

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:	ໄດ. rson				h	e e	Instruc	ctions:		
5 0 0 0 1 2 3 3 4 6 6 6 7 7 8 9 9	4567	3 0 0 0 0 4 5 6 7 8 9	() (1) (2) (3) (4) (5) (6) (7) (8) (9)	Ч ① ① ② ③ ③ ⑥ ⑥ ⑦ ⑧ ⑨	7 ① ① ② ③ ④ ⑥ ⑥ ⑥ ⑨ ③	ろ ① 1 2 ② 4 ⑤ 6 ⑦ 8 9	electr You r	ronic de may use full cre	vices are r one sheet	each problem
1	2		3		4	5	6	7	TOTAL	GRADE



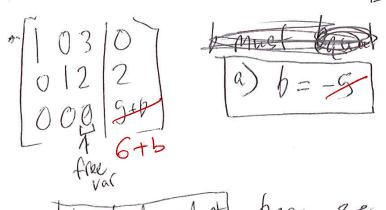
1. (20 points) Consider the following vectors in \mathbb{R}^3 :

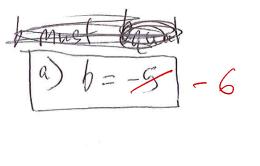
$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix}
1 & -1 & 1 & -2 \\
0 & 1 & 2 & 2 \\
2 & -3 & 0 & 0
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & -1 & 1 & -2 \\
0 & 1 & 2 & 2 \\
0 & -1 & -2 & 4+6
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & -1 & 1 & -2 \\
0 & 1 & 2 & 2 \\
0 & 0 & 0 & 5+6
\end{bmatrix}$$

$$6 + 6$$





b) The set is linearly dependent because V there is not a pivot column in every column



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix}
10 & 1 & 0 & 10 \\
6 & 72 & 2 & 20 \\
0 & 0 & 1 & 2 & 2
\end{bmatrix}$$

$$\begin{vmatrix}
10 & 1 & 0 & 10 \\
0 & 1 & -1 & 10 \\
0 & 0 & 1 & 2 & 2
\end{vmatrix}$$

$$\begin{vmatrix}
10 & 0 & -2 & 3 & -1 \\
0 & 1 & -1 & 1 & 1
\end{vmatrix}$$

$$\begin{vmatrix}
0 & 0 & 1 & 2 & -2 & 1 \\
0 & 0 & 1 & 2 & -2 & 1
\end{vmatrix}$$

$$A^{-1} = \begin{bmatrix} -23 - 1 \\ 1 - 1 \end{bmatrix}$$
 $\begin{bmatrix} 2 - 21 \end{bmatrix}$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\vec{A} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & 12 & 3 \\ -1 & 0 & 2 & | & 45 & 4 \\ 2 & 1 & -1 & | & 32 & 1 \end{bmatrix} \xrightarrow{\begin{bmatrix} 11 & 0 & 12 & 7 \\ -1 & 0 & 2 & 45 & 4 \\ 0 & 1 & -5 & -5 & -7 \end{bmatrix}$$

$$\begin{bmatrix}
-102454 \\
012577 \\
01-5-9-8-7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
-102454 \\
012577 \\
00-3010
\end{bmatrix}
\rightarrow
\begin{bmatrix}
10-2-4-5-4 \\
012577 \\
06-8 0-30
\end{bmatrix}$$



This would nock

Then use A' from problem 2.

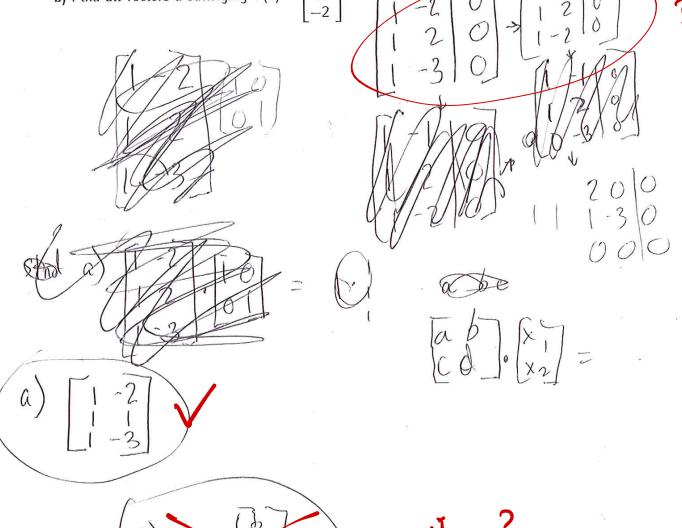


4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

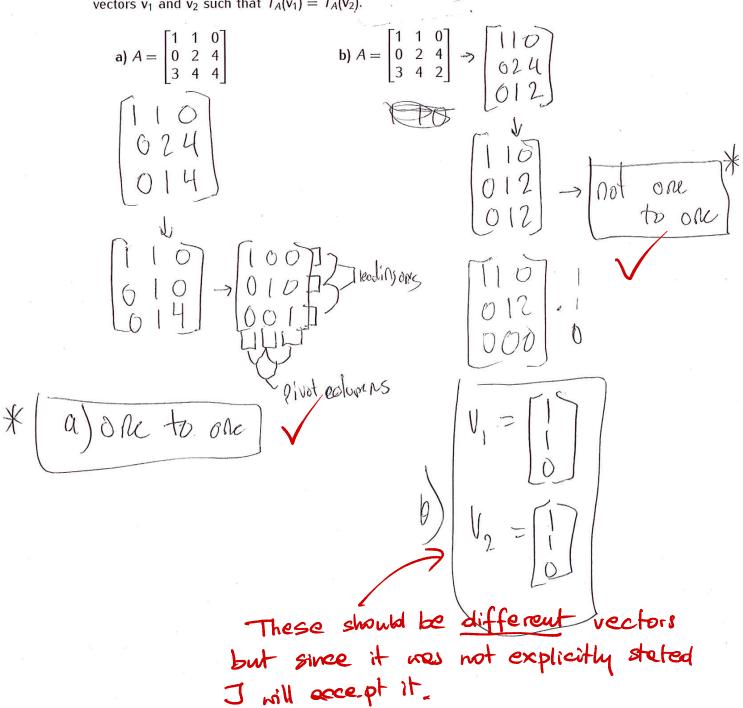
$$3 \times 2 \quad 2 \times 2$$

- a) Find the standard matrix of \mathcal{T} .
- b) Find all vectors u satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.





6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.
ADDE THE THE
RTFASENJANAMANA JA
outside the span when? Not added a to u
We could be dijected for
outside the span when?
not added a to u
b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent) then the
set $\{u,v\}$ must be linearly independent.
Truc because the you will
still have wery column as
a givot column



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

That the matrix A is lancarly dependent not necessarily work

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $(u, v, w \in \mathbb{R}^2)$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True because if the same transformation

True because if the same transformation

15 performed on some u that's in the

Shorn of the some v and w; with

Shorn of T(V), T(W) by the

Hansformation T(V).