

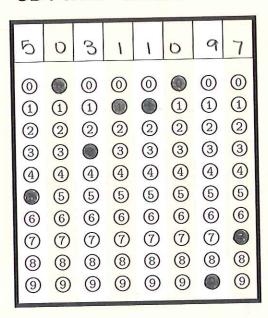
MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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UB Person Number:



Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
		*						

10	5	7	20	4	6		52	nan
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in Span(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ (inearly independent? Justify your answer. $(1, 1)^2 (2, 1)^$

(a)
$$C_1\begin{bmatrix} 0 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

Retained

See iy

each column

has

leading one?

(b)
$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \leftarrow \frac{1}{2} R_1 + R_3 \rightarrow R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ +0 & +1 & +2 & +0 \end{bmatrix} \leftarrow R_2 + R_3 \rightarrow R_3$$

[0 1 2 0] A Vectors V1, V2 & V3 are linearly dependant (NOT linearly Independent) because, V2 & V3 are



2. (10 points) Consider the following matrix:

(10 points) Consider the following matrix:
$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
Compute A^{-1} .
$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

$$3 \times 3$$



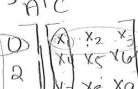
3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} A^{T}C$$



Come back!

flello?



- 4. (20 points) Let $T: \mathbb{R}^3$ be a linear transformation given by $2 \text{ (1)W} T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 2x_2 \\ x_1 + x_2 \\ x_1 3x_2 \end{bmatrix}$
- a) Find the standard matrix of T.

 b) Find all vectors (u) satisfying T(u) = $\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a)
$$T = \begin{bmatrix} 1 & 0 & | 0 \\ 0 & 1 & | 0 \end{bmatrix}$$

(b)
$$U = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} X_1 - 2X_2 & 1 \\ X_1 + X_2 & -10 \\ X_1 - 3X_2 & -2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

 $X_1 = 1 + 2X_2$ $\Rightarrow 1 + 2(3) = \begin{bmatrix} 7 \\ 7 \\ 3 \end{bmatrix} = 7$
 $1 + 2X_2 + X_2 = 10$ $\Rightarrow 1 + 3X_2 = 10 \Rightarrow 3X_2 = 9 = \begin{bmatrix} X_2 = 3 \\ 1 + 2X_2 - 3X_2 & -2 \end{bmatrix} = 7$
 $1 + 2X_2 - 3X_2 = -2 \Rightarrow 1 - X_2 = -2$
 $\begin{bmatrix} X_2 = 3 \end{bmatrix}$



Conce

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

	Γ1	1	0
a) $A =$	0	2	4
o#:	3	4	4

V 1/2/2-> R2

0 1 0 0 0

Pivot Column because column without a

WOT one-to-one).

	Γ1	1	$\lceil 0 \rceil$
b) <i>A</i> =	0	2	4
	3	4	2

-10 One - to - One Pivot Columns - Dulla)?

$$\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 2 & 0 \\
3 & 4 & 2 & 10
\end{bmatrix}$$

$$\begin{bmatrix}
3 & 4 & 2 & 10
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 0 \\
3 & 3 & 3 & 40 & +0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 2 & 0 \\
0 & 1 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
-R_2 + R_3 \rightarrow R_3
\end{bmatrix}$$

0 1 210 10 - K2+R3-R3

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

-10-1-2+0

Decouse

Columns Colify



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True because, with consists of was well and when when Espan(u,v); w must be in Span(u,v).

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

Counterexample:

Let {u,v,w} be set {5,6,8} suspectively Than,

{5,6,8} doesn't mover {5,8} is also linearly
Independent.



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and (u, v) are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, y also must be linearly dependent.

(b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w))

Come back

U E Span (V, W)

then T(U) & Span(T(V), T(W)) ?!

