Systems of linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$a_{ij}, b_j \in \underbrace{\mathbb{R}}$

the set of real numbers

Example:

$$\begin{cases} 2x_1 - 3x_2 = 4 \\ \frac{1}{2}x_1 + x_2 = 0 \end{cases}$$

system of 2 equations in 2 variables

Example:

$$\begin{cases} x_1 - \frac{1}{2}x_2 + 7x_3 = 15 \\ 4x_1 + \sqrt{2}x_2 - x_3 = -6 \end{cases}$$

system of 2 equations in 3 variables

Question: How many solutions a system of linear equations can have?

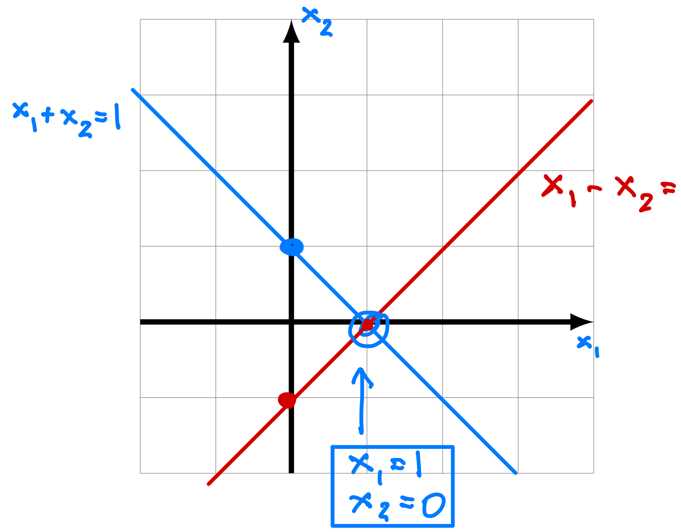
Example: Systems of equations in 2 variables.

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 1 \end{cases}$$

$$2x_1 + 0x_2 = 2$$

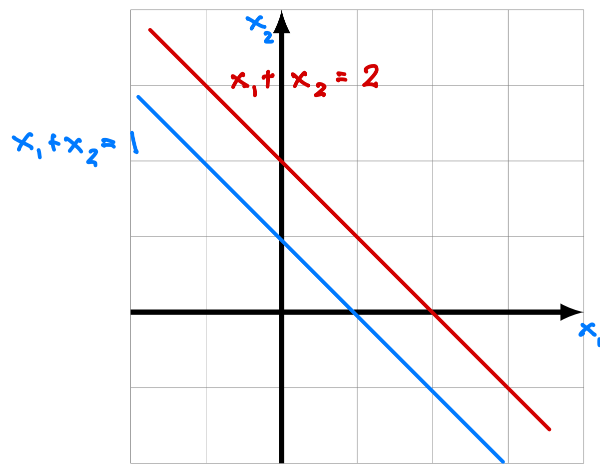
This gives: $\begin{cases} x_1 = 1 \\ x_2 = 0 \end{cases}$

- only one solution



$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + x_2 = 2 \end{cases}$$

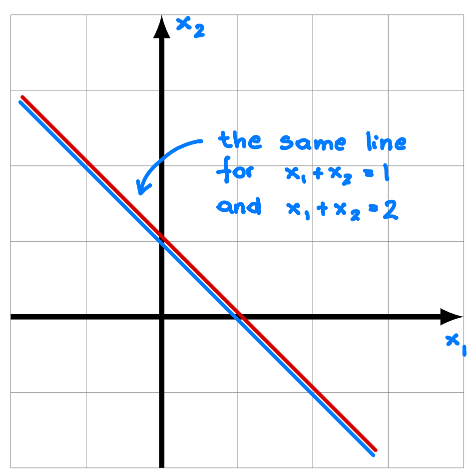
- no solutions



$$\begin{cases} x_1 + x_2 = 1 \\ \cancel{2x_1 + 2x_2 = 2} \end{cases}$$

$$\begin{cases} x_1 = 1 - x_2 \\ x_2 = \text{any number} \end{cases}$$

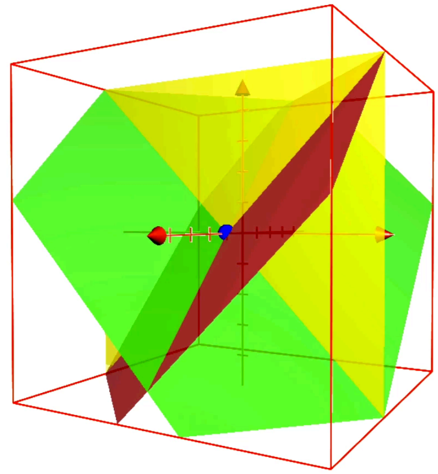
- infinitely many solutions



Example: Systems of equations in 3 variables.

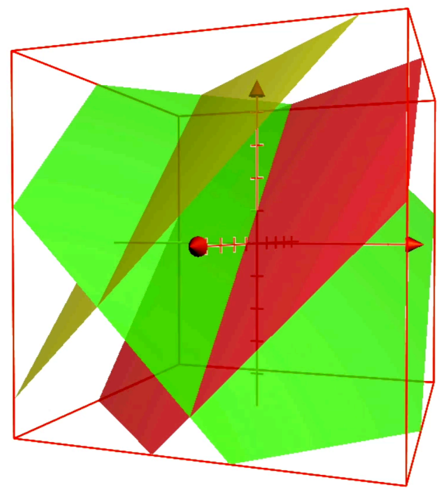
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 = 1 \end{cases}$$

only one solution



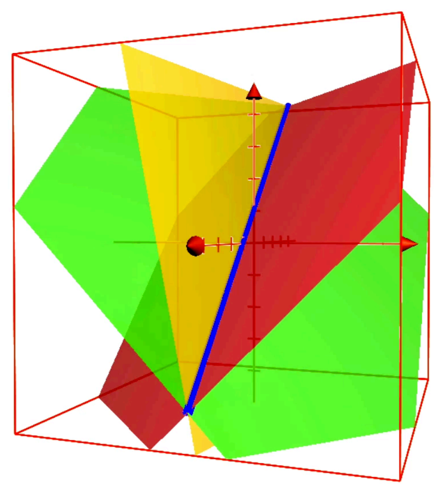
$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 6 \end{cases}$$

no solutions



$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 1 \\ x_1 + 5x_2 + x_3 = 1 \end{cases}$$

infinitely many solutions



In general:

A system of linear equations can have either

- no solutions
- exactly one solution
- infinitely many solutions

Definition

A system of linear equations which has no solutions is called an *inconsistent system*. Otherwise the system is *consistent*.