

## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:  Luke Baldy  UB Person Number:	Instructions:					
5 0 7 7 9 9 8 7  0 0 0 0 0 0 0 0 0  1 1 1 1 1 1 1 1 1  2 2 0 2 2 2 2 2 2  3 3 3 3 3 3 3 3 3  4 4 4 4 4 4 4 4 4  5 5 5 5 5 5  6 6 6 6 6 6 6 6 6 6  7 7 7 9 7 9 7 7 9  8 8 8 8 8 8 8 8 8 8 8  9 9 9 9 9 9 9	<ul> <li>Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.</li> <li>For full credit solve each problem fully, showing all relevant work.</li> </ul>					
1 2 3 4 5	6 7 TOTAL GRADE					

9	10	4	20	20	2	5			70	B-
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \left[ \begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[ \begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[ \begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[ \begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

$$\begin{vmatrix}
1 & -1 & 1 & | & -2 & | \\
0 & 1 & 2 & | & 2 & | \\
2 & -3 & 0 & | & 6
\end{vmatrix}$$

$$C_3 = \frac{b_{-1}}{11}$$
 $C_1 = 2 - 2C_3$ 
 $C_1 = -2 + C_2 - C_3$ 

$$0 = (b-6)$$



## 2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

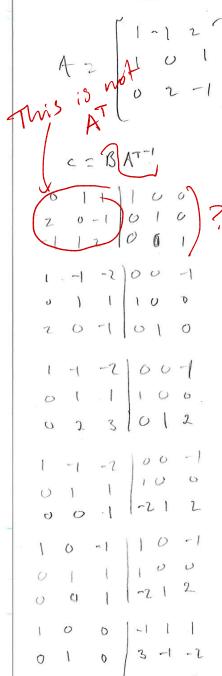
Compute  $A^{-1}$ .

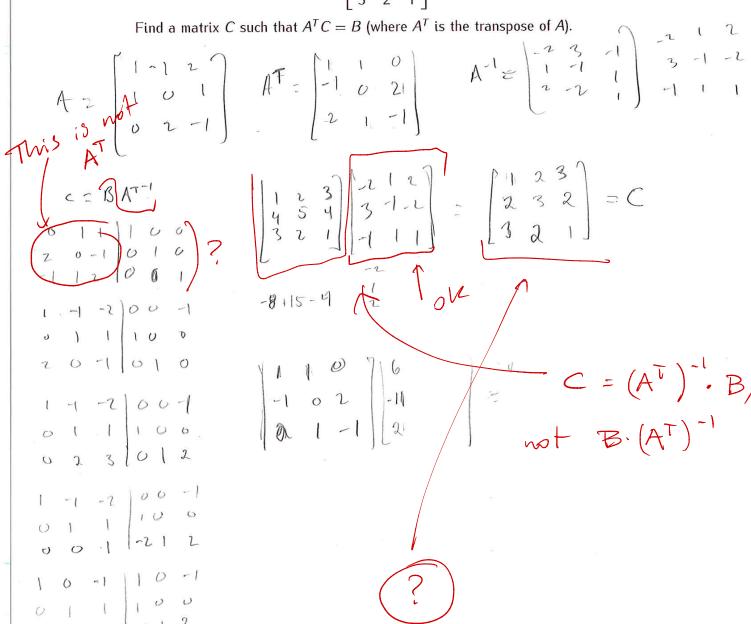
$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$







4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$
 $u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ 

 $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ 



5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A : \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(v) = Av$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$ .

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

b) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$T(V_1) : T(V_2) = 0$$

$$T(V_1 - V_2) : 0$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 6 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} =$$

b) not one-to-one
$$V_{1} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad V_{2} = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$$



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u,v,w are vectors in  $\mathbb{R}^3$  such that  $w+u\in Span(u,v)$  then  $w\in Span(u,v)$ .

United True V

b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

False - why?

b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True CIV+CIW=U

CIT(V)+CIT(W)=T(u) - 30?