Instructions.

In this problem you will be given a few statements. For each statement you need to decide if it is true or not, and justify your answer.

How to justify your answer.

• In order to show that a statement is false, it suffices to give a counterexample. For example, consider the the statement:

The last digit of every even number is either 2, 4, or 8.

To show that this statement is false, it is enough to point out that, for example, 10 is an even number, but its last digit is 0.

• In order to show that a statement is true, you need to provide a reasoning explaining why it is true in all instances. Giving one example when it is true will not suffice, since the statement may not work in some other cases. For example, consider the the statement:

If n is an even number then $n + 2$ is also an even number.

You can justify that this is true as follows. Even numbers are integers which are multiples of 2. If n is even then $n = 2m$ for some integer m. Then $n + 2 = 2m + 2 = 2(m + 1)$, which shows that $n + 2$ is even.

Note

This problem will not be collected or graded. However, problems of this type will appear on exams in this course. Sample solutions are provided at the end.

For each of the statements given below decide if it is true or false. If you decide that it is true, justify your answer. If you think it is false give a counterexample.

a) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in $\mathbb{R}^3$ and $\mathbf{v}_1 \in \mathrm{Span}(\mathbf{v}_2, \mathbf{v}_3)$ then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

b) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in $\mathbb{R}^3$ and the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent, then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.

c) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in $\mathbb{R}^3$ and the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, then the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ is also linearly independent.

d) If A is a 2×3 matrix, and $\mathbf{v}$ is a vector in the column space $\mathrm{Col}(A)$ then $\mathbf{v} \in \mathbb{R}^3$.

e) If A is a matrix, $\mathrm{Nul}(A)$ is the null space of A, and $\mathbf{v}$ is a non-zero vector such that $\mathbf{v} \in \mathrm{Nul}(A)$, then $\mathrm{Nul}(A)$ must contain infinitely many vectors.