

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) if w span
$$(v_{1}, v_{3}, v_{3})$$
 in $w = c_{1} v_{1} + c_{3} v_{3} + c_{3} v_{3}$

$$v = c_{1} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + c_{2} \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix} + c_{3} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v' = -b \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix}$$



because every column of the wan reduced matrix is a pivot column.

This comes from a mistale in the row reduction



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad \begin{bmatrix} (r_1 c_1)(t_1 c_3)(r_1 c_3) \\ (r_2 c_1)(r_3 c_3)(r_3 c_3) \\ (r_3 c_1)(r_3 c_3)(r_3 c_3) \end{bmatrix}$$

Compute A^{-1} .

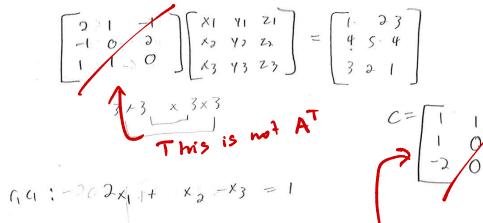
$$A^{-1} = \begin{bmatrix} -\partial & 3 - 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).



$$C_{5} C_{16} = -X_{1} + 3X_{3} = 4$$

$$x_1 + x_2 = 3$$

 $x_1 = 3 - x_2$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{RR} \begin{bmatrix} c & c & c \\ c & c & c \end{bmatrix}$$

There did teris come from?



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T. im Jefining Piss as A.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

 The length of $T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \stackrel{\neq_1}{\neq_1}$.

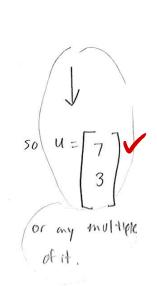
$$T(e_i) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{i=1}^{x_i}$$

a)
$$T(\begin{bmatrix} 0 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 - \theta(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 0 - \theta(1) \\ 0 + 1 \\ 0 - \theta(1) \end{bmatrix} = \begin{bmatrix} -\theta(1) \\ 1 \\ -\theta(2) \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & -\partial \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & -\partial x_2 & = 1 \\ 1 & -\partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & = 1 + \partial x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & x_1 & x_2 \\ 1 & 1 + \partial x_2 \end{bmatrix} * \begin{bmatrix} x_1 & x_1 & x_1 & x_2 \\ 1 &$$



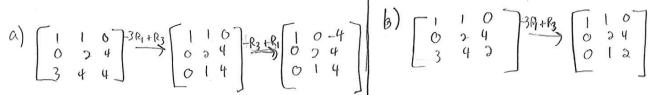
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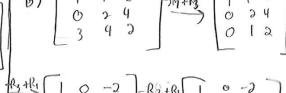


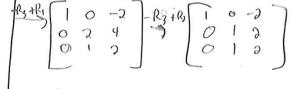
5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

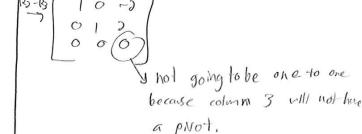
a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

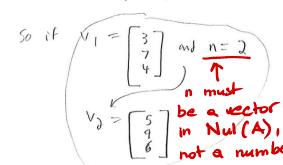








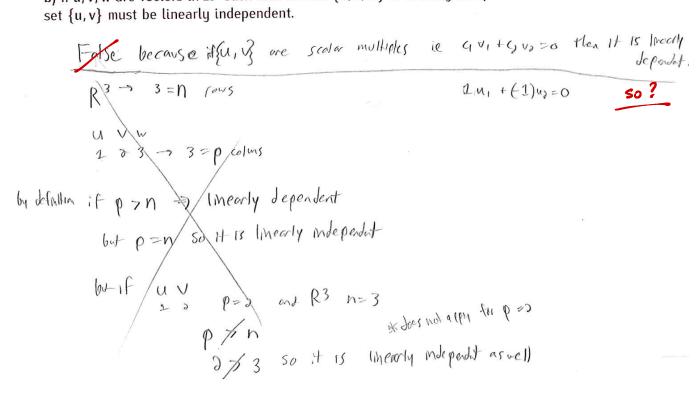
$$T_A(v_1) = T_A(v_2)$$
 if $v_2 = v_1 + n$





- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in Span(u, v)$ then $w \in Span(u, v)$.

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set {u, v} must be linearly independent.





- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

Au, Av morrodes that after multipliation the was a free variable causing on solutions, that makes Au, Av Inearly departed.

This means that the vectors u and v must have use had free variables when multiply by some x value.

Not true.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

Folse, if the transformation changes u, v, w to where u no longor

Span (v, w) that would make T(u) not Span(T(v), T(w))u

[17] $u_1 + 3u_2$ Span(u) but T(u) = [1] that it my not Span(u) but $u_1 + u_2 = u_3$ but $u_2 + u_4 = u_4$ but $u_3 + u_4 = u_4$ but $u_4 + u_5 = u_4$ but $u_5 + u_6 = u_6$ but $u_6 = u_6$ but u_6