5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{v}$ is an eigenvector of A corresponding to an eigenvalue λ then $2\mathbf{v}$ is an eigenvector of A corresponding to the eigenvalue 2λ.

$\begin{matrix} v_{12} \\ v_{22} \end{matrix}\Bigg]$

b) If V is a subspace of $\mathbb{R}^2$ and $\mathbf{w}$ is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then $\mathbf{w}$ must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

a) False, just because there is an eigenvalue λ does not mean there is an eigenvalue 2λ. There is no guarantee $2\lambda = \text{val}$ is another solution to $|A - I(\text{val})| = 0$ Ex) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\lambda_1 = 1$, $\lambda_2 = -1$

+5

b) $\begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \dfrac{w_1 v_{11} + w_2 v_{21}}{v_{11}^2 + v_{21}^2} + \dfrac{w_1 v_{12} + w_2 v_{22}}{v_{12}^2 + v_{22}^2} \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} = \begin{bmatrix} -w_1 \\ -w_2 \end{bmatrix}$ Say $w_1 = w_2 = 1$

$\dfrac{v_{11} + v_{21}}{v_{11}^2 + v_{21}^2}$

$\begin{bmatrix} w_1 + w_2 \\ w_1 + w_2 \end{bmatrix} + \begin{bmatrix} w_1 + w_2 \\ w_1 + w_2 \end{bmatrix}$ $W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

+2

True, the projection of a vector on some space can be visualized as the "shadow" of that vector on that space, ∴ any projection of w_1, w_2 somewhere else should carry the same sign, unless they both are zero

?

c) False, $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $A = A^T$ ✓ Symmetric ✓ $A^2 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) False $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

+1

does not have 2 linearly independent eigenvectors