

Recall:

1) If  $A$  is an  $m \times n$  matrix then the function

$$T_A: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

defined by  $T_A(\mathbf{v}) = A\mathbf{v}$  is called the matrix transformation associated to  $A$ .

2) A function  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation if

(ii)  $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$

(ii)  $T(c\mathbf{v}) = cT(\mathbf{v})$

3) Every matrix transformation is a linear transformation.

4) Every linear transformation  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a matrix transformation:

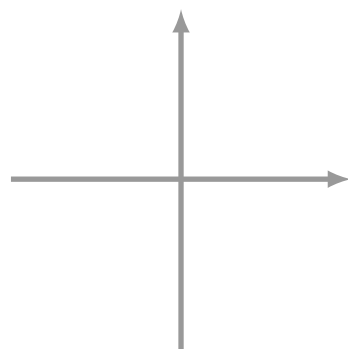
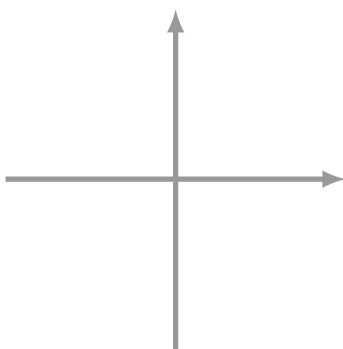
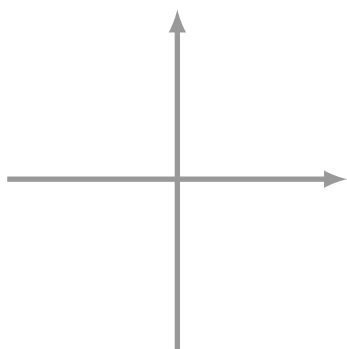
$$T(\mathbf{v}) = A\mathbf{v}$$

where

$$A = [ T(\mathbf{e}_1) \quad T(\mathbf{e}_2) \quad \dots \quad T(\mathbf{e}_n) ]$$

The matrix  $A$  is called the standard matrix of  $T$ .

## Composition of linear transformations



### Theorem

If  $S: \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $T: \mathbb{R}^m \rightarrow \mathbb{R}^k$  are linear transformation then the composition

$$T \circ S: \mathbb{R}^n \rightarrow \mathbb{R}^k$$

is also a linear transformation.

**Upshot.** The function  $T \circ S$  is represented by some matrix  $C$ :

$$T \circ S(\mathbf{v}) = C\mathbf{v}$$

**Question.** Let  $S: \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $T: \mathbb{R}^m \rightarrow \mathbb{R}^k$  be linear transformations, and let

- $B$  is the standard matrix of  $S$
- $A$  is the standard matrix of  $T$

What is the standard matrix of  $T \circ S: \mathbb{R}^n \rightarrow \mathbb{R}^k$ ?

## Definition

Let

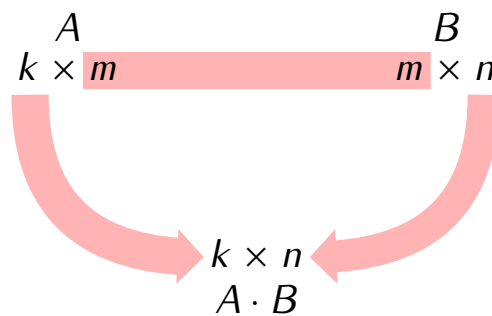
- $A$  be an  $k \times m$  matrix
- $B = [\mathbf{v}_1 \ \mathbf{v}_2 \ \dots \ \mathbf{v}_n]$  be an  $m \times n$  matrix

Then  $A \cdot B$  is an  $k \times n$  matrix given by

$$A \cdot B = [A\mathbf{v}_1 \ A\mathbf{v}_2 \ \dots \ A\mathbf{v}_n]$$

**Note.** The product  $A \cdot B$  is defined only if

(number of columns of  $A$ ) = (number of rows of  $B$ )



**Example.**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ 4 & 5 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$