

Recall:

1) If  $A$  is an upper triangular matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix}$$

then  $\det A = a_{11} \cdot a_{22} \cdot \dots \cdot a_{nn}$ .

**Note.** If  $A$  is a square matrix then the row echelon form of  $A$  is always upper triangular.

2) Let  $A$  and  $B$  be  $n \times n$  matrices.

- If  $B$  is obtained from  $A$  by interchanging two rows (or two columns) then

$$\det B = -\det A$$

- If  $B$  is obtained from  $A$  by multiplying one row (or one column) of  $A$  by a scalar  $k$  then

$$\det B = k \cdot \det A$$

- If  $B$  is obtained from  $A$  by adding a multiple of one row of  $A$  to another row (or adding a multiple of one column to another column) then

$$\det B = \det A$$

### Computation of determinants via row reduction

**Idea.** To compute  $\det A$ , row reduce  $A$  to the row echelon form. Keep track how the determinant changes at each step of the row reduction process.

**Example.** Compute  $\det A$  where

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 4 & 0 & 10 \\ 3 & 4 & 1 & 7 \\ -2 & 5 & 3 & 0 \end{bmatrix}$$

### Theorem

If  $A$  is a square matrix then  $A$  is invertible if and only if  $\det A \neq 0$

**Recall:**  $A$  is invertible if and only if its reduced row echelon form is the identity matrix.

## A direct way of computing the determinant of a $2 \times 2$ matrix

### Proposition

If  $A$  is a  $2 \times 2$  matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then  $\det A = ad - bc$

**Example.**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

## A direct way of computing the determinant of a $3 \times 3$ matrix

Example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

### Some further properties of determinants

1)  $\det(A^{-1}) = (\det A)^{-1}$

2)  $\det(A^T) = \det A$

**Note.** In general  $\det(A + B) \neq \det A + \det B$ .