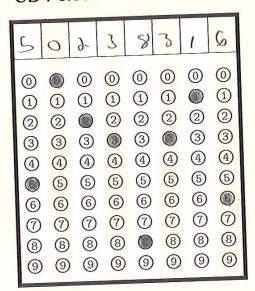


## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
Andrew	Jank	

## **UB Person Number:**



## Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
   You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
		,						

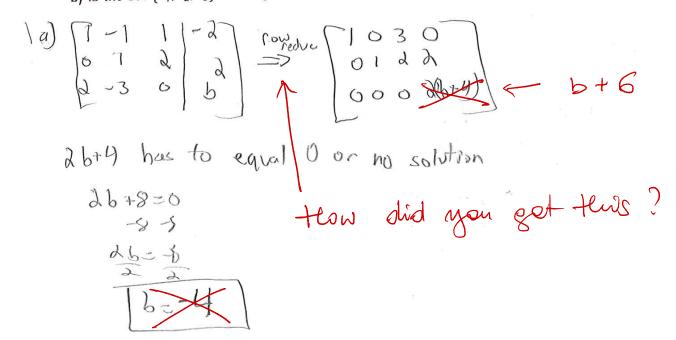
12	4	8	20	20	5	3	2	10	81	В
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \left[ \begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[ \begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[ \begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[ \begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.



16) No since there is not apport column in very column when reduced also three is a free variable w/ x3 00 infinite solutions and dependent.



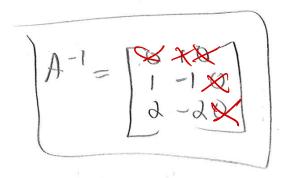
2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$A^{-1} = \begin{bmatrix} 1 & -1 & d & | & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & | & 0 & | & 0 \\ 0 & 2 & -1 & | & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow PDW | 100 | 010 | 1-10 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 001 | 2-20 | 0$$

printineich row and column so one-to-one and onto is invertible





3. (10 points) Let A be the same matrix as in Problem 2, and let

Az 
$$\begin{bmatrix} 1 - 1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 - 1 \end{bmatrix}$$
  
Find a matrix C so

$$B = \left[ \begin{array}{rrr} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{array} \right]$$

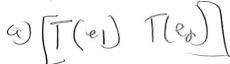
The arrival of the such that  $A^TC = B$  (where  $A^T$  is the transpose of A).  $3 \times 3$   $3 \times 3$  3



4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

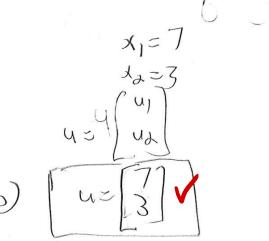
$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of  $\mathcal{T}$ .
- b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .





$$\begin{bmatrix} 1 & -3 & 1 \\ 1 & -3 & -2 \end{bmatrix} \Rightarrow \text{rediction} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

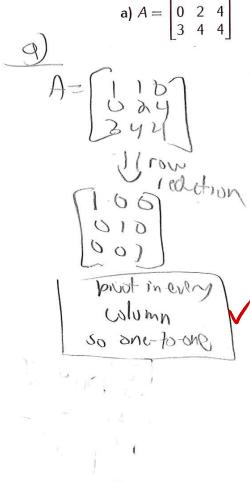


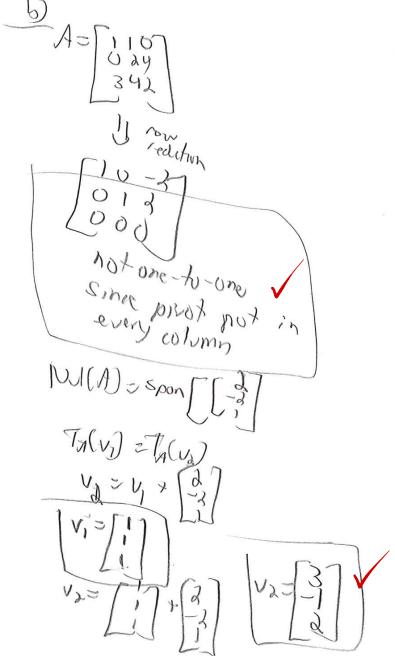


5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A : \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to-one or not. If  $T_A$  is not one-to-one, find two vectors  $v_1$  and  $v_2$  such that  $T_A(v_1) = T_A(v_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$







6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

( = (9) Wy is only in the Span(y)-u?

b) If u,v,w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.

The if yours linearly independent the only solution is zero vector grand his a private cotons in every whom, Usu would be independent as well w/ prot-position in each column and only solution would be zero vector.

This could be explained more clearly...



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

then u, v also must be thearty dependent.

Tork Av and Av have infinite solutions?

So u, and v will also like dependent as they

will be scalar multiples of each other.

A = [0] u= [0] v= [1]

Tindependent

Au = [0] Av = [0]

Tapendent

b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

The it use linear combination of T(W, T(w),

In honemark problems T(v)=T(w)+T(w)+T(w);

meaning it is a linear combination.

My?