3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$A^T \cdot C = \begin{bmatrix} x_1 + y_1 + 0 & x_2 + y_2 + 0 & x_3 + y_3 + 0 \\ -x_1 + 0 + 2z_1 & -x_2 + 0 + 2z_2 & -x_3 + 0 + 2z_3 \\ 2x_1 + y_1 - z_1 & \boxed{2x_2 + y_2 - z_2} & 2x_3 + y_3 - z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$x_1 + y_1 = 1 \rightarrow y_1 = 1 - x_1$

$-x_1 + 2z_1 = 4$

$2x_1 + y_1 - z_1 = 3 \rightarrow 2x_1 + 1 - x_1 - z_1 = 3$

$z_1 = 6$, $x_1 = 8$, $y_1 = -7$ $\Leftarrow \begin{cases} x_1 - z_1 = 2 \\ -x_1 + 2z_1 = 4 \end{cases}$

$x_2 + y_2 = 2 \rightarrow y_2 = 1 - x_2$

$-x_2 + 2z_2 = 5$

$2x_2 + y_2 - z_2 = 2$

$\begin{rcases} x_2 - z_2 = 1 \\ -x_2 + 2z_2 = 5 \end{rcases}$ $z_2 = 6$, $x_2 = 7$, $y_2 = -6$

$x_3 + y_3 = 3 \rightarrow y_3 = 3 - x_3$

$-x_3 + 2z_3 = 4$

$2x_3 + y_3 - z_3 = 1$

$\begin{rcases} x_3 - z_3 = -2 \\ -x_3 + 2z_3 = 4 \end{rcases}$ $z_3 = 2$, $x_3 = 0$, $y_3 = 3$

$$\begin{rcases} x_1 = 8 & x_2 = 7 & x_3 = 0 \\ y_1 = -7 & y_2 = -6 & y_3 = 3 \\ z_1 = 6 & z_2 = 6 & z_3 = 2 \end{rcases} C = \begin{bmatrix} 8 & 7 & 0 \\ -7 & -6 & 3 \\ 6 & 6 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad ?\ A^T C = B \qquad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$C = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

This is good!

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☒ 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix} \cdot C = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{matrix}$$

$$(AB)^T = B^T A^T \qquad 2 \times 4 = 8$$

$$A^T \cdot C = B \qquad \frac{B}{A^T} = C = B \cdot (A^T)^{-1}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = 0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 0\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$-1+2+3=4$

$-4+5+4=5$

$-3+2+1=0$

$$\begin{bmatrix} -1 & 4 & 3 \\ 1 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix} = C$$

0 1 2 3 4 5 6 7 8 9 10

MTH-309T-F19-EX1-004-P03

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$(A^T)^{-1} = (A^{-1})^T \qquad AA^{-1} = I \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$x_1 + y_1 = 1$
$-x_1 + 2z_1 = 4$
$2x_1 + y_1 - z_1 = 3$

$$(A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

$z_1 = 6$
$y_1 = 5 - 2(6)$
$y_1 = -7$
$x_1 = 1 - -7$
$x_1 = 8$

$$C_1 = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -1 & 0 & 2 & 5 \\ 2 & 1 & -1 & 2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

$z_2 = 5$
$y_2 = 7 - 10$
$y_2 = -3$
$x_2 = 2 - -3$
$x_2 = 5$

$$C_2 = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$$

check

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix} = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{matrix} \quad \checkmark$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -5 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$z_3 = 2$
$y_3 = 7 - 4 = 3$
$x_3 = 3 - 3 = 0$

$$C_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\cancel{(A^T)^{-1} A^T} C \; (A^T)^{-1} B$$

$$C = (A^{-1})^T \cdot B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

3 x 3   3 x 3

$$\boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

2+6

-2+4+6=

1+4

-4+5+4

-6+4+2

-2+2

3+4-6

-1+6

6-5-4

1+4

9-4-2

5-2

-1+4+3

3+3

-2+5+2

-3+4+1 · 1·

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} A^T C = B (A^T)^{-1}$$

$$C = B (A^T)^{-1}$$

$$C = B (A^{-1})^T$$

$$C = B \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad A^T C = B \qquad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 2 & 5 & 0 \\ -1 & -3 & 3 \\ 0 & 5 & 2 \end{bmatrix}}$$

$c_{11} + c_{21} = 1$ $\quad c_{11} = 1 - c_{21}$

$-c_{11} + 2c_{31} = 4$

$2c_{11} + c_{21} - c_{31} = 3$ $\quad 2(1-c_{21}) + c_{21} + c_{31} = 3$

$2 - c_{21} + c_{31} = 3$

$c_{31} = 1 + c_{11}$

$2(1-c_{21}) + c_{11} - (1 + c_{11}) = 3$

$2 - 2c_{21} + c_{21} - 1 - c_{21} = 3$

$1 - 2c_{21} = 3$

$-2c_{21} = 2 \quad c_{21} = -1 \Rightarrow$ $\boxed{\begin{matrix} c_{11} = 2 \\ c_{21} = -1 \\ c_{31} = 0 \end{matrix}}$

$c_{12} + c_{22} = 2$ $\quad c_{12} = 2 - c_{22}$

$-c_{12} + 2c_{32} = 5$

$2c_{12} + c_{22} - c_{32} = 2$

$-(2 - c_{22}) + 2(2 - c_{22}) = 5$

$-2 + c_{22} + 4 - 2c_{22} = 5$

$2 - c_{22} = 5$

$-c_{22} = 3$

$c_{22} = -3$

$2(2 - c_{22}) + c_{22} - c_{32} = 2$

$4 - 2c_{22} + c_{22} - c_{32} = 2$

$4 - c_{22} - c_{32} = 2$

$-c_{22} - c_{32} = -2$

$-c_{32} = -2 + c_{22}$

$c_{32} = 2 - c_{22}$

$\Rightarrow c_{12} = 5$

$c_{22} = -3$

$c_{32} = 5$

$c_{13} + c_{23} = 3$ $\quad c_{13} = 3 - c_{23}$

$-c_{13} + 2c_{33} = 4$

$2c_{13} + c_{23} - c_{33} = 1$

$2(3 - c_{13}) + c_{23} - c_{33} = 1$

$6 - 2c_{23} + c_{23} - c_{33} = 1$

$6 - c_{23} - c_{33} = 1$

$-c_{23} - c_{33} = -5$

$-c_{33} = -5 + c_{23}$

$c_{33} = 5 - c_{23}$

$-(3 - c_{23}) + 2(5 - c_{23}) = 4$

$-3 + c_{23} + 10 - 2c_{23} = 4$

$7 + c_{23} - 2c_{23} = 4$

$-c_{23} = -3$

$c_{23} = 3$

$\Rightarrow c_{13} = 0$

$c_{23} = 3$

$c_{33} = 2$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☒ 9 ☐ 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}}^{A^T} \overbrace{\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}}^{C} = \overbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}^{B}$$

$3 \times 3 \quad\quad 3 \times 3 \quad\quad 3 \times 3$

$\cancel{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$

$\cancel{3 \times 2 \quad 2 \times}$

~~Colums have to be same~~ ?

Come back!

$1x_1 - 1x_2 + 2x_3 = 1$

Hello?

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T \cdot B$$

$$C = (A^T)^{-1} \cdot B$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 0 & 0 \\ 0 & 13 & 0 \\ 0 & 0 & 6 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 13 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☒ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$3 \times \underbrace{3 \cdot 3}_{B} \cdot 3$

A C

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

$x_1 = 1 - y_1$

$z_1 = 1 + \frac{x_1}{2}$

$z_1 = 1 + \frac{1-y_1}{2}$

$x_1 + y_1 = 1$

$-x_1 + 2z_1 = 2$

$2x_1 + y_1 - z_1 = 3$

$2(1-y_1) + y_1 - \left(1 + \frac{(1-y_1)}{2}\right) = 3$

$2 - 2y_1 + y_1 - 1 - \frac{1}{2} + \frac{y_1}{2} = 3$

$1 = 2y_1 + y_1 = 6$

$y_1 = -5$

$x_1 = 6$

$z_1 = 4$

$x_2 = 4 - y_2$

$z_2 = \frac{5}{2} + \frac{x_2}{2}$

$x_2 + y_2 = 4$

$-x_2 + 2z_2 = 5$

$2x_2 + y_2 - z_2 = 4$

$4 = 2(4-y_2) + y_2 - \left(\frac{5}{2} + \frac{(4-y_2)}{2}\right)$

$8 = 4(4-y_2) + 2y_2 - 5 - 4 + y_2$

$8 = 16 - y_2 + 2y_2 - 9 + y_2$

$y_2 = 3$

$x_2 = 1$

$z_2 = 1$

$x_3 + y_3 = 3$

$-x_3 + 2z_3 = 2$

$2x_3 + y_3 - z_3 = 1$

$\boxed{C = \begin{bmatrix} 6 & 3 \\ -5 & 1 \\ 4 & 1 \end{bmatrix}}$

$x_3 = 3 - y_3$

$z_3 = 1 - \frac{3}{2} + \frac{y_3}{2}$

$1 = 2(3-y_3) + y_3 - \left(1 - \frac{3}{2} + \frac{y_3}{2}\right)$

$2 = 12 - 4y_3 + 2y_3 - 2 + 3 - y_3$

$2 = 13 - 3y_3 = \frac{11}{3}$

0	1	2	3	4	5	6	7	8	9	10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \cdot 1 \\ \\ \cdot -2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & -6 \\ 13 & -13 & 13 \\ 12 & -12 & 6 \end{bmatrix} = C$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] -2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] -1$$

$$A^{T-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 1 & 2 & -6 \\ 13 & -13 & 13 \\ 12 & -12 & 6 \end{bmatrix}}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☒ 7 ☐ 8 ☐ 9 ☐ 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore C = B(A^{-1})^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \boxed{C = \begin{bmatrix} -2 & 2 & 6 \\ 12 & -5 & -8 \\ -3 & 2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 5 & 2 \\ 3 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{bmatrix}$$

$\Rightarrow$ $a + 4d + 3g = 1$
$2a + 5d + 2g = 4$
$3a + 4d + g = 3$

$\cancel{a = \frac{7}{6}, d = 0, g = \ldots}$

$a = -1, d = 2, g = -2$

$b + 4e + 3h = 2$
$2b + 5e + 2h = 5$
$3b + 4e + h = 4$

$b = \frac{-3}{4}, e = 2, h = \frac{-7}{4}$

$c + 4f + 3i = 3$
$2c + 5f + 2i = 4$
$3c + 4f + i = 1$

$c = -2, f = 2, i = -1$

$$C = \begin{bmatrix} -1 & -\frac{3}{4} & -2 \\ 2 & 2 & 2 \\ -2 & -\frac{7}{4} & -1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad \therefore A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 & c_4 & c_7 \\ c_2 & c_5 & c_8 \\ c_3 & c_6 & c_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

12
34
56

1 3 5
2 4 6

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^{T-1} \cdot B = C$

$-2+4+6$
$3-4-6=-7$
$-1+4+3=6$

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^{T-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

$=$

$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$

1 1 0 1
-1 0 2 0
2 1 -1 0

1 1 0 1
0 1 2 1
0 -1 -1 -2

1 1 0 1
0 1 2 1
0 0 1 -1

1 1 0 1
0 1 0 3
0 0 1 -1

1 0 0 -2
0 1 0 3

$-\cancel{4}+5+\cancel{4}=5$
$6-5-4=-3$
$-2+5+2=5$

$-6+4+2=0$
$9-4-2=3$
$-3+4+1=2$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$A^T \cdot A^{T-1} C = B A^{T-1}$$

$$C = B A^{T-1}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \quad \therefore \; A^T \text{ inverse} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{-1}{-1\times 1} + \overset{4}{1\times 4} + 0\times 3 & \overset{-2}{-1\times 2} + \overset{5}{1\times 5} + 0\times 2 & -1\times 3 + 1\times 4 + 0\times 1 \\ \overset{-1}{-1\times 1} + 0\times 4 + 2\times 3 & -1\times 2 + 0\times 5 + 2\times 2 & -1\times 3 + 0\times 4 + -1\times 1 \\ \overset{2}{2\times 1} + 1\times 4 + -1\times 3 & 2\times 2 + 1\times 5 + -1\times 2 & 2\times 3 + 1\times 4 + -1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 1 \\ 5 & 2 & -4 \\ 3 & 7 & 9 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^T C = B$

$A \; A^T$

$(A^T)(A^T)^{-1} C = B(A^T)^{-1}$.

$C = B(A^T)^{-1}$

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

invertible.

~~$(A^T)^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] = [$~~

$(A^T)^{-1} = (A^{-1})^T$.

$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

~~$\begin{bmatrix} 1\times-2+1\times1+0\times1 & 1\times1+1\times-1+0\times1 & 1\times2+1\times-2+0 \\ -1\times-2+3\times0+2\times-1 & -1\times1+0\times-1+2\times1 & 2\times2+1\times-2+1\times \\ 2\times-2+1\times3+-1\times-1 & 2\times1+1\times-1+-1\times1 & -1\times2+0\times-2+2\times1 \end{bmatrix}$~~

$2\times2+1\times-2+-1\times1$

$C = B(A^{-1})^T$

$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times-2+2\times3+3\times-1 & 1\times1+2\times-1+3\times1 & 1\times2+2\times-2+3\times1 \\ 4\times-2+5\times3+4\times-1 & 4\times1+5\times-1+4\times1 & 4\times2+5\times-2+4\times1 \\ 3\times-2+2\times3+1\times-1 & 3\times1+2\times-1+1\times1 & 1\times2+2\times-2+1\times1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}$.

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$C = B(A^T)^{-1} \qquad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2+6-3 & 1-2+3 & 2-4+3 \\ -8+15-4 & 4-5+4 & 8-10+4 \\ -6+6-1 & 3-2+1 & 6-4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$Ax = b$

$x = A^{-1} \cdot b$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B \Rightarrow C = (A^T)^{-1} \cdot B \qquad (A^T)^{-1} = (A^{-1})^T$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{-1})^T \cdot B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

$A^T C = B$ $\quad$ $AC = \frac{B}{A^{-1}}$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$A+D=1$

12 $-A+2G=4$

16-7-6 $2A+D-G=3$

$2A+D-G=3$

$A+D=1$

$A-G=2$

$-A+2G=4$

$G=6$

$A=8$

$D=-7$

$B+E=2$

$-B+2H=5$

10 -3 -5

$2B+E-H=2$

$2B+E-H=2$

$B+E=2$

$B-H=0$

$-B+2H=5$

$H=5$

$B=5$

$E=-3$

$C+F=3$

$-C+2I=4$

$2C+F-I=1$

$2C+F-I=1$

$C+F=3$

$C-I=-2$

$-C+2I=4$

$I=2$

$-C+4=4$

$-C=0$

$C=0$

$F=3$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$\underset{3\times 3}{A} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad \underset{3\times 3}{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T \boxed{C} = B$ (where A^T is the transpose of A).

$$\frac{3}{1} - \frac{4}{3} \qquad \frac{9}{3} - \frac{4}{3} \qquad \frac{3}{3} - \frac{4}{3} = \frac{-1}{3}$$

$$\frac{15}{1} - \frac{4}{3} = \frac{45}{3} - \frac{4}{3} \qquad \frac{2}{1} + \frac{10}{3}$$

$$\frac{6}{1} - \frac{1}{3} = \frac{18}{3} - \frac{1}{3} \qquad \frac{6}{3} + \frac{10}{3}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T(A^T)^{-1}C = B(A^T)^{-1}$$

$$C = B(A^T)^{-1}$$

$$(A^T)^{-1} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ 3 & \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$B(A^T)^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 3 \\ -\frac{1}{3} \end{bmatrix} = 0 + 3\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + -\frac{1}{3}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ 6 \end{bmatrix} + \begin{bmatrix} -1 \\ -\frac{4}{3} \\ -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{41}{3} \\ \frac{17}{3} \end{bmatrix}$$

$$B \cdot \begin{bmatrix} \frac{1}{3} \\ \frac{5}{3} \\ \frac{1}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + \frac{5}{3}\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{4}{3} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{10}{3} \\ \frac{25}{3} \\ \frac{10}{3} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{4}{3} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} \\ 11 \\ \frac{10}{3} \end{bmatrix}$$

$$B \cdot \begin{bmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} + -\frac{2}{3}\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{8}{3} \\ \frac{2}{3} \end{bmatrix} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{10}{3} \\ -\frac{4}{3} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{4}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\boxed{\therefore C = \begin{bmatrix} 5 & \frac{10}{3} & \frac{5}{3} \\ \frac{41}{3} & 11 & \frac{2}{3} \\ \frac{17}{3} & \frac{10}{3} & -\frac{1}{3} \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A^T \cdot B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = C$$

$$\left(\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}\right) + \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & -2 \end{bmatrix} + \left(\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$\left(1\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) + \left(2\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right)$$

$$+ \left(3\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right)$$

$$2 \times 2 \times 2 \times 2 \times 3 \; \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 7 & 1 \\ -1 & 6 & 5 \\ 9 & 7 & 3 \end{bmatrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}} \qquad A = \underset{3 \times 3}{\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \text{ (calculated in problem 2)}$$

Using the Fact that: $(A^T)^{-1} = (A^{-1})^T$

$$\therefore (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \rightarrow \begin{aligned} (A^T)^{-1}(A^T)C &= B(A^T)^{-1} \\ C &= B(A^T)^{-1} \end{aligned}$$

$$\therefore C = \underset{B}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}} \underset{(A^T)^{-1}}{\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}} = \begin{bmatrix} 1(-2)+3(2)-3, & 1-2+3, & 2-4+3 \\ 4(-2)+5(3)-4, & 4-5+4, & 4(2)-5(2)+4 \\ 3(-2)+2(3)-1, & 3-2+1, & 3(2)-2(2)+1 \\ -6+6 & & 6-4+1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3, & -1+3, & -2+3 \\ -8+15-4, & -5, & 8-10+4 \\ -1, & 0, & 6-4+1 \end{bmatrix} = \begin{bmatrix} 1, & 2, & 1 \\ 3, & -5, & 2 \\ -1, & 0 & 3 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 1, & 2, & 1 \\ 3, & -5, & 2 \\ -1, & 0, & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$C_{11} + C_{21} = 1$

$-C_{11} + 2C_{13} = 4$

$2C_{11} + C_{12} - C_{13} = 3$

$C_{12} = 1 - C_{11}$

$C_{13} = 2 + \frac{1}{2}C_{11}$

$2C_{11} + 1 - C_{11} - 2 - \frac{1}{2}C_{11} = 3$

$-1 + \frac{1}{2}C_{11} = 3$

$\frac{1}{2}C_{11} = 4$

$C_{11} = 8$

$C_{21} = -7$

$C_{31} = 6$

$C_{21} + C_{22} = 2$

$-C_{21} + 2C_{23} = 5$

$2C_{21} + C_{22} - C_{23} = 2$

$C_{22} = 2 - C_{21}$

$C_{23} = \frac{5}{2} + \frac{1}{2}C_{21}$

$\frac{4}{2}$

$2C_{21} + 2 - C_{21} - \frac{5}{2} - \frac{1}{2}C_{21} = 2$

$\frac{1}{2}C_{21} - \frac{1}{2} = 2$

$\frac{1}{2}C_{21} = \frac{5}{2}$

$C_{12} = 5$

$C_{22} = -3$

$C_{32} = 5$

$C_{13} + C_{23} = 3$

$-C_{13} + 2C_{33} = 4$

$2C_{13} + C_{23} - C_{33} = 1$

$C_{23} = 3 - C_{13}$

$C_{33} = 2 + \frac{1}{2}C_{13}$

$2C_{13} + 3 - C_{13} - 2 - \frac{1}{2}C_{13} = 1$

$\frac{1}{2}C_{13} = 0$

$C_{13} = 0$

$C_{23} = 3$

$C_{33} = 2$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad \begin{matrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{matrix}$$

$C = BA^{T^{-1}}$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix} = C$$

$-8+15-4$ $\quad \begin{matrix} -2 \\ 1 \\ 2 \end{matrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ -11 \\ 2 \end{bmatrix} = $$

4

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 2 & 1 & 2 & 1 & 2 & 3 \\ -3 & -1 & -2 & 4 & 5 & 4 \\ 1 & 1 & 1 & 3 & 2 & 1 \end{array}\right]$ $-R_2 - R_1 : R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ -3 & -1 & -2 & 4 & 5 & 4 \\ 1 & 1 & 1 & 3 & 2 & 1 \end{array}\right]$ $3R_1 + R_2 : R_2$

$C = \begin{bmatrix} -5 & -7 & -7 \\ 5 & 2 & -1 \\ 3 & 7 & 9 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & -1 & -2 & -11 & -16 & -17 \\ 1 & 1 & 1 & 3 & 2 & 1 \end{array}\right]$ $R_3 : R_3 - R_1$
$R_2 : -R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & +1 & +2 & +11 & +16 & +17 \\ 0 & 1 & 1 & 8 & 9 & 8 \end{array}\right]$ $R_3 : R_3 - R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & 1 & 2 & 11 & 16 & 17 \\ 0 & 0 & +1 & +3 & +7 & +9 \end{array}\right]$ $R_3 = -R_3$
$R_2 = R_2 - 2R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & 1 & 0 & 5 & 2 & -1 \\ 0 & 0 & 1 & 3 & 7 & 9 \end{array}\right]$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$A^T C = B$

Matrix Algebra Property: $(A^T)^{-1} = (A^{-1})^T$

$C = (A^T)^{-1} \cdot B$

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C = \underbrace{\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}}_{3\times 3} \cdot \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}_{3\times 3}$$

defined with
$C = 3\times 3$

$$C = \begin{bmatrix} -2(1)+1(4)+2(3) & -2(2)+1(5)+2(2) & -2(3)+1(4)+2(1) \\ 3(1)+(-1)(4)+(-2)(3) & 3(2)+(-1)(5)+(-2)(2) & 3(3)+(-1)(4)+(-2)(2) \\ 0(1)+1(4)+1(3) & 0(2)+1(5)+1(2) & 0(3)+1(4)+1(1) \end{bmatrix}$$

$$C = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-4 \\ 4+3 & 5+2 & 4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 1 \\ 7 & 7 & 5 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 3$, $x_3 = 7$

$x_2 = 5 - x_3$

$x_3 = 7$

$5 - 7 = -2$

$1 + 2 = 1$

1
-2
7

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ B

3x3 3x3 = 3x3

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ -1 & 0 & 2 & 5 \\ 2 & 1 & 1 & 2 \end{bmatrix}$$

1 1 0 2
0 1 2 7
0 -1 1 -2

1 1 0 2 $x_1 =$
0 1 2 7 $x_2 = 5$
0 0 3 5

3 -2 0 1
-3 0 2 3
6 -2 -1 = 3

2 0 2
-2 6 = 4
4 -2 -2

1 1
-1 6 = 5
2 3 5

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B$$

$$\Leftrightarrow (A^T)^{-1} \cdot A^T \cdot C = (A^T)^{-1} \cdot B \quad \ulcorner \because (A^T)^{-1} \cdot A^T = I \lrcorner$$

$$\Leftrightarrow C = (A^T)^{-1} \cdot B$$

$$\Leftrightarrow C = (A^{-1})^T \cdot B \quad \ulcorner \because (A^T)^{-1} = (A^{-1})^T \lrcorner$$

$$\therefore C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$3 \times 3 \quad \times \quad 3 \times 3$

$$C = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix}$$

$r_1 c_1 : \quad 2x_1 + x_2 - x_3 = 1$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{RR} \begin{bmatrix} C \end{bmatrix}$$

$r_2 c_1 : \quad -x_1 + 2x_3 = 4$

$r_3 c_1 : \quad x_1 + x_3 = 3$

$x_1 = 3 - x_3$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\cancel{(A^T)}(A^{-1})^T C = B(A^{-1})^T$$

$$C = B(A^{-1})^T$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$A^T \cdot (A^T)^{-1} C = B \cdot (A^T)^{-1}$ since $(A^T)^{-1} = (A^{-1})^T$

$IC = B \cdot (A^{-1})^T$

$$C = B \cdot (A^{-1})^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \qquad \left|\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right|$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] \qquad \begin{array}{cccccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \qquad (A^T)^{-1} = \begin{vmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix}$$

$$(A^T)^{-1}A^T C = (A^T)^{-1}B \qquad \begin{vmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{bmatrix}$$

1 · 2

$(-2 + 4 + 6)$ $(-4 + 5 \ 4)$ $(-6 + 4 + 2)$

2 · 1   2 2

$(3 + -4 - 6)$ $(6 - 5 - 4)$ $(9 - 4 - 2)$

$(-1 + 4 + 3)$ $(2 + 5 + 2)$ $(-3 + 4 + 1)$

$$\boxed{C = \begin{vmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{vmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $\therefore$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \cancel{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

Let $C = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ $(A^T)^{-1} = (A^{-1})^T$

rows $\longleftrightarrow$ columns

$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$ $(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

$A^T C = B$

$C = B(A^T)^{-1}$

$C = B(A^{-1})^T$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -2+6-3 \\ -8+15-4 \\ -6+6-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ $= \begin{bmatrix} 1-2+3 \\ 4-5+4 \\ 3-2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$

$v_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 2-4+3 \\ 8-10+4 \\ 6-4+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$C = [v_1\ v_2\ v_3]$

$\boxed{C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}^{-1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbb{S}_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array}\right] R2+2R3 \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array}\right] +R1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \end{array}\right] R3-R2 \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & -2 & 1 & -1 & 0 \end{array}\right] \cdot -\frac{1}{2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right] -4 \cdot R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right] -R2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 0 \\ 0 & 1 & 0 & -2 & 4 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right]$$

$$\frac{1}{2}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\boxed{C = \begin{bmatrix} -1 & 4 & 0 \\ -4 & 16 & 0 \\ -3 & 4 & 0 \end{bmatrix}}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 & 0 \\ -2 & 4 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

$$3\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + -4\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ -3 \end{bmatrix}$$

$$-2\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + 4\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + 0 = \begin{bmatrix} 4 \\ 10 \\ 4 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

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3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{pmatrix} \qquad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^T C = B \Rightarrow (A^T)^{-1}(A^T)C = B(A^T)^{-1} \Rightarrow C = B((A^T)^{-1})$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ -3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 31 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} -11 & 31 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}}$$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$\underbrace{3\times 3 \quad C\; 3\times 3}$ $\quad B = 3\times 3$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$\downarrow$ row reduction

$$AC = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$A^T x = B$

$C = 3\times 3$

$\underbrace{A^T \cdot C}_{\text{defined}}$ (3×3, 3×3)

$B = 3\times 3$ ✓

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \Rightarrow (A^{-1})^T = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{-1})^T \cdot B = C \qquad \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+8+6 & -4+10+4 & -6+8+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 12 & 10 & 4 \\ -7 & -3 & 3 \\ 6 & 2 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T \cdot C = B$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

8
0
6

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \qquad B \cdot (A^T)^{-1} = C$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right] \div 3 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right] \rightarrow$$

$$(A^T)^{-1} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 4 \\ 5 & 5 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -7 & -7 \\ 8 & 9 & 8 \\ 5 & 7 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & -7 & -7 \\ 8 & 9 & 8 \\ 5 & 7 & 7 \end{bmatrix} =$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$C = B \cdot (A^T)^{-1}$

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$(A^T)^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -2$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] 1$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] \cdot 1$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] -2$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \cdot -1$

$\cancel{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array}\right]}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

$C = B \cdot (A^T)^{-1}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

$C = [Bv_1 \; Bv_2 \; Bv_3]$

$Bv_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -2+6+-3 \\ -8+15+-8 \\ -6+6+-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$Bv_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1+-2+3 \\ -4+-5+4 \\ -3+-2+1 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -4 \end{bmatrix}$

$Bv_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+-4+3 \\ 8+-10+4 \\ 6+-4+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\boxed{C = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -5 & 2 \\ -1 & -4 & 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$3\times 3 \cdot 3\times 3 = 3\times 3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] R_2 = R_2 + R_1 \qquad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] R_3 = -2R_1 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right] \qquad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & -1 & -1 & 1 & -2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array}\right] R_1 = -R_2 + R_1 \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & -1 & -1 & 1 & -2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array}\right] R_2 = -R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 & -2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array}\right] R_3 = \tfrac{1}{2} \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 & -2 & -5 \\ 0 & 0 & 1 & 5/2 & 7/2 & 7/2 \end{array}\right] R_2 = -R_3 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 0 & -3/2 & -\frac{11}{2} & -\frac{17}{2} \\ 0 & 0 & 1 & \frac{5}{2} & \frac{7}{2} & \frac{7}{2} \end{array}\right] \qquad C = \begin{bmatrix} 0 & 4 & 8 \\ -\frac{3}{2} & -\frac{11}{2} & -\frac{17}{2} \\ \frac{5}{2} & \frac{7}{2} & \frac{7}{2} \end{bmatrix}$$

$\frac{2}{2} - \frac{5}{2} = -\frac{3}{2}$

$-\frac{4}{2} - \frac{7}{2}$

10

0	1	2	3	4	5	6	7	8	9	10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Now, $$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

Here, $$[A^T]^{-1}[A^T][C] = [I][C] = [C] = [A^T]^{-1}[B]$$

$$\therefore [C] = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} (-2+4+6) & (-4+5+4) & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}$$

$$\therefore \boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

Ans:-

Check:- $[1 \ 1 \ 0]\begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix} = [8-7] = [1]$ ✓

$[2 \ 1 \ -1]\begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} = [0+2-3] = [1]$

$[2 \ 1 \ -1]\begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix} = \left[16 - \cancel{(8)} - 6\right]$, 7

$\Rightarrow [16-7 \cancel{+}6] = \underline{3}$ $\quad [16-13] = 3?$

24 - 7

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$A^T A^T C = B \rightarrow C = B(A^T)^{-1}$$

$$C = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix} \cdot \begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} (-2+2+6), (3-2-6), (-1+2+3) \\ (-8+5+8), (12-5-8), (-4+5+4) \\ (-6+2+2), (9-2-2), (-3+2+1) \end{vmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ -2 & 5 & 0 \end{vmatrix}$$

$$C = \begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ \cancel{-2}\ -2 & 5 & 0 \end{vmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^T C = B$

$C = (A^{-1})^T B$

Based on problem 2, $A^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 & 0 \end{bmatrix}$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$C = (A^{-1})^T B = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 4 & 6 \\ 4 & -\frac{5}{2} & -2 \\ 0 & 2 & 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

$B \cdot \frac{1}{A^T}$

$B \cdot A^{T^{-1}}$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^T C = B \rightarrow C = \frac{B}{A^T}$ $\quad C = B \cdot (A^T)^{-1}$

$C = B \cdot (A^{-1})^T$

$$A^{-1} = \begin{matrix} 2 & 3 & 1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{matrix}$$

$$(A^{-1})^T = \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} & C_1 & C_2 & C_3 \\ C_1 & 11 & -6 & 9 \\ C_2 & 27 & -13 & 22 \\ C_3 & 13 & -6 & 11 \end{array}$$

$C_{11} = \overset{-2}{1 \cdot 2} + \overset{6}{2 \cdot 3} + \overset{3}{1 \cdot 3}$

$C_{12}\ \overset{-1}{-1 \cdot 1} + \overset{-2}{2 \cdot -1} + \overset{-3}{3 \cdot -1}$

$C_{13} = \overset{2}{2 \cdot 1} + \overset{4}{2 \cdot 2} + \overset{3}{3 \cdot 1}$

$C_{21} = \overset{8}{4 \cdot 2} + \overset{15}{5 \cdot 3} + \overset{4}{4 \cdot 1}$

$C_{22} = \overset{-4}{4 \cdot -1} + \overset{-5}{5 \cdot -1} + \overset{-4}{4 \cdot -1}$

$C_{23} = \overset{8}{4 \cdot 2} + \overset{10}{5 \cdot 2} + \overset{4}{4 \cdot 1}$

$C_{31} = \overset{6}{3 \cdot 2} + \overset{6}{3 \times 2} + \overset{1}{1 \cdot 1} = 13$

$C_{32} = -3 + -2 + -1 = -6$

$C_{33} = 6 + 4 + 1 = 11$

$$C = \begin{bmatrix} 11 & -6 & 9 \\ 27 & -13 & 22 \\ 13 & -6 & 11 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$B_{11} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 1 \Rightarrow$ $2c_1$

$A^T C = B$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{T-1} \cdot A^T C = A^{T-1} \cdot B$$

$$C = A^{T-1} \cdot B$$

$$A^{T-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R2 \to -2R1+R2$
$R3 \to R1+R3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R2 \to -R2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R1 \to -R2+R1$
$R3 \to -R2+R3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$R2 \to -R3+R2$
$R1 \to R3+R1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$$A^{T-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$A^{T-1} \cdot B =$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+0-3 \\ 0+0+6 \\ 0+4-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+0-2 \\ 0+0+4 \\ 0+5-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+0-1 \\ 0+0+2 \\ 0+4-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A^{T-1} \cdot B = \begin{bmatrix} -2 & 0 & 2 \\ 6 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix} = C$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} A^T C = B (A^T)^{-1}$$

$$I C = B (A^{-1})^T$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} (-2+6-3) & (1-2+3) & (2-4+3) \\ (-8+15-4) & (4-5+4) & (8-10+4) \\ (-6+6-1) & (3-2+1) & (6-4+1) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A A^T) C = AB$$
$$I C = AB$$
$$C = AB$$

$$\overset{A}{\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}} \overset{B}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}} = \boxed{\begin{bmatrix} 3 & 1 & 1 \\ 4 & 4 & 4 \\ 5 & 8 & 7 \end{bmatrix} = C}$$

$1(1) - 1(4) + 2(3)$
$= 1 - 4 + 6$
$= 3$

$1(2) - 1(5) + 2(2)$
$= 2 - 5 + 4$
$= 1$

$1(3) - 1(4) + 2(1)$
$= 3 - 4 + 2$
$= 1$

$1(1) + 0 + 1(3) = 4$
$1(2) + 0 + 1(2) = 4$
$1(3) + 0 + 1(1) = 4$

$0 + 2(4) - 1(3) = 8 - 3 = 5$
$0 + 2(5) - 1(2) = 10 - 2 = 8$
$0 + 2(4) - 1(1) = 8 - 1 = 7$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\underset{3\times 3}{A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}} \cdot \underset{3\times 3}{C} = \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}$$

$$C = B \cdot \frac{1}{A^T} = B \cdot A^{T^{-1}}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{T^{-1}} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 2 & 6 & 2 & 2 & 0 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 6 & 0 & -6 & -6 & 0 & 6 \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & -3 & -3 & 3 \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 5/2 & -\frac{1}{2} & -3/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right]$$

Identity Matrix $\qquad A^{T^{-1}}$

$$C = B \cdot A^{T^{-1}} = \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}} \cdot \underset{3\times 3}{\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 5/2 & -\frac{1}{2} & -3/2 \\ -\frac{1}{2} & \frac{1}{2} & 1/2 \end{bmatrix}}$$

$$\boxed{C = \begin{bmatrix} -\frac{1}{2} & 5 & -3/2 \\ -2 & -5/2 & 2 \\ 3/2 & -3 & 1/2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$(A^T)^{-1} = (A^{-1})^T \qquad C = (A^{-1})^T B$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^T)^{-1} \cdot \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+4+6 \\ 3-4-6 \\ -1+4+3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix} \qquad (A^T)^{-1} \cdot \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -4+5+4 \\ 6-5-4 \\ -2+5+2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$$

$$(A^T)^{-1} \cdot \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+4+2 \\ 9-4+2 \\ -3+4+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$3\times 3 \quad 3\times 3$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$3\times 3 \qquad 3\times 3$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{array}{ccc} x_1 & x_2 & x_3 \\ \hline & & \\ & & \\ & & \end{array} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^T \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = B$

$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix}$

$C_{11} = 0.5$ $4 = 2c_{12} - c_{12}$ $4 = c_{12}$ $3 = 2c_{13} + \cancel{c_{13}} - c_{13}$

$\frac{3}{2} = c_{13}$

$C_1 = \begin{bmatrix} 0.5 \\ 4 \\ 1.5 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{21} \\ c_{22} \\ c_{23} \end{bmatrix}$

$2 = 2c_{21}$ $c_{21} = 1$

$5 = 1c_{22}$ $c_{22} = 5$

$2 = 2c_{23}$ $c_{23} = 1$

$C_2 = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{31} \\ c_{32} \\ c_{33} \end{bmatrix}$

$3 = 2c_{31}$ $c_{31} = 1.5$

$4 = 1c_{32}$ $c_{32} = 4$

$1 = 2c_{33}$ $c_{33} = 0.5$

$C_3 = \begin{bmatrix} 1.5 \\ 4 \\ 0.5 \end{bmatrix}$

$$\boxed{C = \begin{bmatrix} 0.5 & 1 & 1.5 \\ 4 & 5 & 4 \\ 1.5 & 1 & 0.5 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^T C = B$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$-1 \cdot \begin{cases} C_1 + C_4 = 1 \\ -C_1 + 2C_7 = 4 \\ 2C_1 + C_4 + C_7 = 3 \end{cases}$ $\quad -1 \cdot \begin{cases} C_2 + C_5 = 2 \\ -C_2 + 2C_8 = 5 \\ 2C_2 + C_5 + C_8 = 2 \end{cases}$ $\quad -1 \begin{cases} C_3 + C_6 = 3 \\ -C_3 + 2C_9 = 4 \\ 2C_3 + C_6 + C_9 = 1 \end{cases}$

$C_1 + C_7 = 2$
$-C_1 + 2C_7 = 4$
$3C_7 = 6$
$C_7 = 2$
$-C_1 + 4 = 4$
$C_1 = 0$
$0 + C_4 = 1$
$C_4 = 1$

$C_2 + C_8 = 0$
$-C_2 + 2C_8 = 5$
$3C_8 = 5$
$C_8 = \frac{5}{3}$
$-C_2 + \frac{10}{3} = \frac{15}{3}$
$C_2 = \frac{-5}{3}$
$\frac{-5}{3} + C_5 = \frac{6}{3}$
$C_5 = \frac{11}{3}$

$C_3 + C_9 = -2$
$-C_3 + 2C_9 = 4$
$3C_9 = 2$
$C_9 = \frac{2}{3}$
$-C_3 + \frac{4}{3} = \frac{12}{3}$
$C_3 = \frac{-8}{3}$
$\frac{-8}{3} + C_6 = \frac{3}{3}$
$C_6 = \frac{11}{3}$

$$C = \begin{bmatrix} 0 & \frac{-5}{3} & \frac{-8}{3} \\ 1 & \frac{11}{3} & \frac{11}{3} \\ 2 & \frac{5}{3} & \frac{2}{3} \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$(A^T)^{-1} A^T C = B (A^T)^{-1}$

$C = B(A^T)^{-1}$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Inverse of A^T

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] R_2 \to R_1 + R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] R_3 \to R_3 - 2R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right] R_2 \to R_3 + R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right] R_1 \to R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right] R_1 \to R_1 + R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right] R_3 \to R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & -1 & 1 & 2 \end{array}\right] R_2 \longleftrightarrow R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] R_2 \to -R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

Inverse $A^T = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$1 \quad 3-4-2$
-10
$-7-2$

$(A^T)^{-1} \cdot B = C$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0+4+6 & 0+5+4 & 0+4+2 \\ 1-4-6 & 2-5-4 & 3-4-2 \\ 0+4+3 & 0+5+2 & 0+4+1 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 10 & 9 & 6 \\ -9 & 7 & -9 \\ 7 & 7 & 5 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$+\left(\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ -1 & 0 & 2 & | & 4 & 5 & 4 \\ 2 & 1 & -1 & | & 3 & 2 & 1 \end{bmatrix}\right.$$

$$(-2)\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 2 & 1 & -1 & | & 3 & 2 & 1 \end{bmatrix}$$

$$+\left(\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & -1 & -1 & | & 1 & -2 & -5 \end{bmatrix}\right.$$

$$+\left(_{-2}\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix}\right.$$

$$+\left(_{(-1)}\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 0 & | & -7 & -3 & 3 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix}\right.$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 8 & 5 & 0 \\ 0 & 1 & 0 & | & -7 & -3 & 3 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix} \to C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} = (A^{-1})^T$$

From Problem 2: $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$$(A^T)^{-1} B = C$$

$$\begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 1 & 0 & -1 & 2 & 0 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 4 & 5 & 4 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -8 & -5 & 0 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & 7 & 3 \\ 0 & 1 & 0 & -8 & -5 & 0 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$C = \begin{bmatrix} 9 & 7 & 3 \\ -8 & -5 & 0 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} A & Q & X \\ B & R & Y \\ C & S & Z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

$$\begin{array}{ccc} X & Y & Z \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{array}\right] +(1),\ +(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -5 \end{array}\right] +(1)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \end{array}\right] +(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] +(-1)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} =X \\ =Y \\ =Z \end{array}$$

$Q(1) + R(1) + S(0) = 2$
$Q(-1) + R(0) + S(2) = 5$
$Q(2) + R(1) + S(-1) = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -1 & 0 & 2 & 5 \\ 2 & 1 & -1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right] +(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] +(-1)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \begin{array}{l} Q=5 \\ R=-3 \\ S=5 \end{array}$$

Same steps

$A_1 + B_1 + C0 = 1$
$A^{-1} + 0B + C2 = 4$
$A2 + B1 + C^{-1} = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{array}\right] +(1)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 2 & 1 & -1 & 3 \end{array}\right] +(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \end{array}\right] +(1)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 6 \end{array}\right] +(-2)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array}\right] +(-1)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array}\right] \begin{array}{l} A=8 \\ B=-7 \\ C=6 \end{array}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^T C = B$$

$$\text{set } C = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 2 \\ -1 & 0 & 2 & 5 \\ 2 & 1 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 & -4 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\therefore C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B \qquad \underline{(A^T)^{-1} = (A^{-1})^T}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B$$

$$C = (A^T)^{-1} B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ 1-4-3 & 2-5-2 & 3-4-1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & +3 \\ 6 & +5 & +2 \end{bmatrix}}$$

check $A^T C = B$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & +3 \\ 6 & +5 & 2 \end{bmatrix} = \begin{bmatrix} 8-7+0 & 5-3+0 & 0+3+0 \\ -8+0+12 & -5+0+10 & 4 \\ 16-7-6 & 10-3-5 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \quad R1+R2$$

this matrix is solution

get this to RRE

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \quad (R1 \times -2) + R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right] \quad R2+R3=R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad R2 \times -1 + R1$$

$2 \times 2 + -4$
$4 + -4 = 0$

$(6 \times 2) - 4$
$12 - 4$
8

$(5 \times 2) + -5$
$10 - 5$
5

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad R3 \times -2 + R2$$

$(6 \times -2) + 5$
$-12 + 5$
-7

$(5 \times -2) + 7$
$-10 + 7$
-3

$(2 \times -2) + 7$
$-4 + 7$
3

$$R3 \times 2 + R1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A)β.

$$A^T C C^{-1} = B C^{-1}$$
$$A^T E = B C^{-1}$$
$$B^{-1} A^T = B^{-1} B C^{-1}$$
$$B^{-1} A^T = E C^{-1}$$
$$C^{-1} = B^{-1} A^T$$

$$B \quad \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 4 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \quad E$$

$$\Rightarrow B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 7 & -4 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 7 & -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$= 1\begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 17 \end{bmatrix}$$

$$= 1\begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix} + 0 + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$$

$$= 0 + 2\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -7 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -5 & -4 & 2 \\ 17 & 10 & -7 \end{bmatrix}$$

$$C^{-1} \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -5 & -4 & 2 & 0 & 1 & 0 \\ 17 & 10 & -7 & 0 & 0 & 1 \end{array}\right] \quad E.$$

$$C = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = B \cdot (A^T)^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 2 & -1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 10 \\ 4 & 5 & 5 & 5 & -9 & 9 \\ 3 & 3 & 2 & 7 & 7 & 7 \end{matrix}$$

$5-14=-9$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{matrix} ① \\ ② \\ ③ \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$R_2 = ② + ①$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$1-6=-5$

$R_3 = ③ - ① \times 2$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 2 & -2 & -5 \end{array}\right]$$

$R_3 = ③ + ②$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array}\right]$$

$R_2 = R_2 - 2R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -9 & -3 & 3 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array}\right]$$

$R_1 = R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & 5 & 0 \\ 0 & 1 & 0 & 9 & -3 & 3 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array}\right]$$

$$C = \begin{bmatrix} 10 & 5 & 0 \\ 9 & -3 & 3 \\ 7 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \to \begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & -5 & -5 & -8 & -7 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 1 & -5 & -5 & -8 & -7 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & -3 & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -5+\frac{1}{3} & -4 \\ 0 & 1 & 0 & 5 & 7+\frac{1}{3} & 7 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \end{array}\right]$$

$$-\frac{15}{3} - \frac{1}{3} = -\frac{16}{3}$$

$$\frac{21}{3} + \frac{1}{3}$$

$$C = \begin{bmatrix} -4 & -\frac{16}{3} & -4 \\ 5 & \frac{22}{3} & 7 \\ 0 & -\frac{1}{3} & 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{bmatrix} \quad \begin{bmatrix} -8x_4 - 5x_5 \\ 3 + 7x_4 + 3x_5 \\ 2 - 6x_4 - 5x_5 \\ x_4 \\ x_5 \end{bmatrix} \quad \begin{matrix} x_3 + 6x_4 + 5x_5 = 2 \\ x_2 - 7x_4 - 3x_5 = 3 \\ x_1 + 8x_4 + 5x_5 = 0 \end{matrix}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B. \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}.$$

$$(A^T)^{-1} B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$C_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

$x_1 + x_2 + 0x_3 = 1$
$-x_1 + 0x_2 + 2x_3 = 4$
$2x_1 + x_2 - x_3 = 3$

$\rightarrow -x_1 + x_3 = -2$
$-x_1 + 2x_3 = 4$

$x_3 = 6 \therefore x_1 = 8$ and $x_2 = -7$

$C_1 = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$

$C_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$

$x_1 + x_2 + 0x_3 = 2$
$-x_1 + 0x_2 + 2x_3 = 5$
$2x_1 + x_2 - x_3 = 2$

$-x_1 + x_3 = 0$
$-x_1 + 2x_3 = 5$

$x_1 = x_3 = 5$
$\therefore x_2 = -3$

$C_2 = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$

$C_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$

$x_1 + x_2 + 0_3 = 3$
$-x_1 + 0x_2 + 2x_3 = 4$
$2x_1 + x_2 - x_3 = 1$

$-x_1 + x_3 = 2$
$-x_1 + 2x_3 = 4$

$x_3 = 2$
and $x_1 = 4$
$\therefore x_2 = -1$

$C_3 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

$\therefore \boxed{C = \begin{bmatrix} 8 & 5 & 4 \\ -7 & -3 & -1 \\ 6 & 5 & 2 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10

$$A=\begin{bmatrix}1 & -1 & 2\\ 1 & 0 & 1\\ 0 & 2 & -1\end{bmatrix}$$

MTH-309T-F19-EX1-073-P03

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 4\\ 3 & 2 & 1\end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T=\begin{bmatrix}1 & 1 & 0\\ -1 & 0 & 2\\ 2 & 1 & -1\end{bmatrix}$$

3x3

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 4\\ 3 & 2 & 1\end{bmatrix}$$

B

3x3

$C = BA^{-1}$

$$C=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 4\\ 3 & 2 & 1\end{bmatrix}\begin{bmatrix}-2 & 3 & 0\\ 1 & 1 & 0\\ 2 & 2 & 0\end{bmatrix}$$

$$C=\begin{bmatrix}-2+2+6 & 3+3+6 & 0\\ -8+5+8 & 12+5+8 & 0\\ -6+2+2 & 9+3+2 & 0\end{bmatrix}$$

$$C=\begin{bmatrix}6 & 12 & 0\\ 5 & 25 & 0\\ -2 & 14 & 0\end{bmatrix}$$

C is a 3x3 matrix

$$\begin{bmatrix}1 & 1 & 0\\ -1 & 0 & 2\\ 2 & 1 & -1\end{bmatrix}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 4\\ 3 & 2 & 1\end{bmatrix}$$

A^T C B

for Matrix division

1) $a/b = a \cdot b^{-1}$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$C = (A^T)^{-1} B \qquad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} (-2+4+6) & \overset{-4+5+4}{(\quad)} & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

2 x 3

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2+4+3 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$c_1 \; c_2 \; c_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \xrightarrow{R_3 = 2R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 3 & 11 & 12 & 9 \end{array}\right] \quad R_2 = R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 1 & 3 & 11 & 12 & 9 \end{array}\right] \quad R_3 = R_3 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad (-2)$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \quad (-1)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \qquad C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} & C & \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} X_{11} \\ X_{12} \\ X_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \rightarrow X = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} X_{21} \\ X_{22} \\ X_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \rightarrow X = \begin{bmatrix} 13 \\ -11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} X_{31} \\ X_{32} \\ X_{33} \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \rightarrow X = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 8 & 13 & 0 \\ -7 & -11 & 3 \\ 6 & 9 & 2 \end{bmatrix}}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^{-1} = [$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$(A^{-1})^T$

$$A^T C = B \qquad A^T \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\cancel{(A^T)^{-1} A^T} C = (A^T)^{-1} B$$

$$C = (A^{-1})^T B \qquad (A^{-1})^T = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$3 \times 3 \qquad 3 \times 3 \qquad = 3 \times 3$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = 1\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+8+6=12 \\ 3-4-6=-7 \\ -1+4+3=6 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = 2\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 5\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4+10+4=10 \\ 6-5-4=-3 \\ -2+5+2=5 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = 3\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} +2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+8+2=4 \\ 9-4-2=3 \\ -3+4+1=2 \end{bmatrix}$$

$$C = \begin{bmatrix} 12 & 10 & 4 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$A^T C = B$$

$$C = (A^T)^{-1} B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$
$$A^{T^{-1}} A^T C = A^{T^{-1}} B$$
$$I C = A^{T^{-1}} B$$
$$C = A^{T^{-1}} B$$

Find inverse of A^T

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

Inverse of $A^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

3 x 3   3 x 3

C = 3x3 matrix

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix} = C$$

$-2 + 4 + 6 = 8$

$-4 + 5 + 4 = 5$

$-6 + 4 + 2 = 0$

$3 + (-4) + (-6) = -7$

$6 + (-5) + (-4) = -3$

$9 + (-4) + (-2) = 3$

$-1 + 4 + 3 = 6$

$-2 + 5 + 2 = 5$

$-3 + 4 + 1 = 2$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$(A^T)^{-1} \cdot A^T C = (A^T)^{-1} \cdot B$$

$$C = (A^T)^{-1} \cdot B$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$C = (A^{-1})^T \cdot B$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = (A^{-1})^T \cdot B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} (-2+4+6) & (-4+5+4) & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot B^{-1} \qquad \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 4 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$=$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \begin{array}{l} \\ ①+② \\ ①\times2-③ \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 1 & 1 & -1 & 2 & 5 \end{array}\right] \begin{array}{l} \\ \\ ②-③ \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \begin{array}{l} \\ ②-③\times2 \\ \\ \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \begin{array}{l} ①-② \\ \\ \\ \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

so $$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{T^{-1}} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

so

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \downarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] \downarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\downarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & -2 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 4 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$C = BA^{-1T}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -5 & 2 & 4 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

so

$$\begin{bmatrix} (-5+6-3) & (2-2+3) & (4-4+3) \\ (-20+15-\cancel{8}4) & (8-5+4) & (16-10+4) \\ (-15+6-1) & (6-2+1) & (12-4+1) \end{bmatrix}$$

so

$$C = \begin{bmatrix} -2 & 3 & 3 \\ -9 & 7 & 10 \\ -10 & 5 & 9 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{array}{ccc|} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{array}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$(A^{-1})^T C = B$$

$$(A^T)^{-1} \cdot A^T C = (A^T)^{-1} B$$

$$C = (A^{-1})^T B$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & 5 & 4 \\ 0 & -1 & 0 \\ -3 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ -5 \\ -10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \xrightarrow{R2 \to R2 + R1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \xrightarrow{R3 \to R3 - 2R1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right]$$

$$\xrightarrow{R3 \to R3 + R2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

12 10 4

$$\xrightarrow{R2 \to R2 - 2R3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \xrightarrow{R1 \to R1 - R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$Ax = B$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10