

MTH 309T LINEAR ALGEBRA EXAM 1

Name: Emery Constoct	er 3, 2019
UB Person Number: 5 0 2 5 2 7 7 2 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1	 Textbooks, calculators and any other electronic devices are not permitted You may use one sheet of notes. For full credit solve each problem fully, showing all relevant work.
1 2 3 4 5	6 7 TOTAL GRADE

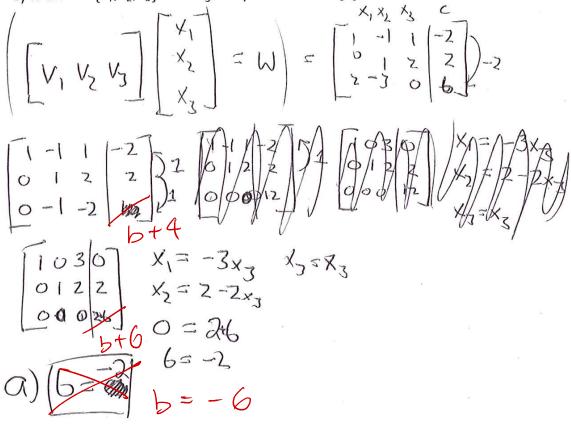
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1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.



6) Linearly dependents because xz is a free variable in the matrix [iv, xvxxv] resulting in a null space with infinite solutions



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$m=n\sqrt{\begin{bmatrix} 1-1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix}
1-17 & 100 \\
01-1-110 \\
02-100 \\
010
\end{bmatrix}$$

$$\begin{bmatrix}
1-12 & 100 \\
01-1-100 \\
001
\end{bmatrix}$$

$$\begin{bmatrix}
1-12 & 100 \\
01-1-100 \\
001
\end{bmatrix}$$

$$\begin{bmatrix}
101 & 010 \\
001 \\
2-21
\end{bmatrix}$$

$$\begin{bmatrix}
101 & 010 \\
010 \\
1-11 \\
001
\end{bmatrix}$$

$$\begin{bmatrix}
100 & 100 \\
010 \\
1-11 \\
001
\end{bmatrix}$$

$$\begin{bmatrix}
100 & 100 \\
010 \\
1-11 \\
001
\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 2 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1.5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2$$

$$C_{11} = 0.5$$

$$C_{12} = C_{12}$$

$$C_{13} = C_{13}$$

$$C_{14} = C_{13}$$

$$C_{15} = C_{15}$$

$$C_{15} = C_{1$$

$$\begin{bmatrix} \frac{7}{4} \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ -1 & 02 \\ 2 & 1-1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{1}{4}$$

$$C = \begin{bmatrix} 8 & 1.5 \\ 4 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} A^T \\ B \end{bmatrix}$$

$$C = \begin{bmatrix} 63 & 1.5 \\ 4 & 5 & 4 \\ 1.5 & 1.6.5 \end{bmatrix}$$

$$C = \begin{bmatrix} AT & B \\ 1.5 & B \end{bmatrix}$$

Then use A' from problem 2.



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-260 \\ 1+360 \end{bmatrix} = \begin{bmatrix} 1 \\ 1-360 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-260 \\ 0-360 \end{bmatrix} = \begin{bmatrix} 32 \\ 32 \end{bmatrix}$$

$$a) \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

vectors
$$v_1$$
 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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c) $V_{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in R3 such that w + u ∈ Span(u, v) then w ∈ Span(u, v).

Vectors encompassed by a

True, edelarate left a Span are linear combinations

of the vectors in that Span, so the linear

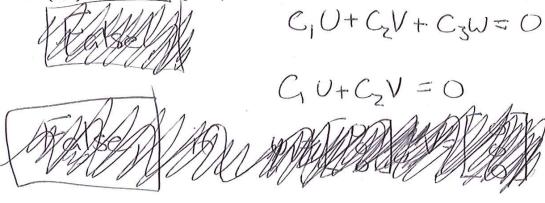
Combination of some vector and a vector in some

span can only be encompassed by that span if

the other vector is also a linear combination of

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

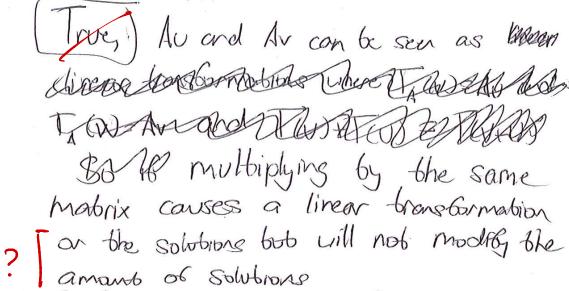
e violage in the spon.



True, because if {U,V} is not linearly dependent then {U,V,W} cannot be linearly independent as multiplying w by a constant of zero would yield all solutions of {U,V}



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

