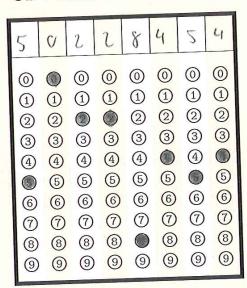


MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
Cortlandt	Chin	

UB Person Number:



Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

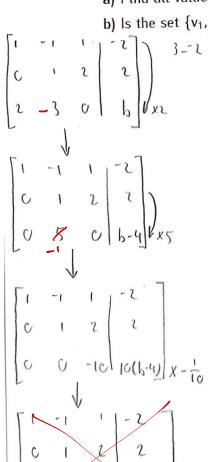
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that w ∈ Span(v1, v2, v3). h = any number but 4
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.



3-2 The set {V, V, V, V,} is tinearly independent hecause each vector is not a scalar multiple or each other

This is not enough to check if three vectors one independent or not.

= b=4 = 4, then W & Span(V, V, V)

shoud be: b+6=0 b=-6

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute
$$A^{-1}$$
.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 3 & 4 & 3 & -1 \\ 0 & 1 & 3 & 5 & 3 & 1 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 & 5 & 5 & 5 \\ 0 & 1 & 3 &$$

$$\begin{bmatrix} 1 & -1 & 0 & | \frac{1}{3} & \frac{1}{3} & | \frac$$



3. (10 points) Let A be the same matrix as in Problem 2, and let 3×3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T[C] = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A'(a')\dot{c} = B(a')'$$

$$c = B(a')'$$

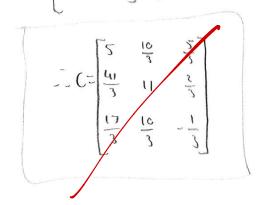
$$\frac{1}{1} - \frac{4}{3} = \frac{4}{3} - \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{1} - \frac{1}{3} = \frac{1}{3} - \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{1} + \frac{1}{3} = \frac{1}{3} + \frac{1}{3}$$

Find a matrix C such that
$$A^{T}C = B$$
 (where A^{T} is the transpose of A).

$$A^{T}C = A^{T}C = A^{$$





4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of
$$T$$
.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$e_{1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 3T(e_{1}) = T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$e_{2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 3T(e_{3}) = T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{cases} X_1 - 2X_2 = 10 \\ X_1 + X_2 = 10 \end{cases}$$

$$\begin{cases} X_1 - 3X_2 & = -2 \\ X_1 - 3X_2 & = -2 \end{cases}$$

$$\begin{bmatrix} 1 & -2 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & | & 1 & |$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 0 \\ 1 & -3 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & -3 \\ 0 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & -6 \\ 0 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & -6 \\ 0 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & -6 \\ 0 & -6 & -6 \end{bmatrix} = 3 \operatorname{Enconsister}$$
System



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 &$$

V,, Vz ?



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u,v,w are vectors in \mathbb{R}^3 such that $w+u\in Span(u,v)$ then $w\in Span(u,v)$. True

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent. True Removing a vector will still make





7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

AUZAV

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent. False 🗸

 $A=\begin{bmatrix}1\\1\end{bmatrix}$ $\overline{U}=\begin{bmatrix}2\\3\end{bmatrix}$ $A\overline{U}:\begin{bmatrix}5\\5\end{bmatrix}$

 $\tilde{V} = \begin{bmatrix} 9 \\ 1 \end{bmatrix} \quad A\tilde{V} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

b) If $T:\mathbb{R}^2\to\mathbb{R}^2$ is a linear transformation and $u,v,w\in\mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)). For the T(v) may no longer he in

 $\overline{U} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \overline{V} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \overline{W} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Span(Flo), T(W)) after transformation