

2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

3×2 2×3

~~$$A^T A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$$~~

$$A^T A x = A^T b$$

$$b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$0 + (-2) + 2$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

~~$$\begin{array}{l} \times 3 \\ \frac{1}{2} \left(\begin{bmatrix} 2 & 0 & 3 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 3 & -1 & 5 & | & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & -1 & -4 & | & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & -1 & -4 & | & 1 \end{bmatrix} \xrightarrow{\times 1} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & -9/2 & | & 2 \end{bmatrix} \xrightarrow{-9/2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & -4/9 \end{bmatrix} \end{array}$$~~

~~$$\rightarrow \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & -4/9 \end{bmatrix} \xrightarrow{-1/2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & 0 & | & 7/9 \\ 0 & 0 & 1 & | & -4/9 \end{bmatrix} \xrightarrow{3/2} \begin{bmatrix} 1 & 0 & 0 & | & 3/2 \\ 0 & 1 & 0 & | & 7/9 \\ 0 & 0 & 1 & | & -4/9 \end{bmatrix}$$~~

$x_1 = 3/2$
 $x_2 = 7/9$
 $x_3 = -4/9$

$$\frac{9}{10} \quad \frac{2}{9} \quad \frac{3}{2}$$

$$\left(\frac{3}{2}, 0 \right) \quad \left(\frac{7}{9}, 2 \right) \quad \left(-\frac{4}{9}, 1 \right)$$

$$a = \frac{1 - 0}{\frac{9}{18} - \frac{36}{18}} = \frac{1}{-\frac{44}{18}}$$

$$b = 0$$

$$f(x) = 0.4x + 0$$

10/20