



DO NOT WRITE OUTSIDE THE MARKED AREA

MTH 309Y LINEAR ALGEBRA
EXAM 3
December 11, 2018

Name: _____

Person Number: _____

- Textbooks and electronic devices (calculators, cellphones etc.) are not permitted.
- You may use one sheet of notes.
- For full credit explain your answers fully, showing all work.
- Each problem is worth 20 points.

1	
2	
3	
4	
5	
Total:	



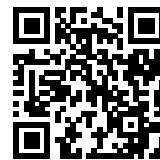
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1. Consider the following vectors in \mathbb{R}^4 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace V of \mathbb{R}^4 .

- a) Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace V .
- b) Compute the vector $\text{proj}_V \mathbf{u}$, the orthogonal projection of \mathbf{u} on V .



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2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.



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3. Consider the following matrix A :

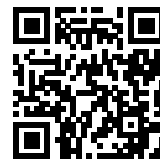
$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A .

a) $\lambda = 0$

b) $\lambda = -1$

c) $\lambda = -2$



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4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1} .



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5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and \mathbf{v} is an eigenvector of A corresponding to an eigenvalue λ then $2\mathbf{v}$ is an eigenvector of A corresponding to the eigenvalue 2λ .

b) If V is a subspace of \mathbb{R}^2 and \mathbf{w} is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then \mathbf{w} must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.