

## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Jon Yalger  UB Person Number:									Instructions:				
5 0 1 2 3 4 6 7 8 9		→ (0) (1) (3) (4) (5) (6) (7) (8) (9)	3 0 1 2 0 4 6 6 7 8 9	0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		(a) (b) (c) (c) (d) (d) (d) (d) (d) (d) (d) (d) (d) (d	5 0 1 3 4 6 6 6 8 9		<ul> <li>Textbooks, calculators and any oth electronic devices are not permitted. You may use one sheet of notes.</li> <li>For full credit solve each problem fully, showing all relevant work.</li> </ul>				
1		2		3		4	5		6	7	TOTAL	GRADE	

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1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

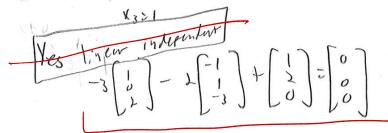
$$\mathbf{v}_1 = \left[ \begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[ \begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[ \begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[ \begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

$$V = \begin{bmatrix} 1 & -1 & 1 & -\lambda \\ 0 & 1 & \lambda & 2 \\ 2 & -3 & 0 & 6 \end{bmatrix} \begin{pmatrix} 2 & 1 & -1 & 1 & -\lambda \\ 0 & 1 & \lambda & 2 \\ 0 & -1 & -\lambda & + + 6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & \lambda & 2 \\ 0 & 0 & 0 & 6 + 6 \end{pmatrix} = \begin{pmatrix} x_1 = -3x_3 \\ x_2 = \lambda - x_3 \\ 6 = -6 \end{pmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
2 & -3 & 0 & 0
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 3 & 0 & 1 & 2 & -3 & 23 \\
0 & 1 & 2 & 0 & 0 & 0 & 23 & 2 & 23
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & -3 & 23 \\
0 & 0 & 1 & 2 & 0 & 0 & 0 & 23 & 2 & 23
\end{bmatrix}$$



This shows that these vectors are linearly dependent



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$\begin{bmatrix} 1 & -1 & 2 & | & 0 & 0 \\ 1 & 0 & 1 & 0 & | & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & | & 0 & 0 \\ 0 & 1 & -1 & | & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 \\ 0 & 1 & | & 1 & | & 0 & | \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & 1 & | & 0 \\ 0 & 0 & 1 & | & & 1 & | & 0 \\ 0 & 0 & 1 & | & & & & & & \\ 0 & 0 & 1 & | & & & & & & \\ 0 & 0 & 1 & | & & & & & & \\ 0 & 0 & 1 & | & & & & & & \\ 0 & 0 & 1 & | & & & & & \\ 0 & 0 & 1 & | & & & & & \\ 0 & 0 & 1 & | & & & & & \\ 0 & 0 & 1 & | & & & & & \\ 0 & 0 & 1 & | & & & & & \\ 0 & 0 & 1 & | & & & & \\ 0 & 0 & 1 & | & & & & \\ 0 & 0 & 1 & | & & & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 & 0 & 1 & | & \\ 0 &$$

$$\begin{bmatrix} 1 & -1 & 9 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 - 7 & 1 \\ 1 & -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

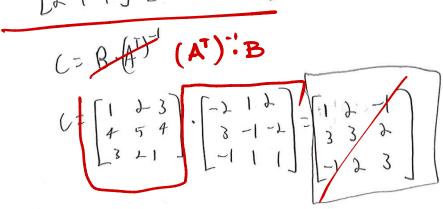


3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
?

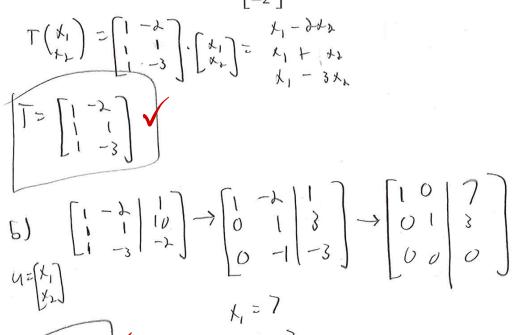




4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .





5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A: \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $v_1$  and  $v_2$  such that  $T_A(v_1) = T_A(v_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ 

b) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & -1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & -1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 \end{bmatrix}$$

$$T_{A} \text{ is not one-to-one}$$

$$X_{3} = \text{free} \quad \begin{cases} V_{1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} & V_{2} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \\ X_{1} = 2X_{3} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} & V_{2} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \end{cases}$$



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in Span(u, v)$  then  $w \in Span(u, v)$ .

Take

$$V = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
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b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

[1] [1] [2] = [1] An and Av are dependent

[1] [1] [2] = [1] [2] [2] are independent

b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True a transform will only move a vector, not change it so it a is in the span it will also be in the transform span why?

 $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$   $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ ?