3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad A^T C = B \qquad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T: \begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix} \cdot C = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{matrix}$$

$(AB)^T = B^T A^T$

$2 \times 4 = 8$

$A^T \cdot C = B$

$\frac{B}{A^T} = C = B \cdot (A^T)^{-1}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = 0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 0\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$-1 + 2 + 3 = 4$
$-4 + 5 + 4 = 5$
$-3 + 2 + 1 = 0$

$$\begin{bmatrix} -1 & 4 & 3 \\ 1 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix} = C$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad \therefore A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 & c_4 & c_7 \\ c_2 & c_5 & c_8 \\ c_3 & c_6 & c_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^T C = B$

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A\ A^T$

$(A^T)(A^T)^{-1} C = B(A^T)^{-1}.$

$C = B(A^T)^{-1}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

invertible.

$\cancel{(A^T)^{-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] = [}$

$(A^T)^{-1} = (A^{-1})^T.$

$\cancel{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}}$

$C = B(A^{-1})^T$

$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \cancel{\begin{bmatrix} 1\times -2 + 1\times 1 + 0\times 1 & 1\times 1 + 1\times -1 + 0\times 1 & 1\times 2 + 1\times -2 + 0 \\ -1\times 2 + 3\times 0 + 2\times -1 & -1\times 1 + 0\times -1 + 2\times 1 & -1\times 2 + 0\times -2 + 2\times 1 \\ 2\times -2 + 1\times 3 + -1\times -1 & 2\times 1 + 1\times -1 + -1\times 1 & 2\times 2 + 1\times -2 + -1\times 1 \end{bmatrix}}$

$= \begin{bmatrix} 1\times -2 + 2\times 3 + 3\times -1 & 1\times 1 + 2\times -1 + 3\times 1 & 1\times 2 + 2\times -2 + 3\times 1 \\ 4\times -2 + 5\times 3 + 4\times -1 & 4\times 1 + 5\times -1 + 4\times 1 & 4\times 2 + 5\times -2 + 4\times 1 \\ 3\times -2 + 2\times 3 + 1\times -1 & 3\times 1 + 2\times -1 + 1\times 1 & 1\times 2 + 2\times -2 + 1\times 1 \end{bmatrix}.$

$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 1 \end{bmatrix}.$

0	1	2	3	4	5	6	7	8	9	10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \qquad \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{vmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array}\right] \qquad \begin{matrix} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{matrix}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \qquad (A^T)^{-1} = \begin{vmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B \qquad \begin{vmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix}$$

$$\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{bmatrix}$$

$(-2 + 4 + 6)$ $(-4 + 5 \;\; 4)$ $(-6 + 4 + 2)$

$(3 + -4 - 6)$ $(6 - 5 - 4)$ $(9 - 4 - 2)$

$(-1 + 4 + 3)$ $(2 + 5 + 2)$ $(-3 + 4 + 1)$

$$\boxed{C = \begin{vmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{vmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \Rightarrow (A^{-1})^T = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{-1})^T \cdot B = C \qquad \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+8+6 & -4+10+4 & -6+8+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 12 & 10 & 4 \\ -7 & -3 & 3 \\ 6 & 2 & 2 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^T C = B$

$C = (A^{-1})^T B$

Based on problem 2, $A^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 & 0 \end{bmatrix}$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$C = (A^{-1})^T B = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 4 & 6 \\ 4 & -\frac{5}{2} & -2 \\ 0 & 2 & 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

$B \cdot \frac{1}{A^T}$

$B \cdot A^{T^{-1}}$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A^T C = B \rightarrow C = \frac{B}{A^T}$ $\quad C = B \cdot (A^T)^{-1}$

$C = B \cdot (A^{-1})^T$

$$A^{-1} = \begin{matrix} 2 & 3 & 1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{matrix}$$

$$(A^{-1})^T = \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline c_1 & 11 & -6 & 9 \\ c_2 & 27 & -13 & 22 \\ c_3 & 13 & -6 & 11 \end{array}$$

$C_{11} = \overset{2}{1 \cdot 2} + \overset{6}{2 \cdot 3} + \overset{3}{1 \cdot 3}$

$C_{12} = \overset{-1}{-1 \cdot 1} + \overset{-2}{2 \cdot -1} + \overset{-3}{3 \cdot -1}$

$C_{13} = \overset{2}{2 \cdot 1} + \overset{4}{2 \cdot 2} + \overset{3}{3 \cdot 1}$

$C_{21} = \overset{8}{4 \cdot 2} + \overset{15}{5 \cdot 3} + \overset{4}{4 \cdot 1}$

$C_{22} = \overset{-4}{4 \cdot -1} + \overset{-5}{5 \cdot -1} + \overset{-4}{4 \cdot -1}$

$C_{23} = \overset{8}{4 \cdot 2} + \overset{10}{5 \cdot 2} + \overset{4}{4 \cdot 1}$

$C_{31} = \overset{6}{3 \cdot 2} + \overset{6}{3 \times 2} + \overset{1}{1 \cdot 1} = 13$

$C_{32} = -3 + -2 + -1 = -6$

$C_{33} = 6 + 4 + 1 = 11$

$$C = \begin{bmatrix} 11 & -6 & 9 \\ 27 & -13 & 22 \\ 13 & -6 & 11 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

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3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right] \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & -5 & -5 & -8 & -7 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 1 & -5 & -5 & -8 & -7 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 & 2 & 4 & 5 & 4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & -3 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -5+\frac{1}{3} & -4 \\ 0 & 1 & 0 & 5 & 7+\frac{1}{3} & 7 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \end{array}\right]$$

$$-\frac{15}{3} - \frac{1}{3} = -\frac{16}{3}$$

$$\frac{21}{3} + \frac{1}{3}$$

$$C = \begin{bmatrix} -4 & -\frac{16}{3} & -4 \\ 5 & \frac{22}{3} & 7 \\ 0 & -\frac{1}{3} & 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \qquad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$1\cdot\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 2 & 1 \end{array}\right]$$

$$-1\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$-1\cdot\left[\begin{array}{ccc|ccc} -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$-2\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$2\cdot\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$A^T C = B$$

$$C = (A^T)^{-1} B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left.\begin{matrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{matrix}\right|$$

0 1 2 3 4 5 6 7 8 9 10