1. Consider the following vectors in $\mathbb{R}^4$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V.

$X_1 = V_1 \quad X_2 = V_2 - \frac{V_2 \cdot V_1}{V_1 \cdot V_1} \cdot V_1 \quad X_3 = V_3 - \left(\frac{V_3 \cdot V_1}{V_1 \cdot V_1}\right)V_1 - \left(\frac{V_3 \cdot V_2}{V_2 \cdot V_2}\right)V_2$

$X_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$V_2 \cdot V_1 = 2+0+1+0 = 3$

$V_1 \cdot V_1 = 1+0+1+1 = 3$

$\frac{V_2 \cdot V_1}{V_1 \cdot V_1} = \frac{3}{3} = 1 \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$V_2 - \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

$X_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

$X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

$V_3 \cdot V_1 = 2+0+1+3 = 6$

$V_1 \cdot V_1 = 3 \quad \frac{V_3 \cdot V_1}{V_1 \cdot V_1} = 2$

$V_3 \cdot V_2 = 4+-2+1+0 = 3 \quad \frac{V_3 \cdot V_2}{V_2 \cdot V_2} = \frac{1}{2}$

$V_2 \cdot V_2 = 4+1+1 = 6$

$X_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - 2 \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \frac{1}{2} \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$

$X_3 = \begin{bmatrix} 2 \\ 2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ -2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3/2 \\ 3/2 \\ 1 \end{bmatrix}$

$\boxed{D = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3/2 \\ 3/2 \\ 1 \end{bmatrix} \right.}$

$\text{Proj}_V U = \left(\frac{U \cdot w_1}{w_1 \cdot w_1}\right) w_1 + \left(\frac{U \cdot w_2}{w_2 \cdot w_2}\right) w_2 + \left(\frac{U \cdot w_3}{w_3 \cdot w_3}\right) w_3$

$U \cdot w_1 = 3+0-3+3 = 3 \qquad \text{Proj}_U V = w_1 + w_2 + \frac{9}{13/2} w_3 = w_1 + w_2 + \frac{18}{13} w_3$

$w_1 \cdot w_1 = 1+0+1+1 = 3$

$U \cdot w_2 = 3+3+0-3 = 3$

$w_2 \cdot w_2 = 1+1+0+1 = 3$

$U \cdot w_3 = -3+9/2+9/2+3 = 9$

$w_3 \cdot w_3 = 1+2.25+2.25+1 = 13/2$

$= \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -18/13 \\ 27/13 \\ 27/13 \\ 18/13 \end{bmatrix} = \begin{matrix} 2-\frac{18}{13} \\ 1+27/13 \\ 1-27/13 \\ \frac{18}{13} \end{matrix} = \begin{bmatrix} 8/13 \\ 40/13 \\ -14/13 \\ 18/13 \end{bmatrix}$

14/20

$\boxed{\text{Proj}_U V = \begin{bmatrix} 8/13 \\ 40/13 \\ -14/13 \\ 18/13 \end{bmatrix}}$

1.5 × 3 = 4.5

18 × 3 = 54

$\frac{27}{13}$

26