$$\begin{bmatrix} -2 & 4 & 2 \\ -1 & 3 & 2 \\ 1 & -4 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 4 & 2 \\ -1 & 3 & 2 \\ 0 & -1 & 5 \end{bmatrix}$$

$$\Rightarrow \cancel{-2 \to 3}$$

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

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$$\lambda = 3 \quad \begin{bmatrix} 1-\lambda & 8 & 4 \\ -2 & 11-\lambda & 4 \\ 2 & -8 & -1-\lambda \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 0 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & -8 & -4 \end{bmatrix}$$

$2x_1 - 8x_2 + 4x_3 = 0$ or $x_1 - 4x_2 + 2x_3 = 0$

$$x_1 = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda = 5 \quad \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \sim \cancel{\begin{bmatrix} -4 & 8 & 4 \\ 0 & -2 & -2 \\ 2 & -8 & -6 \end{bmatrix}} \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ -2 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 0 & -6 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 2 \\ -1 & 3 & 2 \\ 1 & -4 & -3 \end{bmatrix} \sim \begin{bmatrix} -2 & 2 & 0 \\ -2 & 6 & 4 \\ 0 & -6 & -6 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 0 \\ -1 & 3 & 2 \\ 0 & -3 & -3 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & -3 & -3 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -1 \\ -3 & 0 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -3 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_2 = x_3$.

$x_1 = -x_3$

$x_1 = x_3$.

$$\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \quad P_1 = \begin{bmatrix} 4/\sqrt{17} \\ 1/\sqrt{17} \\ 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} -2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{bmatrix} \quad P_3 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20