7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

TRUE → IF A IS 2x2 MATRIX THEN NUL(A) IS SPAN OF SOME VECTORS IN $\mathbb{R}^2$.

NUL(A) = (SET OF SOLUTIONS OF A.V = 0)

→ LINEARLY INDEPENDENT

IF $Au \neq 0$, $AV \neq 0$ THEN u, v LINEARLY DEPENDENT.

$Au \neq 0$
$Av \neq 0$ FOR DEPENDENCE

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

TRUE

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True - Linearly dependent sets have infinetly many solutions

a set of two vectors is linearly independent if and only if one vector is a scalar multiple of the other.

Since $A(v+u) = Au + Av$, if Au and Av are linearly dependent u, v must also be linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True! $T(u+v) = T(u) + T(v)$ $\quad$ $u \in \text{Span}(v, w)$

$T(cv) = cT(v)$ $\quad$ $T(u) \in \text{span}(T(v), T(w))$?

Span holds through transformations

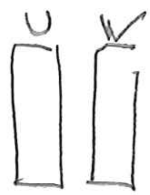
$T \cdot S(v) = (A \cdot B)v = A(B(v))$

$\text{col}(A) = \text{row}(B)$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.



True, if they have a unique solution, it ~~doesn't~~ matter whether they are ~~seperated or not~~ represented in a matrix or not

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True, $\mathrm{Span}(v_1, \dots v_p)$ = set of all linear combination $c_1 v_1 + c_2 v_2$.

thus it holds ~~every~~

than $T(u)$ must be in $\mathrm{Span}(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

true. A matrix is a linear transform, so anything that is linear dependent that is linearly transformed stays linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

true. linear transforms will not change is something is within a span

$u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \overset{u}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{array}{cc} A & u \\ \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 6 & 10 \end{array}\right] \end{array} \quad -2$$

$$(-2)\quad \begin{array}{cc} A & v \\ \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 2 & 6 & 7 \end{array}\right] \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 0 & 3 \end{array}\right]$$

FALSE,

just because Au, Av are linearly dependent does not mean that u, v will also be linearly dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True

Col(A) = set of values for T_A
so if u is in span(v, w) for matrix A then it will also be in the span of the transformation for A.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

false

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

true

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent. False

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$\{u\}, \{v\}$ are only linearly dependent if $u = \vec{0}$, $v = \vec{0}$

Matrix A can be linearly dependent even if u, v are not zero-vectors

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

True by linear combination

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\boxed{\mathbf{u}, \mathbf{v}}$ also must be linearly dependent.

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$Au = \left[\begin{array}{cc|c} 1 & 2 & u_1 \\ 3 & 4 & u_2 \end{array}\right]$

$Av = \left[\begin{array}{cc|c} 1 & 2 & V_1 \\ 3 & 4 & V_2 \end{array}\right]$

linearly independent

True because, Since Au & Av are linearly dependent, u & v must also be.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\boxed{\text{Span}(\mathbf{v}, \mathbf{w})}$ then $T(\mathbf{u})$ must be in $\boxed{\text{Span}(T(\mathbf{v}), T(\mathbf{w}))}$.

Come back.

$u \in \text{Span}(v, w)$

then

$T(u) \in \text{Span}(T(v), T(w))$?!

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $u = [\ ], v = [\ ]$

True

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

This is true

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True! If Au and Av are linearly dependent, then Au = cAv. Therefore u = cv, so u and v must be linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True. If u ∈ Span(v, w) then u = cv or u = cw. If your take T(u), you get A∘u = A∘cv, which shows T(u) ∈ Span(T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

$v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $T = \begin{bmatrix} 5 & 7 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{array}{c} A \\ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \end{array} \quad \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix}$$

$$\begin{array}{cc} A & u \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \end{bmatrix} \end{array} (L.I)$$

$$\begin{array}{cc} A & v \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \end{bmatrix} \end{array} (L.I)$$

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \to L\,I$$

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \to L\,I$$

$$\begin{array}{cc} u & v \end{array}$$
$$\begin{bmatrix} 3 & 2 \\ 4 & 2 \end{bmatrix}$$

False

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \to$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True, every matrix transformation is a linear transformation, making the statement true

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

FALSE

If $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

then $Au \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is linearly dependent ~~but u is linearly independent~~.

~~TRUE, the~~ but $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is linearly independent.

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

TRUE

$u = v + w$

$T(u) = T(v + w)$

$\hookrightarrow T(v) + T(w)$

So, $T(u)$ is in Span $(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

~~False.~~

True.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\boxed{\text{True}}$,

Putting linearly dependent vectors through transformation, they must stay dependent when put together with more dependents.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

True, the linear transformation must be one to one and onto, so the span would be kept it tact.

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent. T

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

T

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False. ~~Au~~ ~~$Au = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $Ab = [$~~

$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\because$ Au, Av is linearly dependent.

$u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\therefore$ u, v, is not dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

False.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$Au = T_A(u) \qquad Av = T_A(v)$

$x_1 T_A(u) + x_2 T_A(v) = 0$

$T_A(x_1 u + x_2 v) = 0$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True

$u \in \text{Span}(v, w)$

$u = x_1 V + x_2 W$

$T(u) = T(x_1 V + x_2 W)$

$T(u) = T(x_1 v) + T(x_2 w)$

$T(u) = x_1 T(v) + x_2 T(w)$

$T(u) \in \text{Span}(T(v), T(w))$

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐
0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\{Au, Av\}$ linearly dependent

False. Multiplying matrix A by vectors u and v does not necessarily preserve linear dependence.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True. If vector $u \in \text{span}(v, w)$ and transformation T is applied to u, v, w then $T(u) \in \text{span}(T(v), T(w))$.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, $a_1 v_1, \ldots, a_n v_n$ = non-zero therefore the vectors u,v. must also be linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, there can be any T(u) within the span of T(v) and T(w) because you can transform T(u) into either T(v, w).

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

false

$$\overset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \quad \overset{u}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \quad \overset{v}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$Au \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$Av \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

u and v are not linearly dependent

no scalar multiple of u could product v

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$$\overset{A}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \quad \overset{u}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \quad \overset{v}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \quad \overset{w}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

True

Yes because linear transformation has some algebra properties as Nector addiction with $u = x_1 v + x_2 w$

$T(u) = x_1 T(v) + x_2 T(w)$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

$A\bar{u} \neq A\bar{v}$

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent. False

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\bar{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $A\bar{u} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

$\bar{v} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$ $A\bar{v} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$). False. $T(\bar{u})$ may no longer be in Span$(T(\bar{v}), T(\bar{w}))$ after transformation

$\bar{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\bar{v} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\bar{w} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True because for what A you are given it'll be on the span of u,v assuming they are dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v})$, $T(\mathbf{w})$).

~~The~~ True because when ~~we are~~ we are doing linear transformati we are just multiplying vectors ~~with~~ which makes it a linear ~~trans~~ combination and a span is a set of liner combination.

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

If Au, Av are linearly dependent that means the homogenus equation has infinite solutions, for both u & v that means u or v are multiples of one another & ∴ not linearly independent

∴ true

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, transformations are linear operators

$T(u+v) = T(u) + T(v)$

∴ if u is in Span(v, w)

the T(u) is in Span(T(v), T, w)

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\boxed{\text{False}}$ ran out of time

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True, transformations are a linear operator so their original relations hold true to the outcomes of a transformation

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True

$C_1 v + C_2 w = u$

$C_1 T(v) + C_2 T(w) = T(u)$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$$

u & v need to be set to 0 and checked to see if a solution exists. i.e. $c_1, c_2, \ldots c_n$ all equal 0.

$Au = 0$

$Av = 0$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True

If the Span(v, w) contains u then the span of (T(v), T(w)) contains T(u)

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$A = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}, \quad u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad v = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Au and Av are LD

No matter what the vectors u, v in $\mathbb{R}^2$ are they will have a pivot column so statement is FALSE

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

TRUE because the properties of linear transformations tells us that if u is in the Span(v, w) then T(u) has to be in the Span(T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\cancel{\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}}$ $\quad u=\begin{bmatrix}1\\0\end{bmatrix}$ $v=\begin{bmatrix}-1\\0\end{bmatrix}$

$\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$

True because if you have inf solution and then transform it is still the same.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$. ~~false~~

~~False~~ $\quad u=\begin{bmatrix}2\\2\end{bmatrix}$ Span$\left(\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}5\\5\end{bmatrix}\right)$

True

$\begin{bmatrix}2\\2\end{bmatrix}\begin{bmatrix}2\end{bmatrix}=\begin{bmatrix}4\\4\end{bmatrix}$ $\left(\begin{bmatrix}2\\2\end{bmatrix}\begin{bmatrix}10\\10\end{bmatrix}\right)$

2×1 1×1

2×1 $\qquad$ 2+2

True because of the example of a linear transformation of [2]

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then

$Au = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $Av = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\rightarrow$ $Au = Av$, so Au, Av are linearly dependent

but u and v are linearly Independent.

Pf: $[u \ v \mid 0]$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 2 & 0 \end{array}\right] \xrightarrow{\text{Row reduction}} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$

False

So $x_1u + x_2v = 0$ has only one solution $x_1 = x_2 = 0$ $\Leftrightarrow$ u, v are linearly Independent. ⌟

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

$u = c_1 v + c_2 w$

$T(u) = T(c_1 v + c_2 w)$

$= T(c_1 v) + T(c_2 w)$

$= c_1 \cdot T(v) + c_2 \cdot T(w)$ is a linear combination of $T(v), T(w)$

$\therefore$ $T(u) \in \mathrm{Span}(T(v), T(w))$ True

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True.

Au, Av indicates that after multiplication there was a free variable causing ∞ solutions, that makes Au, Av linearly depended

This means that the vectors u and v must have also had free variables when multiplied by some x value.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

False, if the transformation changes u, v, w to where u no longer Span(v, w) that would make T(u) not Span(T(v), T(w))

$\overset{u}{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}$ $u_1 + 3u_2$ span (u) but $T(u) = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$ then it my not span for two other vectors if it is not a scaler multiple

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

This is False. For example, let $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Then $Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Au and Av are the same vector, so they are clearly linearly dependent, however, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are linearly independent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

This is true. If $u \in \text{Span}(v, w)$, then $u = c_1 v + c_2 w$ for some $c_1, c_2 \in \mathbb{R}$. So $T(u) = T(c_1 v + c_2 w)$. In order to show that $T(u) \in \text{Span}(T(v), T(w))$, we must show that $T(u) = c_3 T(v) + c_4 T(w)$ for some $c_3, c_4 \in \mathbb{R}$. Let $c_1 = c_3$ and $c_2 = c_4$. By def. of linear transformation, $c_1 T(v) + c_2 T(w) = T(c_1 v) + T(c_2 w)$. Again, by def. of linear transformation, $T(c_1 v) + T(c_2 w) = T(c_1 v + c_2 w)$. Since we already had that $T(u) = T(c_1 v + c_2 w)$, then $T(u) \in \text{Span}(T(v), T(w))$.

QED.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True set of vectors have to be linearly depentent for Au, Av to be linearly dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$C_1V + C_2W = C_3U$

Since linear Trans

$T(C_1V) + T(C_2W) = T(U) \Rightarrow C_1 T(V) + C_2 T(W) = T(U)$

Thus $T(u) \in \text{Span}(T(V), T(W))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

This statement is false

$$\left|\begin{matrix} 1 & 1 & 2 \\ 0 & 2 & 4 \end{matrix}\right| \qquad \left|\begin{matrix} 1 & 1 & 2 \\ 0 & 2 & 9 \end{matrix}\right|$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

T(u) must be in Span(T(v), T(w)), be

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

$\therefore Au = A[u] + A[v] = x_1 u + x_2 v$

can be linearly dependent if u, v & x_1 & $x_2 \neq 0$.

~~Also $c_1 u + c_2 v$ can also be~~

$\therefore c_1 u + c_2 v$ need not be linearly dependant because.

$u, v = 0$.

$\therefore c_1 u + c_2 v$ becomes linearly independent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True.

$T(v) = T(v + w)$

$= T(u) + T(w)$.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

Lin Dependent: Au, Av have ∞ solutions to $Au = 0$, $Av = 0$

$[A \mid 0] \rightarrow \infty$ solutions $\therefore$ u and v aren't necessarily linearly dependent because when any matrix is multiplied by A, the solution is a linearly dependent matrix $\therefore$ u, v are not always lin. dependent.

$\boxed{\text{False}}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$x_1 v + x_2 w = u \rightarrow (u \in \text{Span}(v, w))$

$\downarrow$

$T(x_1 v) + T(x_2 w) = T(u)$

$x_1 T(v) + x_2 T(w) = T(u) \rightarrow (T(u) \in \text{Span}(T(v), T(w)))$

$\boxed{\text{True}}$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

~~false~~ false, u, v can be linearly independent before being multiplied by A.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, T(u) is in span (T(v), T(w)) if u is in span(v, w)

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True. For Au and Av to be Linearly dependent, u and v must have infinitely many solutions. Some combination of u & v other than the trivial solution will result in the 0 vector.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True. If u is in Span(v,w), then it is a linear combination of v & w. Therefore it's transformation must be in the span as well.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, Au and Av have infinite solutions so u, and v will also be dependent as they will be scaler multiples of eachother.

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True if u is a linear combination of v, w. Tu is a linear combination of T(v), T(w). In homework problems $T(v) = T(w_1) + T(w_2) + T(w_3)$ meaning it is a linear combination.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False; the matrix A can cause Au, Av to become lin. dependent even if u,v are lin. independent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True; let $u=\begin{bmatrix}6\\3\end{bmatrix}$ and $v=\begin{bmatrix}2\\1\end{bmatrix}$ and $w=\begin{bmatrix}1\\2\end{bmatrix}$

$A=\begin{bmatrix}1&1\\0&1\end{bmatrix}$

$T(u)=\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}6\\3\end{bmatrix}=\begin{bmatrix}9\\3\end{bmatrix}$

$T(v)=\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$

$T(w)=\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$

T(u) is in Span(T(v), T(w))

So, T(u) must be in Span(T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True

If Au and Av are linearly dependent, then they have infinitely many solutions. Therefore u and V would have to also have infinitely many solutions

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True

Since u is in the span(v, w) taking the linear transformation of each vector is just taking the product of each vector with A. Therefore T(u) would still be in Span(T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, because Au & Av is just a linear transformation so it will still retain the properties of having infinitely many solutions to the vector.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v})$, $T(\mathbf{w})$).

True, because ~~all~~ you are applying the same ~~a~~transformation to ~~both~~ all 3 vectors so the span will stay the same as before meaning T(u) is in Span((T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\cancel{\begin{matrix}1 & 0\\ 0 & 1\end{matrix} \cdot \begin{matrix}3\\ 4\end{matrix} = \begin{matrix}3\\ 4\end{matrix}}$$

False

linear dept

$$\begin{matrix}-1 & 1\\ 1 & -1\end{matrix} \cdot \begin{matrix}-4\\ -4\end{matrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \Leftarrow \text{linear dependent}$$

But $\begin{bmatrix}-4\\ -4\end{bmatrix}$ = is not linear dep b/c $u \neq 0$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

$$\{T(v_1), T(v_2)\} \neq T(v_1)\ T(v_2)$$

$$\begin{matrix}4\\ 2\end{matrix} \cdot \begin{bmatrix}2\\ 1\end{bmatrix} = \begin{matrix}8\\ 2\end{matrix}$$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

$A\cancel{(u-v)=0,\quad u=v}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False.

Transformation doesnt guarantee being in the Span..

~~If $T(v)x_1 + T(w)x_2 = 0$, doesn't have~~ sol'n,

~~they~~

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True

$u = c_1 v + c_2 w$

$T(u) = T(c_1 v + c_2 w)$

$\because$ T is Linear transformation

$\therefore T(c_1 v) + T(c_2 w) = T(c_1 v + c_2 w)$

$\therefore T(u) = T(c_1 v_1) + T(c_2 w_1)$

$\therefore T(u) \in \text{span}(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$u = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad v = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad A = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$

$A \cdot u = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \quad A \cdot v = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

False

True, because if $u = c_1 v + c_2 w$ then $T(u) = c_1 T(v) + c_2 T(w)$ because the transfomation is linear

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

True because Au, Av are a linear combination of A.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True because of the matrix property $c \cdot T_1 = c(T)$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Au, AV are dependent

but U, V are independent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

$U = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $W = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(U) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $T(V) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ $T(W) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$U \in \text{Span}(V, W)$

$T(U) \notin \text{Span}(T(V), T(W))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

0 1 2 3 4 5 6 7 8 9 10

~~$A\begin{bmatrix}-3\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}-4+2\\-6+4\end{bmatrix}=\begin{bmatrix}-2\\-2\end{bmatrix}$~~

~~$A\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1+0\\2+0\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$~~ } dependent

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u},\mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u},\mathbf{v}$ also must be linearly dependent.

$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\4\end{bmatrix}$

$A=\left[\begin{array}{cc|c}1&2&0\\2&4&0\end{array}\right]$ $R_2\to -2R_1+R_2$

$\begin{bmatrix}1&2&0\\0&0&0\end{bmatrix}$

~~$v=\begin{bmatrix}1\\1\end{bmatrix}$ $u=\begin{bmatrix}2\\2\end{bmatrix}$~~

~~$A\cdot\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}-2+2\\-4+4\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$~~

~~$A\circ\begin{bmatrix}-4\\2\end{bmatrix}=\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-4\\2\end{bmatrix}=\begin{bmatrix}-4+4\\-8+8\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$~~

$x_1=-2x_2$

$x_2=x_2$

$\text{Nul}(A)=\text{Span}\left\{\begin{bmatrix}-2\\1\end{bmatrix},\begin{bmatrix}0\\0\end{bmatrix}\right\}$

2 examples $\begin{bmatrix}-3\\2\end{bmatrix}$, [

$A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$

False $v=\begin{bmatrix}1\\0\end{bmatrix}$ $u=\begin{bmatrix}0\\1\end{bmatrix}$

A(v), A(u) are linearly dependent, but v, u is linearly independent

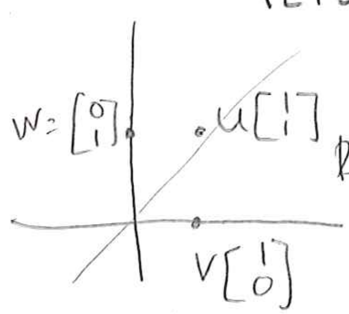
b) If $T:\mathbb{R}^2\to\mathbb{R}^2$ is a linear transformation and $\mathbf{u},\mathbf{v},\mathbf{w}\in\mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\text{Span}(\mathbf{v},\mathbf{w})$ then $T(\mathbf{u})$ must be in $\text{Span}(T(\mathbf{v}),T(\mathbf{w}))$.

$u=\begin{bmatrix}1\\1\end{bmatrix}$ $v=\begin{bmatrix}1\\0\end{bmatrix}$ $w=\begin{bmatrix}0\\1\end{bmatrix}$ $u\in\text{Span}(v,w)$

$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=\begin{bmatrix}-5\\-5\end{bmatrix}$ $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)\to\begin{bmatrix}-5\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)\to\begin{bmatrix}0\\-5\end{bmatrix}$

True because with any reflection, T(u) will be a linear combination of T(v) and T(w) if u is in Span(v, w)



Reflect over x-axis	reflect over line y=x	reflect over y-axis
$T(u)=\begin{bmatrix}1\\-1\end{bmatrix}$	$T(u)=\begin{bmatrix}1\\1\end{bmatrix}$	$T(u)=\begin{bmatrix}-1\\1\end{bmatrix}$
$T(w)=\begin{bmatrix}0\\-1\end{bmatrix}$	$T(w)=\begin{bmatrix}1\\0\end{bmatrix}$	$T(w)=\begin{bmatrix}0\\1\end{bmatrix}$
$T(v)=\begin{bmatrix}1\\0\end{bmatrix}$	$T(v)=\begin{bmatrix}0\\1\end{bmatrix}$	$T(v)=\begin{bmatrix}-1\\0\end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

true because $T(cv) = cT(v)$

if $u = cv$ then $T(u) = cT(v)$

$T(u) = T(cv)$

$T(u) = T(u)$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True

if u is a linear combo of v & w

$u = v + w$

then $T(u) = T(v+w)$

$T(u) = T(v) + T(w)$

$T(u)$ is a linear combo of

$T(v)$ & $T(w)$

therefore

$T(u) \in \text{span}(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

Counter example:

$\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A\vec{u} = T(\vec{u}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A\vec{v} \quad T(\vec{v}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A\vec{u} = A\vec{v}$

$T(\vec{u}) = T(\vec{v})$

$T(\vec{u}) - T(\vec{v}) = 0$

$T(\vec{u} - \vec{v}) = 0$

$\vec{u}, \vec{v} \in Nul(A)$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

$\vec{u} \in Span(\vec{v}, \vec{w}) \Rightarrow T(\vec{u}) \in Span(T(\vec{v}), T(\vec{w}))$

$\vec{u} = c_1\vec{v} + c_2\vec{w}$

$T(\vec{u}) = T(c_1\vec{v} + c_2\vec{w})$

$= T(c_1\vec{v}) + T(c_2\vec{w})$

$= c_1 T(\vec{v}) + c_2 T(\vec{w}) \Rightarrow T(\vec{u}) \in Span(T(\vec{v}), T(\vec{w}))$

$\therefore$ True.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $Au = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$ $Av = \begin{bmatrix} 10 \\ 22 \end{bmatrix}$

2×1 2×1

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ 2×2 $v = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $Au = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ $Av = \begin{bmatrix} 13 \\ 29 \end{bmatrix}$

$Au = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $Av = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$

True

If Au and Av are linearly dependent the u, v must be linearly dependent since the scalar multiple multiplied to either Au or Av to get the other is the same scalar multiple to get c·u to v or c·v to u

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True since T(u) the u is a vector $\mathbb{R}^2$ and v, w is a vector in $\mathbb{R}^2$ ∴ T(u) must be in the span of T(v), T(w).

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\boxed{\text{False}}$ Counterexample: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $u = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $v = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

Au & Av both $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ which is linearly dependent

but u and v are not linearly dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$\boxed{\text{True}}$ If u is in Span(v, w) it is a combination of $cv + dw$ where c and d are some constants

So $T(u) = T(cv + dw) = T(cv) + T(dw) = cT(v) + dT(w)$

Which means that $T(u)$ is a combination of constant values of $T(v)$ & $T(w)$ which is what it means to be in Span$(T(v), T(w))$.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

False A single vector such as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is linearly independent. If $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ then it is linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True. All 3 vectors are transformed by the same transformation, so properties are not changed. -They are still scalar multiples.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\boxed{\text{True,}}$ Au and Av can be seen as ~~linear~~ ~~linear transformations where $T_A(u) = Au$ and $T_A(v) = Av$ and [illegible]~~ ~~So if~~ multiplying by the same matrix causes a linear transformation on the solutions but will not modify the amount of solutions

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$\boxed{\text{True,}}$ because ~~linear transformations ensure that the transformation of the sum of 2 vectors~~

$$\text{Span}(V, W) = C_1 V + C_2 W$$

if: $U = C_1 V + C_2 W$

then:

$$T(U) = T(C_1 V + C_2 W) =$$

$$\checkmark \quad T(U) = T(C_1 V) + T(C_2 W)$$

$$\text{Span}(T(V), T(W)) = C_1 T(V) + C_2 T(W) = T(C_1 V) + T(C_2 W)$$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A_u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $A_v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$2u_1 + u_2 = 1$ $2u_1 + u_2 = 2$

$u_1 + u_2 = 1$ $u_1 + u_2 = 2$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

True; since all matrix transformations are linear transformations, the matrix transformation T_A preserves linear dependence between $\{u, v\}$, and $\{Au, Av\}$.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

True: since all matrix transformations are linear transformations, applying the same transformation to all three vectors preserves the column space and $T(u)$'s existence in it.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\boxed{\text{FALSE}}$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$Au = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$Av = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

then $Av = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $Aw = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Since these two vectors are NOT scalar multiples of each other this statement is false.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

$\boxed{\text{TRUE}}$ If U is in span of V, W this means

$c_1 V + c_2 W = U$

A linear transformation is applied to both sides $T_A(c_1 V + c_2 W) = T_A(u)$

$c_1 T_A(V) + c_2 T_A(W) = T_A(u)$

Any linear combination of $T(V)$ and $T(W)$ will get you $T_A(u)$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, if Au, Av are linearly dependent, then

u, v are always zero vectors, which are linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, linear tranrformation is in span of other

trans. if the vector of lin. trans is in span of

other vectors.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u},\mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u},\mathbf{v}$ also must be linearly dependent.

false

$$\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}=\begin{bmatrix}2\\2\end{bmatrix}$$

$$\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

$u=\begin{bmatrix}2\\0\end{bmatrix}$ & $v=\begin{bmatrix}0\\1\end{bmatrix}$ are linearly independent

$Au=\begin{bmatrix}2\\2\end{bmatrix}$ & $Av=\begin{bmatrix}1\\1\end{bmatrix}$ are linearly dependent

b) If $T:\mathbb{R}^2\to\mathbb{R}^2$ is a linear transformation and $\mathbf{u},\mathbf{v},\mathbf{w}\in\mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v},\mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}),T(\mathbf{w}))$.

true

If u is in the Span(v, w), then it can be found through a combination of multiples of v and w, $c_1v+c_2w=u$

For linear transformations, $T(u)=T(c_1v+c_2w)=T(c_1v)+T(c_2w)=c_1T(v)+c_2T(w)$

This means that the transformation of u can be solved as a linear combination of the multiples of $T(v)$ and $T(w)$.

This means that $T(u)$ must be in the Span$(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True,

since every matrix transformation is a linear transformation, then both of the resulting vectors were linearly transformed. Since they were linearly dependent before transformation, then transforming the line they form does not break their linear dependence since linear transformations keep straight lines intact.

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True,

if the whole span is linearly transformed, then $T(u)$ will be transformed in the same way, preserving its linearity and remaining on the plane of the span.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \underbrace{u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{linearly independent}} \qquad \underbrace{\left\{ Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}}_{\text{linearly dependent}}$$

False.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

False

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$

$[\ \ ]\cdot\begin{bmatrix}1\\0\end{bmatrix}\cdot = [$

$[\ \ ]\cdot\begin{bmatrix}0\\1\end{bmatrix}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True.

$u \in \mathrm{Span}(v, w) \qquad u = c_1 v + c_2 w$

$T(u) = Au$

$T(v) = Av$

$T(w) = Aw$

$Au = c_1 Av + c_2 Aw.$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, since multipying by the same matrix give linear dependance

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True since every matrix transformation is a linear transformation

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\Downarrow$

u and v are linear combinations of eachother

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \overset{Au}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \Rightarrow u_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \overset{Au}{\begin{bmatrix} 2 \\ 4 \end{bmatrix}}$$

$R2 \times {}^{-1}/_2 + R1$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}\begin{matrix} /2 \\ /2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 4 \end{bmatrix}\begin{matrix} /2 \\ /2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$\rhd$ true, solutions are still linearly dependent

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

$$T\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = T\left(c_1\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + c_2\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right)$$

$\rhd$ $T(u) = c_1 T(v) + c_2 T(w)$

$\downarrow$

$$T\left(c_1\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + c_2\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right) = T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) + T\left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}\right)$$

$\Downarrow$

$\boxed{\text{true}}$

For linear transformations, $T(u+v)$ can be written as $T(u) + T(v)$, and here, because u is in the span of v and w, its transformation can be witten as a combination of the transformations for v and w

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

$n = m = 2$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\boxed{\text{False}}$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ Au and Av are dependent

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ $\begin{bmatrix} -1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are independent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$\boxed{\text{True}}$ a transform will only move a vector, not change it so if u is in the span it will also be in the transform span

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

Ture

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot U + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} V$

$X_1 U_1 + X_2 V_2$ are linearly dependent

$U_1 V_2$ are also linear dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

Ture

$T(v) = T(w) = T(u)$

$AV = AW = AU$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}, A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

this may simply mean that the matrix A is linearly dependent not necessarily u or v

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

~~True because~~

True because if the same transformation is performed on some u that's in the span of some v and w, u will be moved into that new span of T(v), T(w) by the transformation T(u).

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

Au will be 2×1 Av will be 2×1.

so Au Av will be 2×2.

only infinate number or no solution will be dependent

$A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $Av = \begin{bmatrix} 11 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 11 & 18 \\ 0 & -5 \end{bmatrix}$ dependent.

$u = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $Au = \begin{bmatrix} 18 \\ -5 \end{bmatrix}$

$u, v = \begin{bmatrix} 3 & 2 \\ 5 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \to$ independ

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True.

u can be represent by v and w.

$u = av + bw$ $\quad T(u) = Au = A(av + bw)$

$= aAv + bAw$

$= aT(v) + bT(w)$

so Tu must be in Span $(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False; Linear dependence/independence is only guaranteed to be preserved if A is a matrix which defines a linear transformation. Since this condition is not specified, then linear dependence cannot be guaranteed after transformation.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True; Since T is a linear operator whose operation preserves the dimensions of the original vectors, then any vector $u \in \text{Span}(v, w)$ must be in the span of $T(v), V(w)$. Additionally, since there are 3 vectors in 2 space, and it is known that $u \in \text{Span}(v, w)$, v and w either are linearly dependent on one another, and u, or are linearly independent. Since T is a linear transformation, these properties are maintained, meaning $T(u) \in \text{Span}\, T(v), T(w)$ by definition (3 vectors in 2 space, 2 are linearly independent; v and w) or because all 3 vectors were linearly dependent to begin with.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A = \begin{bmatrix} \# & \# \\ \# & \# \end{bmatrix}$

true, same transformation done on u and v

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

if transformation is 2×2 matrix it could take u out of span of v and w

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \quad V_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad V_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$AV_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad AV_2 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

V_1, V_2 are independent

AV_1, AV_2 are dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

$u = a_1 V + a_2 w$ True

$\cancel{T(u) = a_1 T(v) + a_2 T(w)}$

u, $a_1 V + a_2 w$

are equivolent

vectors

$\therefore$

Their transformations

are the same

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True Because A Stay the same So inorder to keep Au, Av dependent u V must be dependent

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True if T is a traslation done to all Vectors then the vectors will change in the Same magnitude

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True because

$A(u+v) = Au + Av$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False, $T(u)$ could not be in Span$(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False →

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

$u \in \text{Span}(v, w)$

$T(u) \in \text{Span}(T(v), T(w))$

True because the transformation is applied for each vector, providing the same vectors at different positions

0 1 2 3 4 5 6 7 8 9 10

7. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False,

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

u and v could still be independent and be multiplied with A to make it linearly dependant.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

Yes, true because if u is in the span of (v, w) then the transformation could just be the scalar of u vector and it would still let T(u) be in the span of T(v), T(w)

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False:

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

True: if u is in the span (v, w) then a combination $c_1V + c_2W = u$, so $T(c_1V + c_2W) = T(u)$ as $T(c_1V + c_2W)$ can be split into $c_1T(v) + c_2T(w) = T(u)$ which means that there is a linear combination $T(v)$ and $T(w)$ that equal $T(u)$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Linearly independent

It is false. You could have a linear independent vectors and by multiplying it by the matrix, end up ~~w/ a~~ linearly dependent.

$Au = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$Av = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

It is false. T(u) means A(u) which means multiplying by a matrix that could lead to no linear combinations for T(u).

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

FALSE.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Au, Av are linearly dependent
- u, v are not linearly dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

$$u = c_1 v + c_2 w$$

By linearity: $T(u) = T(c_1 v + c_2 w) = T(c_1 v) + T(c_2 w)$

$$= c_1 T(v) + c_2 T(w)$$

$T(u)$ is a constant multiple of $T(v)$ and $T(w)$

$\therefore T(u) \in \mathrm{Span}(T(v), T(w))$

TRUE

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False. Au, Av are linearly dependent which means they have free variables, but u, v can be having one solution or no solution.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True.

~~False.~~ [illegible] $[v, w \mid u]$ has a solution

$T(u) = \cancel{T(u)}\ Au$ $\quad [T(v), T(w) \mid T(u)]$

$T(v) = \cancel{T(v)}\ Av$ $\quad [Av, Aw \mid Au]$ has a solution.

$T(w) = Aw$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$(c_1 A u + c_2 A v) = 0$

$\downarrow$

$A(c_1 u) + c_2$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

Yes. Since u is in Span (v, w),

then $u = c_1 v_2 + c_2 w$ has solution.

Also, $T(u) = T(c_1 v + c_2 w)$ has solution

Also, $T(u) = T(c_1 v) + T(c_2 w)$ has solution.

This means that $T(u)$ is in Span $(T(v), T(w))$.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

~~$u = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ v~~

~~$u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ then $A_u = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ $A_v = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$~~

$u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

~~but u & v are Not linearly indep. dep.~~

~~False~~

True, thought it was false but couldn't disprove

so $A_u = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $A_v = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

but ~~the~~ u and v

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

$u = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

so $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$ is valid

but

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

is Not

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

TRUE

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Because if Au and AV yield the linearly dependent result, then u v are also dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span(T(v), T(w)).

~~True. Because applying the~~

FALSE

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

false $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Not linearly dependent

$Au = \begin{bmatrix} 1+0 \\ 1+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$Av = \begin{bmatrix} 0+1 \\ 0+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Linearly dependent

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v})$, $T(\mathbf{w})$).

True $c_1 v + c_2 w = u$

$cT(v) + cT(w) = T(u)$

$c(Av) + c(Aw) = T(u)$

$A(cv) + A(cw) = T(u)$

$T_A(cv) + T_A(cw) = T(u)$

$T(cv) + T(cw) = T(u)$; $c_1 v + c_2 w = u$

0 1 2 3 4 5 6 7 8 9 10

7. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

true

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

true, If all the vectors undergo a linear transformation then they will still shared the relationship of being linear combinations.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False,

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True,

0 1 2 3 4 5 6 7 8 9 10