

354.9 up
 $480 - 476 + 13$

1. Consider the following vectors in \mathbb{R}^4 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

$\frac{1}{9}$
 $-\frac{1}{9}$
 $-\frac{1}{18}$
 $\frac{3}{18}$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of \mathbb{R}^4 .

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V .

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V .

Using Gram-Schmidt Process we computed.

$$w_1 = v_1$$

$$w_2 = v_2 - \frac{w_1 \cdot v_2}{v_2 \cdot v_2} \cdot v_2$$

$$= \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{2+0+1+0}{1+0+1+1} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{3}{3} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

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$$w_3 = v_3 - \frac{w_2 \cdot v_3}{v_3 \cdot v_3} v_3 - \frac{w_1 \cdot v_3}{v_3 \cdot v_3} v_1$$

$$= \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{2+0+0-3}{4+4+1+9} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{2+0+0-3}{4+4+1+9} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} + \frac{1}{18} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{1}{18} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 19/9 \\ -19/9 \\ -9/8 \\ 19/6 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1/9 \\ -1/9 \\ -1/8 \\ 1/6 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

Rough.

$$4+4+1+9$$

$$2+0+1+3$$

$$\frac{5}{18} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 10/18 \\ -10/18 \\ -5/18 \\ 5/6 \end{bmatrix}$$