

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:

Patrick Hall

UB Person Number:

-	100						
5	0	2	9	9	1	8	6
0 1 2 3 4 6 7 8 9	(a) (b) (c) (d) (d) (e) (e) (e) (e) (e) (e) (e) (e) (e) (e	(a) (b) (c) (c) (d) (d) (e) (e) (e) (e) (e) (e) (e) (e) (e) (e	0 1 2 3 4 5 6 7 8	0 1 2 3 4 5 6 7 8 •	0 1 2 3 4 5 6 8 9	○ ①○ ②○ ③○ ④○ ⑨○ ⑨	

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

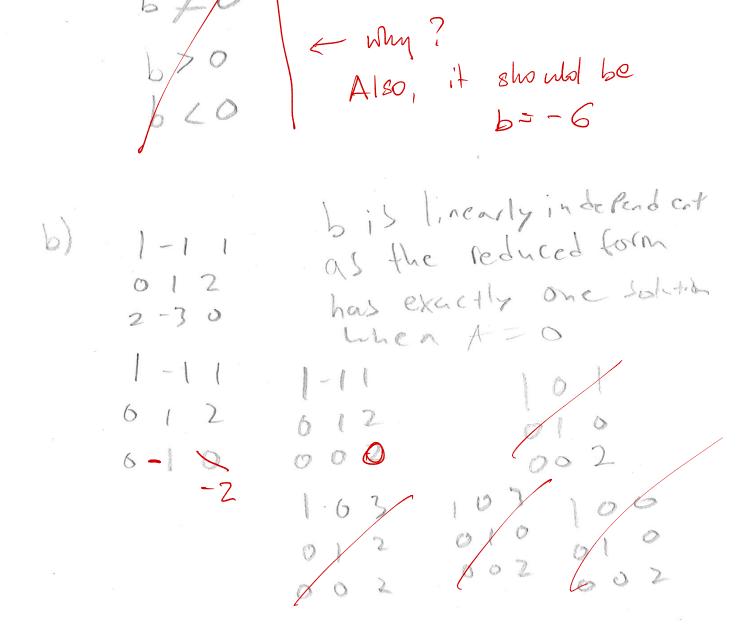
1	2	3	4	5	6	7	TOTAL	GRADE
							10	



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$1 - 12100$$
 $1 0 1010$
 $0 2 - 1001$
 $1 - 12100$
 $0 - 11 - 110$
 $0 2 - 1001$
 $1 - 12100$
 $0 - 11 - 110$
 $0 - 11 - 110$
 $0 - 10 - 10$
 $0 - 11 - 110$
 $0 - 10 - 10$
 $0 - 10 - 10$
 $0 - 10 - 10$
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 $0 - 10 - 10$
 $0 - 10 - 10$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A). ATC=B -> C=B

$$A^{-1} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & -1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{c|c}
-1 & -1 & -1 \\
\hline
-1 & -1 & -1 \\
-1 & -1 & -1 \\
\hline
-1 & -1 & -1 \\$$

$$C = (A^{T})^{-1} B$$

$$= (A^{-1})^{T} B$$

$$C_{11} = 1 \cdot \frac{2}{7} + 2 \cdot \frac{3}{3} + 1 \cdot \frac{3}{3}$$

$$C_{12} = 1 \cdot \frac{1}{7} + 2 \cdot \frac{3}{7} + 1 \cdot \frac{3}{3}$$

$$C_{13} = 2 \cdot \frac{1}{7} + 2 \cdot \frac{1}{7} + \frac{3}{7} \cdot \frac{1}{7}$$

$$C_{13} = 2 \cdot \frac{1}{7} + 2 \cdot \frac{1}{7} + \frac{3}{7} \cdot \frac{1}{7}$$

$$C_{13} = 2 \cdot 1 + 2 \cdot 2 + 3 \cdot 1$$

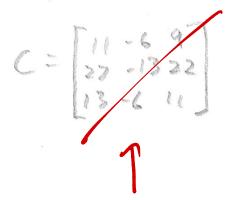
$$C_{21} = 4 \cdot 2 + 5 \cdot 3 + 4 \cdot 1$$

$$C_{22} = 4 \cdot 4 + 5 \cdot 3 + 4 \cdot 1$$

$$C_{23} = 4 \cdot 2 + 5 \cdot 2 + 4 \cdot 1$$

$$C_{23} = 4 \cdot 2 + 5 \cdot 2 + 4 \cdot 1$$

$$C_{23} = 4 \cdot 2 + 5 \cdot 2 + 4 \cdot 1$$



Metrix multiplication is incorrect too.



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$. where C, C C

$$T(e_1) = T([6]) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} A = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$$

 $T(e_2) = T([1]) = \begin{bmatrix} -2 \\ 1 & -3 \end{bmatrix}$

b)
$$T(u) = \begin{bmatrix} 1 & 0 \\ -2 \end{bmatrix}$$

Where did this come from?

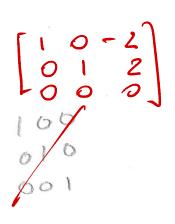


5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

bill one to one





6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

False take W= & U= 1 V= 3 W+V= ? E Spar (U, V) but

W= V-U, so We Spen (u,v)

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

set {u, v} must be linearly independent.

Let X, V = 3

X2, V + X3, V = 3

X3, V = 3

X3, V = 3

X, V + X₂V = 0

X, V = X₂V, SO X, V is a Scalar ?

multiple

ok
X₂V



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

Failse

A=[6] v=[6] v=(1]

Au, Av are dereding
but V, V are inderedged

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

$$V = \begin{bmatrix} 2 \\ 2 \end{bmatrix} V = \begin{bmatrix} 1 \\ 1 \end{bmatrix} V = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

U & Span (V. W) What is There?

T(U) & Span (T(U), T(W))