MTH 309 Practice Exam 1

1. Let A be a matrix and $\mathbf{v}$ be a vector given as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

a) Determine if $\mathbf{v}$ is in $\mathrm{Col}(A)$, where $\mathrm{Col}(A)$ is the column space of A.

b) Determine if $\mathbf{v}$ is in $\mathrm{Nul}(A)$, where $\mathrm{Nul}(A)$ is the null space of A.

c) Find an explicit description of $\mathrm{Nul}(A)$ by listing vectors that span the null space.

2. Consider the following matrices:

$$A = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

For each algebraic expression given below decide if it is defined. If it is, compute it. If it is not, give a reason why.

a) $C^T + 3B$ **b)** $CB + B$ **c)** $A^T BA$ **d)** $A^T C^{-1} A$ **e)** CBC

3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first reflects points through the line $x_1 = x_2$ and then reflects points through the x_1-axis.

a) Find the standard A matrix of T.

b) Find all vectors $\mathbf{v} \in \mathbb{R}^2$ such that $\mathbf{v} \in \mathrm{Nul}(A)$.

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad\qquad T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

a) Compute $T\left(\begin{bmatrix} 3 \\ 6 \end{bmatrix}\right)$.

b) Find a vector $\mathbf{v}$ such that $\mathbf{v} \neq \mathbf{0}$ and $T(\mathbf{v}) = \mathbf{0}$.

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A, B are matrices such that AB is defined and is a square matrix (i.e. it has the same number of rows and columns) then BA is also defined.

b) If A is an 2×2 matrix such that $A\mathbf{v} = \mathbf{0}$ for some non-zero vector $\mathbf{v} \in \mathbb{R}^2$ then A cannot be invertible.

c) If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set of vectors in $\mathbb{R}^2$ and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation then the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2)\}$ must be also linearly independent.

d) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^2$ such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $\mathbf{v}$ must be in $\mathrm{Span}(\mathbf{u}, \mathbf{w})$.