**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False

$w+u \in \text{span}(u,v)$

$A(v+w) = Av + Aw$

$w \in \text{Span}(u,v)$

$w = cu + cv$

$w+u \in \text{Span}(u,v) \not\Rightarrow w \in \text{Span}(u,v)$

$w+u \in \text{Span}(u,v)$

$w+u = cu + cv$

Counter example

$w \in \text{Span}(u,v) + u \notin \text{Span}(u,v)$

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True

$\{u, v, w\}$ is linearly independent, i.e. every column is a pivot column and there is only one solution

$\{u, v\}$ must also be linearly independent because without $w$ there will still be a pivot column in every column

ex)

$$\{u, v, w\} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{pivot columns}} \qquad \{u, v\} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{pivot columns}}$$

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

It is ~~true~~ false. No matter how much you ~~can~~ change it the rule only applies for multiplication

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False. linearly independant means: that there ~~is only one~~ must be infinite solutions. We do not know which vector has a free variable. so we dont know 100 percent of the time.

Exampe
Free variable
u, v, w

u and v can be dependant

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False. because, $w + u = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \in$ span (u,v.

not necessary,

$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \in$ span (u, v).

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False.

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $W = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}$

the set $\{u, v, w\}$ is linearly independent. ~~be~~ cause, $x_1 u + x_2 v + x_3 w = 0$ has only one, trivial solution. on the other, $\{u, v, w\}$ is linearly dependent because $x_1 u + x_2 v + x_3 w = 0$ has non-trival solution

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True. 诗 only $[c_1 u + c_2 v] \in \text{span}(u, v)$

if $w + u \in \text{span}(u, v)$

$w = c_1 u$ or $w = c_2 v$.

$w \in \text{Span}(u, v)$

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False. if $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

~~$2u \quad v + 0w_3 = 0.$ - indep~~

~~$x_1 = 2$~~

~~$x_2 = -1$~~

$u + v + 0w = 0.$ - linear indep.

~~$x_1 u + x_2 v + x_3 w = 0.$ is linear independent.~~

but ~~$x_1 u + x_2 v = 0$ is not linealy independen~~

~~$x_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0.$~~

but $x_1 u + x_2 v$ is not linear idepo.

$x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $x_2 \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ is linear dependent.

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

This statement is false, because $w + u$ will only span $(u, v)$ if $w + u$ is in the null set of $(u, v)$

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

This statement is true.

If $\{u, v, w\}$ is linearly independent then ~~th~~ each column in the aug matrix must have a leading one. If you remove one vector (column), each column will still have a leading one

0 1 2 3 4 5 6 7 8 9 10

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True; since $u \in Span(u,v)$, then if $w+u \in Span(u,v)$ then $w$ must be a multiple of a vector in $Span(u,v)$. Therefore, $w \in Span(u,v)$.

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

A set is lin. ind. when $x_1 u + x_2 v + x_3 w = 0$ has only one solution, where $u, v, w = 0$. If the set $\{u, v, w\}$ is lin. ind., then either combination of two of those vectors must also be lin. ind. Therefore, this statement is true.

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

False take $w = \begin{matrix} 1 \\ 0 \\ 0 \end{matrix}$ $u = \begin{matrix} 1 \\ 1 \\ 0 \end{matrix}$ $v = \begin{matrix} 2 \\ 1 \\ 0 \end{matrix}$

$w + u = \begin{matrix} 2 \\ 1 \\ 0 \end{matrix} \in \text{Span}(u, v)$, but

$w = \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \notin \text{Span}(u, v)$

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$x_1 v + x_2 v + x_3 w = 0$

$3 + 3 + -6 = 0$

Let $x_1 v = 3$

$x_2 v = 3$

$x_3 w = -6$

$x_1 u + x_2 v = 0$

$x_1 u = x_2 v$, so $x_1 u$ is a scalar multiple of $x_2 v$

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

* $\boxed{\text{False}}$

w could be projected outside the span when not added to u

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\boxed{\text{True}}$ because you will still have every column as a pivot column

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

True : because is $w + u$ is
is in Span $(u, v)$ then $u - v = w$
which is also in Span $(u, v)$

**b)** If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True : is $\{u, v, w\}$ are linearly independent
than $\{u, v\}$ is linearly independent because $c_1u + c_2v + c_3w = 0$
would have only one trivial solution, meaning that $c_1u + c_2v + c_3w$
or $c_1u + c_3w$ or $c_1u + c_2v$ or $c_2v + c_3w$ have no combination to sum to 0 other
than multiplying by 0, if they did than $\{u, v, w\}$ would
have another solution to $c_1u + c_2v + c_3w = 0$ which
would make $\{u, v, w\}$ linearly dependent. Since $c_1u + c_2v$
have no combination to 0 other than 0
they are linearly independent

0 1 2 3 4 5 6 7 8 9 10

**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $u, v, w$ are vectors in $\mathbb{R}^3$ such that $w + u \in \mathrm{Span}(u, v)$ then $w \in \mathrm{Span}(u, v)$.

FALSE

**b)** If $u, v, w$ are vectors in $\mathbb{R}^3$ such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

FALSE

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10