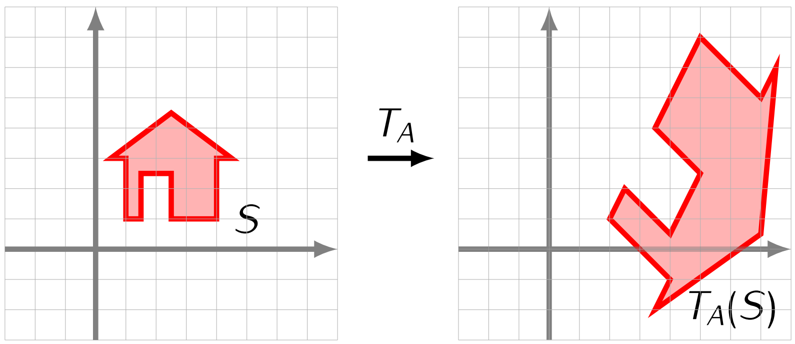
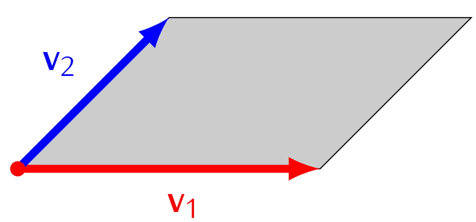
Recall: Determinants indicate how an area (or volume) changes under a matrix transformation:



$$\mathrm{area}(T_A(S)) = |\det A| \cdot \mathrm{area}(S)$$

Note. Any two vectors in $\mathbb{R}^2$ define a parallelogram:



Notation

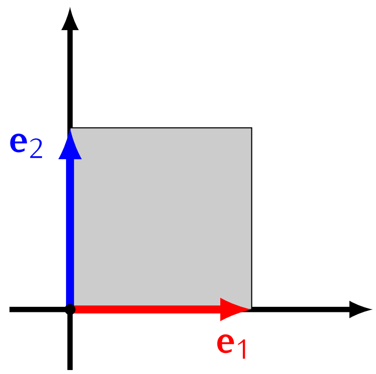
$$\mathrm{area}(\mathbf{v}_1, \mathbf{v}_2) = \begin{pmatrix} \text{area of the parallelogram} \\ \text{defined by } \mathbf{v}_1 \text{ and } \mathbf{v}_2 \end{pmatrix}$$

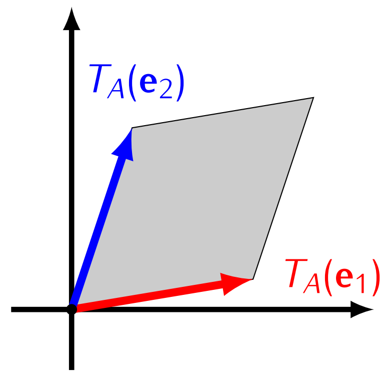
Theorem

If $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^2$ then

$$\operatorname{area}(\mathbf{v}_1, \mathbf{v}_2) = \left|\det\left[\begin{array}{cc} \mathbf{v}_1 & \mathbf{v}_2 \end{array}\right]\right|$$

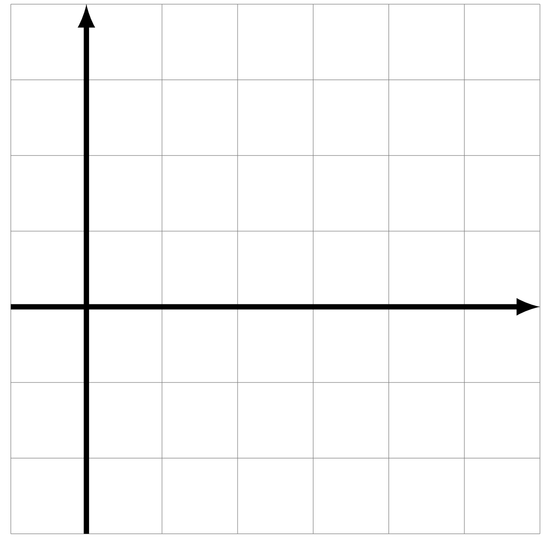
Proof.



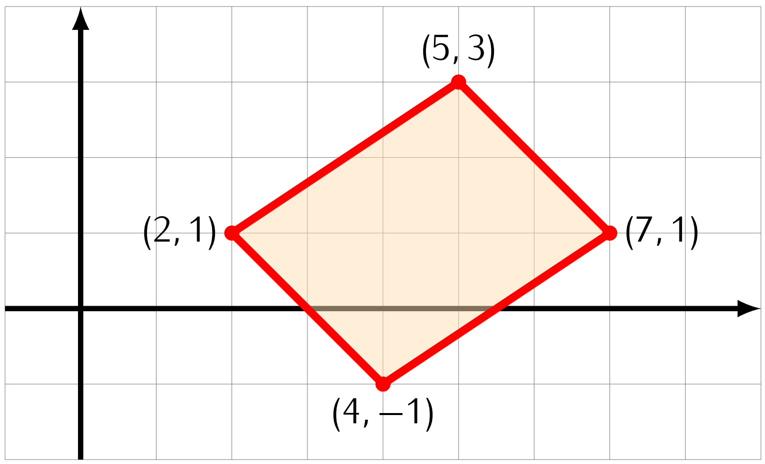


Example.

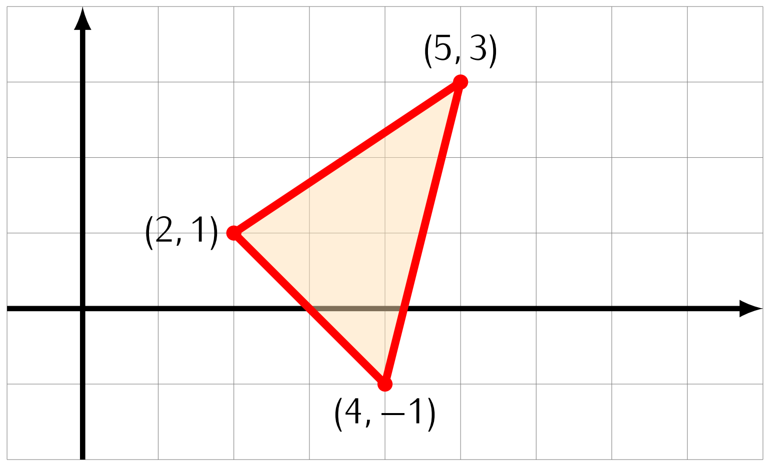
$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$



Example. Calculate the area of the parallelogram with vertices at the points $(2, 1)$, $(5, 3)$, $(7, 1)$, $(4, -1)$.



Example. Calculate the area of the triangle with vertices at the points $(2, 1)$, $(5, 3)$, $(4, -1)$.



Note. In order to compute areas of other polygons, subdivide them into triangles.