$w_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ //

$w_2 = v_2 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1$

$w_3 = v_3 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1 - \left(\frac{w_2 \cdot v_3}{w_2 \cdot w_2}\right) w_2$

1. Consider the following vectors in $\mathbb{R}^4$:

$1+0+1+1$

$2+0+1+1+0$

3

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

W_2 $\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

$\begin{bmatrix} 1 \\ -2 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

a) Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace V.

b) Compute the vector $\mathrm{proj}_V \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ on V.

a) $w_1 = v_1$ //, $w_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \left(\frac{3}{3}\right)\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ // ✓

$w_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{6}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \left(\frac{-3}{3}\right)\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} = \cancel{\begin{bmatrix} 0 \\ -3 \\ 0 \\ 3 \end{bmatrix}}$

$$\mathcal{D} = \begin{array}{c} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -3 \\ -1 & 0 & 0 \\ 1 & -1 & 3 \end{bmatrix} \\ \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \end{array}$$

$9+9$

b) $\mathrm{Proj}_V u = \left(\frac{u \cdot v_1}{v_1 \cdot v_1}\right) v_1 + \left(\frac{u \cdot v_2}{v_2 \cdot v_2}\right) v_2 + \left(\frac{u \cdot v_3}{v_3 \cdot v_3}\right) v_3$

$= \left(\frac{3}{3}\right)\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \left(\frac{3}{3}\right)\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \cancel{\left(\frac{0}{18}\right)\begin{bmatrix} 0 \\ -3 \\ 0 \\ 3 \end{bmatrix}}$

$\mathrm{Proj}_V u = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ //

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