always upper triangular.

2) Let A and B be $n \times n$ matrices.

• If B is obtained from A by interchanging two rows (or two columns) then

$$\det B = -\det A$$

• If B is obtained from A by multiplying one row (or one column) of A by a scalar k then

$$\det B = k \cdot \det A$$

• If B is obtained from A by adding a multiple of one row of A to another row (or adding a multiple of one column to another column) then

$$\det B = \det A$$

Computation of determinants via row reduction

Idea. To compute $\det A$, row reduce A to the row echelon form. Keep track how the determinant changes at each step of the row reduction process.

Example. Compute $\det A$ where

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 4 & 0 & 10 \\ 3 & 4 & 1 & 7 \\ -2 & 5 & 3 & 0 \end{bmatrix}$$

Theorem

If A is a square matrix then A is invertible if and only if $\det A \neq 0$

Recall: A is invertible if and only if its reduced row echelon form is the identity matrix.

A direct way of computing the determinant of a 2×2 matrix

Proposition

If A is a 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then $\det A = ad - bc$

Example.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

A direct way of computing the determinant of a 3×3 matrix

Example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Some further properties of determinants

1) $\det(A^{-1}) = (\det A)^{-1}$

2) $\det(A^T) = \det A$

Note. In general $\det(A + B) \neq \det A + \det B$.