

## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
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## **UB Person Number:**

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## Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
   You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

_		90)									
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	1	2	3	4	5	6	7	TOTAL	GRADE		
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1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

WIKADO

$$x_1 - x_2 + x_3 = -2$$
  
 $x_1 + 2x_3 = 2$ 

 $\frac{1}{b} = -6 \quad \text{only}$ 

$$x_1 - (2 - 2x_3) + x_3 = -2$$
  
 $x_1 + 3x_2 = 0$ 

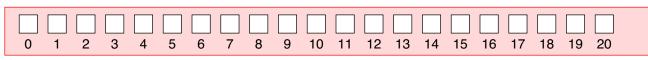
$$b = -6x_3 - 6 + 6x_3$$

be cause

Vy I C, Vy for any SER!

12 # Call for any C3 ER

b) {V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub>} is not linearly independent because 3v<sub>1</sub> = V<sub>3</sub>-2V<sub>2</sub>.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$a_1 - a_{11} + 2a_{11} = 1$$
 $a_1 - a_2 - a_1 + 2a_2 = 1$ 
 $a_1 + a_2 = 0$ 
 $a_1 + a_2 = 0$ 
 $a_1 + a_2 = 0$ 
 $a_1 - a_2 = 0$ 
 $a_1 - a_2 = 0$ 
 $a_1 = 1$ 
 $a_2 - a_2 = 0$ 
 $a_1 = 1$ 
 $a_2 - a_2 = 0$ 

$$a_1 - a_5 + 2a_8 = 0$$
  $2a_5 = a_8$   
 $a_2 + a_8 = 1$   $a_8 = 1 - a_8$   
 $2a_5 - a_8 = 0$ 

$$a_3 - a_6 + 2a_9 = 0$$
 $a_3 + a_9 = 0$ 
 $2a_6 - a_9 = 1$ 

$$|a_5| = a_8$$
  $|-a_8| - \frac{1}{2}a_8 + 2a_8 = 0$   
 $|a_8| = |-a_8|$   $|a_8| = -1$   
 $|a_8| = -2$   
 $|a_5| = -1$   $|a_2| = 3$ 

$$a_3 = -a_q$$
 $a_6 = \frac{1}{2}(1 + a_q)$ 
 $a_6 = \frac{1}{2}(1 + a_q)$ 
 $a_6 = \frac{1}{2}(1 + a_q)$ 
 $a_6 = 1$ 

$$a_3 = -1$$
 $a_6 = \frac{1}{2}(1+1) = 1$ 



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 1 & 2 \\ 3 & -1 & -2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of  $\mathcal{T}$ .
- b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

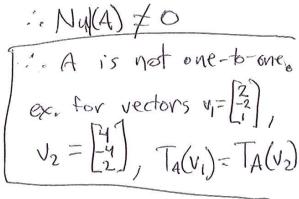
There are no vectors u satisfying T(u)=[10]



5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$





- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in Span(u, v)$  then  $w \in Span(u, v)$ .

This is true. By def. of spath, whu= $Gu+C_2V$  for some  $C_1$ ,  $C_2 \in \mathbb{R}$ . Therefore  $W=C_1u+C_2v-U$ , and compining gives  $W=(C_1-1)U+C_2V$ . However,  $C_1-1$  is just some other constant in  $\mathbb{R}$ , so let  $C_3=C_1-1$ . Therefore  $W=C_3U+C_2V$ , meaning  $W\in Span(U_1V)$  by definition.

QED.

b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.

This is thue. If {u, v, w} is linearly independent,

then  $u \neq c, v + c_w + c_v + c$ 

Since  $U \neq C, v \land for any ciseR, then {u,v} is linearly independent by definition.$ 



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

this is false. For example, let u=[0], v=[0], and A = [ii]. Then A = [ii] and A = [ii]. An and A = [ii]. An are the same vector, so they are clearly linearly disdependent, however, [0] and [0] are linearly independent.

b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

This is true. If  $u \in Spain(v, w)$ , then  $U = C_1 \vee C_2 \vee C_2 \vee C_3 \vee C_4 \vee C$ 

QED.