

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
Andrew	Jank	

UB Person Number:

5	0	4	3	8	3	1	S
0 1 2 3 4 6 7 8 9	① ① ② ③ ④ ⑤ ⑥ ⑦ ③ ⑨	(a) (b) (c) (c) (d) (d) (e) (e) (e) (e) (e) (e) (e) (e) (e) (e	0 1 2 3 4 5 6 7 8 9	○ ①○ ②○ ③○ ④○ ④○ ⑨	0 1 2 4 5 6 7 8 9		

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
		,						

	,							
							0	nan
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

26+4 has to equal 0 or no solution

16) NO since there is not apport column in every column when reduced also three is a free varioble w/ x3 00 infinite solutions and dependent.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} = \begin{bmatrix} 0 & 10 \\ 1 & -10 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \left[\begin{array}{rrr} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{array} \right]$$

Find a matrix C such that
$$A^TC = B$$
 (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

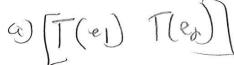
$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors \mathbf{u} satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.





$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1000 \\ 1 & 100 \\$$

$$x_1 = 7$$

$$x_2 = 3$$

$$y = 4$$

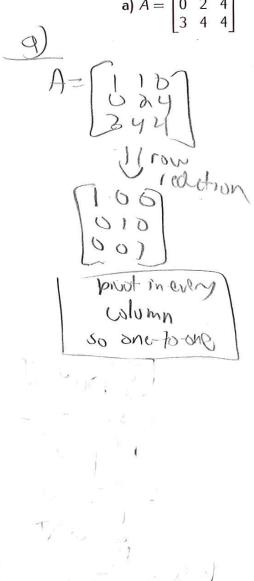
$$y = 7$$

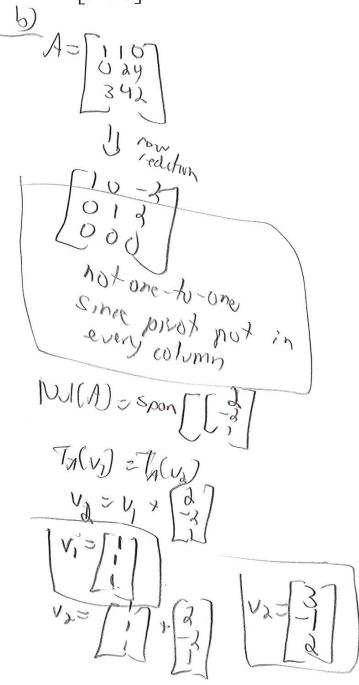


5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$







6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

False we []

w=[]

w=[]

w=[]

wru[]

wru[]

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

The only solution is zero vector gand his a protestion in every whom i Usu would be independent as well w/ priot-position in each column and only solution would be zero vector.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True Avand Av have infinite solutions
so a, and a will also like dependent as they
will be scalar multiples of each other.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

The it used linear combination of TW, Tow.

In hone work problems TWD=T(w)+T(w)+T(w)

Meening it is a linear combination.