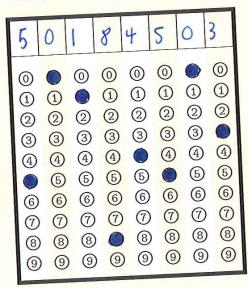


## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:	E CONTRACTOR			
Brandon	Hosken	-	, , , , , , ,	

## **UB Person Number:**



## Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
   You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
								,



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1,v_2,v_3\}$  linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & | & -2 \\ 0 & 1 & 2 & | & 2 \\ 2 & 3 & 0 & | & 6 \end{bmatrix} \xrightarrow{2} \xrightarrow{2}$$

[0-1-2 | 1-2 | 5 | 5+6

4=0

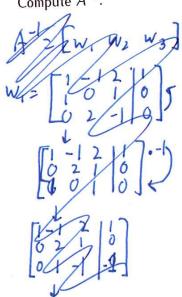
- 4) The only value of b that allows w to bein Spon (v, v2, v3) is 0 6 beauty if it isn't zero there is no solution to the augmented mutax after row reduction.
- b) The Set EVIJV2, V33 is not linearly independent become x3 is a from V voriable meaning of has infinitely many solutions.



## 2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .



$$\begin{bmatrix} 1 & -1 & 2 & | 100 & | \\ 0 & 2 & -1 & | 001 & | \\ 0 & 1 & -1 & | 410 & | \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & | 100 \\ 0 & 2 & -1 & | 000 \\ 0 & 0 & | 010 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & | 100 \\ 0 & 0 & | 010 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & | 100 \\ 0 & 0 & | 010 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & | 1 & -1 & | \\ 0 & 0 & | & 010 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & | & -1 & | \\ 0 & 0 & | & 010 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 13 & -1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$



 $\beta V_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2 + 6 + -3 \\ 9 + 15 + -8 \\ 6 + 6 + -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ 

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix 
$$C$$
 such that  $A^TC = B$  (where  $A^T$  is the transpose of  $A$ ).

$$A^T = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} A^T \\ A^T \end{bmatrix} \quad B$$

Simpler: 
$$(A^T)^{-1} = (A^{-1})^T$$
  
Thou use problem 2.



4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

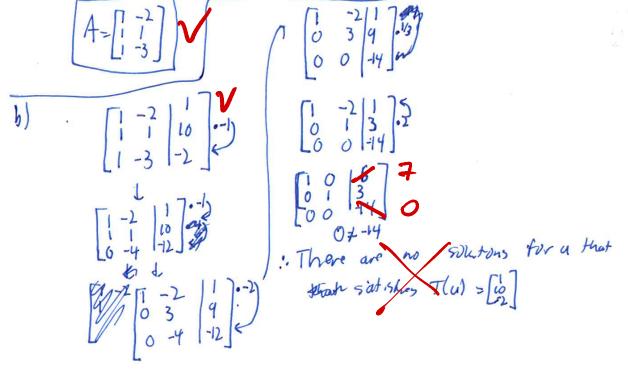
- a) Find the standard matrix of  $\mathcal{T}$ .
- b) Find all vectors u satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a) 
$$A=[T(Q_1) \uparrow (Q_2)]$$
  $Q_1=[0]$   $Q_2=[0]$ 

$$T(Q_1) = T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 & 2(0) \\ 1 & 3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$T(Q_1) = T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 0 & 2(0) \\ 1 & 3(0) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

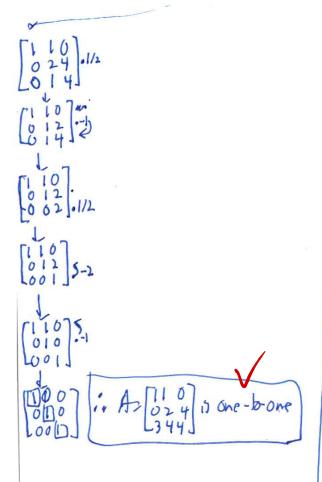
$$T(\ell_2):T(\begin{bmatrix}0\\1\end{bmatrix})=\begin{bmatrix}0-20\\0-30\end{bmatrix}=\begin{bmatrix}-2\\1\\-3\end{bmatrix}$$





5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$



b) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 & 4 \\ 0 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 12 \end{bmatrix}$$

V1 2 V2



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, because why is some linear combination of a that regults in the spon, the new vector being in the spon, for that is occur by the definition of a linear combination we must also bein the spon(i, v).

b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.

The Type, because the now voductor of the vedors the augmented which require nx n to be linearly independent in un augmented with respondent to the only 1 solution,



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent

then u, v also must be linearly dependent.

They because Auts just a linear trong briefly of horny

So it will still robin the properties of horny

Inhaltely wany solutions to the value,

b) If  $T:\mathbb{R}^2\to\mathbb{R}^2$  is a linear transformation and  $u,v,w\in\mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True, because the you are applying the some estrons for notion to thather all 3 vectors so the span will s by the some us before meany T(u) is in Span (LTW), T(w))