



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

TRUE \rightarrow IF A IS 2×2 MATRIX THEN
 $NUL(A)$ IS SPAN OF SOME
 VECTORS IN \mathbb{R}^2 ,

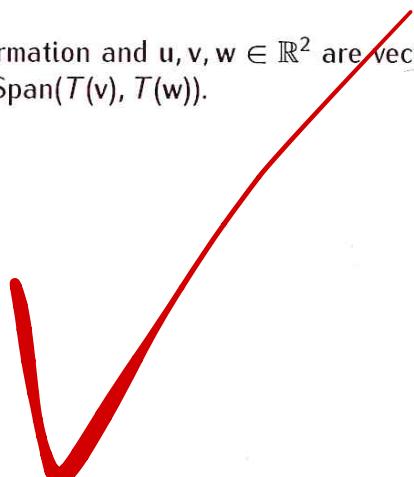
$Au \neq 0$ FOR DEPENDENCE
 $Av \neq 0$

$NUL(A)$ = (SET OF SOLUTIONS OF
 $A.v = 0$)
 \hookrightarrow LINEARLY
 INDEPENDENT

IF $Au \neq 0$
 $Av \neq 0$ THEN u, v LINEARLY DEPENDENT.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

TRUE





7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True - Linearly dependent sets have infinitely many solutions
 a set of two vectors is linearly independent if and only if one vector is a scalar multiple of the other.
 Since $A(v+u) = Au + Av$, if Au and Av are linearly dependent u, v must also be linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True: $T(u+v) = T(u) + T(v)$ $u \in \text{Span}(v, w)$
 $T(cv) = cT(v)$ $T(u) \in \text{span}(T(v), T(w))?$

Span holds through transformation

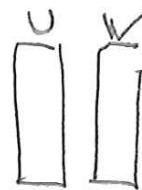
$$T \circ S(v) = (A \cdot B)v = A(B(v))$$

$$\text{col}(A) = \text{row}(B)$$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



True, if they have
a unique solution,
it doesn't matter
whether they are
~~separated or not~~
represented in a
matrix or not

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True; $\text{Span}(v_1, \dots, v_p) = \text{set of all}$
linear combination
 $c_1 v_1 + c_2 v_2 + \dots + c_p v_p$
thus it holds ~~every~~
that $T(u)$ must be in
 $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

true. A matrix is a linear transform so anything that is linear dependent that is linearly transformed stays linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

true. Linear transforms will not change is something is within a span

$$u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{array}{c} u \quad A \quad u \\ | \quad | \quad | \\ 1 \quad 3 \quad | \quad 5 \\ 2 \quad 6 \quad | \quad 10 \end{array} \quad \text{FALSE,}$$

just because Au, Av are linearly dependent does not mean that u, v will also be linearly dependent.

$$(-2) \left(\begin{array}{c} A \quad v \\ | \quad | \\ 1 \quad 3 \quad | \quad 5 \\ 2 \quad 6 \quad | \quad 7 \end{array} \right)$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 0 & 3 \end{array} \right]$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True

$\text{COL}(A) = \text{set of values for } T_A$
 so if u is in $\text{span}(v, w)$ for matrix A then it will also be in the span of the transformation for A .



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

false

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

true



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent. *False*

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \left\{ u_1 \right\}, \left\{ u_2 \right\} \text{ are only linearly dependent}$$

if $u_1 = \vec{0}, u_2 = \vec{0}$

$Matrix A$ can be linearly dependent
even if u, v are not zero-vectors

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True by linear combination



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a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$Au = \left[\begin{array}{cc|c} 1 & 2 & u_1 \\ 3 & 4 & u_2 \end{array} \right]$$

$$Av = \left[\begin{array}{cc|c} 1 & 2 & v_1 \\ 3 & 4 & v_2 \end{array} \right]$$

) linearly independent

True because,

Since Au & Av are linearly dependent, u & v must also be.

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

Come back.

$$u \in \text{Span}(v, w)$$

then

$$T(u) \in \text{Span}(T(v), T(w)) ??$$





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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad u = [1], v = [1]$$

True

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

This is true



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True. If Au and Av are linearly dependent, then $Au = cAv$. Therefore $u = cv$, so u and v must be linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True. If $u \in \text{Span}(v, w)$ then $u = cv + cw$.
If you take $T(u)$, you get $T(u) = A \cdot u = A \cdot cv + A \cdot cw$, which shows $T(u) \in \text{Span}(T(v), T(w))$.

REVIEW
 $T_1(u) + T_2(u)$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False

$$V = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad T = \begin{bmatrix} 5 & 7 \end{bmatrix}$$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \quad \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} \rightarrow L.I$$

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} \rightarrow L.I$$

$$\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} A & u \\ 0 & v \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \rightarrow (L.I)$$

$$\begin{bmatrix} A & v \\ 0 & u \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \rightarrow (L.I)$$

False

$$\begin{bmatrix} u & v \\ 3 & 2 \\ u & 2 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, every matrix transformation is a linear transformation, making the statement true





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a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

FALSE If $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then $Au \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is linearly dependent ~~but u is linearly independent.~~

~~$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ is~~ ~~TRUE,~~ but $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is linearly independent.

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

TRUE

$$u = v + w$$

$$\begin{aligned} T(u) &= T(v + w) \\ &\xrightarrow{\quad} \underline{T(v) + T(w)} \end{aligned}$$

So, $T(u)$ is in $\text{Span}(T(v), T(w))$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False.

True.



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$u, v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

TRUE, The transformation that my 2×2 Matrix keeps linearity.

TRUE,

Putting two linearly dependent vectors through transformation, they must stay dependent when put together with more dependents.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The two vectors are dependent after the transformation, thus putting

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, the transformation must be one to one and onto, so the span would be kept intact.



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a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent. \top

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$. \top



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False. ~~$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$~~ , $Au = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $Av = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ so Au , Av is linearly dependent.

$w = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

u, v is not dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False.



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False $Au = T_A(u)$ $Av = T_A(v)$

$$x_1 T_A(u) + x_2 T_A(v) = 0$$

$$T_A(x_1 u + x_2 v) = 0$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True $u \in \text{Span}(v, w)$

$$u = x_1 v + x_2 w$$

$$T(u) = T(x_1 v + x_2 w)$$

$$T(u) = T(x_1 v) + T(x_2 w)$$

$$T(u) = x_1 T(v) + x_2 T(w)$$

$$T(u) \in \text{Span}(T(v), T(w))$$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$\{Au, Av\}$ linearly dependent

False. Multiplying matrix A by vectors u and v does not necessarily preserve linear dependence.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True. If vector $u \in \text{span}(v, w)$ and transformation T is applied to u, v, w then $T(u) \in \text{span}(T(v), T(w))$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True, $q_1v_1, \dots, q_nv_n = \text{non-zero}$ therefore the vectors u, v must also be linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, there can be any $T(u)$ within the span of $T(v)$ and $T(w)$ because you can transform $T(v)$ into either $T(v, w)$,



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

false

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ If u and v are not linearly dependent
 $Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ no scalar multiple of u could produce v

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True
 $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Yes because linear transformation has some algebraic properties
as vector addition with $u = x_1v + x_2w$
 $T(u) = x_1T(v) + x_2T(w)$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

$$A\bar{u} \neq A\bar{v}$$

- a) If A is a 2×2 matrix and \bar{u}, \bar{v} are vectors in \mathbb{R}^2 such that $A\bar{u}, A\bar{v}$ are linearly dependent then \bar{u}, \bar{v} also must be linearly dependent. False

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \bar{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad A\bar{u} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\bar{v} = \begin{bmatrix} 9 \\ 1 \end{bmatrix} \quad A\bar{v} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\bar{u}, \bar{v}, \bar{w} \in \mathbb{R}^2$ are vectors such that \bar{u} is in $\text{Span}(\bar{v}, \bar{w})$ then $T(\bar{u})$ must be in $\text{Span}(T(\bar{v}), T(\bar{w}))$. False. $T(\bar{u})$ may no longer be in $\text{Span}(T(\bar{v}), T(\bar{w}))$ after transformation

$$\bar{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \bar{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \bar{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True because for what A you are given

Ab' will be on the span of u, v assuming they are dependent

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

The True because when we are doing linear transformations we are just multiplying vectors with which makes it a linear combination and a span is a set of linear combinations.



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

If Au, Av are linearly dependent that means the homogeneous equation has infinite solutions, for both $u \neq v$. That means u or v are multiples of one another $\neq \therefore$ not linearly independent

\therefore true

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, transformations are linear operators

$$T(u+v) = T(u) + T(v)$$

\therefore if u is in $\text{Span}(v, w)$

the $T(u)$ is in $\text{Span}(T(v), T(w))$



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- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

False ran out of time

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, transformations are a linear operator so their original relations hold true to the outcomes of a transformation



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- a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

False

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True

$$c_1\mathbf{v} + c_2\mathbf{w} = \mathbf{u}$$

$$c_1T(\mathbf{v}) + c_2T(\mathbf{w}) = T(\mathbf{u})$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

$\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ u & v need to be
set to 0 and checked
 $Au = 0$ to see if a solution
 $Av = 0$ exists i.e. c_1, c_2, \dots, c_n
all equal 0.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True

If the $\text{Span}(v, w)$ contains u
then the Span of $(T(v), T(w))$
contains $T(u)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}, \quad u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad v = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Au and Av are LD

No matter what the vectors u, v in \mathbb{R}^2 are they will have a pivot column so statement

is FALSE

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

TRUE because the properties of linear transformation tells us that if u is in the $\text{Span}(v, w)$ then $T(u)$ has to be in the $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\text{True} \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

True because if you have
inf solution and then transform it is still
the same.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$. ~~False~~

$$\text{True} \quad v = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$$\begin{smallmatrix} 2 \times 1 & 1 \times 1 \\ 2 \times 1 \end{smallmatrix}$$

$$\begin{smallmatrix} 2+2 & 1 \\ 2+1 & 1 \end{smallmatrix}$$

The ~~because~~ hence

of the example

of a linear transformation of $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then

$$Au = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad Av = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow Au = Av, \text{ so } Au, Av \text{ are linearly dependent}$$

$\therefore [u \ v | 0]$ but u and v are linearly independent.

$$\begin{bmatrix} 1 & 1 & | & 0 \\ 1 & 2 & | & 0 \end{bmatrix} \xrightarrow{\substack{\text{Row reduction} \\ }} \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

So $x_1u + x_2v = 0$ has only one solution $\Leftrightarrow u, v$ are linearly independent.
 $x_1 = x_2 = 0$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$\begin{aligned} u &= c_1v + c_2w \\ \therefore T(u) &= T(c_1v + c_2w) \\ &= T(c_1v) + T(c_2w) \\ &= c_1 \cdot T(v) + c_2 \cdot T(w) \text{ is a linear combination of } T(v), T(w) \end{aligned}$$

$$\therefore T(u) \in \text{Span}(T(v), T(w)) \quad \underline{\text{True}}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True.

Au, Av indicates that after multiplication there was a free variable causing oo solutions, that makes Au, Av linearly dependent. This means that the vectors u and v must have also had free variables when multiplied by some x value.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False, if the transformation changes u, v, w to where u no longer

$\text{Span}(v, w)$ that would make $T(u)$ not $\text{Span}(T(v), T(w))$

$$\begin{matrix} u \\ \left[\begin{matrix} 1 \\ 3 \end{matrix} \right] \end{matrix} \quad u_1 + 3u_2 \quad \text{span}(u) \quad \text{but} \quad T(u) = \left[\begin{matrix} 1 \\ 9 \end{matrix} \right] \quad \text{then it not span}$$

for two other vectors
if it is not a
scalar multiple



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

This is false. For example, let $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Then $Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Au and Av are the same vector, so they are clearly linearly ~~dependent~~ independent, however, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are linearly independent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

This is true. If $u \in \text{Span}(v, w)$, then

$u = c_1 v + c_2 w$ for some $c_1, c_2 \in \mathbb{R}$. So $T(u) = T(c_1 v + c_2 w)$.

In order to show that $T(u) \in \text{Span}(T(v), T(w))$, we must show that $T(u) = c_3 T(v) + c_4 T(w)$ for some $c_3, c_4 \in \mathbb{R}$.

Let $c_1 = c_3$ and $c_2 = c_4$. By def. of linear transformation, $c_1 T(v) + c_2 T(w) = T(c_1 v) + T(c_2 w)$. Again, by def. of linear transformation, $T(c_1 v) + T(c_2 w) = T(c_1 v + c_2 w)$. Since we already had that $T(u) = T(c_1 v + c_2 w)$, then $T(u) \in \text{Span}(T(v), T(w))$.

QED.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

(a) True set of vectors have to be linearly dependent
for Au, Av to be linearly dependent

$c_1 u + c_2 v = 0$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$c_1 v + c_2 w = c_3 u$$

$$c_1 T(v) + c_2 T(w) = T(c_3 u)$$

Since linear trans

$$T(c_1 v) + T(c_2 w) = T(u) \Rightarrow c_1 T(v) + c_2 T(w) = T(u)$$

Thus $T(u) \in \text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

This statement is false

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$\left\{ \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & q \end{pmatrix} \right\}$$

$$\begin{pmatrix} 6 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$T(u)$ must be in $\text{Span}(T(v), T(w))$, be



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False. $\therefore Au = A[u] + A[v] = n_1 u + n_2 v$
can be linearly dependent of c_1, c_2 $n_1, n_2 \neq 0$.

~~All $c_1 u + c_2 v$ can also be 0~~
~~so $c_1 u + c_2 v$ need not be linearly dependant because~~
~~not be~~

$$\text{if } u, v = 0.$$

$c_1 u + c_2 v$ becomes linearly independent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True.

$$\begin{aligned} T(v) &= T(v+w) \\ &= T(u) + T(w). \end{aligned}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

Lin Dependent: Au, Av have ∞ solutions to $Au=0, Av=0$

$[A | 0] \rightarrow \infty$ solutions $\therefore u$ and v aren't necessarily linearly independent
because ^{when} any matrix multiplied by A , the solution is a linearly dependent matrix $\therefore u, v$ are not always lin. dependent.

False

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$x_1v + x_2w = u \rightarrow (u \in \text{Span}(v, w))$$



$$T(x_1v) + T(x_2w) = T(u)$$

$$x_1T(v) + x_2T(w) = T(u) \rightarrow (T(u) \in \text{Span}(T(v), T(w)))$$

True



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

~~False~~ false, u, v can be linearly independent before being multiplied by A .

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, $T(u)$ is in $\text{span}(T(v), T(w))$
if u is in $\text{span}(v, w)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True. For Au and Av to be linearly dependent, u and v must have infinitely many solutions. Some combination of u & v other than the trivial solution will result in the 0 vector.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True. If u is in $\text{Span}(v, w)$, then it is a linear combination of v & w . Therefore its transformation must be in the same space.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True, Au and Av have infinite solutions
so u , and v will also be dependent as they
will be scalar multiples of each other.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

If u is a linear combination of v, w . T_u is a linear combination of $T(v), T(w)$.
In homework problems $T(v) = T(w_1) + T(w_2) + T(w_3)$
meaning it is a linear combination.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False; the matrix A can cause Au, Av to become lin. dependent even if u, v are lin. independent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True; let $u = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad T(u) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix} \quad T(u) \text{ is in } \text{Span}(T(v), T(w))$$

$$T(v) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$T(w) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

So, $T(u)$ must be in $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True

If Au and Av are linearly dependent, then they have infinitely many solutions. Therefore u and v would have to also have infinitely many solutions

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True

Since u is in the $\text{span}(v, w)$ taking the linear transformation of each vector is just taking the product of each vector with A . Therefore $T(u)$ would still be in $\text{span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

*True, because A is just a linear transformation
so it will still return the properties of having
infinitely many solutions to the vector.*

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

*True, because all you are applying the same transformation
to both all 3 vectors so the span will be the
same as before meaning $T(u)$ is in $\text{Span}(T(v), T(w))$*



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

False

\downarrow linearly dependent

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

\leftarrow linearly dependent
But $\begin{bmatrix} -4 \\ -4 \end{bmatrix}$ is not linear dep b/c $w \neq 0$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.



False

$$\{T(v_1), T(v_2)\} \neq T(v_1) T(v_2)$$

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False.

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$$

$$A(u-v) = 0, u-v \neq 0$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False. *being in the span..*
Transformation doesn't guarantee the span..

~~If $T(v) x_1 + T(w) x_2 = 0$, doesn't have soln,
they~~



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$\begin{aligned}
 & u = c_1 v + c_2 w \\
 \text{True } & T(u) = T(c_1 v + c_2 w) \\
 & \because T \text{ is Linear transformation} \\
 & \therefore T(c_1 v) + T(c_2 w) = T(c_1 v + c_2 w) \\
 & \therefore T(u) = T(c_1 v) + T(c_2 w) \\
 & \therefore T(u) \in \text{Span}(T(v), T(w))
 \end{aligned}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$A \cdot u = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, A \cdot v = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False

True, because if $u = c_1v + c_2w$ then

$T(u) = c_1T(v) + c_2T(w)$ because the transformation is linear



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

True because Au, Av are a linear combination of A .

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True because of the matrix property $C \cdot T_1 = C(T)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

' Au, Av are dependent

but u, v are independent

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$u = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{False}$$

$$T(u) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad T(v) = \begin{bmatrix} -2 \\ -1 \end{bmatrix} \quad T(w) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$u \in \text{Span}(v, w)$

$T(u) \notin \text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.



$$A \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4+2 \\ 6+4 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 2+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad R_2 \rightarrow -2R_1 + R_2 \quad V = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x_1 = -2x_2$$

$$x_2 = x_2$$

$$\text{Nul}(A) = \text{Span}\left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$$

$$\text{False} \quad V = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$u = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad u \in \text{Span}(v, w)$$

$$T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

$$T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$$

True because

with any

reflection,

$T(u)$ will

be a linear

combination

of $T(v)$ and

$T(w)$

If u is in

$\text{span}(v, w)$

$$w = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

reflect over

x-axis

reflect over

line $y=x$

reflect over

y-axis

$$T(u) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$T(w) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$T(w) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(w) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(v) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\text{true because } T(v) = cT(u)$$

$$\text{if } u = cv \text{ then } T(u) = cT(v)$$

$$T(u) = T(cv)$$

$$T(u) = T(v)$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True

If u is a linear comb of $v + w$

$$u = v + w$$

$$\text{then } T(u) = T(v + w)$$

$$T(u) = T(v) + T(w)$$

$T(u)$ is a linear comb of

$$T(v) + T(w)$$

Therefore

$$T(u) \in \text{span}(T(v), T(w))$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

False.

Counter example :

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A\vec{u} = T(\vec{u}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A\vec{v} = T(\vec{v}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A\vec{u} = A\vec{v}$$

$$T(\vec{u}) = T(\vec{v})$$

$$T(\vec{u}) - T(\vec{v}) = \mathbf{0}$$

$$T(\vec{u} - \vec{v}) = \mathbf{0}$$

$$\vec{u}, \vec{v} \in \text{Null}(A)$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

$$\vec{u} \in \text{Span}(\vec{v}, \vec{w}) \Rightarrow T(\vec{u}) \in \text{Span}(T(\vec{v}), T(\vec{w}))$$

$$\vec{u} = c_1 \vec{v} + c_2 \vec{w}$$

$$T(\vec{u}) = T(c_1 \vec{v} + c_2 \vec{w})$$

$$= T(c_1 \vec{v}) + T(c_2 \vec{w})$$

$$= c_1 T(\vec{v}) + c_2 T(\vec{w}) \Rightarrow T(\vec{u}) \in \text{Span}(T(\vec{v}), T(\vec{w}))$$

True.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad Au = \begin{bmatrix} 5 \\ 11 \end{bmatrix} \quad Av = \begin{bmatrix} 10 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{rank } 2 \times 2 \quad u = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \quad \text{rank } 2 \times 1 \quad Au = \begin{bmatrix} 4 \\ 10 \end{bmatrix} \quad Av = \begin{bmatrix} 13 \\ 24 \end{bmatrix}$$

$$Au = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad Av = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

True

If Au and Av are linearly dependent then u, v must be linearly dependent since the scalar multiple multiplied to either Au or Av to get the other is the same scalar multiple to get u to v or v to u .

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True since $T(u)$ the u is a vector in \mathbb{R}^2 and v, w is a vector in \mathbb{R}^2 so $T(u)$ must be in the span of $T(v), T(w)$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False Counter example: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $v = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$Au \& Av$: both $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ which is linearly dependent

but u and v are not linearly dependent

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True If u is in $\text{Span}(v, w)$ it is a combination of $cv + dw$ where c and d are some constants

so $T(u) = T(cv + dw) = T(cv) + T(dw) = cT(v) + dT(w)$

which means that $T(u)$ is a combination of constant values of $T(v)$ & $T(w)$ which is what it means to be in $\text{Span}(T(v), T(w))$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

~~False~~

~~True~~

~~False~~

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

False

A single vector such as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

is linearly independent. If $A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
then it is linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True. All 3 vectors are transformed by the same transformation, so properties are not changed.
They are still scalar multiples.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

True.) Au and Av can be seen as linear combinations where $T_A(u) = Au$ and $T_A(v) = Av$. So if multiplying by the same matrix causes a linear transformation on the solutions but will not modify the amounts of solutions.

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, because Uibxon transformations ensure that the order of the sum of vectors

$$\text{Span}(v, w) = c_1 v + c_2 w$$

$$\text{if } U = C_1 V + C_2 W \leftarrow$$

ther:

$$T(U) = T(G_1 V + G_2 W) =$$

$$\checkmark T(U) = T(C_1V) + T(C_2W)$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

~~$Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$~~
 $Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
 ~~$Av = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$~~
 $Av = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
 $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$2u_1 + u_2 = 1$

$2u_1 + u_2 = 2$

$u_1 + u_2 = 1$

$u_1 + u_2 = 2$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

True; since all matrix transformations are linear transformations, the matrix transformation T_A preserves linear dependence between $\{u, v\}$, and $\{Au, Av\}$.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

True; since all matrix transformations are linear transformations, applying the same transformation to all three vectors preserves the column space and $T(u)$'s existence in it.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

FAISE

$$Au = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$Av = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{then } Av = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } Au = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Since these two vectors are NOT scalar multiples of each other this statement is false.

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

TRUE If U is in Span of V, W this means

$$c_1V + c_2W = U$$

$$\text{A linear transformation } T_A(c_1V + c_2W) = T_A(U)$$

$$\text{is applied to both sides } c_1T_A(V) + c_2T_A(W) = T_A(U)$$

Any linear combination of $T(V)$ and $T(W)$

will get you $T_A(U)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True, if Au, Av are linearly dependent, then

u, v are always zero vectors, which are linearly dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True, linear transformation is in span of other trans. if the vector of lin. trans is in span of other vectors.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

false

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$u = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ & $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are linearly independent

$Au = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ & $Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are linearly dependent

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

true

If u is in the $\text{Span}(v, w)$, then it can be found through a combination of multiples of v and w , $c_1v + c_2w = u$

For linear transformations, $T(u) = T(c_1v + c_2w) = T(c_1v) + T(c_2w) = c_1T(v) + c_2T(w)$

This means that the transformation of u can be solved as a linear combination of the multiples of $T(v)$ and $T(w)$.

This means that $T(u)$ must be in the $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True,

since every matrix transformation is a linear transformation,
then both of the resulting vectors were linearly transformed.
Since they were linearly dependent before transformation,
then transforming the line they form does not break their
linear dependence since linear transformations keep straight
lines intact.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True,

if the whole span is linearly transformed, then $T(u)$ will
be transformed in the same way, preserving its linearity
and remaining on the plane of the span.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \left\{ \begin{array}{l} Au = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ Av = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array} \right. \quad \left. \begin{array}{l} \text{linearly independent} \\ \text{linearly dependent} \end{array} \right\}$$

False.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False.

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True.

$$u \in \text{Span}(v, w) \quad u = c_1v + c_2w$$

$$T(u) = Au$$

$$T(v) = Av$$

$$T(w) = Aw$$

$$Au = c_1Av + c_2Aw.$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

True, since multiplying by the same matrix gives linear dependence

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True since every matrix transformation is a linear transformation



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$R2 \times 1/2 + R1$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} / 2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & c \\ c & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} / 2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow u_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

► true, solutions are still linearly dependent

u and v are linear combinations of each other

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$T\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = c_1 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + c_2 \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$T(u) = c_1 T(v) + c_2 T(w)$$

$$T\left(c_1 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + c_2 \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right) = T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) + T\left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right)$$

$$\Downarrow \quad \quad \quad + T\left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right)$$

True

For linear transformations,

$T(u+v)$ can be written as $T(u) + T(v)$,

and here, because u is in the span of v and w , its transformation can be written as a combination of the transformations for v and w



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False

$n=m=2$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$\boxed{\text{False}}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{Au and Av are dependent}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{u and v are independent}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$\boxed{\text{True}}$ a transform will only move a vector, not change it so if u is in the span it will also be in the transform span

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True

$$\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} \cdot u + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot v$$

$x_1 u_1 + x_2 v_2$ are linearly dependent

u_1, v_2 are also linear dependent.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True

$$T(v) = T(w) = T(u)$$

$$AV = AW = AU$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and $\underline{u}, \underline{v}$ are vectors in \mathbb{R}^2 such that $\underline{Au}, \underline{Av}$ are linearly dependent then $\underline{u}, \underline{v}$ also must be linearly dependent.

~~True~~ ~~False~~ ~~True~~ ~~False~~ ~~True~~ ~~False~~

~~This~~ this may simply mean that the matrix A is linearly dependent not necessarily \underline{u} or \underline{v}

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\underline{u}, \underline{v}, \underline{w} \in \mathbb{R}^2$ are vectors such that \underline{u} is in $\text{Span}(\underline{v}, \underline{w})$ then $T(\underline{u})$ must be in $\text{Span}(T(\underline{v}), T(\underline{w}))$.

~~True because~~ True because if the same transformation is performed on some \underline{u} that's in the span of ~~the~~ some \underline{v} and \underline{w} , ~~it will~~ \underline{u} will be moved into that new span of $T(\underline{v}), T(\underline{w})$ by the transformation $T(\underline{u})$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False.

Au will be 2×1 . Av will be 2×1 .

so Au Av will be 2×2 .

only infinite number or no solution will be dependent

$$A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad Av = \begin{bmatrix} 11 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 11 & 18 \\ 0 & -5 \end{bmatrix} \text{ dependent.}$$

$$u = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad Au = \begin{bmatrix} 18 \\ -5 \end{bmatrix}$$

$$u, v = \begin{bmatrix} 3 & 2 \\ 3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \text{independent}$$

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True.

u can be represented by v and w .

$$u = av + bw \quad T(u) = Au = A(av + bw) \\ = aAv + bAw \\ = aT(v) + bT(w)$$

so Tu must be in $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False; Linear dependence/independence is only guaranteed to be preserved if A is a matrix which defines a linear transformation. Since this condition is not specified, then linear dependence cannot be guaranteed after transformation.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True; Since T is a linear operator whose operation preserves the dimensions of the original vectors, then any vector $u \in \text{Span}(v, w)$ must be in the span of $T(v), T(w)$. Additionally, since there are 3 vectors in 2 spaces, and it is known that $u \in \text{Span}(v, w)$, v and w either are linearly dependent on one another, and u , or are linearly independent. Since T is a linear transformation, these properties are maintained, meaning $T(u) \in \text{Span}(T(v), T(w))$ by definition (3 vectors in 2 spaces, 2 are linearly independent; v and w) or because all 3 vectors were linearly dependent to begin with.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$A = \begin{bmatrix} \# & \# \\ \# & \# \end{bmatrix}$$

true , same transformation
done on u and v

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False



if transformation is 2×2 matrix
it could take u out of
 Span of v and w



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$Av_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad Av_2 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

v_1, v_2 are independent

Av_1, Av_2 are dependent

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$u = a_1 v + a_2 w$$

True

$T(u) = a_1 T(v) + a_2 T(w)$

$$u, a_1 v + a_2 w$$

are equivalent
vectors

∴

Their transformations
are the same



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True Because A stay the same so in order to keep Au, Av dependent u, v must be dependent +

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True if T is a translation done to all vectors then the vectors will change in the same magnitude



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True because

$$A(u+v) = Au + Av$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

False, $T(u)$ could not be
in $\text{Span}(T(v), T(w))$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False \rightarrow

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$u \in \text{Span}(v, w)$$

$$T(u) \in \text{Span}(T(v), T(w))$$

True because the transformation is applied for each vector, providing the same vectors at different positions



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

u and v could still be independent and be multiplied with A to make it linearly dependant.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

Yes, true because if u is in the span of (v, w) then the transformation could just be the scalar of a vector and it would still let $T(u)$ be in the span of $T(v), T(w)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True: if u is in the $\text{span}(v, w)$

then a combination $c_1 v + c_2 w = u$, so

$T(c_1 v + c_2 w) = T(u)$ as $T(c_1 v + c_2 w)$

can be split into $c_1 T(v) + c_2 T(w) = T(u)$
which means that there is a linear combination

$T(v)$ and $T(w)$ that equal $T(u)$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Av, Au are linearly dependent then u, v also must be linearly dependent.

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ Linearly independent}$$

It is false. You could have a linear independent vectors and by multiplying it by the matrix, end up ~~not~~ a linearly dependent.

$$Au = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Av = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

It is false. $T(u)$ means $A(u)$ which means multiplying by a matrix that could lead to no linear combinations for $T(u)$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

FALSE

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Au, Av are linearly dependent
- u, v are not linearly dependent

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

$$u = c_1 v + c_2 w$$

$$\begin{aligned} \text{By linearity: } T(u) &= T(c_1 v + c_2 w) = T(c_1 v) + T(c_2 w) \\ &= c_1 T(v) + c_2 T(w) \end{aligned}$$

$T(u)$ is a constant multiple of $T(v)$ and $T(w)$

$$\therefore T(u) \in \text{Span}(T(v), T(w))$$

TRUE



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False. Au, Av are linearly dependent which means they have free variables, but u, v can be having one solution or no solution.

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

~~False~~ True.
~~False~~ $\Leftrightarrow [v, w | u]$ has a solution

$$T(u) = \cancel{T(v)} Au$$

$$[T(v), T(w) | T(u)]$$

$$T(v) = \cancel{T(w)} Av$$

$$[Av, Aw | Au] \text{ has a solution.}$$

$$T(w) = Aw$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False

$$\begin{aligned} & \text{If } (Au, Av) \text{ is linearly dependent} \\ & \downarrow \\ & \text{Then } \{u, v\} \text{ is linearly dependent} \end{aligned}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

Yes. Since u is in $\text{Span}(v, w)$,

then $u = c_1v + c_2w$ has solution.

Also, $T(u) = T(c_1v + c_2w)$ has solution

Also, $T(u) = T(c_1v) + T(c_2w)$ has solution,

This means that $T(u)$ is in $\text{Span}(T(v), T(w))$.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

10

~~4- []~~

$$u = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\text{then } A_n = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

but u & v are
~~not linearly indep.~~

~~True~~
True, thought it
was false but couldn't
disprove

50

$$A_u = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad A_v = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

but
~~the~~ n and v

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$. ✓

$$U = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, V = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, W = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

False

so $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is valid

but

[1], [0], [0]

is not



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

TRUE

Because if Au and Av yield the linearly dependent result, then u and v are also dependent.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

~~True. Because applying the~~

FALSE



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False: $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$Au = \begin{bmatrix} 1+0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ↗ Not linearly dependent

$Av = \begin{bmatrix} 0+1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ↗ Linearly dependent

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

True $c_1v + c_2w = u$

$$(T(c_1v) + T(c_2w)) = T(u)$$

$$(Ac_1v) + (Aw) = T(u)$$

$$A(c_1v) + A(c_2w) = T(u)$$

$$T_A(c_1v) + T_A(c_2w) = T(u)$$

$$T(c_1v) + T(c_2w) = T(u) \quad ; \quad c_1v + c_2w = u$$



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

true

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

true, if all the vectors undergo a linear transformation then they will still share the relationship of being linear combinations.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If A is a 2×2 matrix and \mathbf{u}, \mathbf{v} are vectors in \mathbb{R}^2 such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then \mathbf{u}, \mathbf{v} also must be linearly dependent.

False,

- b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that \mathbf{u} is in $\text{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True,