6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

TRUE → IF U, V, W ARE VECTORS IN $\mathbb{R}^3$

THEN SPAN (U V W) = { SET OF ALL LINEAR COMBINATIONS }

?

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

FALSE → ONLY IF $u x_1 + v x_2 = 0$ $\{u, v\}$ LINEARLY INDEPENDENT

$u \cdot x_1 + v \cdot x_2 + w \cdot x_3 = 0$ → LINEARLY INDEPENDENT

THIS EQUATION DOES NOT GURANTEED $u x_1 + v x_2 = 0$

?

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False

$w+u \in \text{span}(u,v)$ $\quad A(v+w) = Av+Aw$

$w \in \text{Span}(u,v)$ $\quad w+u \in \text{Span}(u,v) \not\Rightarrow w \in \text{Span}(u,v)$

$w = cu + cv$

$w+u \in \text{Span}(u,v)$ $\quad$ Counter example

$w+u = cu+cv$ $\quad w \in \text{Span}(u,v) + u \notin \text{Span}(u,v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True $\{u,v,w\}$ is linearly independent, i.e. every column is a pivot column and there is only one solution

$\{u,v\}$ must also be linearly independent because without w there will still be a pivot column in every column

ex) $\{u,v,w\} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{pivot columns}}$ $\quad \{u,v\} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ & \end{bmatrix}}_{\text{pivot columns}}$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

It is ~~true~~ false. No matter how much you ~~can~~ change it the rule only applies for multiplication

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False. linearly independant means: that there ~~is only one~~ must be infinite solutions. We do not know which vector has a free variable. so we dont know 100 percent of the time.

Exampe
Free variable
u, v, w

u and v
can be dependant

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True. lin. combinations of vectors in a span are still within a span and vice versa.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. In the set of 3, they are all lin. ind. from each other, meaning if one vector was taken away, the remaining two would still be lin. ind.

0 1 2 3 4 5 6 7 8 9 10

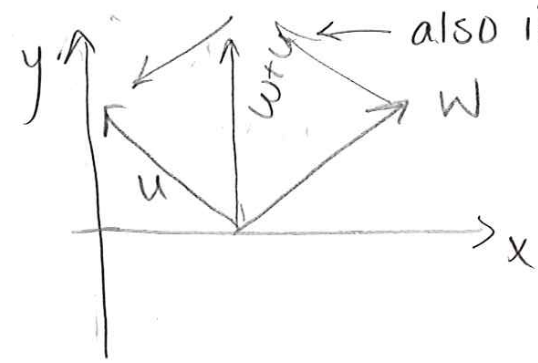
6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True:

if $x = w$, then $X_0 = w + u$

graphical interpretation



also in the span.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

$$A = \begin{array}{c} \begin{array}{ccc} u & v & w \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \end{array}$$

is linear independent

[after row reduction each value of u,v,w corresponds to a unique solution]

but

$$A = \left[\begin{array}{cc|c} 1 & 1 & 9 \\ 0 & 2 & 3 \\ 3 & 4 & 5 \end{array}\right]$$

is inconsistent after row reduction because last row is

$$\left[\begin{array}{cc|c} x & y & z \\ a & b & c \\ \boxed{0} & \boxed{0} & \boxed{\#} \end{array}\right]$$

No solution; linearly dependent

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in $\mathbb{R}^3$ such that $w + u \in \mathrm{Span}(u, v)$ then $w \in \mathrm{Span}(u, v)$.

true

$\mathrm{Span}(u, v) = \underbrace{c_1 u}_{w} + c_2 v$

if $w + u \in \mathrm{Span}(u, v)$, then w must be lin. combination of u, v

b) If u, v, w are vectors in $\mathbb{R}^3$ such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

true - each vector must be linearly independent from each other vector for the whole set to be linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, linear combination

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, if set of vectors is linearly independent,
then $\{u, v\}$ are also linearly independent is long
is u or v are not a scaler multiple of the other

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True because, w+u consists of w as well and when w+u ∈ Span(u,v); w must be in Span(u,v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

Counterexample:

let {u,v,w} be set {5,6,8} respectively. Then, {5,6,8} doesn't mean {5,8} is also linearly independent.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, u = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix}$

True

$V_1 + 0V_2 = 1$
$0V_1 + V_2 = 2$ ✓
$0V_1 + 0V_2 = 0$

$u + w = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$V_1 + 0V_2 = 4$
$0V_1 + V_2 = 8$
$0V_1 + 0V_2 = 0$

$V_1 + 0V_2 = 4$
$0V_1 + V_2 = 6$ ✓
$0V_1 + 0V_2 = 0$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True because in order for a set to be linearly independent they must have 1 solution (Homogeneous equation) so that means that u, v, w all only have 1 solution which ~~means~~ means that the set $\{u, v\}$ is linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False.

$u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad w + u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \in \text{Span}(v)$

$\in \text{Span}(u, v)$

$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$v = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\quad w \neq cu$, $w \neq cv$, so $w \notin \text{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. If set $\{u, v, w\}$ are indepentant, that means $u \neq cv$, $u \neq cw$, $v \neq cu$, $v \neq cw$, $w \neq cu$, $w \neq cv$.

Show linear independence between u and v.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False,

Counter example

$$u = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \quad w = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$$u + w = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \in \mathrm{span}\left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}\right)$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \notin \mathrm{span}\left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}\right)$$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True because in order to be independent there must be no dependency on another vector, so if a vector were to leave the group that vector would still be independent.

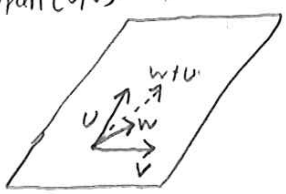
0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, if w was not within the span(u,v) then the resultant vector would not be within the plane

Span (u,v)



b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False, only linearly independent if u is a scalar multiple of v, or vice versa.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

TRUE

If $w + u = v$, then $w = v - u$

v is in the span(u, v) and $(v - u)$ is also in the span(u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

TRUE

The set of u, v, w, in that each column is a pivot column

So, in the set of $\{u, v\}$ each column will also be a pivot column,

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False. because, $w + u = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \in$ span $(u, v.$

not necessary,

$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \in$ span (u, v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False.

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}$

the set $\{u, v, w\}$ is linearly independent. ~~be~~ cause, $x_1 u + x_2 v + x_3 w = 0$ has only one, treival solution. on the other, $\{u, v, w\}$ is linearly dependent because $x_1 u + x_2 v + x_3 w = 0$ has non-treival solution

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$
$u \quad v \quad w$

$$w + u \in \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

True,
if double plus u kept it the span of u, v, the w by it self should be in the span.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True,
because if, $\{u, v, w\}$ was linerly independent, then making w zero didn't change it to be dependent, so removing w entirely changes nothing.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

F

$u \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ $w = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

linearly independent $\rightarrow$

F

linearly independent mean have exactly one solution.

if $\{u, v, w\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\{u, v\} = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\leftarrow$ have infinite solution.

$\therefore$ It's False.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True. i ~~t~~ only $[c_1 u + c_2 v] \in \text{Span}(u, v)$

if $w + u \in \text{Span}(u, v)$

$w = c_1 u$ or $w = c_2 v$.

$w \in \text{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False. if $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$. ~~$2u - v + 0w = 0$. - indep~~ ~~$x_1 = 2$~~ ~~$x_2 = -1$~~

$u + v + 0w = 0$. - linear indep ~~$x_1 u + x_2 v + x_3 w = 0$.~~ is ~~linear independent~~.

but ~~$x_1 u + x_2 v = 0$ is not linealy independen~~

~~$x_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$.~~

~~t~~ ~~i~~ but $x_1 u + x_2 v$ is not linear ~~indepe~~.

$x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $x_2 \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ is linear dependent.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

True   If $w + u \in \operatorname{Span}(u, v)$

$$w + \cancel{u} = x_1 u + x_2 V$$
$$\quad -\cancel{u} \qquad -u$$

$$W = (x_1 - 1)u + x_2 V$$

Let $x_1 - 1 = c_1$, $x_2 = c_2$

$$W = c_1 u + c_2 V$$

Therefore $w \in \operatorname{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True $x_1 u + x_2 V + x_3 W = 0$

Linear independence states that $x_1 = 0, x_2 = 0, x_3 = 0$

Let u, v, and w be standard basis vectors.

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$c_1 u + c_2 V = 0$$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0$$ also only has the trivial solution

$c_1 = 0$, $c_2 = 0$.

Therefore, the set $\{u, v\}$ must also be linearly independent.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True. Vector u must be in the span(u,v), and given that w∈span(u,v) then w+u must also be in the span(u,v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. In order for {u,v,w} to be linearly independent, it must have a leading one in every column. This means {u,v} also has a leading one in every column, so {u,v} is linearly independent.

0 1 2 3 4 5 6 7 8 9 10

$w \quad u \quad v$

$\begin{bmatrix}1\\1\\0\end{bmatrix} \begin{bmatrix}-1\\0\\2\end{bmatrix} \begin{bmatrix}2\\1\\-1\end{bmatrix}$

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

false,

$w+u = c_1 u + c_2 v$

$w \neq d_1 u + d_2 v$

$u = \begin{bmatrix}1\\1\\0\end{bmatrix}, \quad v = \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad w = \begin{bmatrix}2\\2\\0\end{bmatrix}$

$w+u = \begin{bmatrix}2+1\\2+1\\0+0\end{bmatrix} = \begin{bmatrix}3\\3\\0\end{bmatrix} = c_1 \underbrace{\begin{bmatrix}1\\1\\0\end{bmatrix}}_{u} + c_2 \begin{bmatrix}0\\1\\0\end{bmatrix} \checkmark \quad c_1 = 3, \; c_2 = 0$

$\begin{bmatrix}2\\2\\0\end{bmatrix} \neq d_1 \begin{bmatrix}1\\1\\0\end{bmatrix} + d_2 \begin{bmatrix}0\\1\\0\end{bmatrix}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False, u, v, w can all be ~~linearly~~ linearly independent but if you form the matrix of vectors, $\mathrm{Nul}(A) \neq \{0\}$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

false

$w + u = x_1 u + x_2 v$

$w + u - u = x_1 u + x_2 v - u$

$w = x_1 u + x_2 v - u$

since $w + u \in \mathrm{span}(u, v)$

so $w + u = x_1 u + x_2 v$

and $w = x_1 u + x_2 v - u$

does not equal to $w = x_1 u + x_2 v$

which w is not linear combination of $x_1 u + x_2 v$

which w not in Span (u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$$\begin{array}{ccc} u & v & w \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$ True

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$ because u v have pivot position at every column

$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$. True

$$U = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad W = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad V = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$W + U = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \therefore W \in \text{Span}(U)$$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent. True. Removing a vector will still make the set linearly independent

$$U = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad V = \begin{bmatrix} 17 \\ 19 \\ 21 \end{bmatrix} \quad W = \begin{bmatrix} 33 \\ 33 \\ 33 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True because if $w+u$ is in the Span(u,v) then $w \in$ Span(u,v) shows it is a part of that span. Graphically it would be on the same line because w is only being added by u

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False you can have $\left\{ \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 10 \\ 14 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix} \right\}$

when it is dependent while these two are dependent.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \mathrm{Span}(\mathbf{u},\mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u},\mathbf{v})$.

False if $w + u \in \mathrm{Span}(u, v)$

that means w is linearly dependent for

u, $W \notin \mathrm{Span}(u, v)$

b) If u, v, w are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u},\mathbf{v},\mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u},\mathbf{v}\}$ must be linearly independent.

False, $\{u, v, w\}$ is only inlinear independent if it has only one solution to the homogenus equation $x_1 u + x_2 v + x_3 w = 0$

Counter Example: $\begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} u_1 & 0 & * \\ 0 & v_2 & * \\ 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} u_1 & 0 & * \\ 0 & v_1 & * \\ 0 & * & * \end{array}\right]$

$\therefore$ $\{u, v\}$ is not neccacirly linearly independent

0	1	2	3	4	5	6	7	8	9	10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$$\cancel{\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right)}$$

$\boxed{\text{True}}$ because if w+u is in the span(u,v), then w must be some linear combination of u or v.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, if the whole set is linearly independent, then its parts must hold the relation.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$\begin{bmatrix} u_1+w_1 \\ u_2 \; w_2 \\ u_3 \; w_3 \end{bmatrix}$ True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False

$w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

w + u is a linear combination so just w itself may not be in span

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

L. I. depends on the values $c_1, c_2, \dots c_n$

Set $\{u, v, w\} \in \mathbb{R}^3$
$c_1 = c_2 = c_3 = 0$

Set $\{u, v\} \in \mathbb{R}^2$
$c_1 = c_2 = 0$

$\therefore$ Set gets dropped by one dimension solution remains the same

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad w = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$

TRUE because if $w + u$ is in span(u,v) then w will also be in Span (u,v) because they are linear combinations of one another

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad v = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad w = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

- Since the set is linearly independent the matrix would have to be

$$\begin{bmatrix} 1 & b_1 & c_1 \\ a_2 & 1 & c_2 \\ a_3 & b_3 & 1 \end{bmatrix}$$

in order for every column to have a pivot column

set $\{u, v\}$

$$\begin{bmatrix} 1 & b_1 \\ a_2 & 1 \\ a_3 & b_3 \end{bmatrix}$$

this set is still linearly independent so the statement is TRUE

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$U = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

$w = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \cancel{\begin{bmatrix} 5 \\ \end{bmatrix}}$ span $\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$

The statement is true

~~False~~

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$ ~~This is false because just because~~

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ This is false beause just becuse

$u + v + w = 0$

does,n't men

$u + v = 0$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$w+u$ is a linear combination of u, v.

$\Leftrightarrow$ $w+u = C_1 u + C_2 v$

$w = (C_1 - 1) u + C_2 v$ is a linear combination of u, v.

$\therefore$ $w \in \mathrm{Span}(u, v)$

True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

① $X_1 u + X_2 v + X_3 w = 0$ has only one and trivial solution $X_1 = X_2 = X_3 = 0$.

Then, $X_1 u + X_2 v = 0$ also has only one and trivial solution $X_1 = X_2 = 0$.

② $A = [u \ v \ w]$

A has a pivot position in every column.

$B = [u \ v]$

Then, B also has a pivot position in every column.

True

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True

Span (u, v) = all vecks / linear combinations $c_1u + c_2v$

or $c_1w + c_2u$ spanning (u, v)

therefor $c_1 w$ span (u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False because if $\{u, v\}$ are scalar multiples ie $c_1v_1 + c_2v_2 = 0$ then it is linearly dependent.

$R^3 \to 3 = n$ rows

$1u_1 + (-1)u_2 = 0$

u v w

1 2 3 $\to$ 3 = p colums

by definition if $p > n \Rightarrow$ linearly dependent

but $p = n$ so it is linearly independent

but if u v

1 2

$p = 2$ and R3 $n = 3$

* does not apply for $p = 2$

$p \not> n$

$2 \not> 3$ so it is linearly independent as well

0 1 2 3 4 5 6 7 8 9 10

MTH-309T-F19-EX1-032-P06

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

This is true. By def. of span, $w + u = c_1 u + c_2 v$ for some $c_1, c_2 \in \mathbb{R}$. Therefore $w = c_1 u + c_2 v - u$, and combining gives $w = (c_1 - 1) u + c_2 v$. However, $c_1 - 1$ is just some other constant in $\mathbb{R}$, so let $c_3 = c_1 - 1$. Therefore $w = c_3 u + c_2 v$, meaning $w \in \text{Span}(u, v)$ by definition.

QED.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

This is true. If $\{u, v, w\}$ is linearly independent, then

$u \neq c_1 v + c_4 w$ for any $c_1, c_4 \in \mathbb{R}$

$v \neq c_2 w + c_5 u$ for any $c_2, c_5 \in \mathbb{R}$

$w \neq c_3 u + c_6 v$ for any $c_3, c_6 \in \mathbb{R}$

Since $u \neq c_1 v$ and $v \neq c_5 u$ for any $c_1, c_5 \in \mathbb{R}$, then $\{u, v\}$ is linearly independent by definition.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True

If $w + u = c_1 u + c_2 v$

$w = c_1 u + c_2 v - u = c_3 u + c_2 v$

Thus $w \in \text{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

if $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\{u, v\}$ has to be lin ind for $\{u, v, w\}$ to be lin ind

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

This statement is false, because W + U will only span (u, v) if W + U is in the null set of (u, V)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $V = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$ $W = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

This statement is true.

If {u, v, w} is linearly independent then ~~th~~ each column in the aug matrix must have a leading one. If you remove one vector (column), each column will still have a leading one

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True.

$\therefore \quad T(w+u) = T(w) + T(u) = T(w) + T(v).$

$\therefore \quad T(w) = T(u) + T(v).$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True:

If ~~ucvw~~

$c_1 u + c_2 v + c_3 w = 0$ is linearly independant

it means $\exists\, u, v \,\&\, w = 0$.

$\therefore$ The set $\{u, v\}$ has to be linearly independent

because $x_1 u + x_2 v = 0$.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$\mathrm{Span}(u,v) = x_1 u + x_2 v$

For $w \in \mathrm{Span}(u,v)$, $w = x_1 u + x_2 v$

$w + u = x_1 u + x_2 v$

$w = (x_1 - 1)u + x_2 v$ $\therefore$ $w \in \mathrm{span}(u,v)$ because w is still a linear combination of u and v.

$\boxed{\text{True}}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

Lin. Independant: only 1 solution to $x_1 u + x_2 v + x_3 w = 0$

$0u + 0v + 0w = 0$

$\downarrow$

$0 + 0 + 0 = 0$

$\downarrow$

$0u + 0v = 0$

$\downarrow$

$0 + 0 = 0$ $\therefore$ No matter which vector is removed from the set, the set of the other two is always linearly independent.

$\boxed{\text{True}}$

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True, so tn is in the span

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

false, $\{u, v\}$ could have a differente solution with nothing trivial

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False. If u,v,w are vectors in $\mathbb{R}^3$ s.t. w u $\in$ Span, w must be a linear combination of u+v not w+u.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. ~~If Span(u,v,w)~~ ~~If~~ If $\{u,v,w\}$ is linearly independent, only have trivial solution as the solution to homogeneous equation. $\therefore$, $c_1u + c_2v = 0$ must also only have the trivial solution as the answer.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in $\mathbb{R}^3$ such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

False $w = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$u = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

w is only in the Span$(u, v) - u$

$w + u \begin{bmatrix} \\ \\ \\ \end{bmatrix}$

b) If u, v, w are vectors in $\mathbb{R}^3$ such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

True if u, v, w is linearly independent the only solution is zero vector and has a pivot ~~column~~ position in every column. u, v would be independent as well w/ pivot position in each column and only solution would be zero vector.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True; since $u \in Span(u,v)$, then if $w+u \in Span(u,v)$ then w must be a multiple of a vector in $Span(u,v)$. Therefore, $w \in Span(u,v)$.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

A set is lin. ind. when $x_1 u + x_2 v + x_3 w = 0$ has only one solution, where $u, v, w = 0$. If the set $\{u, v, w\}$ is lin. ind., then either combination of two of these vectors must also be lin. ind. Therefore, this statement is true.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True

if $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ then Span(u, v) is all vectors of the form $\begin{bmatrix} c_1 \\ c_2 \\ 0 \end{bmatrix}$ where c_1 and c_2 are any constants,

If $w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ then $w + u = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ therefore w and w+u are both in the Span(u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

If $x_1 u + x_2 v + x_3 w = 0$ has one solution, then $x_1 u + x_2 v$ also has to have one solution, therefore $\{u, v\}$ is also linearly independent

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ then $x_1 = x_2 = x_3 = 0$

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ then $x_1 = x_2 = 0$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True because U is not needed for W to be in the span if it was given that all 3 were in $\mathbb{R}^3$.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False it would depend on what vectors U and V are because the homogeneous solution may be different when using only the 2 vectors out of the 3.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

~~False, because~~

True, because w+u is some linear combination of u that results in the ~~span~~ the new vector being in the span, for that to occur by the definition of a linear combination w must also be in the span(u,v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

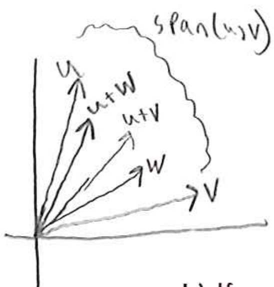
~~True,~~ False, because the row reduction of the vectors in an augmented matrix require nx n to be linearly independent so without the 3rd vector in $\mathbb{R}^3$, it is impossible to have only 1 solution.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True Because Span(u, v) is the span of all vectors inbetween vector u and vector v. In $\mathbb{R}^2$ the graph below explains this concept which still holds true for an extra dimension. To scale this problem up a dimension we could just make the 3rd row of values in u, v, w = 1



therefore if
w+u ∈ span(u,v)
then w ∈ span(u,v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True because in order for {u v w} to be linear independent every column must be a pivot column therefore the subset {u, v} must also have every column be a pivot column.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True.

Since, in set $\{u, v, w\}$ are already indep.

as subset $\{u, v\}$ will also be indep.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True

$w+u = c_1 u + c_2 v \quad (c_1, c_2 \in \mathbb{R})$

$w = (c_1 - 1)u + c_2 v$

$w \in \mathrm{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True.

$\{u, v, w\}$ is Linearly independent means they're not multiple of each other.

then $\{u, v\}$ u, v won't be multiple of each other

then they're Linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False, counter example

$w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

w, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ but $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False - Counter example~~

True, because something in linerly independent when the vectors are not scalar multiples of each other, and taking out one vector of a set wont make it that the other 2 are now scalar multpoles of each other.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False take $W = \begin{matrix}1\\0\\0\end{matrix}$ $U = \begin{matrix}1\\1\\0\end{matrix}$ $V = \begin{matrix}2\\1\\0\end{matrix}$

$W + U = \begin{matrix}2\\1\\0\end{matrix} \in \text{Span}(U, V)$, but

$W = \begin{matrix}1\\0\\0\end{matrix} \notin \text{Span}(U, V)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$X_1 V + X_2 V + X_3 W = 0$

$3 + 3 + -6 = 0$

Let $X_1 V = 3$

$X_2 V = 3$

$X_3 W = -6$

$X_1 U + X_2 V = 0$

$X_1 U = X_2 V$, so $X_1 U$ is a Scalar multiple of $X_2 V$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$w + u \in \text{Span}(u,v) \quad w \in \text{Span}(u,v)$

$w = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$w + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$w = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$w + \begin{bmatrix} 2 \\ 3 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$w = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \ldots\right)$

$\begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$\left[\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{array}\right] \xrightarrow[R2 \to -2R1+R2]{R3 \to -3R1+R3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & -6 & -3 \end{array}\right] \xrightarrow{R3 = -2R2+R3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{R2}{-3}} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right]$

True because w + u linear combination of u (w to be in span) therefore it will be in span(u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix}$

True because if set {u, v, w} is all linearly independent from each other, then u and v has to be linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True because if $w + 1(u)$ is a linear combo of span(u,v), then $w + 0(u)$ is also a linear combo of span(u,v) $\Rightarrow$ therefore w is an element of span(u,v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

$u = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

because the only way for 2 vectors to be linearly dependent is if they are scalar multiples of one another, but if 2 of the 3 vectors are scalar multiples $Ax = 0$ will have a free variable therefore it is not linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$\boxed{\text{True.}}$

$$\left\{\begin{aligned} w+u &= c_1u + c_2v \\ w &= c_1u + c_2v - u \\ w &= (c_1-1)u + c_2v \end{aligned}\right.$$

$w \in Span(u,v)$ $\Longleftarrow$
if $w+u \in Span(u+v)$
because w can be written as linear combination of u and v.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False.~~

Let $\vec{u} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\vec{v} = \begin{bmatrix}0\\1\\0\end{bmatrix}$, $\vec{w} = \begin{bmatrix}0\\0\\1\end{bmatrix}$, $\vec{b} = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$$\begin{array}{cccc} u & v & w & \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{cc} u & v \\ \end{array}\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

$A\vec{x} = [b]$

Let $\{\vec{u}, \vec{v}, \vec{w}\}$ be exact slt'n. Then $\{\vec{u}, \vec{v}\}$ is not a sltn to the eqn and ∴ not linearly independent.

True, $\vec{u}, \vec{v}, \vec{w}$ cannot be written as a linear combination of each other so $\vec{u}, \vec{v}$ cannot be written as a linear combination of each other.
∴ $\{u, v, w\}$ is linearly independent
$\Rightarrow \{\vec{u}, \vec{v}\}$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False w+u can be in the span of u,v
but w E has no correlation with
the span of u,v since w is added
to u.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True since {u,v,w} in $\mathbb{R}^3$ are all vectors
with leading ones and in row reduced e form
to be linearly independent, also to be linearly independent
u,v,w cannot be multiples of each other so
{u,v} must be linearly independent because
they are not multiples of each other as proven
by {u,v,w} linear independence

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$\boxed{\text{True}}$ Since $u \in \text{span}(u, v)$
for $u + w \in \text{span}(u, v)$ w would have to be some combination of $cu + dv$ where c & d are some constants.
Therefore $w \in \text{Span}(u+v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly <u>independent</u> then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent. *Linearly dependent - homogenous vector equation has more than 1 solution

$\boxed{\text{False}}$ let $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

~~$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is linearly independent~~

~~$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ is linearly dependent~~

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is linearly independent

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is linearly dependent

~~True For a set of p vectors (2 in this case) of $\mathbb{R}^n$ ($\mathbb{R}^3$ in this case) can only be linearly dependent if $p > n$. $2 \not> 3$ so the set $\{u, v\}$ would be linearly independent.~~

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False

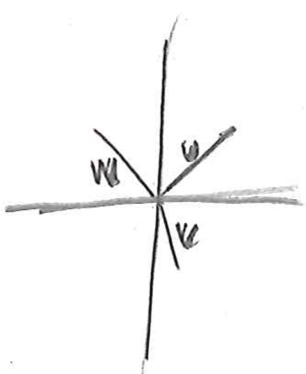
$w = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ $\quad \text{Span}\left(\underset{u}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}, \underset{v}{\begin{bmatrix} 7 \\ 4 \end{bmatrix}}\right)$

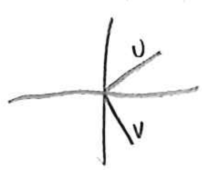
$w + u = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

$\begin{bmatrix} 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ is in Span(U, V), but $\overset{\begin{bmatrix} 6 \\ 2 \end{bmatrix}}{W}$ not is Span

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. U, V, W are 3 unique vectors, so even if only 2 of them were in the set, it would still be linearly independent





still independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True, vectors encompassed by a ~~elements of a~~ span are linear combinations of the vectors in that span, so the linear combination of some vector and a vector in some span can only be encompassed by that span if the other vector is also a linear combination of the vectors in the span.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False~~

$$C_1U + C_2V + C_3W = 0$$

$$C_1U + C_2V = 0$$

~~False~~

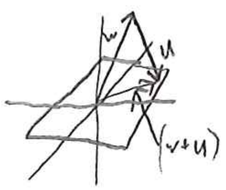
True, because if $\{U, V\}$ is not linearly dependent then $\{U, V, W\}$ cannot be linearly independent as multiplying W by a constant of zero would yield all solutions of $\{U, V\}$

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

~~comb... of two vectors with coefficients of 2 ... combination with a ... span.~~

True; since the span of two vectors in $\mathbb{R}^3$ can be visualized as a plane in 3D space, and u is in the span of (u, v), then w must also lie in that plan if w+u is to be in the span as well.



w+u is in

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True; $\{u, v\}$ is linearly independent only if u and v are scalar multiples, which would not allow $\{u, v, w\}$ to be linearly independent.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\boxed{\text{True}}$

$w + u \in \text{Span}(u, v)$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$ True

$w \in \text{Span}(u, v)$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$ True

Since $w + u$ is in the span of u, v their combination of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ can be made from $c_1 u + c_2 v = w + u$. If you let $c_1 = 1$ and $c_2 = 1$

$\cancel{u} + v = w + \cancel{u}$

$v = w$

Since $v = w$, w is in span of v which makes $w \in \text{Span}(u, v)$ True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Lin independent means one trivial solution to $Ax = b$

$\boxed{\text{TRUE}}$

Since u, v, and w are linearly independent that means all columns on A are pivot columns. So if you were to remove a vector w for example and solve for independence with u and v you are left with a 3×2 matrix with pivot positions in every column so they are Independent.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True, if a vector is in span of other vectors when modified, then that vector is in span itself.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, in $\mathbb{R}^3$ if $\{u, v, w\}$ has one particular solution, then the vectors are linearly independent with one another always.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

true

The span of (u, v) is the set of all linear combinations of u and v. If $w+u \in \text{Span}(u,v)$, that means $w+u$ is a linear combination of u and v. If we subtract a multiple of u, the result should still be obtainable from a combination of u and v since we are essentially adding $-u$ to a combination of u and v

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

true

A linearly independent set means u v and w all cannot be produced from a linear combination of the other elements in the set. This means any combination of any multiples of u and w cannot produce v. If this is the case, we still cannot produce v from any multiples of just u. The same logic applies in the opposite case of v producing u.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False,

if $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True

Since $w + u$ is in the span of u, v,

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True.

To be a linearly dependent set, for $X_1 u + X_2 v + X_3 w = 0$ $X_1, X_2, X_3 = 0$ must be the only solution.

If $\{u, v\}$ was a linearly dependent set, $\{u, v, w\}$ would also be linearly dependent, since $X_3 w$ could just set $X_3 = 0$.

Therefore, since $\{u, v, w\}$ is linearly independent, $\{u, v\}$ MUST be, too.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False

~~set~~ $\cancel{u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}}$

$\cancel{w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$

$\cancel{w+u = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in span(u,v)}$

$\cancel{w \notin Span(u,v)}$

True

$w+u \in Span(u,v)$

then $w \in$ ~~Span~~ $Nul(u,v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False~~

~~assume~~ $\cancel{u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$

~~$\{u,v,w\}$ is linearly independent.~~

~~while $\{u,v\} = \{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\}$ is linearly dependent~~.

True.

$\{u,v,w\}$ is linearly independent.

then $x_1 u + x_2 v + x_3 w = 0$. has only one solution $x_1 = x_2 = x_3 = 0$

$x_1 u + x_2 v = -x_3 w$

$x_1 u + x_2 v = 0$

$\Rightarrow$ every column of the matrix is a pivot column.

since $\{u, v, w\}$ every column of u, v, w is pivot column.

$\{u, v\}$ also has pivot column is every column.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$W \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad u \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad v \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\underset{W}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} + \underset{u}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} \in \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$

<u>false</u>

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \notin \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, if u, v, w are all lin. independent, u, v must be linearly independent

$$\underbrace{\underbrace{u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{u,\, v \Rightarrow \text{lin. ind.}} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{all 3 lin. ind.}}$$

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$\Downarrow$ w+u can be written as a linear comb. of u and v

w can be written as a linear comb. of u and v

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_3 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$\Downarrow$ then...

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_2 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_3 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} - \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$\therefore$ $\boxed{\text{true}}$ w is in the span (u,v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\boxed{\text{True,}}$ if u, v, and w are already linearly independent and are not linear combinations of eachother, the the set $\{u, v\}$ must also already be linearly independent

$$\left\{ \underset{u}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}, \underset{v}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}, \underset{w}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \right\} \Rightarrow \text{linearly independent}$$

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \Rightarrow \text{still linearly independent}$$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

false

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

false

let set $\{u, v, w\}$ called $S = \{u, v, w\}$

$R(S) = 3 =$ the number of vectors

$\Rightarrow$ at least, one of $w = k_1 u + k_2 v + k_3 w$ ..... is right.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

False

$w + u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ but there is no x value where $xw = \text{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

True linear independent means vectors are not multiples so removing one will not affect linear independence

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

Ture

u. v are in span

w+u is span

w in ∈ span

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

$u \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} v \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} w \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

u v w are linearly independent

But u v are linear dependent.

$x_1 u + x_2 v_2 + x_3 w = 0$ has solution of 0

$x_1 u + x_2 v_2$ will have solution.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

$\boxed{\text{False}}$

w could be projected outside the span when not added to u

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\boxed{\text{True}}$ because you will still have every column as a pivot column

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$$\begin{array}{ccc} u & v & w \end{array}$$
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix} \quad u+w = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right)$$

True

$w+u \in \text{Span}(u,v)$. suppose that $w+u = a\,u + bv$ a.b are sclar.

$w = (a-1)u + bv$

$w \in \text{Span}(u.v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

if it is linearly independent. True

they must have pivot in every column.

it also in $\mathbb{R}^3$. so the reduce form should be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

so the set $\{u.v\}$ must be linerly independent.

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True; If there is a linear combination of vectors v and u that equal w, w must be in the span of u and v. Since addition and subtraction of one vector from another is a linear operator, the problem can be considered as the following:

$w = (w+u) - u \therefore$ If $w+u = x_1 v + x_2 u$, then $w = x_1 v + (x_2 - 1)u$, which is a linear combination of u and v $\therefore$ $w \in \text{Span}(u,v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, any subset of a linearly independent set of vectors must also be linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True, u is in the span of u

So adding it does not take it out

of span.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

true. there would be no free variables

when reducing the set to $\{u, v, w\}$ to $\{u, v\}$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True

For any equation $w+u = a_1 u + a_2 v$, $w = a_1 u - u + a_2 v = cu + a_2 v$

$w \in \mathrm{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

If $\{u, v\}$ were dependent, then $a_1 u + a_2 v + 0w = 0$ would prove $\{u, v, w\}$ to be dependent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True if u is in the span as well then $w+u \in \text{span}(u,v)$

Because

$w = -u + v$

$w + u = v$

B $-u, v$ are in the span of u, v

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

false

$$\begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} & \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

u v are dependent

u v w are independent

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, because $w+v = w'$ so

w has to be in Span(u,v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

TRUE,

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

because $x_1u + x_2v + x_3w = 0$ has only one solution, $x_1u + x_2v$ will only have one solution as well.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True because u is in the span of it self, making it equal to $w \in \mathrm{Span}(u, v)$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False → two vectors can be linearly independent but the third can be linearly dependent and vise versa.

$V_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ $V_2 = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$ $V_3 = \begin{bmatrix} 1 \\ 3 \\ -12 \end{bmatrix}$

$V_1, V_2, V_3 \rightarrow$ linearly dependent

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -2 & 4 & -12 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & \underset{\text{free}}{0} & 0 \end{array}\right]$$

However, V_1 and V_2 are linearly independent

$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 5 & 0 \\ -2 & 4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}\right\}$ trivial solution.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, because since $u+w$ is in the span (u,v) then it must be true that $w \in \mathrm{Span}(u,v)$ because the sum of the two vectors $(u+w)$ are in the span (u,v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

~~False~~, for example

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

then the set $\{u,v\}$ $\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right)$ is also linearly independent because it has only one solution and pivot col in every column. If u,v,w are independent then it has pivot col. in every columnn that means with two vectors it still holds true.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True : because is w + u is is in Span (u, v) then u - v = w which is also in Span (u, v)

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True : is $\{u, v, w\}$ are linearly independent than $\{u, v\}$ is linearly independent because $c_1u + c_2v + c_3w = 0$ would have only one trivial solution, meaning that $c_1u + c_2v + c_3w$ or $c_1u + c_3w$ or $c_1u + c_2v$ or $c_2w + c_3w$ have no combination to sum to 0 other than multiplying by 0, if they did than $\{u, v, w\}$ would have a nother solution to $c_1u + c_2v + c_3w = 0$ which would make $\{u, v, w\}$ linearly dependent. Since $c_1u + c_2v$ have no combination to 0 other then 0 they are linearly independent

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$\begin{bmatrix}1\\0\\0\end{bmatrix} = u \quad \begin{bmatrix}0\\1\\0\end{bmatrix} = v \quad \begin{bmatrix}0\\1\\0\end{bmatrix} = w$

$\begin{bmatrix}1\\1\\0\end{bmatrix} = w + u \in \text{Span}\left(\overset{u}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \overset{v}{\begin{bmatrix}0\\1\\0\end{bmatrix}}\right)$

It is true

is $\begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow w \in \text{Span}\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right)$?

So $0\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{bmatrix}0\\1\\0\end{bmatrix}$

$0u + 1v = w$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad v = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad w = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$\rightarrow$ set $\{u, v, w\}$

linearly independent $\Rightarrow \left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$

set $\{u, v\} \Rightarrow \begin{array}{cc}x_1 & x_2\end{array} \left[\begin{array}{cc|c}1&0&0\\0&1&0\\0&0&0\end{array}\right] \Rightarrow \begin{array}{l}x_1 = 0\\x_2 = 0\\0 = 0\end{array}$

doesn't matter!

It is true. Since, linear in dependence means having a trivial sol. So by decreasing the set to $\{u, v\}$, it won't change to infinite amount of sol.

$x_1 = 0, x_2 = 0, x_3 = 0$

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$w + u = c_1 u + c_2 v$

$w = (c_1 - 1)u + c_2 v$

TRUE; w is a linear combination of u and v

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

By definition, u, v, w are not constant multiples of each other: $u \neq c_1 v$ for any c_1

u, v are not constant multiples of each other

$\therefore$ u, v are linearly independent

TRUE

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False.~~ True. $\{u, v, w\}$ is linearly independent which means every column of matrix is pivot column. If cancel w vector, ~~It~~ the matrix still ~~have~~ has columns that are pivot column.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

Yes, Since $w+u \in \mathrm{Span}(u,v)$, then $w+u = c_1u + c_2v$ has solutions,
$w = c_1u + c_2u - u = (c_1-1)u + c_2u$ has solutions

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

* **a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

$\cancel{u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \; v = \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix} \; w =}$ $w = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $V = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

False,

$w + u = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ so

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right]$ is valid

but $\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{array}\right]$ is invalid

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False, $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$

$u + v = w$ so $\{u, v, w\}$ is linearly independent

but u and v alone $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ are linearly dep. because neither vector can represent the other.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

FALSE

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

FALSE

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

MTH-309T-F19-EX1-086-P06

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$w+u = c_1 u + c_2 v$

$w = c_1 u + c_2 v$

True because if $w \in \mathrm{Span}(u, v)$, according to $x_1v_1 + \ldots x_pv_p = w$, the set of vectors is linearly dependent. All the vectors are just linear combinations of each other so w is in the span of u, v!

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True because if they are all linearly independent, the vectors can't be changed to be linearly dependent. There is no relationship between u, v, w, so if you take one out, it would not change the linear independently.

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, because if we add the vectors together in a vector equation an the resulting vector is in the span, then that means that the vector that was added is in the span as it is a linear combination.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

true, because if one of the vectors is linearly dependent then the whole set would have been linearly dependent but it's not.

0 1 2 3 4 5 6 7 8 9 10

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True. In order to be linearly independent, each column is a pivot column. So if you do set $\{u, v\}$ they will both still have a pivot column in each column.

0 1 2 3 4 5 6 7 8 9 10