

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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UB Person Number:

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0 1 2 3 4 6 6 7 8 9	① ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	0 1 2 4 5 6 7 8 9	0 1 3 4 5 6 7 8 9	① ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨	0 1 0 3 4 5 6 7 8 9	0 1 2 3 4 6 6 7 8 9	

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

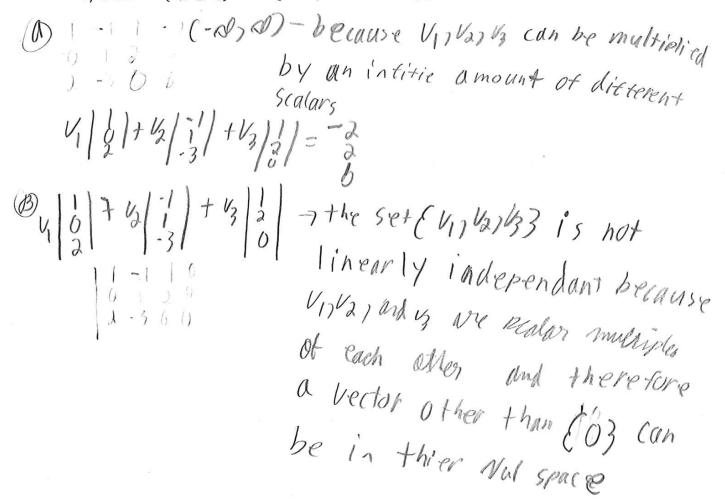
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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.





2. (10 points) Consider the following matrix:

$$A = \left[\begin{array}{rrr} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right]$$

Compute A^{-1} .

$$\begin{vmatrix}
1 - 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 - 1 & 0 & 0 & 1 & - 1 - 1 & 0 \\
0 & 2 - 1 & 0 & 0 & 1 & - 1 & - 1 & 0 \\
0 & 2 - 1 & 0 & 0 & 1 & - 1 & - 1 & 0 \\
0 & 1 & - 1 & 1 & 0 & 0 & 1 & - 1 & 1 \\
0 & 1 & 0 & 1 & - 1 & 1 & 0 & 0 & 1 & - 1 & 1 \\
0 & 1 & 0 & 1 & - 1 & 1 & 0 & 0 & 1 & - 1 & 1 \\
0 & 0 & 1 & 2 & - 2 & 1 & 0 & 0 & 1 & 2 & - 2 & 1 \\
0 & 0 & 1 & 2 & - 2 & 1 & 0 & 0 & 1 & 2 & - 2 & 1 \\
0 & 1 & 2 & 3 - 1 & 0 & 0 & 1 & 2 & - 2 & 1 \\
0 & 1 & 2 & 3 - 1 & 0 & 0 & 1 & 2 & - 2 & 1$$

$$R_3 \times -2 + R_1$$
 $A^{-1} = \begin{vmatrix} -2 & 3 - 1 \\ 1 - 1 & 1 \\ 2 - 21 \end{vmatrix}$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T_2} \begin{vmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$
 $A^{T_2} \begin{vmatrix} -1 & 0 & 2 \\ 2 & 1 & -1 \end{vmatrix}$
 $A^{T_1} = A^{-1/2}$
 $A^{T_1} = A^{-1/2}$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

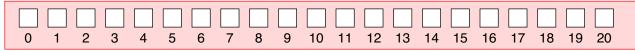
a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

1 + has a pivot columnia every column and the Null of A

O's not one to one because it dosn't have a pivot in every column and it has infinite solution

* NulCA)=U proof





- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in Span(u, v)$ then $w \in Span(u, v)$.

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

True, because some thing in linerly inderen

When ithe vectors are not scalar multiples of each other, and taking out one vector of a set wont make it that the other 2 are now scalar multiples of each other.



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False
$$4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} V = \begin{bmatrix} 0 \\ 1 \end{bmatrix} A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} A = \begin{bmatrix} 1 \\ 0 \end{bmatrix} A = \begin{bmatrix} 0 \\ 0 \end{bmatrix} A = \begin{bmatrix} 0$$

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).