2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

3×2 $\quad 2 \times 3$

$$A^TA = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$$

$A^TAx = A^Tb$ ✓

$$b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

2×3 $\quad 3 \times 1$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$ ✓

$$\begin{bmatrix} 2 & 0 & 3 & 0 \\ 0 & 2 & -1 & 2 \\ 3 & -1 & 5 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & -1 & -4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 1 \\ 0 & -1 & -4 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 1 \\ 0 & 0 & -9/2 & 2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 1 \\ 0 & 0 & -1 & 4/9 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & 0 & 7/9 \\ 0 & 0 & -1 & 4/9 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 7/9 \\ 0 & 0 & 1 & 4/9 \end{bmatrix}$$

$x_1 = 3/2$

$x_2 = 7/9$

$x_3 = 4/9$

$(3/2, 0)$ $(7/9, 2)$ $(-4/9, 1)$

$$a = \frac{1 - 0}{-\frac{8}{18} - \frac{36}{18}} = \frac{1}{-\frac{44}{18}}$$

$b = 0$

$f(x) = 0.\overline{4}x + 0$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20