5. Pivot positions and pivot columns

$$\begin{bmatrix} 0 & 4 & -8 & 0 & | & 4 \\ 2 & 6 & -6 & -2 & | & -4 \\ 2 & 7 & -8 & 0 & | & -1 \end{bmatrix} \xrightarrow{row} \begin{bmatrix} 1 & 0 & 3 & 0 & | & -4 \\ 0 & 1 & -2 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Definition

A *pivot position* in a matrix is a position that after row reduction contains a leading one.

A pivot column of a matrix is a column that contains a pivot position.

Theorem

- 1) A system of linear equations is inconsistent if and only if the last column of its augmented matrix is a pivot column.
- 2) Free variables of the system correspond to non-pivot columns of the coefficient matrix.
- 3) The system has only one solution if and only if every column of its augmented matrix is a pivot column, except for the last column.

Theorem

A system of linear equations can have either 0, 1, or infinitely many solutions.

Proof.

