

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:	
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UB Person Number:	Instructions:
50177958	 Textbooks, calculators and any other electronic devices are not permitted.

5	0	1	7	7	9	5	8
0	6	0	0	0	0	0	0
1	1		1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
6	(5)	(5)	(5)	(5)	(5)	(5)	(5)
6	6	6	6	6	6	6	6
7	7	7	0	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

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 For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.

a)
$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix}$$

$$R_3 = R_3 - 2R_1$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix}$$

$$b+6=0$$

$$b=-6$$

b)
$$X | V_1 + X_2 V_2 + X_3 V_3 = 0$$

aug matrix

$$\begin{bmatrix} V_1 & V_2 & V_3 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$
The Set $\mathcal{E}_{V_1 V_{e_1} V_3 S}$ is not indiverdent



2. (10 points) Consider the following matrix:

$$A = \left[\begin{array}{rrr} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right]$$

Compute A^{-1} .

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A). $C = BA^{-1}$

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 - 1 \end{bmatrix}$$
3×3

C is a
$$3\times3$$
 matrix
$$\begin{bmatrix} 1 & 10 \\ -1 & 02 \\ 2 & 1-1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{T} C B$$

$$C = \begin{bmatrix} 6 & 12 & 0 \\ 5 & 25 & 0 \\ -2 & 14 & 0 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 3 \times 2 \end{bmatrix} \begin{bmatrix} 2 \times 1 \end{bmatrix} = \begin{bmatrix} 3 \times 1 \end{bmatrix}$$

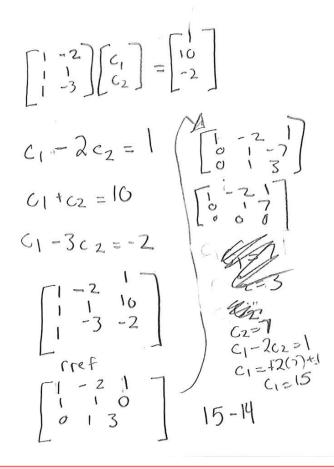
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$
 $3 \times 2 \quad 2 \times 1 \quad 3 \times 1$

Mere $f(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

Would solve it $U = \begin{bmatrix} 77 \\ 3 \end{bmatrix}$





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

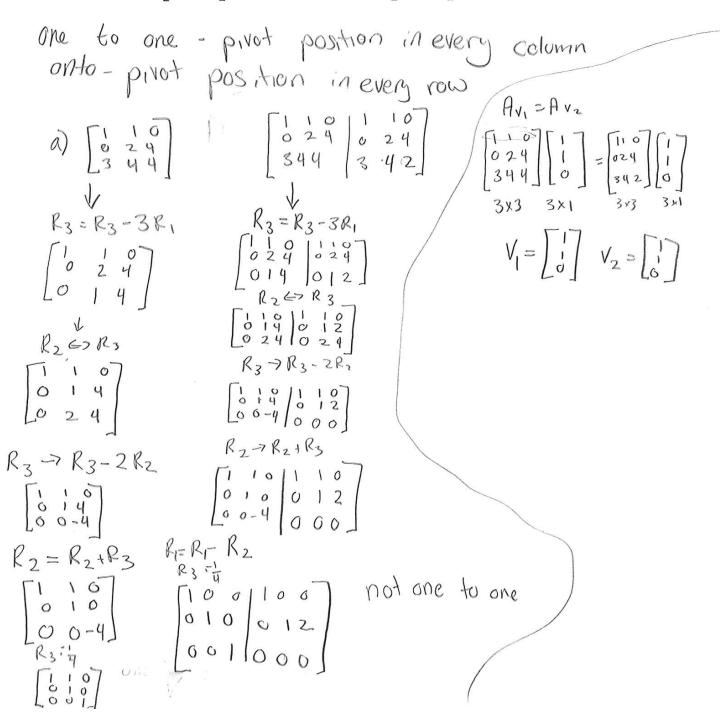
1

2

3

6

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$



10 11 12 13 14 15 16

17 18

19



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u,v,w are vectors in \mathbb{R}^3 such that $w+u\in Span(u,v)$ then $w\in Span(u,v).$

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

true. there would be no free variables when reducing the set to Eu, v, w} to Eu, v, w} to Eu, v, w}



- **7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

False

if transformation is 2×2 matrix

it could take u out of

Appen of vandor