

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then $2v$ is an eigenvector of A corresponding to the eigenvalue 2λ .

+5 False. The eigenvector $2v$ still corresponds to the eigenvalue λ . Consider (A) below.

b) If V is a subspace of \mathbb{R}^2 and w is a vector such that $\text{proj}_V w = -w$ then w must be the zero vector.

True, see (B) below.

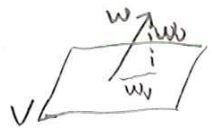
c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

+5 True by definition to determine if a sym. matrix is orthogonal, you compute $A^T A$ and if you get identity matrix it's orthogonal. Because $A^T = A$ then $A^2 = A^T A$ therefore it will be identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

+5 By definition a matrix is orthogonally diagonalizable if it is symmetric and every symmetric matrix is diagonalizable, so the answer is true because any 2×2 symmetric matrix for example: $\begin{bmatrix} A & C \\ C & B \end{bmatrix} + \begin{bmatrix} D & F \\ F & E \end{bmatrix} = \begin{bmatrix} A+D & C+F \\ C+F & B+E \end{bmatrix}$ added to another symmetric matrix is still symmetric. TRUE.

A) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ $\lambda^2 - 5\lambda + 6$ $\begin{bmatrix} -1 & 2 & 0 \\ 1 & 2 & 0 \end{bmatrix} \rightarrow x_1 = -2x_2$
 $\lambda = 2, 3$ but $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ is still an eigenvector $\rightarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix} \leftarrow$ eigenvector
 that corresponds to λ , not 2λ . FALSE



In order for $\text{proj}_V w$ to be $-w$ we would have to have $w \in -w$. That isn't possible unless w is the zero vector because $(w - \text{proj}_V w)$ must be in V^\perp and not in V . Unless it's the zero vector. TRUE.