1. Consider the following vectors in $\mathbb{R}^4$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V.

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a) Orthogonal Basis $D = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$

$C_1 = \frac{UV_1}{V_1V_1} = \frac{3+0-3+3}{1^2+0^2+(-1)^2+1^2} = \frac{3}{3} = 1$

$C_2 = \frac{UV_2}{V_2V_2} = \frac{6+3-3+0}{2^2+1^2+(-1)^2+0^2} = \frac{6}{6} = 1$

$C_3 = \frac{UV_3}{V_3V_3} = \frac{6-6-3+9}{2^2+(-2)^2+(-1)^2+3^2} = \frac{6}{18} = \frac{1}{3}$

$4+4+1+9$

Orthogonal Basis D of subspace V $= \begin{bmatrix} 1 \\ 1 \\ 1/3 \end{bmatrix}$

$\text{Proj}_V U = \frac{UV_1}{V_1V_1}V_1 + \frac{UV_2}{V_2V_2}V_2 + \frac{UV_3}{V_3V_3}V_3$

$UV_1 = 3+0-3+3 = 3$

$V_1V_1 = 1^2+0^2+(-1)^2+1^2 = 3$

$UV_2 = 6+3-3+0 = 6$

$V_2V_2 = 2^2+1^2+(-1)^2+0^2 = 6$

$UV_3 = 6-6-3+9 = 6$

$V_3V_3 = 2^2+(-2)^2+(-1)^2+3^2 = 18$

$\text{Proj}_V U = \frac{3}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \frac{6}{6}\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} + \frac{6}{18}\begin{bmatrix} 2 \\ -2 \\ -2 \\ 3 \end{bmatrix}$

$\text{Proj}_V U = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2/3 \\ -2/3 \\ -2/3 \\ 1 \end{bmatrix}$

$\text{Proj}_V V = \begin{bmatrix} 11/3 \\ 1/3 \\ -8/3 \\ 2 \end{bmatrix}$

$\text{Proj}_V U = \begin{bmatrix} 11/3 \\ 1/3 \\ -8/3 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20