$$A = \begin{bmatrix} a_{11} & \dots & a_{1m} \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{km} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$$

$$AB = \begin{bmatrix} c_{11} & \dots & c_{1m} \\ \vdots & & \vdots \\ c_{k1} & \dots & c_{km} \end{bmatrix}$$

$$c_{ij} = \begin{bmatrix} a_{i1} & a_{i2} & \dots & a_{im} \end{bmatrix} \cdot \begin{bmatrix} b_{1j} \\ b_{1j} \\ \vdots \\ b_{1j} \end{bmatrix} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{im}b_{mj}$$

the entry in i^{th} row j^{th} column of AB

i^{th} row of A

j^{th} column of B

Example.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 & 2 & 1 \\ 4 & 5 & 1 & 0 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

2×3 $\qquad$ 3×4

AB is defined and it is a 2×4 matrix

$$AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{bmatrix}$$

$$c_{11} = [1^{st} \text{ row of } A] \cdot \begin{bmatrix} 1^{st} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} = 0 \cdot 0 + 1 \cdot 4 + 2 \cdot 1 = 6$$

$$c_{12} = [1^{st} \text{ row of } A] \cdot \begin{bmatrix} 2^{nd} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} = 0 \cdot (-1) + 1 \cdot 5 + 2 \cdot 2 = 9$$

$\vdots$

$$c_{24} = [2^{nd} \text{ row of } A] \cdot \begin{bmatrix} 4^{th} \\ \text{column} \\ \text{of} \\ B \end{bmatrix} = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 3 \cdot 1 + 4 \cdot 0 + 5 \cdot 1 = 8$$

$$AB = \begin{bmatrix} 6 & 9 & 7 & 2 \\ 21 & 27 & 25 & 8 \end{bmatrix}$$

Example.

- Acme Inc. makes two types of widgets: **WG1** and **WG2**.
- Each widget must go though two processes: **assembly** and **testing**.
- The number of hours required to complete each process is as follows:

	assembly	testing
WG1	3	1
WG2	7	3

$= A$

- Acme Inc. has three plans in New York, Texas, and Minnesota.
- Hourly cost (in dollars) of each process in each plant is as follows:

	NY	TX	MN
assembly	10	15	12
testing	15	20	15

$= B$

Problem. What is the cost of producing each type of widgets in each plant?

cost of WG1 in TX: $3 \cdot 15 + 1 \cdot 20 = \$65$

$$[3 \ \ 1] \cdot \begin{bmatrix} 15 \\ 20 \end{bmatrix} = [1^{st} \text{ row of } A] \cdot \begin{bmatrix} 2^{nd} \\ \text{column} \\ \text{of } B \end{bmatrix}$$

cost of WG2 in NY: $7 \cdot 10 + 3 \cdot 15 = \$115$

$$[7 \ \ 3] \cdot \begin{bmatrix} 10 \\ 15 \end{bmatrix} = [2^{nd} \text{ row of } A] \cdot \begin{bmatrix} 1^{st} \\ \text{column} \\ \text{of } B \end{bmatrix}$$

$$AB = \begin{array}{c} \\ \text{WG1} \\ \text{WG2} \end{array} \begin{array}{c} \begin{array}{ccc} \text{NY} & \text{TX} & \text{MN} \end{array} \\ \begin{bmatrix} 45 & 65 & 51 \\ 115 & 165 & 129 \end{bmatrix} \end{array}$$