$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ. Assumption: false.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $z = w - w = 0$

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix. Assump: false.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

~~a.) $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ $\det(A - \lambda I) = \det\begin{bmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{bmatrix} = \lambda^2 - 4\lambda + 4$~~

~~$(\lambda-2)(\lambda-2)$~~

~~$\lambda_1 = 2$ $\lambda_2 = 2$~~

~~$\text{Null}(A - 2I) = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] = \begin{bmatrix} x_1 = 0 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$~~

$2v = [$

+1 a.) this is false since for a given Matrix $P = [v_1, \dots v_n]$ where $v_1 \dots v_n$ are eigenvectors, and $D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$ when the eigenvalues are found on diagonal

The Eq $A^k = PD^kP^{-1}$ doesn't involve a change in P at all so why should 2v correspond to 2λ.

?

b.) if $\text{proj}_V w = -w$

+3 then where z is orthogonal to V space

$z = w - \text{proj}_V w$

$z = w + w$

? $z = 2w$ by definition z cannot be in V unless $w = 0$

$\therefore$ True.

c.) True ~~if A is symmetric and orthogonal~~

+3 ~~then~~ consider

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ → it is symmetic because the 0's match up

→ ~~Orthogonal because dot Product = 0~~

because $1^2 = 1$

$6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ → $\therefore$ orthogonal

Identity

In order to have a Orthogonally symmetric Matrix the entries on the diagonal Must be 1 $\therefore$ A^2 will result in the Identity

Do on Back

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20