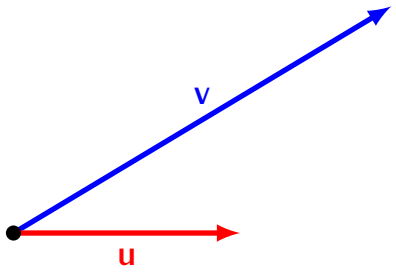
Goal: Given vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ compute the angle between $\mathbf{u}$ and $\mathbf{v}$.



Proposition

If $\mathbf{u}, \mathbf{v}$ are non-zero vectors in $\mathbb{R}^n$ and θ is the angle between $\mathbf{u}$ and $\mathbf{v}$ then

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\cdot||\mathbf{v}||}$$

Example. Compute $\cos\theta$, where θ is the angle between the following vectors in $\mathbb{R}^3$:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Note. In data science the number

$$\frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\cdot||\mathbf{v}||}$$

is called the *cosine similarity* between vectors $\mathbf{u}$ and $\mathbf{v}$. The number

$$1 - \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\cdot||\mathbf{v}||}$$

is called the *cosine distance* between $\mathbf{u}$ and $\mathbf{v}$.