

2. Find the equation  $f(x) = ax + b$  of the least square line for the points  $(1, 0)$ ,  $(-1, 2)$ ,  $(2, 1)$ .

$$\begin{matrix} A \\ \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \end{matrix} \begin{matrix} \begin{bmatrix} a \\ b \end{bmatrix} \end{matrix} = \begin{matrix} b \\ \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \end{matrix}$$

$$A^T A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array} \right] \xrightarrow{-2/6 R_1 + R_2} \left[ \begin{array}{cc|c} 6 & 2 & 0 \\ 0 & 7/3 & 3 \end{array} \right] \xrightarrow{3/7 R_2} \left[ \begin{array}{cc|c} 6 & 2 & 0 \\ 0 & 1 & 9/7 \end{array} \right] \xrightarrow{-2R_2 + R_1} \left[ \begin{array}{cc|c} 6 & 0 & 18/7 \\ 0 & 1 & 9/7 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 6 & 0 & 18/7 \\ 0 & 1 & 9/7 \end{array} \right] \xrightarrow{1/6 R_1} \left[ \begin{array}{cc|c} 1 & 0 & -3/7 \\ 0 & 1 & 9/7 \end{array} \right]$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3/7 \\ 9/7 \end{bmatrix}$$

$$f(x) = -3/7 x + 9/7$$

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