MTH-309T-F19-EX1-051-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	6	0	8	1	3
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

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1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 \to -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$R_1 \to R_1 + R_2$
$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$b = -6$ If b equals anything other than -6, there will be a pivot posinon in the last column leading to no solutions

b) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \Rightarrow -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right]$$

$R_3 \Rightarrow R_2 + R_3$
$R_1 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The set $\{v_1, v_2, v_3\}$ is not linearly dependent because $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$ does not have only the trivial solution. x_3 is a free variable giving infinite solutions making the set linearly dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] R2 \leftrightarrow R3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] R3 \to -R1+R3 \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$R2 = \frac{R2}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$R1 \to R2+R1, \quad R3 \to -R2+R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{-1}{2} & 0 & 0 & -\frac{1}{2} \end{array}\right]$$

$$R3 \to \frac{-2}{1} \cdot R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R2 \to \frac{1}{2}R3+R2 \\ R1 \to \frac{-3}{2}R3+R1 \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{T-1} \cdot A^T C = A^{T-1} \cdot B$$

$$C = A^{T-1} \cdot B$$

$$A^{T-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to -2R_1 + R_2$
$R_3 \to R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to -R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \to -R_2 + R_1$
$R_3 \to -R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$R_2 \to -R_3 + R_2$
$R_1 \to R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$$A^{T-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^{T-1} \cdot B =$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+0-3 \\ 0+0+6 \\ 0+4-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+0-2 \\ 0+0+4 \\ 0+5-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+0-1 \\ 0+0+2 \\ 0+4-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A^{T-1} \cdot B = \begin{bmatrix} -2 & 0 & 2 \\ 6 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix} = C$$

0 1 2 3 4 5 6 7 8 9 10

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard Matrix of T:

$$A = [T(e_1)\ T(e_2)] = \left[T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)\ T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)\right] = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \begin{array}{l} R_2 \to -R_1 + R_2 \\ R_3 \to -R_1 + R_3 \end{array} \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] R_2 \to \frac{R_2}{3}$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \begin{array}{l} R_3 \to R_2 + R_3 \\ R_1 \to 2R_2 + R_1 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$u = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

there is a unique solution

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \; R3 \to -3R1 + R3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \; R2 \to \frac{R2}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \; \begin{array}{l} R1 \to -R2 + R1 \\ R3 \to -R2 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \; R3 \to \frac{R3}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \; \begin{array}{l} R2 \to -2R3 + R2 \\ R1 \to 2R3 + R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

← A IS one to one w/ a pivot pos. in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \; R3 \to -3R1 + R3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \; \begin{array}{l} R1 = -\frac{1}{2}R2 + R1 \\ R3 = -\frac{1}{2}R2 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2x_3$
$x_2 = -4x_3$
$x_3 = x_3$

$$\mathrm{Nul}(A) = \mathrm{Span}\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix} \right\}$$

T(A) is not one to one because there is not a pivot position in every column

$$V_1 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right) = 0$$

$$T\left(\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}\right) = 0$$

$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ is in Nul(A)

$TA(V_1) = TA(V_2)$

$$TA\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}\right) = TA\left(\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right)$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$w + u \in \text{Span}(u, v) \quad w \in \text{Span}(u,v)$

$w = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$w + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$w = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$w + \begin{bmatrix} 2 \\ 3 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$w = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \ldots\right)$

$\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$$\left[\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{array}\right] \xrightarrow[R2 \to -2R1+R2]{R3 \to -3R1+R3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & -6 & -3 \end{array}\right] \xrightarrow{R3 = -2R2+R3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{R2}{-3}} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{True}}$ because w + u linear combination of u (w to be in span therefore it will be in span(u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix}$

$\boxed{\text{True}}$ because if set $\{u, v, w\}$ is all linearly independent from each other, then u and v has to be linearly independent

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

$A\begin{bmatrix}-3\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}-4+2\\-6+4\end{bmatrix}=\begin{bmatrix}-2\\-2\end{bmatrix}$

$A\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1+0\\2+0\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$ } dependent

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\4\end{bmatrix}$

$A=\left[\begin{array}{cc|c}1&2&0\\2&4&0\end{array}\right]$ $R_2\to-2R_1+R_2$ $v=\begin{bmatrix}1\\1\end{bmatrix}$ $u=\begin{bmatrix}2\\2\end{bmatrix}$

$\begin{bmatrix}1&2&0\\0&0&0\end{bmatrix}$

$A\cdot\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}-2+2\\-4+4\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$A\cdot\begin{bmatrix}-4\\2\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-4\\2\end{bmatrix}=\begin{bmatrix}-4+4\\-8+8\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$x_1=-2x_2$

$x_2=x_2$

$\text{Nul}(A)=\text{span}\left\{\begin{bmatrix}-2\\1\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}\right\}$

2 examples $\begin{bmatrix}-3\\2\end{bmatrix}$, [

False $v=\begin{bmatrix}1\\0\end{bmatrix}$ $u=\begin{bmatrix}0\\1\end{bmatrix}$

$A(v)$, $A(u)$ are linearly dependent, but v,u is linearly independent

$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

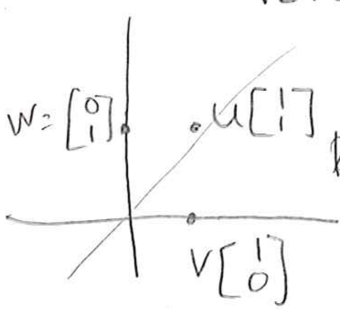
b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$u=\begin{bmatrix}1\\1\end{bmatrix}$ $v=\begin{bmatrix}1\\0\end{bmatrix}$ $w=\begin{bmatrix}0\\1\end{bmatrix}$ $u\in \text{Span}(v,w)$

$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=\begin{bmatrix}-5\\-5\end{bmatrix}$ $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)\to\begin{bmatrix}-5\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)\to\begin{bmatrix}0\\-5\end{bmatrix}$

$\boxed{\text{True}}$ because with any reflection, $T(u)$ will be a linear combination of $T(v)$ and $T(w)$ if u is in $\text{span}(v,w)$



Reflect over x-axis	reflect over line y=x	reflect over y-axis
$T(u)=\begin{bmatrix}1\\-1\end{bmatrix}$	$T(u)=\begin{bmatrix}1\\1\end{bmatrix}$	$T(u)=\begin{bmatrix}-1\\1\end{bmatrix}$
$T(w)=\begin{bmatrix}0\\-1\end{bmatrix}$	$T(w)=\begin{bmatrix}1\\0\end{bmatrix}$	$T(w)=\begin{bmatrix}0\\1\end{bmatrix}$
$T(v)=\begin{bmatrix}1\\0\end{bmatrix}$	$T(v)=\begin{bmatrix}0\\1\end{bmatrix}$	$T(v)=\begin{bmatrix}-1\\0\end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10