

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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UB Person Number:

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Instructions:

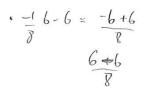
- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$



a) Find all values of b such that
$$w \in Span(v_1, v_2, v_3)$$
.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)
$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 6 \end{bmatrix} \Rightarrow R_3 - 2R_1 \Rightarrow \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 6 \end{bmatrix} \Rightarrow R_3 - 2R_1 \Rightarrow \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -2 & 6 + 4 \end{bmatrix} \Rightarrow R_1 - 5R_2 \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -2 & 6 + 4 \end{bmatrix} \Rightarrow R_3 - 2R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 2 & 2 \\ 3 & 2 & 2 & 2 \\ 2 & 0 & 0 & 1 & 6 \\ 8 & 8 & 8 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 3 & 2 & 2 & 2 \\ 4 & 2 & 2 & 8 \\ 8 & 3 & 3 & 2 \\ 8 & 3 & 3 & 3 \\ 8 & 3 &$$

6) The set { v, va, va} is linearly independent because there is a pivot Column in every column as shown in part A where we reduced the nontrix in row echelon form.

Correct conclusion, but it comes from our incorrect row roduction.

Again, this is correct given the reduced metrix you obtained, but the matrix itself is mong.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad \begin{array}{c} 6 - 2(-1) = 2 \\ 6 - 2(1) = -2 \end{array}$$

$$\begin{array}{c} \text{Compute } A^{-1}. \\ \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix} & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 \end{array} \rightarrow \begin{array}{c} R_1 + R_2 \\ 0 & 1 & -1 \\ 0 & 2 & -1 & 0 \end{array} \rightarrow \begin{array}{c} R_2 - R_1 \Rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 2 & -1 & 0 \end{array} \rightarrow \begin{array}{c} R_3 - 2R_2 \\ 0 & 0 & 1 \end{array} \rightarrow \begin{array}{c} 0 & 1 & 0 \\ 2 - 2 & 1 \end{array}$$

$$\Rightarrow \begin{array}{c} R_1 - R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} -2 & 3 & -1 \\ 2 & -2 & 1 \end{array}$$



3. (10 points) Let
$$A$$
 be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix
$$C$$
 such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$
 (calculated in problem 2)

Using the Fact that:
$$(AT)^{-1} = (A^{-1})^{T} \vee (A^{T})^{-1} \subset C$$

$$(AT)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \rightarrow (AT)^{-1}(AT)C = B(AT)^{-1}$$

$$C = B(AT)^{-1}$$

$$(A^{\mathsf{T}})^{-1}(A^{\mathsf{T}}) C = B(A^{\mathsf{T}})^{-1}$$

$$C = B(A^{\mathsf{T}})^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 1(-2) + 3(2) - 5 & 1 + 2 + 3 & 2 - 4 + 3 \\ 4(-2) + 5(3) - 4 & 4 - 5 + 4 & 4(2) - 5(2) + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 - 3 & -1 + 3 & -2 + 3 \\ -8 + 15 - 4 & -5 & 8 - 10 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -5 & 2 \\ -1 & 0 & 6 - 4 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -5 & 2 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3(-2)+2(3)-1, 3-2+1 \\ -6+6 \end{bmatrix}$$

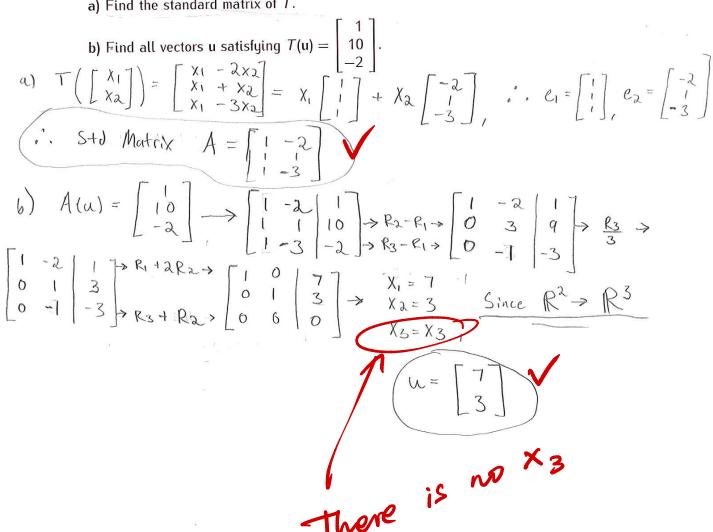
$$= \begin{bmatrix} -8 + 15 - 4, & -5, & 8 - 10 + 4 \\ -1, & 0, & 6 - 4 + 1 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.





(5.) (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} = \frac{1}{6}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} = \frac{1}{6}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \Rightarrow R_3 - 3R_1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \Rightarrow R_2 \cdot \frac{1}{5} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \Rightarrow R_3 - R_3 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Since A has a pivot position in every

column it is $1 + 0 + 1$

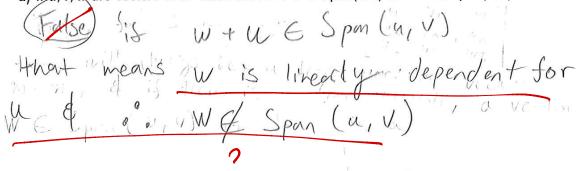
b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \Rightarrow R_3 - 3R_1 \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_2 \cdot \frac{1}{5} = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow R_3 - R_2 \Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1$

 v_1, v_2 ?



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.



b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

one False, {u, v, w} is only in inent independent if it has only solution to the homogenus equation X, u + Xav + X3w = 0

Counter Example: $\begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \rightarrow \begin{bmatrix} -w & 0 & | * \\ 0 & v_2 & | * \\ 0 & 0 & 0 \end{bmatrix}$



(... {u, v3 is necessify linearly independent



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

If Au, Av are linearly dependent that means the homogenes equation has infinite solutions, for both we are multiples of one another & i. not linearly independent in true.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True, transformations are true or operators

T(u+v) = T(u) + T(v)

