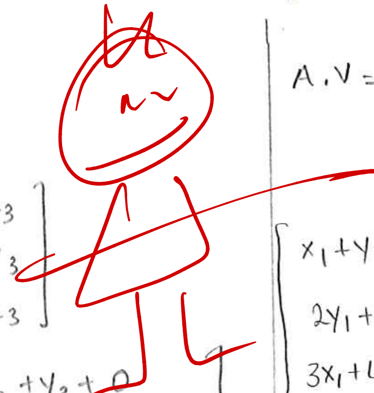
5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

SINCE $\mathbb{R}^3 \to \mathbb{R}^3$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$V = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$



$T_A(V) = A.V$

$A.V = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$

$\begin{bmatrix} x_1+y_1+0 & x_2+y_2+0 & x_3+y_3+0 \\ 2y_1+4z_1 & 2y_2+4z_2 & 2y_3+4z_3 \\ 3x_1+4y_1+4z_1 & 3x_2+4y_2+4z_2 & 3x_3+4y_3+4z_3 \end{bmatrix}$

$= \begin{bmatrix} x_1+y_1 & x_2+y_2 & x_3+y_3 \\ 2y_1+4z_1 & 2y_2+4z_2 & 2y_3+4z_3 \\ 3x_1+4y_1+4z_1 & 3x_2+4y_2+4z_2 & 3x_3+4y_3+4z_3 \end{bmatrix}$

$A.V = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$

$\begin{bmatrix} x_1+y_1+0 & x_2+y_2+0 & x_3+y_3+0 \\ 2y_1+4z_1 & 2y_2+4z_2 & 2y_3+4z_3 \\ 3x_1+4y_1+2z_1 & 3x_2+4y_2+2z_2 & 3x_3+4y_3+2z_3 \end{bmatrix}$

$\begin{bmatrix} x_1+y_1 & x_2+y_2 & x_3+y_3 \\ 2y_1+4z_1 & 2x_2+4z_2 & 2y_3+4z_3 \\ 3x_1+4y_1+2z_1 & 3x_2+4y_2+2z_2 & 3x_3+4y_3+2z_3 \end{bmatrix}$

FOR $T_A(V_1) = T_A(V_2)$

$\to 3x_{31} + 4y_{31} + 4z_{31} = 3x_{32}\ 4y_{32} + 2z_{32}$

$4z_{31} = 2z_{32}$

$2z_{31} = 1z_{32}$ $\qquad \dfrac{z_{31}}{z_{32}} = \dfrac{1}{2}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 12 & 16 & 16 \\ 15 & 22 & 24 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

One-to-one pivot position in every column

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Pivot position in every column

One-to-one ✓

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

no pivot position

$T_A(v_1) = T_A(v_2)$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $\qquad T_A(v) = Av$

$T_A\left(\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}\right) \quad T\left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}\right)$

$c\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = c\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

9

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{bmatrix}$ $R_2 = R_2 \cdot \frac{1}{2}$

$R_3 = -3R_1 + R_3$

R

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{matrix}$

$R_3 = -3R_1 + R_3$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & & \end{matrix}$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 4 & -0 \end{matrix}$ $R_3 = R_3 \times -1$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{matrix}$ $R_3 = R_2 - R_3$

b) $\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{matrix}$ $R_3 = R_2 + R_3$

a) $\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$ $R_3 = R_2 + R_1$

Onto Pivot column

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$

One to one

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

One to one if pivot position in every column.

a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \to$

$\boxed{\text{A is one to one}}$

b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

not a pivot column

A is ~~not~~ 1-1.

$T_A(v_1) = A \cdot v_1$

$A \cdot v_1 = A \cdot v_2$

let $x_3 = 2$

$x_3 = 2$

$x_2 = -2(2) = -4$

$x_1 = 0 - -4 = 4$

$\begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix} = v_1$ $v_2 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

~~$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$~~

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_3 = x_3$ $x_3 = 1$

$x_2 + 2x_3 = 0$ $x_2 = 0 - 2$

$= -2$

$x_1 + x_2 = 0$

$x_1 = 0 - -2$

$= 2$

let $x_3 = 1$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \quad (-3)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \quad (-1)$$

$$-2+4 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\boxed{\text{(a) ONE-TO-ONE}}$

$$-3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$$

$$(-1) \quad \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$-2+2=$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{(b) NOT ONE-TO-ONE}}$

$X_1 = -X_2$

$X_2 = -2X_3$

$X_3 = \text{free}$

$$\boxed{V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\downarrow$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A has pivot in every column $\rightarrow$ A is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\downarrow$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

not one-to-one

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $T = \begin{bmatrix} x_1 & x_2 & 0 \\ 0 & 2x_2 & 4x_3 \\ 3x_1 & 4x_2 & 2x_3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \tfrac{1}{2} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \tfrac{1}{2}$

$\to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

pivot position in every col. so YES, one-to-one

111

$x_1 + x_2 =$

$x_2 + 2x_3 =$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 32 \\ 28 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \tfrac{1}{2} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ Pivot position NOT in every column, so NOT one-to-one

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} 2+1+0 & 3 \\ 0+2+0 & 2 \\ 6+4+0 & 10 \end{array}\right]$

$x_1 + x_2 = 2$ $x_1 = 2 - x_2$

$2x_2 + 4x_3 = 6$

$3x_1 + 4x_2 + 2x_3 = 9$

$3(2-x_2) + 4x_2 + 2x_3 = 9$

$6 - 3x_2 + 4x_2 + 2x_3 = 9$

$x_2 + 2x_3 = 3$

$\frac{2x_3}{2} = \frac{3-x_2}{2}$

$x_3 = \frac{3-x_2}{2}$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 2 & 4 & 6 \\ 3 & 4 & 2 & 9 \end{array}\right] \tfrac{1}{2} \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 2 & 9 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 9 \end{array}\right]$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Come back.

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

→ One-to-one
pivot columns
→ Nul(A) ?

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$$

$\downarrow$ $\tfrac{1}{2}R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \leftarrow -3R_1 + R_3 \to R_3$$

$-3 \quad -3 \quad +0 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

pivot column because column without a leading 1.

$\boxed{\text{NOT one-to-one}}$.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \leftarrow \tfrac{1}{2}R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \leftarrow -3R_1 + R_3 \to R_3$$

$-3 \quad -3 \quad +0 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \leftarrow -R_2 + R_3 \to R_3$$

$+0 \quad -1 \quad -2 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow$$

One to one because no pivot columns?

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} (-3) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot 1/2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} (-1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -4 \end{bmatrix} \cdot(-1/4) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (-1) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (-2) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

A. is one.to one
because every column has a pivot position and the matrix is a homogeneous eavation

b) $$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} (-3) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot 1/2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} (-1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot 1/2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (-1) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} (-2) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} -2$$

A is one-to one
because every column has a pivot position and the matrix is a homogeneous eavation.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\downarrow$ $r_3 = r_3 - 3r_1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

All column are pivot columns, therefore

$\boxed{\text{all } T_A(v) \text{ are one-to-one.}}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\downarrow$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

not one to one

$$\boxed{v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{bmatrix}}$$

$$T_A(v_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$T_A(v_2) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -1+1 \\ 2-2 \\ -3+4-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{T_A(v_1) = T_A(v_2)}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot -3$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot 1/2$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \cdot -1$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{matrix} \cdot -1 \\ \cdot 1/2 \end{matrix}$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot -1$

$\downarrow$

$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is one to one}}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot -3$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot -1/2$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \cdot -1/2 \\ \cdot 1/2 \end{matrix}$

$\downarrow$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

free

$x_1 = 2x_3$

$x_2 = -2x_3$

$x_3 = x_3$

1

3

Not one to one

b

$\downarrow$

Two vectors

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 6 \\ -6 \\ 3 \end{bmatrix}$

☐	☐	☐	☐	☐	☐	☐	☐	8	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + (-3R_1)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + (-2R_2)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 + (R_3)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right] \xrightarrow{R_3 \to -\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 + (-R_2)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

Matrix is one to one because there is a pivot position in every column.

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b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + (-3R_1)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + (-2R_3)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

not one-to-one

using $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T_A(V_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0 \\ 0+0+0 \\ 3+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

now: $T_A(V_2) = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 3 \end{array}\right] \Longrightarrow$ following same row reduce seen above

$\to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} V_1 = -V_2 + 1 \\ V_2 = -2V_3 \\ V_3 = V_3 \end{array}$

$V_3 = 1 \Rightarrow \begin{array}{l} V_3 = 1 \\ V_2 = -2 \\ V_1 = -1 \end{array} \Rightarrow V_2 \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

$\Rightarrow \boxed{V_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_2 = R_2/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 = R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$R_3 = R_3 - R_2 \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ $\xrightarrow{R_2 = R_2 - R_3 \text{ and } R_3 = R_3/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We have pivot pos. in every row so matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ is one-to one.

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_2 = R_2/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$R_1 = R_1 - R_2 \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ Since every col is not a pivot col matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ is not one-to one.

$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $V_2 = \begin{bmatrix} -5 \\ 8 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 - R_3 - 3R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_3} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ a is one-to one. but b is not one-to-one

Now, $T_A(V_1) = T_A(V_2)$

$\therefore T_A(V_1 - V_2) = 0$

$$\therefore \begin{array}{c} V_1 \\ \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \end{array} - \begin{array}{c} V_2 \\ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{array}{c} V_1 \\ \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} \end{array} - \begin{array}{c} V_2 \\ \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ Pivot, Pivot, Pivot

one to one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 1 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}$ - Not Pivot

$\boxed{\text{Not one-to-one}}$

0
0
0

2
-2
1

$x_1 = 2x_3$
$x_2 = -2x_3$

$\begin{matrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}$

$$\boxed{v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 3 & 4 & 4 & 3 & 4 & 2 \end{bmatrix} \times\tfrac{1}{2} \to \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 3 & 4 & 4 & 3 & 4 & 2 \end{bmatrix} \times -3$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 1 & 4 & 0 & 1 & 2 \end{bmatrix} \times -1$$

It's not one to one.

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 2 \end{bmatrix} \times -2$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \end{bmatrix} \times\tfrac{1}{2}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \end{bmatrix} -1$$

$$V_1 = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

$x_1 = -2$

$x_2 + x_5 = 2$

$x_3 = 0$

$x_4 =$ free

$x_5 =$ free

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a. $A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

pivot in each column, One-to-One

b. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ free.

$x_1 = 2x_3$
$x_2 = -2x_3$ $x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$
$x_3 = x_3$.

$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$V_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

$T_A(V_1) = T_A(V_2)$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

$$\text{a) } A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \qquad \text{b) } A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

(a)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} (\tfrac{1}{2}) \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} (-3) + \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} (-1) + \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} + (-1)$$

$$\to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + (-1) \; (\tfrac{1}{2}) \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

T_A is one-to-one because $Nul(A) = \{0\}$.

(b)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} (\tfrac{1}{2}) \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} (-3) + \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} (-1) + \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} + (-1) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

T_A is not one-to-one becuase $Nul(A) \neq \{0\}$.

$$T_A(v_1) = T_A(v_2)$$
$$T_A(v_1) - T_A(v_2) = 0$$
$$T_A(v_1) - T_A(v_2) = T_A(x)$$
$$T_A(v_1 - v_2) = T_A(x)$$
$$v_1 - v_2 = x$$
$$v_1 - v_2 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{v_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}}$$

$$T_A(x) = Ax = 0$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 2x_3$$
$$x_2 = -2x_3$$
$$x_3 = x_3$$
$$x = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3$$

Let $x_3 = 1$

$$x = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{\cdot(-3)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{\cdot\frac{1}{2}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{\cdot(-1)} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{\cdot(-1)} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{\cdot\frac{1}{2}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot(-2)$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\cdot 2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

since A has a pivot position in every column, $T_A(v)$ is one-to-one.

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{\cdot(-3)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{\cdot 1/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{\cdot(-1)} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{\cdot(-1)} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

since A does not have a pivot position in every column, $T_A(v)$ is not one-to-one.

$T_A(v_1) = T_A(v_2)$

let $T_A(v_1) = T_A(v_2) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$

$\begin{cases} x_1 = 1 + 2x_3 \\ x_2 = 2 - 2x_3 \\ x_3 = x_3 \end{cases}$

if $x_3 = 1$:
$x_1 = 1 + 2(1) = 3$
$x_2 = 2 - 2(1) = 0$
$x_3 = 1$

if $x_3 = -1$:
$x_1 = 1 + 2(-1) = -1$
$x_2 = 2 - 2(-1) = 4$
$x_3 = -1$

$V_1 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3+(-3)R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\xrightarrow{R_3+(-1/2)R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

$0 \neq 2$

Not one-to-one

no solution

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_3+(-3)R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\xrightarrow{R_3+(-1/2)R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 + x_2 = 0$

$2x_2 = 4$

$x_2 = 2$ $\quad T_A$ = one-to-one

$x_1 = -2$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \times\tfrac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \times -3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \times -1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \times\tfrac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \times -2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \times -1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Since every column of matrix A has pivot position

TA is one to one

$Nul(A) = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \times\tfrac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \times -3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \times -1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$Nul(A) = \left\{\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}\right\}$

$x_1 + x_2 = 0$ $\quad x_1 = -x_2$

$x_2 + 2x_3 = 0$ $\quad x_2 = -2x_3$

$x_3 = x_3$ $\quad x_3 = x_3$

$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} x_3$

Since not all column of matrix A has pivot column

TA is not one to one.

Since Nu(A) in part a is $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$

and Nu(A) in part b is $\left\{\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}\right\}$

and, $T_A(v_1) = T_A(v_2)$

v_1 has to equal v_2

which v_1 and v_2 are both $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

because each Null space contain $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \times 3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \times 2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{bmatrix} \times -\frac{1}{4}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ A is one-to-one because there is a pivot position in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \times 3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \times 2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ B is not one-to-one

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} (-3) \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & -2 \end{bmatrix}$$

Both ~~B~~ is not one to one since every row is not a pivot ~~column~~ row

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ \cancel{1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

$v_1 \to$

$$= \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$$

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot 2$$

two =

The vectors are such where the last number makes both of them equal,

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{matrix} = 2 \\ = 6 \\ = 9 \end{matrix}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \begin{matrix} \to R_1 - R_2 \to \\ \\ \to R_3 - R_2 \to \end{matrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{matrix} \to R_1 + R_3 \\ \to R_2 - R_3 \\ \\ \end{matrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{R_3 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since A has a pivot position in every column it is 1 to 1

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \cdot \frac{1}{2}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} \to R_1 - R_2 \to \\ \\ \to R_3 - R_2 \to \end{matrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

A is not one to one since no pivot position in every column

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$T_A(v) = T_A(\omega)$

if $\omega = v + n$

$n \in Nul(A)$

$Nul(A): \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \cdot(-3)$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \cdot(-1)$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \cdot(-1)$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$ $\boxed{\text{One}}$

$\boxed{\text{One to - One}}$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \cdot(-3)$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 2x_3$

$x_2 = -2x_3$ infinite

$x_3 = x_3$

$x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$Nul(A) = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$\boxed{v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}}$

$V_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$T(V_1) = T(V_2)$

$T(V_1) - T(V_2) = 0$

$T(V_1 - V_2) = 0$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix}$$

$$\left[\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}\;\middle|\;\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right]$$

$x_1 = -x_2$ $\qquad$ $x_1 = 2$

$x_2 = -2x_3$ $\qquad$ $x_2 = -2$

$x_3 = x_3$ $\qquad$ $x_3 = 1$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} =$$

a) One-to-one

b) not one-to-one

$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $\quad$ $V_2 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ Not one-to-one **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ one-to-one

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $R_3 : R_3 - 3R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$ $R_2 : R_2 - R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$ $R_1 : -R_2 + R_1$

$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $R_3 : R_3 - 3R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $R_1 : R_1 - R_3$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $R_2 : R_2 - R_3$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$R_3 = -3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_2 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_3 = -2R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_3 = -\frac{1}{4}R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right]$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

T_A is one-to-one because there is a pivot position in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$R_3 = -3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$R_2 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$R_3 = -2R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$R_1 = -R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

T_A is not one-to-one because only columns 1 and 2 have pivot positions

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x_1 = 2x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases}$

• For v_1, $x_3 = 1$

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

• For v_2, $x_3 = 2$

$v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$

4-3 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

4-2 $\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

This is ~~onto~~ one to one because there is a pivot in every ~~row~~ col.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}$

$x_1 = -x_2$
$x_2 = -2x_3$
$x_3 = \text{free}$

2
-2
1

-4
2

This isn't one to one cuse there isn't a pivot in every col.

$\mathrm{v}_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $\mathrm{v}_2 \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

If T_A is one-to-one, A has a pivot position in every column.

$\Leftrightarrow \text{Nul}(A) = \{0\}$

(a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \xrightarrow[R_3-3R_1]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$

$\xrightarrow{R_3-R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\therefore$ A has a pivot position in every column,

So $\underline{T_A(v) \text{ is one-to-one}}$.

(b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \xrightarrow[R_3-3R_1]{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$$\xrightarrow{R_3-R_2} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \text{free} \end{array}$$

$\therefore$ A doesn't have a pivot position in 3rd column,

So $\underline{T_A(v) \text{ is not one-to-one}}$.

sol $\begin{cases} x_1 = -x_2 = 2x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases} \to x = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3 \to \text{Nul}(A) = \left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$

• $T_A(v_1) = T_A(v_2)$ if and only if $v_1 - v_2 \in \text{Nul}(A)$

if, $v_1 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, then

$T_A(v_1) = T_A(v_2)$. $\because v_1 - v_2 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \in \text{Nul}(A)$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{-3R_1+R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{-R_3+R_1} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\xrightarrow{-R_3+R_2} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_3-R_3} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \xrightarrow{R_3+R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

$\xrightarrow{-\frac{1}{4}R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ it is one-to-one as every column has 1 pivot position.

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{-3R_1+R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\xrightarrow{-R_3+R_1} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{-R_3+R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\xrightarrow{R_3-R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

not going to be one to one because column 3 will not have a pivot.

$v_1 = \begin{bmatrix} \ \end{bmatrix}$ $v_2 = \begin{bmatrix} \ \end{bmatrix}$

$T_A(v_1) = T_A(v_2)$ if $v_2 = v_1 + n$

So if $v_1 = \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix}$ and $n = 2$

$v_2 = \begin{bmatrix} 5 \\ 9 \\ 6 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

1-1 if $Nul(A) = \mathbf{0}$ $\longrightarrow$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ only solution to $A\mathbf{v} = \mathbf{0}$ is $\mathbf{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore Nul(A) = \mathbf{0}$

$\boxed{\therefore A \text{ is one-to-one}}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2x_3$
$x_2 = -2x_3$

$\therefore Nul(A) \neq 0$

$\boxed{\therefore A \text{ is not one-to-one. ex. for vectors } v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix},\ v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix},\ T_A(v_1) = T_A(v_2)}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{\text{row red}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since $Nul(A) = \{0\}$

A is one to one

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{\text{row red}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x_1 - 2x_3 = 0$ $x_2 + 2x_3 = 0$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3$

$Nul(A) = \text{Span}\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right)$

$\boxed{v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$ $T_A(v_1) = 0$

$T_A(v_2) = 0$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{matrix} \xrightarrow{R_3 = R_3 - 3R_1} \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{matrix} \quad \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{matrix} \quad \begin{matrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix} \quad \begin{matrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix}$$

A is one-to one, because there is a leading one in each column.

b. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \quad \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix} \quad \begin{matrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}$

The matrix is not one-to-one, because the third column does not have a leading one.

$T_A(V_1) = T_A(V_2) \qquad T_A(V_1 - V_2) = 0$

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{matrix} \quad \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \quad \begin{matrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$x_1 - 2x_3 = 0$ $x_1 = 2x_3$

$x_2 + 2x_3 = 0$ $x_2 = -2x_3$

$x_3 = x_3$ $x_3 = x_3$

x_3 Nul A $\in$ Span $\left(\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}\right)$

$x_3 \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

$$\boxed{V_1 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a). ~~for T_A to be~~

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

∴ There is a pivot pos in every column.
$T_A(v) = Av$ is one to one

for $T_A(v)$ to be one to one the $Nul\{A\} = \{0\}$

$\therefore \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 3 & 4 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$Nul(A) = \{0\}$

$\therefore \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$\therefore \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

∴ Every column does not have pivot pos. $T_A(v) = Av$ is not one to one.

If $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ 17 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 2 & 4 & 16 \\ 3 & 4 & 2 & 17 \end{array}\right] = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$\therefore V_2 = \begin{bmatrix} -5 \\ 8 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ -3

$$\begin{array}{ccc} -3 & -3 & 0 \\ 3 & 4 & 4 \\ \hline 0 & 1 & 4 \end{array} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{array}{ccc} 0 & -1 & -2 \\ 0 & 1 & 4 \\ \hline 0 & 0 & 2 \end{array} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} -1$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & -1 & -2 \\ \hline 1 & 0 & -2 \end{array} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} -1$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

A has a pivot pos. in every column
$\therefore T_A(v)$ is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

A does not have a pivot pos. in every column $\therefore$
$T_A(v)$ is not one-to-one

$Nul(A) = T_A(v) = 0$

$Av = 0 \longrightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

free var.

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$\underline{x_3 = 1}$ $\quad x_1 = 2$, $x_2 = -2$, $x_3 = 1$ $\to v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$\underline{x_3 = 2}$ $\quad x_1 = 4$, $x_2 = -4$, $x_3 = 2$ $\to v_1 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right) \cdot \frac{1}{2} \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right) R_3 \circ R_2 \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 4 & 8 & 0 \end{array}\right) R_3 - \tfrac{1}{2}R$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ a) not one to one

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right) \cdot \frac{1}{2} \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right) R3 - 4 \cdot R2 \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 0 & -2 & 0 \end{array}\right) R3 \circ R2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right) \cdot -\frac{1}{4} \quad \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$ b) one to one

✓

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 2 & 4 \end{pmatrix} (\cdot -1) \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 0 & 0 \end{pmatrix} \cdot 1/3 \quad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 1 & 0 & 0 \end{pmatrix} \cdot (-1) \Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 2 & 4 \\ 1 & 0 & 0 \end{pmatrix} (\cdot -2)$

$\Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 4 \\ 1 & 0 & 0 \end{pmatrix} \cdot \frac{1}{4} \Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Pivot position in evry column $\therefore$ A is one to one.

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b) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{pmatrix} (\cdot -3) \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix} \cdot 1/2 \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} (\cdot -1) \Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

Not one to one, no pivot position in every column.

$T_A(v_1) = T_A(v_2)$
$T(v_1 - v_2) = 0$
$v_1 - v_2 \in Nul(A)$

$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & \underbrace{0}_{\text{free}} & 0 \end{array}\right)$

$x_1 + x_2 = 0$
$x_2 + 2x_3 = 0$
$x_3 = x_3$

$\underline{x_3 = 1}$: $x_2 = -2(1)$, $x_2 = -2$; $x_1 = -x_2$, $x_1 = 2$; $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$\underline{x_3 = 2}$: $x_2 = -2(2)$, $x_2 = -4$; $x_1 = -(2)$, $x_1 = -2$; $\begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}$

$\boxed{v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -2 \\ -4 \\ 2 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a)

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\Downarrow$ row reduction

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

pivot in every column so one-to-one

b)

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\Downarrow$ row reduction

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

not one-to-one since pivot not in every column

$\mathrm{Nul}(A) = \mathrm{span}\left[\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right]$

$T_A(v_1) = T_A(v_2)$

$v_2 = v_1 + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 3 & 4 & 4 & 0 \\ 0 & 2 & 4 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] + \cdot(-3)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] + \cdot(-1) \Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] + \cdot(-1) \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Pivot position in every col., so $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ is one-to-one.

b)

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] + \cdot(-3) \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} + \cdot(-2) \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

not one-to-one

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \Rightarrow \begin{bmatrix} c_1 + c_2 \\ c_2 + 2c_3 \\ 0 \end{bmatrix}$$

Let $c_1 = 1$
$c_2 = 2$
$c_3 = 3$

$$\Rightarrow \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix}$$

$v_1 = \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 9 \\ 24 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since after row reduction, matrix A has a pivot position in every row, it is one-to-one therefore T_A is one-to-one

b. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Since A does not have a pivot position in every row, T_A is not one-to-one

$Av_1 = Av_2$

$A \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 2x_2 + 4x_3 \\ 3x_1 + 4x_2 + 2x_3 \end{bmatrix}$

If $x_1 = x_2 = 1$ then $v = \begin{bmatrix} 2 \\ 2 + 4x_3 \\ 7 + 2x_3 \end{bmatrix}$

Setting $x_3 = 0$ Setting $x_3 = 1$

$v_1 = \begin{bmatrix} 2 \\ 2 \\ 7 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 6 \\ 9 \end{bmatrix}$

$v_1 = \begin{bmatrix} 2 \\ 2 \\ 7 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 6 \\ 9 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\cdot -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot -\tfrac{1}{2}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{matrix} \\ \div 2 \\ \div 2 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot -1$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot 2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot -2$$

A is One to one

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix} \cdot -\tfrac{1}{3}$$

Also one to one

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Pivot Position in every colum

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot 1/2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \cdot -1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot 1/2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} S\ -2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} S\ \cdot -1$$

$$\begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$

$\therefore A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot 1/2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \cdot -1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} S\ \cdot -1$$

$$\begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ is not one-to-one as it does not have pivot position in every row.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot -3$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{matrix} \; 2 \cdot -\frac{1}{2}$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \cdot 1/2 \\ 0 & 0 & 2 \end{matrix}$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \cdot 1/2 \end{matrix} \to \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix} \; -1$ $\quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \cdot -1 \end{matrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

T_A is one to one b/c there is a piv position in every column of A

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} -3$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{matrix} \; \cdot -\frac{1}{2}$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \cdot 1/2 \\ 0 & 0 & 0 \end{matrix}$ $\quad \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix} \; \cdot -1$ $\quad \begin{array}{ccc} x_1 & x_2 & x_3 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}$ free

Not One to one

$x_1 = 0$
$x_2 = -2x_3$
$x_3 = x_3$

$x_3 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}$

$V_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 11 \end{bmatrix}$

$3 \times 3 \cdot 3 \times 1$

$V_2 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{matrix} 1 \\ 1 \\ 4 \end{matrix} = \begin{bmatrix} 2 \\ 18 \\ 15 \end{bmatrix}$

7+6

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) Aug. mat. of A: RREF

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_1 \to -R_2 + R_1} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3/2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_3]{R_1 \to R_1 + 2R_3} \begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$

Every column is a pivot column.

$\therefore$ $\boxed{T_A(v) \text{ is one-to-one}}$

b)

Aug. mat: RREF $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2/2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow[R_3 \to \frac{R_3}{2}]{\cancel{R_1 \to R_1 + R}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_2 \to R_2 - 2R_3]{R_1 \to R_1 + 2R_3} \begin{bmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix} \to$$ every column is a pivot column.

$\therefore$ $\boxed{T_A(v) \text{ in one-to-one}}$

Ans:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $Nul(A) = \{0\}$

every column has pivot position

Thus, It's one-to-one

b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

the third column has no pivot postion

Thus, It's not one-to-one

$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $V_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$T_A(V_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$T_A(V_2) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{vmatrix}$

$R_1 \times -3 + R_3$ $R_2 \times \frac{1}{2}$

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ ✓

$R_2 \times -1$ $R_3 \times \frac{1}{2}$

a) A is one to one because it has a pivot column in every column and the Null of A is equal to zero

$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$

* Nul(A) = 0 proof

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$

$R_1 \times -3 + R_3$ R_2

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix} \to$ b) $T_A(v) = Av$

is not one to one because it doesn't have a pivot in every column and it has infinite solutions for it's Nul space

$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 2 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}$

$\begin{vmatrix} 2 \\ 8 \\ 10 \end{vmatrix} = \begin{vmatrix} 2 \\ 6 \\ 9 \end{vmatrix}$

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & 3 & 4 & 0 \\ 3 & 2 & 0 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 2 & 4 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\boxed{\text{one-to-one}}$

b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 3 & 0 & 0 \end{array}\right] \to$

$\boxed{\text{Not one to one.}}$

$V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & -1 & 4 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & -1 & 4 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 4 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a is
One to one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & -1 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 6 \\ 0 & -1 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 8 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & -2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

b is
one to one

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot -3 \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot -\frac{1}{2} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \cdot \frac{1}{2} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot \frac{1}{2} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot -2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot -1 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ pivot in all columns - one-to-one

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot -3 \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot \frac{1}{2} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \cdot -1 \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot -1 \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

some columns missing pivots - not one-to-one

$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{cases} x_1 = 2x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases}$ $Nul(A) = span\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right)$

let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T_A(v_1) = T_A(v_2)$ if and only if $v_1 = v_2 + n$ for $n \in Nul(A)$

$v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] R3 \to -3R1 + R3$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] R2 \to \frac{R2}{2}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \begin{array}{l} R1 \to -R2 + R1 \\ R3 \to -R2 + R3 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] R3 \to \frac{R3}{2}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R2 \to -2R3 + R2 \\ R1 \to 2R3 + R1 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow$$

A IS one to one w/ a pivot pos. in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] R3 \to -3R1 + R3$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \begin{array}{l} R1 = -\frac{1}{2}R2 + R1 \\ R3 = -\frac{1}{2}R2 + R3 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2x_3$
$x_2 = -4x_3$
$x_3 = x_3$

$\mathrm{Nul}(A) = \mathrm{Span}\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix} \right\}$

T(A) is not one to one because there is not a pivot position in every column

$V_1 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ $V_2 = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right) = 0$

$T\left(\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}\right) = 0$

$TA(V_1) = TA(V_2)$

$TA\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}\right) = TA\left(\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right)$

$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ is in Nul(A)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

one to one

b/c Nul(A) = $\{0\}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Nul(A) = $\left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$

Not one to one

$x_1 = 2x_3$

$x_2 = -2x_3$

$x_3 = x_3$

$v_1 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

1-1: Pivot Pos. in every column

(a)

$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \overset{R2\cdot\frac{1}{2}}{=} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$ R1·-3 +R3

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$ R2·-1 +R3 $= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot \frac{1}{2}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

T_A is one-to-one because A has a pivot position in every column.

(b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \overset{R2\cdot\frac{1}{2}}{=} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix}$ R1·-3 +R3

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

T_A is not one-to-one because A does not have a pivot position in every column.

$T_A(\vec{v}_1) = T_A(\vec{v}_2)$

$T_A(\vec{v}_1) - T_A(\vec{v}_2) = 0$

$T_A(\vec{v}_1 - \vec{v}_2) = 0$

$A\vec{w} = 0$

Let $\vec{w} = \vec{v}_1 - \vec{v}_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \overset{R2\cdot-1+R1}{=} \begin{array}{c} x_1 \; x_2 \; x_3 \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow \begin{array}{c} x_1 \; x_2 \; x_3 \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow \begin{array}{l} x_1 - 2x_3 = 0 \\ x_2 + 2x_3 = 0 \end{array}$

$x_1 = 2x_3$

$x_2 = -2x_3$

$x_3 = x_3$

$\mathrm{Nul}(A) = \mathrm{Span}\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right)$

Let $\vec{w} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \vec{v}_1 - \vec{v}_2$

$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + \vec{v}_2 = \vec{v}_1$

Let $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \vec{v}_1$

$\boxed{\vec{v}_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

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5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\cdot -3$

$\cdot \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\cdot -1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$\cdot -1 \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

$\cdot 1 \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\cdot \frac{1}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

∴ A has a pivot position in every column so it is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $\cdot -3$

$\cdot \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\cdot -1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\cdot -1 \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$
$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 2x_3$

$x_2 = -2x_3$

$x_3 = x_3$

vector form

$\begin{bmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{bmatrix} \to x_3\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

∴ A is not one-to-one since there is not a pivot position in every column, so

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ & $V_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

s.t. $T_A(v_1) = T_A(v_2)$

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

* one-to-one
pivot pos in ever col

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \cdot -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \cdot 2 \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{bmatrix}$$

a) is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot -3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \cdot -2 \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot -1 \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 2x_3 = 0$
$x_2 + 2x_3 = 0$
$x_3 = x_3$
Nul(A)

b) not one-to-one

$$\boxed{T\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}\right)}$$

v_1 v_2

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$(-1)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 2 & 0 \end{bmatrix}$$

$$\to (-3)\begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & 0 \\ 0 & 2 & 4 \end{bmatrix} \to (2)\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} \to \begin{matrix}(1) \\ (\tfrac{1}{4})\end{matrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \to (-1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}(-3)$

$$\to (\tfrac{1}{2})\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}(-1) \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

· not one-to-one

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -3 \\ -16 \\ -17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+2+0 \\ 0+4+12 \\ 3+8+6 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ 17 \end{bmatrix}$$

$$T_A(v_1) - T_A(v_2) = 0$$

$$\begin{bmatrix} 3 \\ 16 \\ 17 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{-3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} (\tfrac{1}{2})$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} 2 \cdot -1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} (\tfrac{1}{2})$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{\text{a) One to one}}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{-3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} (\tfrac{1}{2})$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} 2 \cdot -1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Not one to one

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] -1$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3$

b) $\boxed{\text{Not one to one}}$

$\boxed{v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & \cancel{8}\ 1 \end{bmatrix}$

Pivot position every column

T_A is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $\longrightarrow$ $v_1 - 2v_5 = v_2 - 2v_6$

$v_3 + 2v_5 = v_4 + 2v_6$

$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

Not every column has a pivot position

T_A is not one-to-one

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} v_2 \\ v_4 \\ v_6 \end{bmatrix}$

$v_1 + v_3 = v_2 + v_4$

~~$2v_3 + 4$~~

$v_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

$A \cdot V = b$ $\quad T_A(V_1) = T_A(V_2)$

$T_A(V_1) - T_A(V_2) = 0$

$T_A(V_1 - V_2) = 0$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ $\qquad$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

one-to-one pivot position in every column

A) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow \frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_2 - R_3} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{R_3 \to -\frac{1}{2}R_3} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to 2R_3 + R_2}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to 2R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ This is one-to-one

So $T_A(V_1) - T_A(V_2) = 0$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $\quad x_1 = 0$, $x_2 = 0$, $x_3 = 0$ $\quad$ Nul A: Span(0)

V_1 can be $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ V_2 can be $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_3} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \to \frac{1}{2}R_2}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 \to R_2 - R_3} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

We can see not every column will be a pivot so $T_A(v) = AV$ is not one-to-one.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $(1/2)$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $(1/2)$

a)

$(-3)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$

$(-1)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$(1/2)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

$(-2)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$(-1)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

One-to-one

b)

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix}$

$(-1)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ not one-to-one

b) $T_A(v_1) = T_A(v_2)$

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

Try

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

$Av_1 = Av_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2-2+0 \\ 0-4+4 \\ 6-8+2 \end{bmatrix} = \begin{bmatrix} 4-4+0 \\ 0-8+8 \\ 12-16+4 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pivot position in every column, so it is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

every column doesn't have pivot position so not one-to-one

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - 2x_3 = 0$

$x_2 + 2x_3 = 0$

$x_3 = x_3$

if $x_3 = 1$

$x_1 - 2 = 0$

$x_1 = 2$

$x_2 + 2 = 0$

$x_2 = -2$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$T_A\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right) = T_A\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \cdot -3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \cdot -1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \cdot -1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \begin{array}{l} \cdot -1 \\ \\ \cdot \frac{1}{4} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\mathrm{Nul}(A) = \{0\}$

A has a pivot in every col

So T_A is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \cdot -3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \cdot \frac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \cdot -1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot -1 \qquad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A does not have a pivot in every col, so T_A is not one-to-one

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

$$Av_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Av_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 12 \end{bmatrix} + \begin{bmatrix} -4 \\ -8 \\ -16 \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} +(-3)\ [-3\ -3\ 0]$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \times(\tfrac{1}{2})$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} +(-1)$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \to \times(\tfrac{1}{2})$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

One-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} +(-3)\ [-3\ -3\ 0]$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} +(-1)$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

No piv pos

NOT one-to-one

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot v_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot v_2$$

-2 2 -1 | -4 4 -2

-2 2 0 | -4 -4 0

0 4 -4 | 0 8 -8

-6 8 -2 | 12 16 4

$$v_1 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -4 \\ 4 \\ -2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 4 \\ 0 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -4 \\ 0 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

T_A is not one-to-one.

$T_A(V_1) = T_A(V_2)$

$T_A(V_1 - V_2) = 0$

$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix}$

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_2 = -4x_3$

there is no solution.

$\Rightarrow$ there is no two vectors V_1 and V_2 such that $T_A(V_1) = T_A(V_2)$.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

T_A is not one-to-one

$T_A(V_1) = T_A(V_2)$

$T_A(V_1 - V_2) = 0$

$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 3 & 4 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3.$

$V_1 - V_2 = X$

assume $x_3 = 1$ $X = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

assume $V_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

then $V_1 = X + V_2$

$= \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

$\therefore V_1 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ $V_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\xrightarrow{-3}$ $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{array}\right]$ -1

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $\xrightarrow{-3}$ $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{array}\right]$

yes pivot position in every col.

$A = \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{array}\right]$

yes one to one
pivot position in every col.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

R1x-3 +R3 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$ /2

R2x-1 +R3 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ /2

R3x-2 +R2 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Downarrow$

one-to-one and onto b/c pivot pos. in every column and row, respectively.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ (R1x-3)+R3

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ /2

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ (R3x-1)+R2

$\begin{bmatrix} \boxed{\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}} & \boxed{\begin{matrix} 1 \\ 1 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ 2 \\ 0 \end{matrix} \end{bmatrix}$

$\Downarrow$

not one-to one b/c can only have a pivot position in the first two columns, here we have only two.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$n = 3 = m = 3$

$T_A(v) = Av$ is not one-to one.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ - pivot in every column

$\boxed{T_A(v) = Av \text{ is one-to-one}}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

T_A is not one-to-one

$x_3 =$ free

$x_1 = 2x_3$

$x_2 = -2x_3$

$$\boxed{V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}}$$

$x_3 = 1$ $\qquad$ $x_3 = -1$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$R_3 = R_3 - 3R_1$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$

$R_3 = 2R_3 - R_2$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right]$

$x_3 = 4$

$x_2 = 2$

$x_1 = -2$

It is one to one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$R_3 = R_3 - 3R_1$

$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -x_2$

$x_2 = x_2$

$x_3 = -\frac{1}{2}x_2$

$x_2 + 2x_3 = 0$

$2x_3 = -x_2$

$x_3 = \frac{1}{2}x$

$\begin{bmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{bmatrix} x_2$

$v_1 = \begin{bmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{bmatrix}$ $v_2 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

It is not one to one

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

leading ones

pivot columns

$\boxed{\text{a) one to one}}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \boxed{\text{not one to one}}$

~~P0~~

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 1 \\ 1 \\ 0 \end{matrix}$$

b) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Yes. it is one to one.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

no. one to one.

$T_A(V_1) = AV_1 \quad T_A(V_2) = AV_2$

$AV_1 = AV_2$

$V_1 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$

$AV_1 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$AV_2 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$T_A(V_1) = T_A(V_2)$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\xrightarrow{R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{bmatrix}$$

This is not one to one because there is not a pivot in every column

$V_1 = \begin{bmatrix} \quad \end{bmatrix}$

$V_2 = \begin{bmatrix} \quad \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\xrightarrow{R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

This is not one to one because there is not a pivot in every column

$V_1 = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$

$V_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

One to one - pivot position in every column

onto - pivot position in every row

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\downarrow$

$R_3 = R_3 - 3R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$

$\downarrow$

$R_2 \Leftrightarrow R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix}$

$R_3 \to R_3 - 2R_2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{bmatrix}$

$R_2 = R_2 + R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

$R_3 \cdot \frac{-1}{4}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 3 & 4 & 4 & 3 & 4 & 2 \end{array}\right]$

$\downarrow$

$R_3 = R_3 - 3R_1$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 1 & 4 & 0 & 1 & 2 \end{array}\right]$

$R_2 \Leftrightarrow R_3$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & 4 & 0 & 2 & 4 \end{array}\right]$

$R_3 \to R_3 - 2R_2$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 & 1 & 2 \\ 0 & 0 & -4 & 0 & 0 & 0 \end{array}\right]$

$R_2 \to R_2 + R_3$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & -4 & 0 & 0 & 0 \end{array}\right]$

$R_1 = R_1 - R_2$

$R_3 \cdot \frac{-1}{4}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$

not one to one

$Av_1 = Av_2$

$\underset{3\times3}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}$

$V_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) One to one -> pivot in every column

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ A is one-to-one

b) $$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ not one-to-one

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 2x_2 + 4x_3 \\ 3x_1 + 4x_2 + 2x_3 \end{bmatrix}$$

=

no such vectors exist, since $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$ are linearly independent

$x_2 = x_3$

$\begin{bmatrix} 1 & 1 \\ 0 & 6 \\ 3 & 6 \end{bmatrix}$ $\quad x_1 = 1$, $x_2 = 1$ $\quad \begin{bmatrix} 2 \\ 6 \\ 9 \end{bmatrix}$

$x_1 = 2$, $x_2 = 1$ $\quad \begin{bmatrix} 3 \\ 6 \\ 12 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$. $V_1 \neq V_2$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{2R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$-3R_1 + R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{-R_3+R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow[-R_2+R_1]{-R_2+R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\frac{R_3}{4}$

$-2R_3+R_2$ $\quad -R_3+R_1$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(a) is onto one

(b) is not one to one

$X_1 = -2$

$X_2 = 2$

$V_1 = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

$\mathrm{Nul}(A) = \{0\}$

pivot position in every column

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \cdot \frac{1}{2} \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \xrightarrow{R_3 = -3R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] R_3 = -R_2 + R_3 \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] R_2 = R_2 - R_3 \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] R_1 = R_1 - R_2$

$\to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $\therefore$ one-to-one

b) $A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \cdot \frac{1}{2} \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] R_3 = -3R_1 + R_3 \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \cdot (-1) \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

not one-to-one

$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \hline \end{array}$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] V_1 = T_A(V_1)$

$V_1 + V_2 = 0$

$V_2 + 2V_3 = 0$

$\therefore$ when $v_2 = 2$

$V_1 = -2$; $V_3 = -1$

$V_1 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

$V_2 = \begin{bmatrix} -4 \\ 4 \\ -2 \end{bmatrix}$

when $v_2 = 4$

$V_1 = -4$ $V_2 = -2$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

one to one → pivot in every column

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\cdot(-3)$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot(1/2) \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \cdot(-1) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \cdot(-1) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot(1/2)$$

$$\to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot(-2) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot(2) \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a.) Pivot in every column, therefore it is one to one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $\cdot(-3)$

$$\to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \cdot(1/2) \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \cdot(-1) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \cdot(-1) \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free}} \end{bmatrix}$$

b.) Not one to one because there is a free variable on the last column.

$T_A(V_1) = T_A(V_2)$

$AV_1 = AV_2$

$$Nul(A) = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \to \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & \underbrace{0}_{\text{free}} & 0 \end{array}\right] \end{array} \qquad \begin{aligned} x_1 &= 2x_3 \\ x_2 &= -2x_3 \\ x_3 &= x_3 \end{aligned}$$

$$Nul(A) = \text{Span}\left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$$

$$V_2 = V_1 + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \to \boxed{V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $-(3R_1)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} - \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & -1 & 0 \end{bmatrix} \cdot (\tfrac{1}{2})$

$\downarrow$

$\cdot(\tfrac{1}{2}) \cdot (-1) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \leftarrow + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 0 \end{bmatrix} +$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ yes A is one-to-one

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} -2R_2 \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 0 & -6 \end{bmatrix} \begin{matrix} \cdot \frac{1}{2} \\ \cdot -\frac{1}{3} \end{matrix} \to + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 3 \end{bmatrix}$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} -(\cdot 2) \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} -$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ yes, A is one to one.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\cdot -3$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \cdot \tfrac{1}{2}$$

$$-1 \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \cdot \tfrac{1}{2}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot -2$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot -1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution is one to one as every row ~~lead~~ column has a pivot ~~po~~ position or leading one

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

MTH-309T-F19-EX1-080-P05

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix}1&1&0\\0&2&4\\3&4&4\end{bmatrix}$ $\cdot -3$

$$C(1/2)\begin{bmatrix}1&1&0\\0&2&4\\0&1&4\end{bmatrix}$$

$$\Rightarrow \cdot -1 \begin{bmatrix}1&1&0\\0&1&2\\0&1&4\end{bmatrix}$$

$$\Rightarrow \cdot -1 \begin{bmatrix}1&0&-2\\0&1&2\\0&1&4\end{bmatrix}$$

$$\Rightarrow C(1/2)\begin{bmatrix}1&0&-2\\0&1&2\\0&0&2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1&0&-2\\0&1&2\\0&0&1\end{bmatrix} \cdot -2$$

$$\begin{bmatrix}1&0&-2\\0&1&0\\0&0&1\end{bmatrix} \cdot 2$$

$$\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}$$

one-to-one

b) $A = \begin{bmatrix}1&1&0\\0&2&4\\3&4&2\end{bmatrix}$ $\cdot -3 \Rightarrow \begin{bmatrix}1&1&0\\0&2&4\\0&1&2\end{bmatrix} C(1/2)$

$$\cdot -1 \begin{bmatrix}1&1&0\\0&1&2\\0&1&2\end{bmatrix} \Rightarrow \cdot -1 \begin{bmatrix}1&0&-2\\0&1&2\\0&1&2\end{bmatrix}$$

$$\begin{bmatrix}1&0&-2\\0&1&2\\0&0&0\end{bmatrix}$$

Not one-to-one: pivot position not present in every row.

Find the null space $\Rightarrow$ set of vectors v such that $T_A(v) = 0$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c}1&0&-2&0\\0&1&2&0\\0&0&0&0\end{array}\right]$$

$x_1 - 2x_3 = 0$

$x_2 + 2x_3 = 0$

$x_3 = x_3$

$x_1 = 2x_3$

$x_2 = -2x_3 \Rightarrow \begin{bmatrix}2\\-2\\1\end{bmatrix}$

$x_3 = x_3$

scalar: $2\begin{bmatrix}2\\-2\\1\end{bmatrix}$ to create $\Rightarrow \begin{bmatrix}4\\-4\\2\end{bmatrix}$

$v_1 = \begin{bmatrix}2\\-2\\1\end{bmatrix}$

$v_2 = \begin{bmatrix}4\\-4\\2\end{bmatrix}$

Null space: $\left\{ x_3 \begin{bmatrix}2\\-2\\1\end{bmatrix} \right\}$ span$\left(\begin{bmatrix}2\\-2\\1\end{bmatrix}\right)$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T_A(V)$ is 1-to-1; pivot columns in all columns.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Let $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$V_2 = V_1 + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

Not 1-to-1 ; not all columns are pivot columns

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$X = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \times -3$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \times \frac{1}{2}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \times -1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \times \frac{1}{2}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix ~~have~~ has pivot column in every column.

It's one to one.

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \times -3$

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\cancel{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \times -2$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

It's not one to one.

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & a_1 \\ 0 & 1 & 2 & a_2 \\ 0 & 0 & 0 & a_3 \end{array}\right]$

$x_1 + x_2 = a_1$

$x_2 + 2x_3 = a_2$

$a_3 = 0$

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ ③-①×3

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$ ②÷2

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$ ③-②

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$ ③÷2

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Since A has a pivot position in every column, $T_A(v) = Av$ is one to one.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ ③-①×3

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ ②÷2

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ ②-③

$\downarrow$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ ①-②

$\downarrow$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Since A does not have a pivot column in every column, $T_A(v) = Av$ is not one to one

From the matrix, we can get $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \in \text{Nul}(A)$

Let v_1 be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$v_2 = v_1 + \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ V   b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

one to one if all Col(A) are lin. dep.

SO NO

a) $\cancel{x_1}\begin{bmatrix}1\\0\\3\end{bmatrix} \xrightarrow{\text{NO}} \cancel{x_2}\begin{bmatrix}1\\2\\4\end{bmatrix} \cancel{x_3}\begin{bmatrix}0\\4\\4\end{bmatrix}$ $\boxed{\text{So lin. dep.}}$ ✓

$\boxed{\text{So One to One}}$

NO

b) $x_1\begin{bmatrix}1\\0\\3\end{bmatrix} \xrightarrow{\text{NO}} x_2\begin{bmatrix}1\\2\\4\end{bmatrix} \quad x_3\begin{bmatrix}0\\4\\2\end{bmatrix}$

- All lin. dep

$\boxed{\text{So One to One}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $\begin{array}{r} \\ \\ -3R1 \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \overset{:R3}{=} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}_{-R2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix} = {}_{:4}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \overset{-R2}{=} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a) is one-to-one

b) $\begin{array}{r} \\ \\ -3R1 \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} -2R3 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

b) is not one-to-one

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1/4 & 1 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 1/2 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

One-to-one b/c pivot position in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Not one-to-one:

$T_A(v_1) = T_A(v_2)$

$T_A(v_1) - T_A(v_2) = 0$

$Av_1 - Av_2 = 0$

$A(v_1 - v_2) = 0$

$T_A(v_1 - v_2) = 0$

$$\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$v_1 = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \xrightarrow{R3-3R1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[R3 \to R3 - R2]{R2 \to 1/2 R2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

T_A not one-to-one

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R3 \to R3-3R1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20