4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of $A$ are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix $D$ and an invertible matrix $P$ such that

$$A = PDP^{-1}$$

Note: you do not need to compute $P^{-1}$.

For $\lambda = 3$, $A - \lambda I = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$Nul(A-3I) = \text{Span}\left\{\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right\}$ = eigenspace of $A$ corresponding to $\lambda_1$

$A - 5I = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 2 \\ -1 & 3 & 2 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 \\ -2 & 4 & 2 \\ -1 & 4 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -3 & 2 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$Nul(A-5I) = \text{Span}\left\{\begin{bmatrix} \cancel{0}\ -1 \\ -1 \\ 1 \end{bmatrix}\right\}$

$$\boxed{D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \qquad P = \begin{bmatrix} 4 & 2 & \cancel{0}\ -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}}$$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20