MTH 528 TOPOLOGY II

MIDTERM EXAM

March 28, 2019

Name:

UB Person Number:

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Each problem is worth 20 points.
- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1. Let A be a matrix and $\mathbf{v}$ be a vector given as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

a) Determine if $\mathbf{v}$ is in $\mathrm{Col}(A)$, where $\mathrm{Col}(A)$ is the column space of A.

b) Determine if $\mathbf{v}$ is in $\mathrm{Nul}(A)$, where $\mathrm{Nul}(A)$ is the null space of A.

c) Find an explicit description of $\mathrm{Nul}(A)$ by listing vectors that span the null space.

2. Consider the following matrices:

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 3 \\ 0 & 2 & 2 \end{bmatrix}$$

a) Compute the inverse of B.

b) Find the matrix A such that $(BA)^T = C$.

3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which first reflects points through the line $x_1 = x_2$ and then reflects points through the x_1-axis.

a) Find the standard A matrix of T.

b) Find all vectors $\mathbf{v} \in \mathbb{R}^2$ such that $\mathbf{v} \in \mathrm{Nul}(A)$.

4. Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Let $b = 7$. Express $\mathbf{w}$ as a linear combination of vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

c) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A, B are 2×2 matrices such that $AB = 0$ is the zero matrix (e.i. the matrix with all entries equal to zero) then either A or B must be the zero matrix.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v} + \mathbf{w}\}$ must be linearly independent.

c) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

d) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $\mathbf{u}$ must be also in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.