



# MTH 309T LINEAR ALGEBRA

## EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	9	0	3	5	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1 2 3 4 5 6 7 TOTAL GRADE

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19

10

10

20

18

3

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2

10

92

A-

1

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3

4

5

6

7

PIAZZA

HILL

TOTAL

GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of  $b$  such that  $w \in \text{Span}(v_1, v_2, v_3)$ .

b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

$$\begin{aligned} & \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{\substack{R_3 - R_1 \\ R_2 + R_1}} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 4 & b+6 \end{bmatrix} \end{aligned}$$

$w \in \text{Span}(v_1, v_2, v_3)$  as long as  $b = -6$  if  $b \neq -6$  then  
no solution

the set of vectors is linearly dependent  
because even if you had  $b = -6$  then there  
would still be a free variable ✓

if  $w \in \{v_1, v_2, v_3\}$  then  $w = c_1 v_1 + c_2 v_2 + c_3 v_3$   
and there is a nontrivial answer to this equation



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$\begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 + R_1 \\ R_2 + R_1}} \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{2R_2 + R_3} \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 1 & 0 & | & -1 & 2 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & -1 & 2 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 & | & 2 & 3 & -1 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}$$



3. (10 points) Let  $A$  be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad 2 \times 3$$

Find a matrix  $C$  such that  $A^T C = B$  (where  $A^T$  is the transpose of  $A$ ).

~~$A^T$~~   $C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$  ✓

$(A^{-1})^T$   $B$

$$\begin{bmatrix} -2+4+3 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$C = \begin{bmatrix} 5 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \checkmark \quad u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

b)  $\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{-R_1+R_3} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 1 \end{array} \right] \xrightarrow{-R_1+R_2} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 1 \end{array} \right]$

$\begin{cases} x_1 = 7 \\ x_2 = 3 \end{cases} \rightarrow$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix} =$

$7 - 2(3) = 1$

$7 + 3 = 10$

$7 - (3)(3) = -2$



5. (20 points) For each matrix  $A$  given below determine if the matrix transformation  $T_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by  $T_A(v) = Av$  is one-to-one or not. If  $T_A$  is not one-to-one, find two vectors  $v_1$  and  $v_2$  such that  $T_A(v_1) = T_A(v_2)$ .  $v_1 \neq v_2$

a)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$\xrightarrow{3R_1 + R_3}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{-R_2 + R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\frac{R_3}{4}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Ⓐ is one-to-one ✓

b)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{-2R_3 + R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\downarrow} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Ⓑ is not one-to-one ✓

$$x_1 = -2$$

$$x_2 = 2$$

$$x_3 = 1$$

$$v_1 = \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

2                      1



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True <sup>✓</sup> if  $u$  is in the span as well then  $w + u \in \text{Span}(u, v)$

Because

$$\begin{aligned} w &= \underline{-u + v} \quad ? \\ w + u &= \underline{v} \quad ? \end{aligned} \quad \S \quad -u, v \text{ are in the span of } u, v$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

~~false~~

$$\begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} & \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

$u, v$  are dependent

$u, v, w$  are  
~~independent~~  
dependent





7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $A$  is a  $2 \times 2$  matrix and  $u, v$  are vectors in  $\mathbb{R}^2$  such that  $Au, Av$  are linearly dependent then  $u, v$  also must be linearly dependent.

~~True~~ Because  $A$  stay the same so in order to keep  $Au, Av$  dependent  $u, v$  must be dependent  
 ↑  
 Not necessarily

b) If  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that  $u$  is in  $\text{Span}(v, w)$  then  $T(u)$  must be in  $\text{Span}(T(v), T(w))$ .

True<sup>✓</sup> is  $T$  is a translation done to all  
vectors then the vectors will change  
 in the same magnitude  
 ↑  
 Translation is not a linear transformation.