2. **(10 points)** Consider the following matrix:

$$A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \end{array}$$

Compute A^{-1}.

$$A^{-1} = \frac{1}{|A|} \cdot A^T$$

$$|A| = 1.[0.(-1) - 2.1] - (-1)[(1)(-1) - \cancel{1.0}] + 2[1.2 - \cancel{0.0}]$$

$$= -2 - 1 + 4 = 1$$

DETERMINENT A $\Rightarrow |A| = 1$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad A^{-1} = \frac{1}{1} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

TO VERIFY $A . A^{-1} = I$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1+1+2 & 1+0+2 & 0-2-2 \\ 1+0+2 & 1+0+1 & 0+0-1 \\ 0-2-2 & 0+0-1 & 0+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & -4 \\ 3 & 2 & -1 \\ -4 & -1 & 5 \end{bmatrix} \xrightarrow{\text{ROW REDUCTION (ON SCRAP PAPER)}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

☐ 0 ☐ 1 ☑ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^3$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A \mid e_1 e_2 e_3] = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & -1 & 1 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

$(-1)(-2) = 2$ $\quad 2(-1)+1 =$

$-2(1) = -2$ $\quad -2$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$(-1)(-2) = 2 - 1 = 1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = -2(R_2) + R_3$

$R_2 = -1R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$R_1 = 2R_2 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = -R_2 + R_1$

$$\boxed{A^{-1} = \begin{array}{ccc} 0 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{array}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

?

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\frac{1}{2}R_3]{-R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & 0 & 1/2 \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1/2 & 1 & -1 & 1/2 \end{array}\right]$$

$$\xrightarrow{2R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow[-2R_3+R_1]{R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

√

☒ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

(-1)
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-2+1$
$1 \cdot -1 = -1$
(-2)
$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$(1)\cdot$
$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$-1+2$
$-1+1$
$(1)\cdot$
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$(-1)\cdot$
$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

-2
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\substack{(-1)\cdot r1 \\ +r2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{r2+r1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\substack{(-2)\cdot r2 \\ +r3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{r3+r2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{-r3+r1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & -2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot 1$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -2 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot 1 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot -1$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & -3 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ -3 & 3 & -1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3x3

A^T or A^{-1}?

Compute $\boxed{A^{-1}}$.

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

3x3.

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-1) = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-2) = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= (1) \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] = (-2) \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] = (1) \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

~~$x_1 - x_2 = 1$~~

~~$-x_1 + x_2 = 0$~~

~~$x_2 = x_1$~~

~~$x_1 - x_1 = 1$~~

~~$0 \neq 1$~~

~~$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$~~

~~$x\begin{bmatrix} 1 \\ -1 \end{bmatrix} + x\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$~~

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1}\quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_2 = r_2 - r_1$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_1 = r_1 + r_2$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad r_3 = r_3 - 2r_2$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \quad \begin{array}{l} r_1 = r_1 - r_3 \\ r_2 = r_2 + r_3 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{matrix}}_{A^{-1}} \\ 0 & 0 & 1 & \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -1$$

$$\downarrow$$

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -2$$

$$\downarrow$$

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot 1 \quad \cdot -2$$

$$\downarrow$$

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot 1$$

$$\downarrow$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{\begin{bmatrix} -2 & 2 & -1 \\ 1 & -2 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}}$$

0 1 2 3 4 5 6 7 8 9 10

7

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A \cdot I = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & -1 & 2 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 = R_3 + (-R_1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + (-R_3)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-R_1+R_2]{R_2=} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow R_1 = R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xleftarrow{R_3 = -2R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow R_2 = R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_1 = R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

$-1-2(-1)$
$=-1+2$
$=1$
$0-2(-1)$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - R_1} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_3} = \begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 + R_3} = \begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

Reduction

$$\begin{matrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 0 & -3 & 4 & 2 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & -2 & 3 & 2 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{matrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

Check:

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{matrix} \qquad \begin{matrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{matrix} \qquad \begin{matrix} 1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{matrix} \qquad \begin{matrix} -2 \\ 1 \\ 2 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{matrix} \qquad \begin{matrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{matrix} \qquad \begin{matrix} 1 & -1 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{matrix} \qquad \begin{matrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{matrix} \qquad \begin{matrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{matrix} \qquad \begin{matrix} 1 & -1 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{matrix} \qquad \begin{matrix} -1 \\ 1 \\ 1 \end{matrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix}^{2} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 2 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}^{\times -2}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{\times -1} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ A is inverstible.

$$C_1 : \begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix}^{\cdot -2} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}^{-1} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$C_2 : \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 0 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 2 & -1 \\ 2 & 0 & 3 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ \cancel{2} & \cancel{0} & \cancel{3} & \cancel{0} \end{bmatrix} - \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & \\ \cancel{1} & \cancel{0} & \cancel{2} & \cancel{-1} & \\ 0 & 0 & 1 & 0 & \end{bmatrix} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]^{-1}$$

C_2

$$C_3 \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix}^{2} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 0 & 3 & 1 \end{bmatrix}^{\times -2}$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix} + \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}^{\times -2} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ — invertible}$$

To find A^{-1},
$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$4+2 \quad 4-1 \quad -2+0$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

check

$$A \cdot A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1\times-2+-1\times1+2\times2 & 1\times3+-1\times-1+2\times-2 & 1\times-1+-1\times1+2\times1 \\ 1\times-2+0\times1+1\times2 & 1\times3+0\times-1+1\times-2 & 1\times-1+0\times1+1\times1 \\ 0\times-2+2\times1+-1\times2 & 0\times3+2\times-1+-1\times-2 & 0\times-1+2\times1+-1\times1 \end{bmatrix}$$

$1 \quad 0 \quad 0$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-1) + \longrightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-2) +$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] + (-2) \longleftarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] + (1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] + (1) \longrightarrow \left[\begin{array}{c|c} \underbrace{\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}}_{\text{identity matrix}} & \underbrace{\begin{array}{ccc} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array}}_{A^{-1}} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\cdot(-1)\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\cdot 1\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\cdot(-2)\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\cdot 1\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$\cdot(-1)\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Longrightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

check: $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

0 1 2 3 4 5 6 7 8 9 10

(ad - bc)

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 + (-1)R_1} \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 + (-2)R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_1 + (1)R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ & & & & & \\ & & & & & \end{bmatrix}$$

(ad - bc)

$$A = \begin{bmatrix} \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} \end{bmatrix} \rightarrow \begin{bmatrix} -2 & -1 & 2 \\ -3 & -1 & 2 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow \boxed{A^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}}$$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A^{-1}\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 4 & -2 & 1 \end{bmatrix}$$

$A - D + 2G = 1$
$A + G = 0$
$2D - G = 0$

$-D + G = 1$
$2D - G = 0$

$-2D + 2G = 2$
$2D - G = 0$

$G = 2$
$A = -2$
$D = 1$

$B - E + 2H = 0$
$B + H = 1$
$2E - H = 0$

$2B - 2E + 4H = 0$
$2E - H = 0$

$2B + 3H = 0$
$2B + 2H = 2$

$-H = 2$
$H = -2$

$B = 3$
$E = -1$

$C - F + 2I = 0$
$C + I = 0$
$2F - I = 1$

$2C - 2F + 4I = 0$
$2F - I = 1$

$2C + 3I = 1$
$2C + 2I = 0$

$I = 1$
$C = -1$
$F = 1$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \times 1$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \times 2$$

$-1\cdot 2 = -2$

$1 - -2 = 3$

$-1 \cdot 2 = -2$

$0 - -2 = 2$

$1 \cdot 2 = 2$

$0 - 2 = -2$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{array}\right] \times \frac{1}{3}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array}\right] \, -1$$

$\frac{2}{3} \cdot {}^{+}1 = +\frac{2}{3}$

$-\frac{3}{3} + +\frac{2}{3} = -\frac{1}{3}$

$-\frac{3}{3} + \frac{2}{3} = \frac{5}{3}$

$\frac{3}{3} - \frac{4}{3} = -\frac{1}{3}$

$0 - -\frac{4}{3} = \frac{4}{3}$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array}\right] \times 2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -\frac{1}{3} & \frac{4}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array}\right] \times -1$$

$\frac{1}{3} \times -1 = -\frac{1}{3}$

$-\frac{1}{3} + +\frac{1}{3} = 0$

$\frac{5}{3} \times -1 = -\frac{5}{3}$

$\frac{4}{3} + \frac{5}{3} = \frac{9}{3}$

$\frac{1}{3} \times -1 = -\frac{1}{3}$

$-\frac{2}{3} + +\frac{1}{3} = -\frac{1}{3}$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 3 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array}\right]$$

$$\boxed{\therefore A^{-1} = \begin{bmatrix} 0 & 3 & -\frac{1}{3} \\ \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}}_{3\times 3} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3\times 3}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A^{-1}$$

$$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 1 \\ 0 & & \end{bmatrix}$$

We needed to find a matrix s.t it is equal to the Identiy matrix.

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$0-2(-1) = 2$
$0-2(1) = -2$

$1-2$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to R_2 - R_1 \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \to R_1 + R_2 \\ \\ \to R_3 - 2R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{array}{l} \to R_1 - R_3 \to \\ \to R_2 + R_3 \to \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a-d+2g & b-e+2h & c-f+2i \\ a+g & b+h & c+i \\ 2d-g & 2e-h & 2f-i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a-d+2g=1$ $\quad$ $b-e+2h=0$ $\quad$ $c-f+2i=0$

$a+g=0$ $\quad$ $b+h=1$ $\quad$ $c+i=0$

$2d-g=0$ $\quad$ $2e-h=0$ $\quad$ $2f-i=1$

$a=-g$ $\quad$ $b=1-h$ $\quad$ $c=-i$

$d=\frac{1}{2}g$ $\quad$ $e=\frac{1}{2}h$ $\quad$ $f=\frac{1}{2}+\frac{i}{2}$

$-g+\frac{1}{2}g+2g=1$ $\quad$ $1-h-\frac{1}{2}h+2h=0$ $\quad$ $-i-\frac{1}{2}-\frac{i}{2}+2i=0$

$\frac{3}{2}g=1$ $\quad$ $\frac{1}{2}h=-1$ $\quad$ $\frac{1}{2}i=\frac{1}{2}$

$g=\frac{2}{3}$ $\quad$ $h=-2$ $\quad$ $i=1$

$a=-\frac{2}{3}$ $\quad$ $b=3$ $\quad$ $c=-1$

$d=\frac{4}{3}$ $\quad$ $e=-1$ $\quad$ $f=1$

$$\boxed{A^{-1} = \begin{bmatrix} -2/3 & 3 & -1 \\ 4/3 & -1 & 1 \\ -2/3 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 & -2 & 1 \end{array}\right] = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] R_2 - R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] R_3 - 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] R_2 + R_1 : R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] R_1 - R_3 : R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] R_2 : R_2 + R_3$$

$$A^{-1} = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$R_2 = R_1 - R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$R_2 = -R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$R_3 = -2R_1 + R_3 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$R_1 = R_2 + R_1 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$R_2 = R_3 + R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$R_1 = -R_3 + R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Longrightarrow A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & +2 & -2 & 0 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}$$

-2
2-2
-1+$^+$2

2 - 2
2.

-1-2

1-1
1-2
0-2

$$A^{-1} = \begin{bmatrix} -2, & 3, & -1 \\ 1, & -1, & 1 \\ 2, & -2, & 1 \end{bmatrix}$$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 + 2 \cdot R_2]{(-1) \cdot R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow[R_2 + R_3]{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] = [I_3 \mid A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} (r_1c_1) & (r_1c_2) & (r_1c_3) \\ (r_2c_1) & (r_2c_2) & (r_2c_3) \\ (r_3c_1) & (r_3c_2) & (r_3c_3) \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$a_1 - a_4 + 2a_7 = 1$

$a_1 + a_7 = 0$

$2a_4 - a_7 = 0$

$-a_7 - a_4 + 2a_7 = 1$

$1 + a_4 = a_7$

$1 + a_4 = 2a_4$

$a_4 = 1 \quad a_7 = 2 \quad a_1 = -2$

$a_2 - a_5 + 2a_8 = 0$

$a_2 + a_8 = 1$

$2a_5 - a_8 = 0$

$2a_5 = a_8$

$a_8 = 1 - a_2$

$1 - a_8 - \frac{1}{2}a_8 + 2a_8 = 0$

$\frac{1}{2}a_8 = -1$

$a_8 = -2$

$a_5 = -1 \quad a_2 = 3$

$a_3 - a_6 + 2a_9 = 0$

$a_3 + a_9 = 0$

$2a_6 - a_9 = 1$

$a_3 = -a_9$

$a_6 = \frac{1}{2}(1 + a_9)$

$-a_9 - \frac{1}{2}(1 + a_9) + 2a_9 = 0$

$-\frac{1}{2} + \frac{1}{2}a_9 = 0$

$a_9 = 1$

$a_3 = -1$

$a_6 = \frac{1}{2}(1+1) = 1$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}. $A \mid I$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(2)-(1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$(3)-2(2)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xleftarrow{(1)+(2)-(3)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$(2)+(3)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \quad I \mid A^{-1}$$

Check

$A \cdot A^{-1} = I$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$ Ans

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = R_2 - R_1} \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}$$

$R_1 = R_1 + R_2$

$$\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \xrightarrow{R_3 = R_3 - 2R_2} \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}$$

$R_2 = R_2 + R_3$

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}$$

$R_1 = R_1 - R_3$

$$\left|\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right|$$

$$\boxed{A^{-1} = \begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$A \cdot A^{-1} = I$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (-2)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right] (-2)\ (1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right] (1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right]$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

A aug. w/ identity matrix:

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} -1$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} 2 \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} 1 \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} -1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

identity matrix

A^{-1}

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$-1 \; 0 \; -1 \; 0 \; -1 \; 0$
$1 \; -1 \; 2 \; 1 \; 0 \; 0$
———
$0 \; -1 \; 1 \; 1 \; -1 \; 0$

$0 \; -2 \; 2 \; 2 \; -2 \; 0$
$0 \; 2 \; -1 \; 0 \; 0 \; 1$
———
$0 \; 0 \; 1 \; 2 \; -2 \; 1$

$0 \; 1 \; -1 \; -1 \; 1 \; 0$
$0 \; 0 \; 1 \; 2 \; -2 \; 1$
———
$0 \; 1 \; 0 \; 1 \; -1 \; 1$

$1 \; 0 \; 1 \; 0 \; 1 \; 0$
$0 \; 0 \; -1 \; -2 \; 2 \; -1$
———
$1 \; 0 \; 0 \; -2 \; 3 \; -1$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] R2 \cdot \tfrac{1}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] R3 \cdot -2 \cdot R2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] +\tfrac{1}{2}R3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] R1 + R2 \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] R1 - 2R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (\cdot -1) \Rightarrow \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] (\cdot 2)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] (\cdot -1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] (\cdot -1) \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot(-1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{row reduction}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}\right]$$

pivot in each row and column so one-to-one and onto

A is invertible

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot 2 \Rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array}\right] \cdot(-1) \Rightarrow \left[\begin{array}{ccc|ccc} -1 & -1 & -1 & -2 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} -1 & -1 & -1 & -2 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot(-1) \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot(-1)$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot(-1) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$A \cdot A^{-1} = I$ where I is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\leftarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{\cdot +1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{+} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \cdot -1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \cdot 1$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \cdot -1$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \cdot -2$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] R_2 = -R_1 + R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] R_3 = 2R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \cdot -1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] R_2 = R_3 + R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] R_1 = -R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$\det(A) = 1(0-2) - (-1)(-1-0) + 2(2-0)$

$= -2 - 1 + 4 = 4 - 3 = \boxed{1} \rightarrow A^{-1}$ is possible

~~$a_{11} = \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = \boxed{-2}$ $a_{12} = \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = \boxed{-1}$ $a_{13} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = \boxed{2}$~~

~~$a_{21} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 4-1 = \boxed{3}$ $a_{22} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = \boxed{-1}$ $a_{23} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2-0 = \boxed{2}$~~

~~$a_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = \boxed{-1}$ $a_{32} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1-2 = \boxed{-1}$ $a_{33} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 0-(-1) = \boxed{1}$~~

~~Now, $\text{adj}(A) = \begin{bmatrix} -2 & -1 & 2 \\ 3 & -1 & 2 \\ -1 & -1 & 1 \end{bmatrix} \times [$~~

$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$a_{11} = \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = \boxed{-2}$ $a_{12} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 1-4 = \boxed{-3}$ $a_{13} = \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = \boxed{-1}$

$a_{21} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = \boxed{-1}$ $a_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = \boxed{-1}$ $a_{23} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1-2 = \boxed{-1}$

$a_{31} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = \boxed{2}$ $a_{32} = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = \boxed{2}$ $a_{33} = \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -(-1) = \boxed{1}$

$\therefore \text{adj}(A) = \begin{bmatrix} -2 & -3 & -1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{\text{adj}(A)}{\det(A)} = \dfrac{1}{1} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$ Ans

Quick check: $\left[(1\ -1\ 2)\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}\right] = [-2-1+4] = [-3+4] = [1]$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$AA^{-1} = I$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \times 2 \\ )+ \end{array} \longrightarrow$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right])- \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right])+ \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 2 & 0 & 2 & -2 & 2 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left|\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right| \xrightarrow[R_1 \times -1 + R_2]{} \left|\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right| \xrightarrow[R_2 \times -2 + R_3]{} \left|\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right|$$

$R_3 \times 1 + R_2$

$$\left|\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right| \xrightarrow[R_3 \times -2 + R_1]{} \left|\begin{array}{cccccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right| \xrightarrow[R_2 \times 1 + R_1]{} \left|\begin{array}{cccccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right|$$

$$A^{-1} = \left|\begin{array}{ccc} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array}\right|$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{①+③} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{①\cdot 2} \left[\begin{array}{ccc|ccc} 2 & 2 & 2 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{①+③(-1)} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{①+(-1)③} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(-1)①+2②} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{②+③} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 1 & 2 & 0 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{②+(-1)①} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 & -1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{③+(-1)②} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 & -1 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{②\cdot\frac{1}{2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right] \xrightarrow{③\cdot(-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 & 0 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{matrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 2 & 3 & 1 \\ 0 & -1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad R2 \leftrightarrow R3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R3 \to -R1+R3 \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$R2 = \frac{R2}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{-1}{2} & 0 & 0 & -\frac{1}{2} \end{array}\right] \leftarrow \begin{array}{l} R1 \to R2+R1 \\ R3 \to -R2+R3 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$R3 \to \frac{-2}{1} \cdot R3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R2 \to \frac{1}{2}R3+R2 \\ R1 \to \frac{-3}{2}R3+R1 \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{array}{cc} A & I \\ \end{array}$$

$R1 \cdot -1 + R2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R2 \cdot -2 + R3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R2+R1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R3+R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R3 \cdot -1 + R1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

Identity Matrix

$$[\,A \mid I\,] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -1$$

$$\xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array}\right]$$

Identity Matrix

$\therefore$ A is invertible and

$$A^{-1} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & -1 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{\cdot -2,\ \cdot 1}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$\downarrow$

$$(-1)\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$\downarrow$

$$(-\tfrac{1}{2})\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array}\right]$$

$\downarrow$

$$(\tfrac{1}{2})\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{2} & -1 & 1 & -\frac{1}{2} \end{array}\right]$$

$\downarrow$

$$(-2)\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -1 & 1 & -\frac{1}{2} \end{array}\right]$$

$\downarrow$

$$(\tfrac{1}{2})\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right](-2)$$

$\uparrow$

$$(1)\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & -4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$\uparrow$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -5 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$\downarrow$

$$A^{-1} = \begin{bmatrix} -2 & -5 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$m = n$ ✓

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] -1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] -2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] 1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] -1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_1^{-1} & A_2^{-1} & A_3^{-1} \\ A_4^{-1} & A_5^{-1} & A_6^{-1} \\ A_7^{-1} & A_8^{-1} & A_9^{-1} \end{bmatrix}$$

$$A_1^{-1} - A_4^{-1} + 2A_7^{-1} = 1$$
$$A_1^{-1} + A_7^{-1} = 0$$
$$2A_4^{-1} - A_7^{-1} = 0$$
$$2A_1^{-1} + 3A_7^{-1} = 1$$
$$A_1^{-1} + A_7^{-1} = 0$$
$$A_7^{-1} = 1$$
$$\cancel{A_7^{-1} = \frac{1}{2}}$$
$$A_1^{-1} + 1 = 0$$
$$A_1^{-1} = -1$$
$$2A_4^{-1} - 1 = 0$$
$$A_4^{-1} = \frac{1}{2}$$

$$A_2^{-1} - A_5^{-1} + 2A_8^{-1} = 0$$
$$A_2^{-1} + A_8^{-1} = 1$$
$$2A_5^{-1} - A_8^{-1} = 0$$
$$2A_2^{-1} + 3A_8^{-1} = 0$$
$$A_2^{-1} + A_8^{-1} = 1$$
$$A_8^{-1} = -2$$
$$A_2^{-1} - 2 = 1$$
$$A_2^{-1} = 3$$
$$2A_5^{-1} + 2 = 0$$
$$A_5^{-1} = -1$$

$$A_3^{-1} - A_6^{-1} + 2A_9^{-1} = 0$$
$$A_3^{-1} + A_9^{-1} = 0$$
$$2A_6^{-1} - A_9^{-1} = 1$$
$$2A_3^{-1} + 3A_9^{-1} = 1$$
$$A_3^{-1} + A_9^{-1} = 0$$
$$A_9^{-1} = 1$$
$$A_3^{-1} + 1 = 0$$
$$A_3^{-1} = -1$$
$$2A_6^{-1} - 1 = 1$$
$$A_6^{-1} = 1$$

$$A^{-1} = \begin{bmatrix} -1 & 3 & -1 \\ \frac{1}{2} & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$A \cdot A^{-1} = I_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_2 \to R_2 - R_1 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_1 \to R_2 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right] R_3 \to R_3 - 2R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{array}\right] R_3 \to \frac{1}{3} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2/3 & -2/3 & 1/3 \end{array}\right] R_2 \to R_2 + R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 2/3 & -2/3 & 1/3 \end{array}\right] R_1 \to R_1 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & 5/3 & -1/3 \\ 0 & 1 & 0 & -1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 2/3 & -2/3 & 1/3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2/3 & 5/3 & -1/3 \\ -1/3 & 1/3 & 1/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A = \begin{array}{r} (-1) \\ + \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ (-2) \\ + \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ \\ + \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ + \\ (-2) \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{array}{r} + \\ \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -1 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot -2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot 1 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \cdot -2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

Add R2 to R1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$[-1 \ 1 \ -2 \quad -1 \ 0 \ 0]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] +(-1) \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] +(\cdot -2)$$

$[0 \ -2 \ 2 \ 2 \ -2 \ 0]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] +(\cdot 1) \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] +(\cdot -2)$$

$[0 \ 0 \ -2 \ -4 \ 4 \ \cdot]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] +(\cdot 1) \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \times -2 + (3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right] \xrightarrow{R_1 = R_1 + R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right] \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right] \xrightarrow{R_2 = R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$(R1 \times -1) + R3$ $$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$(R3 \times -1) + R2$ $$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array}\right]$$

$(R2 \times -1) + R3$ $$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array}\right]$$

$R2 + R1$ $$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array}\right] \begin{matrix} \\ \\ 1 \quad -1 \end{matrix}$$

$R3 \times -2 + R1$ $$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-2 - 1 + 4 = 1$

$3 + 1 - 4 = 0$

$-1 - 1 \; 2 = 0$

$-2 \; 0 \; 2 = 0$

(-2×-2)
$4 - 1 = 3$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{array}{cc} A & A^{-1} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

So. $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{array}{c} ① \\ ② \\ ③ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = ② - ①$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = ③ - ② \times 2$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$0 - -1 \times 2$
$= 0 + 2$

$R_2 = ② + ③$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$R_1 = ① - ③ \times 2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & 2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$R_1 = ① + ②$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 3 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 3 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 2 & -2 & 4 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & 2 & 2 & -2 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & 2 & 2 & -2 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] = \left[\begin{array}{cccccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

First, append Identity matrix, I, then find rref(A)

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{R_1+R_2,\ (-1)R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \checkmark$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array}\right]$$

$R_2 = R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array}\right]$$

$R_2 = R_2(-1)$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array}\right]$$

$R_1 = R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array}\right]$$

$R_1 = R_1 - 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & -3 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array}\right] \to \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$[A | I] \to [I | A^{-1}]$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{-R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 = -R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 = R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = -2R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 = -R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xrightarrow{R_2 = R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot(-1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \cdot(-2)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \cdot(1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \cdot(1)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \cdot(-1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{A^{-1}}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

← this is the answer

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = R_3 + 2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array}\right]$$

$\downarrow R_1 = R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_3 = R_3 - 3R_2} \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 2 & -3 & 1 \end{array}\right]$$

$\downarrow R_2 = R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ -1 & 0 & 0 & 2 & -3 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 0 & -1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ -1 & 0 & 0 & 2 & -3 & 1 \end{array}\right] \begin{array}{l} \cdot(-1) \\ \\ \cdot(-1) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ 1 & 0 & 0 & -2 & 3 & -1 \end{array}\right]$$

1 $R_3 \leftrightarrow R_1$

2 $R_2 \leftrightarrow R_3$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Compute A^{-1}.

$$-1 \cdot \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$-2 \cdot \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$1 \cdot \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$-2 \cdot \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$1 \cdot \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\cdot -1 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \cdot 1 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \Rightarrow \cdot -2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \cdot -1 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Rightarrow \cdot 1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \times -1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \times -1$$

$$= \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \times -2$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \times -2$$

$$= \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \times 1$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \times 1$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \times -1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ -1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} (2)-(1)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} (3)-(2)\times 2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} (2)+(3)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} (1)+(2)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} (1)-(3)\times 2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}$$

$$\downarrow$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

There is also another way that requires you to find a Cofactor by doing

$$\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} \; \end{bmatrix}$$

ect.

finding the cross product of each box to find a ~~mult~~ factor, X then

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$X(\text{this}) = A^{-1}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{array}{c} -R2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right] = \begin{array}{c} \\ \\ +2R1 \end{array}\left[\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{c} -R3 \\ -R3 \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] = \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]}_{1}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{array}\right] \begin{array}{c} -R2 \\ \\ \end{array} = \begin{array}{c} \\ \\ +2R1 \end{array}\left[\begin{array}{ccc|c} 0 & -1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & -1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{c} -R3 \\ -R3 \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] = \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right]}_{2}$$

$$\begin{array}{c} -R2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] = \begin{array}{c} \\ \\ +2R1 \end{array}\left[\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{c} -R3 \\ -R3 \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] = \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]}_{3}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R2 \to R2 - R1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R1 \to R1 + R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R3 \to R3 - 2R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \xleftarrow[R1 \to R1 - R3]{R2 \to R2 + R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-1+1$

$1-1=0$

$0+0=0$

$$\begin{array}{l|l} x_1 - x_2 + 2x_3 & 1 \\ x_1 + 0x_2 + x_3 & 0 \\ 0x_1 + 2x_2 + 2x_3 & 0 \end{array} \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array}\right] \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \xrightarrow{R_2 + (-R_1)} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{array}\right] \begin{array}{c} 0 \\ 1 \\ 1 \end{array}$$

$-1+1=0$

$$\xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \xrightarrow{R_3 + (-2R_2)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -3 & 2 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \times 1/3 \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -2/3 \end{array}\right] \begin{array}{c} 0 \\ 0 \\ -1/3 \end{array}$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -5/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right] \begin{array}{c} \\ -5/3 \\ -2/3 \end{array} \xrightarrow{R_1 + (-R_3)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -5/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -5/3 & -5/3 & 1/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10