3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$ b) $\lambda = -1$ c) $\lambda = -2$

$$\det(A - \lambda I) = \det \begin{bmatrix} 0-\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix} = \det \begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix} \to \text{Pick this row.}$$

$$\det = (1)\det\begin{bmatrix} 1 & 2 \\ 2 & 2-\lambda \end{bmatrix}(-1^3) + (1-\lambda)\det\begin{bmatrix} -\lambda & 2 \\ 4 & 2-\lambda \end{bmatrix}(-1^4) + 0$$

$$= (2-\lambda-4)(1)(-1) + (1-\lambda)(\lambda^2 - 2\lambda - 8)$$

$-\lambda^3 + 2\lambda^2 + 8\lambda + \lambda^2 - 2\lambda - 8$

$-\lambda^3 + 3\lambda^2 + 6\lambda - 8$

$$= (-2-\lambda)(-1) + (-\lambda^3 + 2\lambda^2 + 8\lambda + \lambda^2 - 2\lambda - 8)$$

$$= (\lambda + 2) + (-\lambda^3 + 3\lambda^2 + 6\lambda - 8) = -\lambda^3 + 3\lambda^2 + 7\lambda - 6 = 0 \checkmark$$

Check: $\lambda = 0$

$-0 + 0 + 0 - 6 = 0$

$-6 = 0$ x

check: $\lambda = -1$

$1 + 3 - 7 - 6 = 0$

$-9 = 0$

Check $\lambda = -2$

$8 + 12 + (-14) - 6 = 0$

$0 = 0 \checkmark$

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Only $\lambda = -2$ is an eigen val for this one.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20