4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$$A-3I = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \to \left[\begin{array}{ccc|c} 2 & -8 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} 4x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix} \to \underset{x_2=1,\, x_3=-1}{\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}}, \underset{x_2=-1,\, x_3=1}{\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}}$$

$$A-5I = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \to \left[\begin{array}{ccc|c} 2 & -8 & -6 & 0 \\ -2 & 6 & 4 & 0 \\ -4 & 8 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -4 & -3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -8 & -8 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} \xrightarrow{x_3=1} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

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$$P = \begin{bmatrix} 2 & \cancel{-2} & -1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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