

## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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UB Person Number:								<ul> <li>Instructions:</li> <li>Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.</li> </ul>				
0 1 2 3 4 6 7 8 9	(a) (b) (c) (c) (d) (d) (e) (e) (e) (e) (e) (e) (e) (e) (e) (e	0 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8	(a) (b) (c) (c) (d) (d) (e) (e) (e) (e) (e) (e) (e) (e) (e) (e	0 1 2 3 4 5 6 7 8 9	0 1 3 4 5 6 7 8 9	0 1 2 3 4 5 6 8 9	<ul> <li>For full credit solve each problem fully, showing all relevant work.</li> </ul>				
1		2		3		4	5	6	7	TOTAL	GRADE	
		25								54 1		



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \left[ \begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[ \begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[ \begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[ \begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1,v_2,v_3\}$  linearly independent? Justify your answer.

b) The set isn't linearly independent because since the span is made up of linear combinations of the vectors all the vectors in the span are linearly dependent



## 2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
1 & 0 & 1 & | & 0 & 1 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & 0 & | & 0 & | & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & | & -1 & 1 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -1 & | & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -12 & | & 100 \\
0 & 1 & -1 & | & -110
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 1 & -1 & | & -110 \\
0 & 2 & -1 & | & 001
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -1 & | & 001
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 2 & -1 & | & 001
\end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & Z \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
 $A^{T} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ 

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
-1 & 0 & 2 & | & 4 & 5 & 4
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
2 & 1 & -1 & | & 3 & 21
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
2 & 1 & -1 & | & 3 & 21
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
0 & 1 & 2 & | & 5 & 77 \\
0 & -1 & -1 & | & 2 & -5
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
0 & -1 & -1 & | & 2 & -5
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
0 & -1 & -1 & | & 2 & -5
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 1 & 23 \\
0 & 1 & 2 & | & 5 & 77 \\
0 & -1 & -1 & | & 2 & -5
\end{bmatrix}$$

$$\begin{bmatrix}
100 & 850 \\
610 & -7-33 \\
001 & 652
\end{bmatrix}$$
RIARI-RZ  $\begin{bmatrix} 10 & 123 \\
010 & -7-33 \\
601 & 652
\end{bmatrix}$ 
RZ>RZ-203  $\begin{bmatrix} 10 & 123 \\
012 & 577 \\
001 & 652
\end{bmatrix}$ 

Simpler: 
$$C = (A^{-1})^{-1} \cdot B = (A^{-1})^{-1} \cdot B$$
  
Then use  $A^{-1}$  from problem 2.



4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

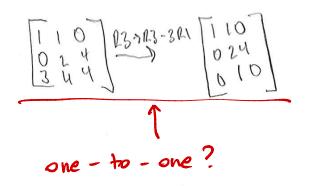


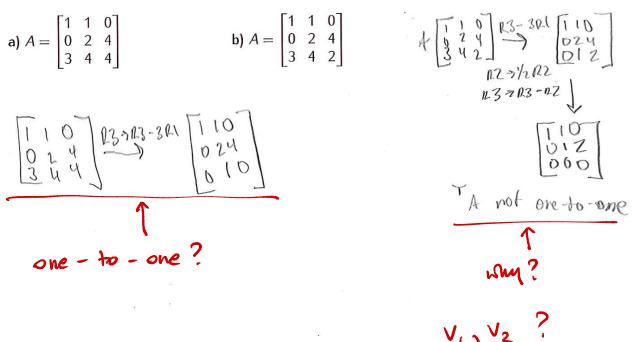


5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to-one or not. If  $T_A$  is not one-to-one, find two vectors  $v_1$  and  $v_2$  such that  $T_A(v_1) = T_A(v_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$





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6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in Span(u, v)$  then  $w \in Span(u, v)$ .

True, because it we add the vectors together in a vector equation on the resulting vector is in the span, other that means that the vector that was added is in the span as it is a linear combination.

May ?

b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, y\}$  must be linearly independent.

the whole set would have been linearly dependent but it's not.

This should be explained more clearly...



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

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b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

trie if all the rectors undergo a tree fransformation.

Then they will still showed the relationship of being treer combinations. why?