2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 1 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 2 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{array}\right]$$

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1+4 & 1-1+2 \\ 1-1+2 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-2+2 \\ 0+2+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$(A^TA)\hat{x} = (A^Tb)$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} x = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 7/3 & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -3/7 \\ 0 & 1 & 9/7 \end{array}\right]$$

$f(x) = -\frac{3}{7}x + \frac{9}{7}$ ✓

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20