

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name: William Hitz UB Person Number:	Instructions:
5 0 1 9 3 1 7 7 0 0 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 6 6 6 6 7 7 7 7 7 7 7 7 7	 Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes. For full credit solve each problem fully, showing all relevant work.
1 2 3 4 5	6 7 TOTAL GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.



(10.60)

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

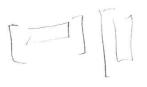
$$A = \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & | & 0 \\ 0 & 2 & -1 & | & 0 & 0 & | \end{bmatrix} \xrightarrow{R_2 + (-1)R_1} \begin{bmatrix} 1 & -1 & 2 & | & 0 & 0 \\ 0 & 1 & -1 & -1 & | & 0 \\ 0 & 2 & -1 & 0 & 0 & | \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \left[\begin{array}{rrr} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{array} \right]$$



Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 1 & 1 & C \\ -1 & C & Z \\ 2 & 1 & -1 \end{bmatrix}.$$

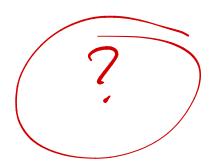


4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of \mathcal{T} .
- b) Find all vectors u satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x^2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 42 \end{bmatrix} \xrightarrow{R_3 + (-3)R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{R_3 + (3)R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 + (3)R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c} \chi_1 + \chi_2 = \zeta \\ 2\chi_2 = \zeta \\ \chi_1 = -2 \end{array}$$

$$\begin{array}{c} \chi_1 = -2 \\ \chi_1 = -2 \end{array}$$

$$\begin{array}{c} \chi_1 = -2 \\ \chi_2 = -2 \end{array}$$

- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$, then $w \in \text{Span}(u, v)$.

$$U = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad V = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad W = \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}$$

$$w+u=c_1u+c_2v$$

 $w\neq d_1u+d_2v$

Whu =
$$\begin{bmatrix} 2+1 \\ 2+1 \\ 0 \end{vmatrix} = \begin{bmatrix} 3 \\ 3 \\ c \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \sqrt{C_1 = 3}$$

$$C_2 = 0$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \pm d_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + d_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 2 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

Fight, U, V, w con all by Knowly Inearly independent but If you form the matrix of vectors, Nulla 7 (0)



- **7.** (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

There, a, VI, ..., 9, Vn = non-zero therefore the vectors U, v. must also be linearly dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

? The, there can be any T(u) within the span of T(v) and T(w) becase you can trons form T(u) into either T(v, w),