MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Mohammedanas Tai

UB Person Number:

5	0	2	3	2	6	6	5
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE
12	1	3	17	5	4	2	2	--	46	D

MTH-309T-F19-EX1-023-P01

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & \cancel{-5}\ -1 & 2 & +4+b \end{array}\right] 5$$

$$\downarrow$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{6+b}{12} \end{array}\right] \leftarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 12 & 6+b \end{array}\right] \frac{1}{12}$$

Span(b) = $\mathbb{R}^3$ since it'd be a solution to any real number

OK, but this is due to a mistake in the row reduction

b)

$$\left.\begin{array}{l} x_1 - x_2 + x_3 = -2 \\ x_2 + 2x_3 = 2 \\ x_3 = \frac{6+b}{12} \end{array}\right\} \quad \begin{array}{l} x_1 = -2 - x_2 + x_3 \\ x_2 = 2 - 2x_3 \\ x_3 = \frac{6+b}{12} \end{array}$$

Since the equation has a solution any value of b ~~be~~ given
it is not a linearly independent set

Linear independence deals with uniqueness of solutions, not with existence.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☒	☐	☐	☐	☐	☐	☐	☐	☐

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}}_{3\times 3} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3\times 3}$$?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$?

$$= \cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}} = A^{-1}$$

$$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We needed to find a matrix s.t it is equal to the Identity matrix.

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$C = (A^T)^{-1} \cdot B$$

$$A^T \cdot B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad \cancel{X} \; C$$

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\left[1\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right] + \left[2\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right]$$

$$+ \left[3\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} + 4\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right]$$

$$C = \begin{bmatrix} 7 & 7 & 1 \\ -1 & 6 & 5 \\ 9 & 7 & 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$A = [T(x_1), T(x_2)]$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

Standard Matrix = $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ ✓

b)

$$x_1 - 2x_2 = 1$$
$$x_1 + x_2 = 10$$
$$x_1 - 3x_2 = -2$$

✓

$$\Rightarrow \begin{array}{ccc} x_1 & x_2 & \\ \end{array}$$
$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 9 \\ 0 & 1 & -3 \end{bmatrix}$$

✓

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 9 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}\right) = \cancel{\begin{bmatrix} -19 \\ 11 \\ -29 \end{bmatrix}}$$

✓

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} (-3) \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & -10 \end{bmatrix}$$

This matrix is not fully reduced yet.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & -2 \end{bmatrix}$$

Both ~~is~~ is not one to one since every row is not a pivot ~~column~~ row

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$$

v_1

$$A \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot 2 \qquad = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$$

two =

The vectors are such when the last number makes both of them equal.

? $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 1 \\ 1/2 \end{bmatrix}$

$T_A(v_1) \neq T_A(v_2)$

b) ?

☐	☐	☐	☐	☐	☒	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True ✓ because if $w+u$ is in the Span(u,v) then $w \in \mathrm{Span}(u,v)$ shows it is a part of that span. Graphically it would be on the same line because w is only being added by u

why?

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False~~ you can have $\left\{ \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 10 \\ 14 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix} \right\}$ when it is dependent while these two are dependent.

This set is lin. dependent

0 1 2 3 4 5 6 7 8 9 10

7. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

?

True because for what A you are given it'll be on the span of u,v assuming they are dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

?

~~The~~ True ✓ because ~~we are~~ when we are doing linear transformation we are just multiplying vectors ~~with~~ which makes it a linear ~~trans~~ combination and a span is a set of linear combination.

0 1 2 3 4 5 6 7 8 9 10