1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\underline{w} \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$V_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ $\quad (-2)\cdot \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$ $\quad 2-3 = 2+b$

$V_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$ $\quad (1)\cdot \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$ $\quad 2-2$

$V_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow (1) \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad 2-(-1)$

$x_1 = x_3 \rightarrow -2$

$x_2 = -2x_3 \rightarrow (2)$

$x_3 = \text{free} \rightarrow b\ (-1)$

(a) $b = -1$

(b) No, dependent because it has infinite solutions

x_3 is a free variable