2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

A $\quad$ b

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1+4 & 1-1+2 \\ 1-1+2 & 1+1+1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A^TA\,x = A^Tb$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array}\right] \leftrightarrow \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 6 & 2 & 0 \end{array}\right] \downarrow -3 \left[\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & -7 & -9 \end{array}\right] \uparrow +3/7 \rightarrow \left[\begin{array}{cc|c} 2 & 0 & -6/7 \\ 0 & -7 & -9 \end{array}\right] \cdot -\frac{1}{7} \rightarrow \left[\begin{array}{cc|c} 2 & 0 & 6/7 \\ 0 & 1 & 9/7 \end{array}\right] \cdot \frac{1}{2}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 3/7 \\ 0 & 1 & 9/7 \end{array}\right] \quad z_1 = -3/7 \quad z_2 = 9/7$$

$$9/3 - 2/3$$

$$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array}\right] \downarrow -\frac{1}{3} \quad \left[\begin{array}{cc|c} 6 & 2/3 & 0 \\ 0 & 7/3 & 3 \end{array}\right] \cdot \frac{1}{6} = \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 7/3 & 3 \end{array}\right] \cdot 3/7 = \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 1 & 9/7 \end{array}\right] \uparrow -\frac{1}{3}$$

$$\left[\begin{array}{cc|c} 1 & 0 & -3/7 \\ 0 & 1 & 9/7 \end{array}\right] \quad \begin{matrix} z_1 \\ z_2 \end{matrix}$$

$z_1 = -3/7$

$z_2 = 9/7$

$$-\frac{3}{7}x + \frac{9}{7}$$

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