



Recall:How to solve a system of linear equations

system of equations

$$\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_3 = 9 \\ 4x_1 - x_2 - 3x_3 = 0 \end{cases}$$



*make
a matrix*



augmented
matrix



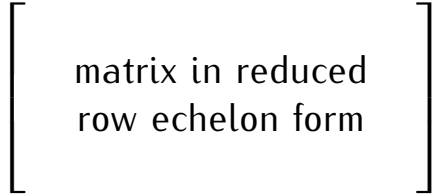
"row reduction"
*Gauss-Jordan
elimination*

solutions

$$\begin{cases} x_1 = \dots \\ x_2 = \dots \\ x_3 = \dots \end{cases}$$

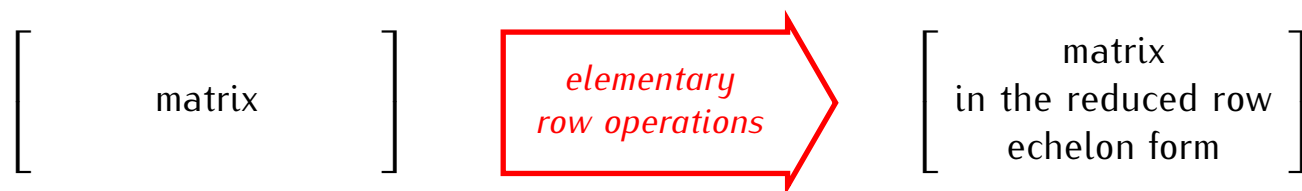


*read off
solutions*



matrix in reduced
row echelon form

Gauss-Jordan elimination process (= row reduction)



- ① Interchange rows, if necessary, to bring a non-zero element to the top of the first non-zero column of the matrix.
- ② Multiply the first row so that its first non-zero entry becomes 1.
- ③ Add multiples of the first row to eliminate non-zero entries below the leading one.
- ④ Ignore the first row; apply steps 1-3 to the rest of the matrix.
- ⑤ Eliminate non-zero entries above all leading ones.

Example.

$$\begin{bmatrix} 0 & 4 & -8 & 0 & 4 \\ 2 & 6 & -6 & -2 & -4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix} \begin{matrix} \curvearrowright \\ \curvearrowright \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 2 & 6 & -6 & -2 & -4 \\ 0 & 4 & -8 & 0 & 4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix} \cdot \frac{1}{2}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 4 & -8 & 0 & 4 \\ 2 & 7 & -8 & 0 & -1 \end{bmatrix} \cdot (-2)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 4 & -8 & 0 & 4 \\ 0 & 1 & -2 & 2 & 3 \end{bmatrix} \cdot \frac{1}{4}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 1 & -2 & 2 & 3 \end{bmatrix} \cdot (-1)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix} \cdot \frac{1}{2}$$

$$\begin{bmatrix} 1 & 3 & -3 & -1 & -2 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \cdot (1)$$

matrix in the row echelon form
(not reduced yet)

$$\downarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & 0 & -1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \cdot (-3)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

matrix in the reduced
row echelon form

(THE END)

How to solve systems of linear equations: example

$$\begin{cases} 4x_2 - 8x_3 = 4 \\ 2x_1 + 6x_2 - 6x_3 - 2x_4 = -4 \\ 2x_1 + 7x_2 - 8x_3 = -1 \end{cases}$$

① \rightarrow

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 0 & 4 & -8 & 0 & 4 \\ 2 & 6 & -6 & -2 & -4 \\ 2 & 7 & -8 & 0 & -1 \end{array}$$

② \downarrow row reduction
(see the previous page)
(for computations)

$$\begin{cases} x_1 + 3x_3 = -4 \\ x_2 - 2x_3 = 1 \\ x_4 = 1 \end{cases}$$

③ \leftarrow

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}$$

\uparrow
free variable

Simplify:

$$\begin{cases} x_1 = -4 - 3x_3 \\ x_2 = 1 + 2x_3 \\ x_3 = \text{free} \\ x_4 = 1 \end{cases}$$

(infinitely many solutions)