$\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$ $(1-\lambda)(1-\lambda)$ $\lambda = 1$

$1-\lambda \quad 1$

$0 \quad -\lambda$ $\lambda = 0$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{v}$ is an eigenvector of A corresponding to an eigenvalue λ then $2\mathbf{v}$ is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and $\mathbf{w}$ is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then $\mathbf{w}$ must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$ $\begin{bmatrix}1 & 3\\3 & 2\end{bmatrix}$ [

a False, consider $\begin{bmatrix}2 & 3\\1 & 5\end{bmatrix}$ $\lambda = 2$.

+2 ?

b True, because $\text{proj}_V w$ is 0 if w is orthogonal to V. and projection cannot reverse a direction. ?

+3

The only case when $\text{proj}_V w = -w$ would be if $w = 0$ because $-w = 0$

c) True, $A^T = A^{-1}$ (orthogonal) $A = A^T$ (symmetric)

+5

~~$A \cdot A^{-1} = I$~~ $A^T \cdot A = I$ (orthogonal)

$(A) \cdot A = I$ since symmetric $A^2 = I$

d) True, since both A & B are $n \times n$ matrices, which are orthogonally diagonizable, which means they have to be symmetric if $A = \begin{bmatrix}x & m\\m & y\end{bmatrix}$ & $B = \begin{bmatrix}p & r\\r & q\end{bmatrix}$

+5

then $A+B = \begin{bmatrix}x+p & m+r\\m+r & y+q\end{bmatrix}$

$A+B$ is symmetric & $n \times n$ so orthogonally diagonizable.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20