5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and **w** is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then **w** must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

False. let $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$. $\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ 2 & -\lambda \end{vmatrix} = -\lambda(1-\lambda) - 4 = -\lambda + \lambda^2 - 2 = 0 \leftarrow (\lambda-2)(\lambda+1)$

$\lambda = 2 \quad \lambda = -1$

+3

For $\lambda = -1 \rightarrow \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

For $\lambda = -2 \rightarrow \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \neq 2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

why?

True. For any u such that $\text{proj}_V u = v$, $u \cdot v \geq 0$. $\therefore w \cdot -w \leq 0$ so the only way for this to be true is if $w = 0$

+4

True. If A is orthogonal and symmetric then it is of the form PDP^T. $A^2 = PD^2P^T$. Since P is an orthogonal basis for A, PD^2P^T is going to be the identity matrix

~~False~~. let $A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$. Both A and B are orthogonaly diagonalizable, let $W = A + B \rightarrow \begin{bmatrix} 1+1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -2\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1+1/\sqrt{2} \end{bmatrix}$. Though W is symmetrical, the columns of W arent orthogonal to each other so W isn't orthogonally diagonalizable.