

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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1	2	TRO .	3	I	4		ō	6	7	TOTAL	GRADE	



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

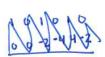
4)
$$x_1v_1 + x_2v_2 + x_3v_3 = v$$
 $x_1\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + x_2\begin{bmatrix} v_1 \\ v_3 \end{bmatrix} + x_3\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$
 $\frac{2 - 3 - 0 \cdot 0}{2 \cdot 2 \cdot 4}$
 $\frac{1 - 2 \cdot 2 \cdot 4}{0 - 1 - 2 \cdot 4 \cdot 4}$
 $\frac{1 - 2 \cdot 2}{0 - 1 - 2 \cdot 4 \cdot 4}$
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2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .



M

Compute
$$A^{-1}$$
.

A any of identity matrix:

$$\begin{bmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & -1 & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & -1 & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & -1 & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & -1 & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 2 & -2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 2 & -2 & 1
\end{bmatrix}$$

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0 & 1 & 2 & -2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & 0 &$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

and a matrix C such that
$$A'C = B$$
 (where A' is the transpose of A')
$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\vec{A}' = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (\vec{A}')^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{T} \cup B$$

$$C = (A^{T})^{T}$$

$$C = (A^{T})^{T}$$

$$A^{T} (:B)$$
 $C = (A^{T})^{T} \cdot B$
 $C = (A^$

$$V_{1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} \qquad V_{2} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 - 3 \\ -8 + 15 - 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \qquad \begin{bmatrix} 1 - 2 + 3 \\ 4 - 5 + 4 \\ 3 - 2 + 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} : \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \\ C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

$$v_{3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4+3 \\ 8-10+4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

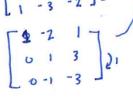
- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(e_1) = T([0]) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 & 1 \end{bmatrix}$$

$$T(e_2) = T([0]) = \begin{bmatrix} 0 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 & 3 \end{bmatrix}$$

b)
$$T(u) = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ -2 & 0 \end{bmatrix}$$
 $\begin{bmatrix} 0 & -2x & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 2x & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 2x & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 6 \\ 1 & 0 & 7 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 6 \\ 1 & 0 & 7 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$$u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$
 (only 1 solution)





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 $)$ -3

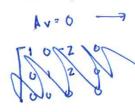
A has a pivot post in every column .. Ta(v) is one-to-one

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

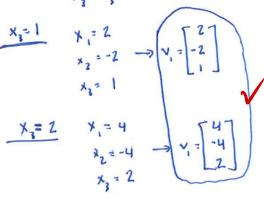
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

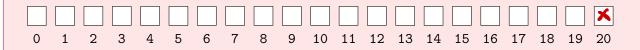
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

A does not have a pivot por in every column : Ta(v) is not one-to-one



$$N_{u}I(A) = T_{a}(v) = 0$$
 $Av = 0$
 $V = 0$







- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.



b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

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from the set, the set of the other two is always linearly independent.

True



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

[A|O] -> 00 solutions: u and v ann't necessarily linearly dependent because any matrix, multiplied by A, the solution is a linearly dependent matrix: u, v are not always lin. dependent.

[False]

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

x, v + x2 w = u -> (u & Span (v, w)) T(x,v)+T(x,w)=T(u) x,T(v) + x2T(w) = T(u) → (T(u) & Span (T(v),T(u)))