$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ. Assumption: false.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $z = w - w = 0$

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix. Assump: false.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

a.) $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ $\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{bmatrix} = \lambda^2 - 4\lambda + 4$
$(\lambda - 2)(\lambda - 2)$
$\lambda_1 = 2 \quad \lambda_2 = 2$

$\text{Null}(A - 2I) = \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \times 1 \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] = \begin{bmatrix} x_1 = 0 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$2v = [$

+1 a.) this is false since for a given Matrix P = [v1 .... vn] where v1 ... vn are eigenvectors, and $D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$ where the eigenvalues are found on diagonal. The Eq $A^k = PD^kP^{-1}$ doesn't involve a change in P at all so why should 2v correspond to 2λ. ?

b.) if $\text{proj}_V w = -w$

+3 then $\rightarrow$ where z is orthogonal to V space

$z = w - \text{proj}_V w$

$z = w + w$

$\therefore$? ($z = 2w$ by definition z cannot be in V unless $w = 0$

$\therefore$ True.

c.) True if ~~A is symmetric and orthogonal~~

+3 ~~then~~ consider

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\rightarrow$ it is symmetric because the 0's match up $\rightarrow$ ~~Orthogonal because dot Product = 0~~

$6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \rightarrow \therefore$ orthogonal

because $1^2 = 1$

In order to have a Orthogonally symmetric Matrix the entries on the diagonal Must be 1 $\therefore$ A^2 will result in the Identity

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