



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad C = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$$

$$A^T \cdot C = \begin{bmatrix} x_1 + y_1 + 0 & x_2 + y_2 + 0 & x_3 + y_3 + 0 \\ -x_1 + 0 + 2z_1 & -x_2 + 0 + 2z_2 & -x_3 + 0 + 2z_3 \\ 2x_1 + y_1 - z_1 & 2x_2 + y_2 - z_2 & 2x_3 + y_3 - z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$x_1 + y_1 = 1 \rightarrow y_1 = 1 - x_1$$

$$-x_1 + 2z_1 = 4$$

$$2x_1 + y_1 - z_1 = 3 \rightarrow 2x_1 + 1 - x_1 - z_1 = 3$$

$$\begin{aligned} z_1 &= 6 \\ x_1 &= 8 \\ y_1 &= -7 \end{aligned} \quad \left\{ \begin{array}{l} x_1 - z_1 = 2 \\ -x_1 + 2z_1 = 4 \end{array} \right.$$

$$\begin{aligned} x_2 + y_2 &= 2 \rightarrow y_2 = 1 - x_2 \\ -x_2 + 2z_2 &= 5 \\ 2x_2 + y_2 - z_2 &= 2 \\ \left(\begin{array}{l} x_2 - z_2 = 1 \\ -x_2 + 2z_2 = 5 \end{array} \right) \quad \left. \begin{array}{l} z_2 = 6 \\ x_2 = 7 \\ y_2 = -6 \end{array} \right. \end{aligned} \quad \begin{aligned} x_3 + y_3 &= 3 \rightarrow y_3 = 3 - x_3 \\ -x_3 + 2z_3 &= 4 \\ 2x_3 + y_3 - z_3 &= 1 \\ \left(\begin{array}{l} x_3 - z_3 = -2 \\ -x_3 + 2z_3 = 4 \end{array} \right) \quad \left. \begin{array}{l} z_3 = 2 \\ x_3 = 0 \\ y_3 = 3 \end{array} \right. \end{aligned}$$

$$x_1 = 8$$

$$x_2 = 7$$

$$x_3 = 0$$

$$y_1 = -7$$

$$y_2 = -6$$

$$y_3 = 3$$

$$z_1 = 6$$

$$z_2 = 6$$

$$z_3 = 2$$

$$C = \begin{bmatrix} 8 & 7 & 0 \\ -7 & -6 & 3 \\ 6 & 6 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B \quad \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(AB)^T = B^T A^T \quad 2 \times 4 = 8$$

$$A^T \cdot C = B \quad \frac{B}{A^T} = C = B \cdot (A^T)^{-1}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = 0 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{aligned} -1+2+3 &= 4 \\ -4+5+4 &= 5 \\ -3+2+1 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 1 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$0 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 0 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 3 \\ 1 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix} = C$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$(A^T)^{-1} = (A^{-1})^T \quad AA^{-1} = I \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad x_1 + y_1 = 1 \\ -x_1 + 2z_1 = 4 \\ 2x_1 + y_1 - z_1 = 3$$

$$(A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 \\ -1 & 0 & 2 & | & 4 \\ 2 & 1 & -1 & | & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 1 & 2 & | & 5 \\ 0 & -1 & -1 & | & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 1 & 2 & | & 5 \\ 0 & 0 & 1 & | & 16 \end{bmatrix} \quad z_1 = 6 \\ y_1 = 5 - 2(6) \\ y_1 = -7$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix} \quad x_1 = 1 - (-7) \\ x_1 = 8$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 2 \\ -1 & 0 & 2 & | & 5 \\ 2 & 1 & -1 & | & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 2 \\ 0 & 1 & 2 & | & 7 \\ 0 & -1 & -1 & | & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 2 \\ 0 & 1 & 2 & | & 7 \\ 0 & 0 & 1 & | & 5 \end{bmatrix}$$

$$z_2 = 5 \\ y_2 = 7 - 10 \\ y_2 = -3 \quad l_2 = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$$

$$x_2 = 2 - (-3)$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 3 \\ -1 & 0 & 2 & | & 4 \\ 2 & 1 & -1 & | & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 3 \\ 0 & 1 & 2 & | & 7 \\ 0 & -1 & -1 & | & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 3 \\ 0 & 1 & 2 & | & 7 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$$

$$z_3 = 2 \\ y_3 = 7 - 4 = 3 \quad l_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \\ x_3 = 3 - 3 = 0$$

$$C = \boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

check

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad \checkmark$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\cancel{(A^T)^{-1}} A^T C = \cancel{(A^T)^{-1}} B$$

$$C = (A^{-1})^T \cdot B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$3 \times 3 \qquad \qquad \qquad 3 \times 3$

$$\begin{aligned} & 2+4 \\ & -2+4+6 \\ & -4+5+4 \\ & -6+4+2 \\ & \quad -2+2 \\ & 3+4-6 \\ & \quad -1+6 \end{aligned}$$

8	5	0
-7	-3	3
6	5	2

$$4 - 5 - 4$$

$$1-4$$

$$-1+4+3$$

$$3+3$$

$$9-4-2$$

$$5-2$$

$$-2+5+2$$

$$-3+4+1-1$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} A^T C = B (A^T)^{-1}$$

$$C = B (A^T)^{-1}$$

$$C = B (A^{-1})^T$$

$$C = B \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad A^T C = B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$c_{11} + c_{21} = 1 \quad c_{11} = 1 - c_{21}$$

$$-c_{11} + 2c_{31} = 4$$

$$2c_{11} + c_{21} - c_{31} = 3 \quad 2(1 - c_{21}) + c_{21} + c_{31} = 3$$

$$2 - c_{21} + c_{31} = 3$$

$$2(1 - c_{21}) + c_{21} - (1 - c_{21}) = 3$$

$$2 - 2c_{21} + c_{21} - 1 + c_{21} = 3$$

$$\begin{aligned} 1 - 2c_{21} &= 3 \\ -2c_{21} &= 2 \quad c_{21} = -1 \Rightarrow \begin{cases} c_{11} = 2 \\ c_{21} = -1 \\ c_{31} = 0 \end{cases} \end{aligned}$$

$$c_{12} + c_{22} = 2 \quad c_{12} = 2 - c_{22}$$

$$-c_{12} + 2c_{32} = 5 \quad -(1 - c_{22}) + 2(1 - c_{22}) = 5$$

$$2c_{12} + c_{22} - c_{32} = 2 \quad -2 + c_{22} + 4 - 2c_{22} = 5$$

$$2(1 - c_{22}) + c_{22} - c_{32} = 2 \quad 2 - c_{22} = 5$$

$$4 - c_{22} + c_{22} - c_{32} = 2 \quad -c_{22} = 3$$

$$4 - c_{22} - c_{32} = 2 \quad -2 - c_{12} = 5$$

$$-c_{22} - c_{32} = -2$$

$$-c_{32} = -2 + c_{22}$$

$$c_{32} = 2 - c_{22}$$

$$\boxed{C = \begin{bmatrix} 2 & 5 & 0 \\ -1 & -3 & 3 \\ 0 & 5 & 2 \end{bmatrix}}$$

$$c_{13} + c_{23} = 3 \quad c_{13} = 3 - c_{23}$$

$$-c_{13} + 2c_{33} = 4 \quad -(3 - c_{23}) + 2(5 - c_{23}) = 4$$

$$2c_{13} + c_{23} - c_{33} = 1 \quad -3 + c_{23} + 10 - 2c_{23} = 4$$

$$7 + c_{23} - 2c_{23} = 4$$

$$2(3 - c_{23}) + c_{23} - c_{33} = 1 \quad -c_{23} = -3$$

$$6 - 2c_{23} + c_{23} - c_{33} = 1 \quad c_{23} = 3$$

$$6 - c_{23} - c_{33} = 1 \quad 27 - c_{13} = 0$$

$$-c_{23} - c_{33} = -5 \quad c_{23} = 3$$

$$-c_{33} = -5 + c_{23} \quad c_{33} = 2$$

$$c_{33} = 5 - c_{23}$$



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$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $\underline{A^T C = B}$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$A^T C$

$$= B \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

3×3

~~$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 8 & 2 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 & 7 \\ 2 & 1 \end{bmatrix}$~~

Columns have to be same!

Come back!

$$1x_1 + 1x_2 + 2x_3 = 1$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T \cdot B$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 0 & 0 \\ 6 & 13 & 0 \\ 0 & 0 & 6 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$



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$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\underbrace{3 \times 3}_{A} \cdot \underbrace{3 \times 3}_{B} \cdot \underbrace{3 \times 3}_{C}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 1 - y_1 \\ z_1 &= 1 + \frac{x_1}{2} \\ z_1 &= 1 + \frac{1 - y_1}{2} \end{aligned}$$

$$\begin{aligned} x_2 &= 4 - y_2 \\ z_2 &= \frac{5}{2} + \frac{x_2}{2} \end{aligned}$$

$$y_1 = 2(1 - y_1) + y_2 - \left(\frac{5}{2} + \frac{4 - y_2}{2}\right) x_2 + y_2 - z_2 = 4$$

$$y_2 = 4(4 - y_2) + 2y_2 - 5 - 4 + y_2 x_3 + 2z_3 = 2$$

$$y_3 = 16 - y_2 + 2y_2 - 9 + y_2 2x_3 + y_3 - z_3 = 1$$

$$y_2 = 3$$

$$y_2 = 1$$

$$z_2 = 1$$

$$x_3 = 3 - y_2$$

$$z_3 = 1 - \frac{3}{2} + \frac{y_2}{2}$$

$$1 = 2(3 - y_2) + y_2 - \left(1 - \frac{3}{2} + \frac{y_2}{2}\right)$$

$$2 = 12 - 4y_2 + 2y_2 - 2 + 3 - y_2$$

$$2 = 13 - 3y_2 = \frac{11}{3}$$

$$C = \begin{bmatrix} 6 & 3 \\ -5 & 1 \\ 4 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ -1 & 0 & 2 & | & 0 & 1 & 0 \\ 2 & 1 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R1} \rightarrow 1} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{R1} \rightarrow \frac{1}{2}} \begin{bmatrix} 1 & 2 & -6 \\ 13 & -13 & 13 \\ 12 & -12 & 6 \end{bmatrix} = C$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & 1 & 1 & 0 \\ 0 & -1 & -1 & | & -1 & -2 & 1 \end{bmatrix} \xrightarrow{\text{R2} \rightarrow 1} \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & 1 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & -1 & 1 \end{bmatrix} \xrightarrow{\text{R3} \rightarrow 2}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 3 & -1 & -2 \\ 0 & 0 & 1 & | & -1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{R1} \rightarrow 1}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 3 & -1 & -2 \\ 0 & 0 & 1 & | & -1 & 1 & 1 \end{bmatrix} \xrightarrow{\text{R2} \rightarrow -1}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \boxed{\begin{bmatrix} 1 & 2 & -6 \\ 13 & -13 & 13 \\ 12 & -12 & 6 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore C = B (A^T)^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow C = \boxed{\begin{bmatrix} -2 & 2 & 6 \\ 12 & -5 & -8 \\ -3 & 2 & 1 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 5 & 2 \\ 3 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{array}{l} \begin{array}{l} a + 4d + 3g = 1 \\ 2a + 5d + 2g = 4 \\ 3a + 4d + g = 3 \end{array} \quad \left| \begin{array}{l} b + 4e + 3h = 2 \\ 2b + 5e + 2h = 5 \\ 3b + 4e + h = 4 \end{array} \right. \quad \left| \begin{array}{l} c + 4f + 3i = 3 \\ 2c + 5f + 2i = 4 \\ 3c + 4f + i = 1 \end{array} \right. \\ \begin{array}{l} \cancel{a=2} \cancel{d=0} \cancel{g=0} \\ a = -1, d = 2, g = -2 \end{array} \quad \left| \begin{array}{l} b = -\frac{3}{4}, e = 2, h = -\frac{7}{4} \\ c = -2, f = 2, i = -1 \end{array} \right. \end{array} \end{array}$$

$$C = \begin{bmatrix} -1 & -\frac{3}{4} & -2 \\ 2 & 2 & 2 \\ -2 & -\frac{7}{4} & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 & c_4 & c_7 \\ c_2 & c_5 & c_8 \\ c_3 & c_6 & c_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$



$$\begin{matrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{matrix} \quad \begin{matrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{matrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T \cdot \boxed{\text{Matrix } C}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 2 & 1 & -1 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -1 & -2 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{matrix}$$

$$A^{T-1} \cdot B = C$$

$$A^{T-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{matrix} -2+4+6 & =-1 \\ 3-4-6 & =6 \\ -1+4+3 & \end{matrix}$$

$$\boxed{A^{T-1}} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} =$$

$$\boxed{C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

$$-6+4+2=0$$

$$9-4-2=3$$

$$-3+4+1=2$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} -1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$A^T \cdot A^{T-1} C = B A^{T-1}$$

$$C = B A^{T-1}$$

$$A^T = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \quad ; \quad A^T \text{ inverse} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} -1 & 4 & -2 & 5 \\ -1x1 + 1x4 + 0x3 & -1x2 + 1x5 + 0x2 & -1x3 + 1x4 + 0x1 \\ -1 & 0 & 2 & 1 \\ -1x1 + 0x4 + 2x3 & -1x2 + 0x5 + 2x2 & -1x3 + 0x4 + -1x1 \\ 2 & 1 & -1 & 1 \\ 2x1 + 1x4 + -1x3 & 2x2 + 1x5 + -1x2 & 2x3 + 1x4 + -1x1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 & 1 \\ 5 & 2 & -4 \\ 3 & 7 & 9 \end{bmatrix} \end{aligned}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(A^T)(A^T)^{-1} C = B(A^T)^{-1}$$

$$C = B(A^T)^{-1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Invertible.

$$(A^T)^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = B(A^{-1})^T$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} & \cancel{\begin{bmatrix} 1x-2 + 1x+1 + 0x+1 \\ -1x+2 + 3x+0 + 0x-1 \\ 2x-2 + 1x+3 + 1x-1 \end{bmatrix}} \quad \cancel{\begin{bmatrix} 1x+1x-1+0x+1 \\ -1x+0x+1+2x-1 \\ 2x+1x+1x-1x \end{bmatrix}} \quad \cancel{\begin{bmatrix} 1x+2x+1x-2+1x+1 \\ 2x-2+1x-2+1x-1 \\ -1x+2+0x-1x+2x-1 \end{bmatrix}} \\ & \cancel{\begin{bmatrix} 1x-2 + 2x+3x+3x-1 \\ 4x-2+3x+4x-1 \\ 3x-2+2x+1x-1 \end{bmatrix}} \quad \cancel{\begin{bmatrix} 1x+1x+2x-1+0x+1 \\ 4x+1+5x-1+4x+1 \\ 3x+1+2x-1+1x+1 \end{bmatrix}} \quad \cancel{\begin{bmatrix} 1x+2x-2+3x+1 \\ 4x+2+5x-2+4x+1 \\ 1x+2+2x-2+1x+1 \end{bmatrix}} \end{aligned}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 15 & -4 \\ 6 & 6 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 15 & -4 \\ 6 & 6 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 15 & -4 \\ 6 & 6 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 15 & -4 \\ 6 & 6 & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$C = B(A^T)^{-1} \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2+6-3 & 1-2+3 & 2-4+3 \\ -8+15-4 & 4-5+4 & 8-10+4 \\ -6+6-1 & 3-2+1 & 6-4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$\begin{aligned} Ax &= b \\ x &= A^{-1} \cdot b \end{aligned}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B \Rightarrow C = (A^T)^{-1} \cdot B \quad (A^T)^{-1} = (A^{-1})^T$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{-1})^T \cdot B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

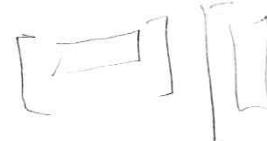
$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



$$A^T C = B \quad AC = \frac{B}{A^{-1}}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$



Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{array}{l} A^T \left(\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \end{array}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}, \quad []$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$$A+D=1$$

$$12 \quad A+2G=4$$

$$16 \cdot 7 \cdot 6 \quad 2A+D-G=3$$

$$2A+D-G=3$$

$$A+\bar{D}=1$$

$$A-G=2$$

$$-A+2\bar{G}=4$$

$$G=6$$

$$A=8$$

$$D=-7$$

$$B+E=2$$

$$-B+2\bar{I}=5$$

$$2B+E-H=2$$

$$2B+E-H=2$$

$$\bar{B}+\bar{E}=2$$

$$B-\bar{H}=0$$

$$-\bar{B}+2H=5$$

$$H=5$$

$$B=5$$

$$E=-3$$

$$C+F=3$$

$$-C+2\bar{I}=4$$

$$2C+F-I=1$$

$$2C+\bar{F}-\bar{I}=1$$

$$C+\bar{F}=3$$

$$C-I=-2$$

$$-C+2\bar{I}=4$$

$$I=2$$

$$-C+4=4$$

$$-C=0$$

$$C=0$$

$$F=3$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & c & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\frac{3}{1} - \frac{4}{3}$$

$$\frac{9}{3} - \frac{4}{3}$$

$$\frac{3}{3} - \frac{4}{3} = -\frac{1}{3}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T (A^T)^{-1} = B(A^T)^{-1}$$

$$C = B(A^T)^{-1}$$

$$\frac{15}{1} - \frac{4}{3} = \frac{45}{3} - \frac{4}{3} = \frac{2+10}{1}$$

$$\frac{6}{1} - \frac{1}{3} = \frac{18}{3} - \frac{1}{3} = \frac{6+10}{3}$$

$$(A^T)^{-1} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{5}{3} & \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$B(A^T)^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{5}{3} & \frac{5}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = 0 + 3 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} + -\frac{1}{3} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ 6 \end{bmatrix} + \begin{bmatrix} -1 \\ -\frac{4}{3} \\ -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{41}{3} \\ \frac{17}{3} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & 1 & 2 \\ 4 & 5 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \frac{4}{3} + \frac{25}{3} + \frac{4}{3} = \frac{10}{3} + \frac{10}{3} = 11$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} & 3 & 2 \\ 2 & 3 & 2 \\ -\frac{2}{3} & 4 & 5 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{8}{3} \\ \frac{2}{3} \end{bmatrix} + \begin{bmatrix} -\frac{4}{3} \\ -\frac{10}{3} \\ -\frac{4}{3} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{6}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ 2 \\ -\frac{1}{3} \end{bmatrix}$$

$$\therefore C = \boxed{\begin{bmatrix} 5 & \frac{10}{3} & \frac{5}{3} \\ \frac{41}{3} & 11 & \frac{2}{3} \\ \frac{17}{3} & \frac{10}{3} & -\frac{1}{3} \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\left(\begin{smallmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{smallmatrix} \right) \times \left(\begin{smallmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{smallmatrix} \right)$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A^T \cdot B = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = C$$

$$\begin{aligned} & \left[4 \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} - 1 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \right] + \left[3 \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + 1 \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} + 1 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \right] \\ & \left[1 \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} + 4 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \right] + \left[2 \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} + 7 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \right] \\ & + \left[3 \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} + 1 \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \right] \end{aligned}$$

$$C = \begin{bmatrix} 7 & 7 & 1 \\ -1 & 6 & 5 \\ 9 & 7 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (\text{calculated in problem 2})$$

Using the fact that: $(A^T)^{-1} = (A^{-1})^T$

$$\therefore (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \rightarrow (A^T)^{-1}(A^T)C = B(A^T)^{-1}$$

$$C = B(A^T)^{-1}$$

$$\therefore C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1(-2) + 3(2) - 3, & 1 - 2 + 3, & 2 - 4 + 3 \\ 4(-2) + 5(3) - 4, & 4 - 5 + 4, & 4(2) - 5(2) + 4 \\ 3(-2) + 2(3) - 1, & 3 - 2 + 1, & 3(2) - 2(2) + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 - 3, & -1 + 3, & -2 + 3 \\ -8 + 15 - 4, & -5, & 8 - 10 + 4 \\ -1, & 0, & 6 - 4 + 1 \end{bmatrix} = \begin{bmatrix} 1, & 2, & 1 \\ 3, & -5, & 2 \\ -1, & 0, & 3 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 1, & 2, & 1 \\ 3, & -5, & 2 \\ -1, & 0, & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C_{11} + C_{21} = 1$$

$$C_{11} + C_{31} = 2$$

$$C_{13} + C_{23} = 3$$

$$-C_{11} + 2C_{13} = 4$$

$$-C_{21} + 2C_{23} = 5$$

$$-C_{13} + 2C_{33} = 4$$

$$2C_{11} + C_{12} - C_{13} = 3$$

$$2C_{21} + C_{22} - C_{23} = 2$$

$$2C_{13} + C_{32} - C_{33} = 1$$

$$C_{12} = 1 - C_{11}$$

$$C_{22} = 2 - C_{21}$$

$$C_{23} = 3 - C_{13}$$

$$C_{13} = 2 + \frac{1}{2}C_{11}$$

$$C_{32} = \frac{5}{2} + \frac{1}{2}C_{31}$$

$$2C_{13} + 3 - C_{13} - 2 - \frac{1}{2}C_{33} = 1$$

$$2C_{11} + 1 - C_{11} - 2 + \frac{1}{2}C_{11} = 3$$

$$2C_{21} + 2 - C_{21} - \frac{5}{2} - \frac{1}{2}C_{21} = 2$$

$$\frac{1}{2}C_{13} = 0$$

$$-1 + \frac{1}{2}C_{11} = 3$$

$$\frac{1}{2}C_{21} - \frac{1}{2} = 2$$

$$C_{13} = 0$$

$$\frac{1}{2}C_{11} = 4$$

$$\frac{1}{2}C_{21} = \frac{5}{2}$$

$$C_{23} = 3$$

$$C_{11} = 8$$

$$C_{12} = 5$$

$$C_{33} = 2$$

$$C_{21} = -7$$

$$C_{22} = -3$$

$$C_{31} = 6$$

$$C_{32} = 5$$

$$C = \boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{pmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{pmatrix} \quad \begin{matrix} -2 & 1 \\ 3 & -1 & -2 \\ 1 & 1 & 1 \end{matrix}$$

$$C = B A^{-1} \quad \left| \begin{array}{ccc|cc} & & & 1 & 2 & 3 \\ 1 & 2 & 3 & 2 & 1 & 2 \\ 4 & 5 & 4 & 3 & -1 & -2 \\ 3 & 2 & 1 & -1 & 1 & 1 \end{array} \right| \xrightarrow{\text{Row operations}} \left| \begin{array}{ccc|ccc} & & & 1 & 2 & 3 & 1 \\ & & & 2 & 3 & 2 & 2 \\ & & & 3 & 2 & 1 & 3 \end{array} \right| = C$$

$$\begin{array}{ccc|cc} 0 & 1 & 1 & 1 & 0 \\ 2 & 0 & -1 & 0 & 1 \\ -1 & 1 & 2 & 0 & 0 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{pmatrix} = C$$

$$1 \cdot -1 \cdot -2 \Big) 0^{\circ} \cdot -1$$

$$-8 + 15 - 4 = 2$$

o 1 1 1 0 o

$$2 \quad 0 \quad -1 \quad | \quad 0 \quad 1 \quad 0$$

Y 1 1 0 716

1-1-7100-1

-1 o 2 -11

8 1 1 | 1 0 6

1 - 1 2

0 2 3 } 0 1 2

1. *Uvula* 2. *Uvula*

$$\left| \begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 10 & 0 & 0 \\ 0 & 0 & 1 & -21 & 2 & 2 \end{array} \right.$$

$$\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 & -1 & -7 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 2 & (& 2 & 3 \\ -3 & -1 & -2 & | & 4 & 5 & 4 \\ 1 & 1 & 1 & | & 2 & 1 & 1 \end{array} \right] \xrightarrow{-R_2 - R_1 + R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ -3 & -1 & -2 & | & 4 & 5 & 4 \\ 1 & 1 & 1 & | & 2 & 1 & 1 \end{array} \right] \xrightarrow{3R_1 + R_2 + R_3}$$

$$C = \begin{bmatrix} -5 & -7 & -7 \\ 5 & 2 & -1 \\ 3 & 7 & 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & -1 & -2 & | & 11 & 16 & 17 \\ 1 & 1 & 1 & | & 3 & 2 & 1 \end{array} \right] \xrightarrow{R_2: R_2 - R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & 1 & 1 & | & 11 & 16 & 17 \\ 0 & 1 & 1 & | & 8 & 9 & 8 \end{array} \right] \xrightarrow{R_3: R_3 - R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & -7 & -7 \\ 0 & 1 & 2 & | & 11 & 16 & 17 \\ 0 & 0 & 1 & | & 3 & 7 & 9 \end{array} \right] \xrightarrow{\begin{matrix} R_2 \leftarrow R_2 - 2R_3 \\ R_1 \leftarrow R_1 - R_3 \end{matrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^T C = B$$

Matrix Algebra Property: $(A^T)^{-1} = (A^{-1})^T$

$$C = (A^T)^{-1} \cdot B$$

$$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

3x3 3x3

defined with
 $C = 3 \times 3$

$$C = \begin{bmatrix} -2(1) + 1(4) + 2(3) & -2(2) + 1(5) + 2(2) & -2(3) + 1(4) + 2(1) \\ 3(1) + (-1)(4) + (-2)(3) & 3(2) + (-1)(5) + (-2)(4) & 3(3) + (-1)(4) + (-2)(2) \\ 0(1) + 1(4) + 1(3) & 0(2) + 1(5) + 1(2) & 0(3) + 1(4) + 1(1) \end{bmatrix}$$

$$C = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-4 \\ 4+3 & 5+2 & 4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 1 \\ 7 & 7 & 5 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 5 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{aligned} x_1 + x_2 &= 1 & x_1 &= 0 \\ x_2 + x_3 &= 5 & x_2 &= 5 - x_3 \\ x_3 &= 7 & x_3 &= 7 \end{aligned}$$

$$\begin{aligned} 5 - 7 &= -2 \\ 1 + 2 &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 7 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$3 \times 3 \quad 3 \times 3 = 3 \times 3$$

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ -3 & 0 & 2 & 0 \\ 6 & -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ -2 & 6 & -2 \\ 4 & -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 & 7 \\ -2 & 6 & -2 \\ 4 & -2 & -2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B$$

$$\Leftrightarrow (A^T)^{-1} \cdot A^T C = (A^T)^{-1} \cdot B \quad \therefore (A^T)^{-1} \cdot A^T = I_3$$

$$\Leftrightarrow C = (A^T)^{-1} \cdot B$$

$$\Leftrightarrow C = (A^{-1})^T \cdot B \quad \therefore (A^T)^{-1} = (A^{-1})^T$$

$$\therefore C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 0 \\ -1 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$\underbrace{3 \times 3}_{\text{ }} \times \underbrace{3 \times 3}_{\text{ }}$

$$C = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix}$$

$$r_1 C_1 : -2x_1 + x_2 - x_3 = 1$$

\downarrow

$$r_1 C_1 : -x_1 + 2x_3 = 4$$

$$r_3 C_1 : x_1 + x_2 = 3$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{RR}} \left[\begin{array}{c} c \\ \vdots \\ \vdots \end{array} \right]$$

$$x_1 = 3 - x_2$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\cancel{(A^T)(A^{-1})^T} C = B(A^{-1})^T$$

$$C = B(A^{-1})^T$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^T \cdot (A^T)^{-1} C = B \cdot (A^T)^{-1} \text{ since } (A^T)^{-1} = (A^{-1})^T$$

$$\therefore C = B \cdot (A^{-1})^T$$

$$C = B \cdot (A^{-1})^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & 2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \quad \left[\begin{array}{cccccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right] \quad \left[\begin{array}{cccccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \quad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B \quad \left[\begin{array}{ccc|ccc} -2 & 1 & 2 & 1 & 2 & 3 \\ 3 & -1 & -2 & 4 & 5 & 4 \\ -1 & 1 & 1 & 3 & 2 & 1 \end{array} \right]$$

$$\begin{array}{l} (-2+4+6) \\ (-3+4-6) \end{array} \quad \begin{array}{l} (1+2) \\ (6-5-4) \end{array} \quad \begin{array}{l} (1+2+3) \\ (9-4-2) \end{array}$$

$$\begin{array}{l} (-1+4+3) \\ (2+5+2) \end{array} \quad \begin{array}{l} (-3+4+1) \end{array}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 9 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad \therefore \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\text{Let } C = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 6 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad (A^T)^{-1} = (A^{-1})^T$$

rows \leftrightarrow columns

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^T C = B$$

$$C = B (A^T)^{-1}$$

$$C = B (A^{-1})^T$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} v_1 &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} & v_2 &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} -2 + 6 - 3 \\ -8 + 15 - 4 \\ -6 + 6 - 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} & &= \begin{bmatrix} 1 - 2 + 3 \\ 4 - 5 + 4 \\ 3 - 2 + 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} v_3 &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 - 4 + 3 \\ 8 - 10 + 4 \\ 6 - 4 + 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned}$$

$$C = [v_1 \ v_2 \ v_3]$$

$$C = \boxed{\begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 2 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_2 + 2R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array} \right) \xrightarrow{R_2 + 2R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array} \right) + R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \end{array} \right) \xrightarrow{R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & -2 & 1 & -1 & 0 \end{array} \right) \xrightarrow{-\frac{1}{2}} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 2 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) - 4 \cdot R_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 4 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) \xrightarrow{-R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 0 \\ 0 & 1 & 0 & -2 & 4 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) \quad \frac{1}{2} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -4 \\ 16 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -3 \\ 4 \end{pmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 & 0 \\ -2 & 4 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \quad \begin{aligned} 3 \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + -4 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} &= \begin{pmatrix} -1 \\ -4 \\ -3 \end{pmatrix} \\ -2 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} &= \begin{pmatrix} 4 \\ 16 \\ 4 \end{pmatrix} \end{aligned}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T C = B \Rightarrow (A^T)^{-1} (A^T) C = B (A^T)^{-1} \Rightarrow C = B (A^T)^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -11 & 3 & 1 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} -11 & 3 & 1 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$AC = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$\begin{array}{c|ccc} & 1 & 0 & 0 \\ \xrightarrow{\text{row reduction}} & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array} \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 8 & 2 \end{bmatrix}$$

$$A^T C = B$$

$$\begin{array}{c|cc} A^T & C \\ \hline 3 \times 3 & 3 \times 3 \\ \text{defined} & \end{array}$$

$$B = 3 \times 3 \checkmark$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 8 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \Rightarrow (A^{-1})^T = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{-1})^T \cdot B = C$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \cancel{-2+8+6}^{12} & \cancel{-4+10+4}^{10} & \cancel{-6+8+2}^{0} \\ \cancel{3-4-6}^{-1} & \cancel{6-5-4}^{-3} & \cancel{9-4-2}^{3} \\ \cancel{-1+4+3}^{6} & \cancel{-2+5+2}^{12} & \cancel{-3+4+1}^{2} \end{bmatrix}$$

$$C = \boxed{\begin{bmatrix} 12 & 10 & 4 \\ -7 & -3 & 3 \\ 6 & 2 & 2 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$\left[\begin{array}{cc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array} \right] \leftarrow \left[\begin{array}{cc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array} \right]$$

$$\left[\begin{array}{cc|ccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \rightarrow \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$\left[\begin{array}{cc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$B \cdot (A^T)^{-1} = C$$

$$\left| \begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right|$$

8
0
6

$$\left| \begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array} \right|$$

$$(A^T)^{-1} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\left| \begin{array}{ccc|ccc} 1 & 2 & 3 & -2 & 0 & -1 \\ 4 & 5 & 4 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 & 0 & 1 \end{array} \right| = \begin{bmatrix} 6 & 2 & 4 \\ 5 & 5 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -7 & -7 \\ 8 & 9 & 8 \\ 5 & 7 & 7 \end{bmatrix}$$

$$\left| \begin{array}{ccc} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{array} \right| \cdot \begin{bmatrix} -5 & -7 & -7 \\ 8 & 9 & 8 \\ 5 & 7 & 7 \end{bmatrix} =$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).
 $(\Rightarrow B \cdot (A^T)^{-1})$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^T)^{-1} = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 \\ 2 & 1 & -1 & 0 \end{array} \right]^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right] \xrightarrow{\text{R2} + R1}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \end{array} \right] \xrightarrow{\text{R3} + R2}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{R2} \leftrightarrow \text{R3}}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{R1} - R3}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 & 2 \\ 0 & 1 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 \end{array} \right]$$

$$(\Rightarrow B \cdot (A^T)^{-1})$$

$$BV_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+6+3 \\ -8+15+8 \\ 6+6+1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$BV_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1+2+3 \\ -4+5+4 \\ 3+2+1 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 4 \end{bmatrix}$$

$$BV_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+4+3 \\ 8+10+4 \\ 6+4+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\boxed{(\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ -1 & -5 & 2 \\ -1 & -4 & 3 \end{bmatrix})}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$3 \times 3 \cdot 3 \times 3 = 3 \times 3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] R_2 = R_2 + R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] R_3 = -2R_1 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & 2 & -5 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & -1 & -1 & 1 & 2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array} \right] R_1 = -R_2 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & -1 & -1 & 1 & -2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array} \right] R_2 = -R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 & -2 & -5 \\ 0 & 0 & 2 & 5 & 7 & 7 \end{array} \right] R_3 = \frac{1}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 1 & 1 & -2 & -5 \\ 0 & 0 & 1 & 5 & 7 & 7 \end{array} \right] R_2 = -R_3 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 4 & 8 \\ 0 & 1 & 0 & -3 & 1 & -\frac{17}{2} \\ 0 & 0 & 1 & \frac{5}{2} & \frac{7}{2} & \frac{7}{2} \end{array} \right]$$

$$C = \begin{bmatrix} 0 & 4 & 8 \\ -\frac{3}{2} & -\frac{11}{2} & -\frac{17}{2} \\ \frac{5}{2} & \frac{7}{2} & \frac{7}{2} \end{bmatrix}$$

$\frac{2}{2} - \frac{5}{2} = -\frac{3}{2}$

$$-\frac{11}{2} + \frac{7}{2}$$

-10



3. (10 points) Let A be the same matrix as in Problem 2, and let ✓

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\text{Now, } (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\text{Here, } [A^T]^{-1} [A]^T [C] = [I][C] = [C] = [A^T]^{-1} [B]$$

$$\therefore [C] = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} (-2+4+6) & (-4+5+4) & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}$$

$$\therefore C = \boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$

Ans:-

Check:- $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix} = \begin{bmatrix} 8-7 \\ 6 \end{bmatrix} = [1]$

$$\begin{bmatrix} 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}, \begin{bmatrix} 16-13 \\ 7 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16-7+6 \\ 24-7 \end{bmatrix} = 3 \quad \begin{bmatrix} 16-13 \\ 7 \end{bmatrix} = 3?$$

$$\begin{bmatrix} 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+2-3 \\ 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{vmatrix} \quad ((A^T)^{-1}) = (A^{-1})^T$$

$$A^T A^T C = B \rightarrow C = B (A^T)^{-1}$$

$$C = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix} \circ \begin{vmatrix} -2 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

$$C = \begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ -2 & 5 & 0 \end{vmatrix} \quad \begin{aligned} & (-2+2+0), (3-2-6), (-1+2+3) \\ & (-8+5+8), (12-5-8), (-4+5+4) \\ & (-6+2+2), (9-2-2), (-3+2+1) \end{aligned}$$

$$\downarrow$$

$$\begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ -2 & 5 & 0 \end{vmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B$$

$$C = (A^{-1})^T B$$

Based on problem 2, $A^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 & 0 \end{bmatrix}$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

$$C = (A^{-1})^T B = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -\frac{1}{2} & -1 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 4 & 6 \\ 4 & -\frac{5}{2} & -2 \\ 0 & 2 & 0 \end{bmatrix}}$$



$$B \cdot \frac{1}{A^T} \rightarrow$$

$$B \cdot A^T$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$A^T C = B \rightarrow C = \frac{B}{A^T} \quad C = B \cdot (A^{-1})^T$$

$$C = (B \cdot (A^{-1}))^T$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 2 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

$$c_{11}, c_{12}, c_{13}$$

$$c_{11}, 11 -6 9$$

$$c_{22}, -13 22$$

$$c_{13} -6 11$$

$$c_{11} = 1 \cdot 2 + 2 \cdot 3 + 1 \cdot 3$$

$$-2 \quad 6 \quad 3$$

$$-1 \quad -2 \quad -3$$

$$c_{12} = 1 \cdot 1 + 2 \cdot -1 + 3 \cdot -1$$

$$c_{13} = 2 \cdot 1 + 2 \cdot 2 + 3 \cdot 1$$

$$8 \quad 15 \quad 4$$

$$c_{21} = 4 \cdot 2 + 5 \cdot 3 + 4 \cdot 1$$

$$-4 \quad -5 \quad -4$$

$$c_{22} = 4 \cdot 1 + 5 \cdot -1 + 4 \cdot -1$$

$$c_{23} = 4 \cdot 2 + 5 \cdot 2 + 4 \cdot 1$$

$$8 \quad 10 \quad 4$$

$$C = \begin{bmatrix} 11 & -6 & 9 \\ 22 & -13 & 22 \\ 13 & -6 & 11 \end{bmatrix}$$

$$c_{31} = 3 \cdot 2 + 3 \cdot 2 + 1 \cdot 1 = 13$$

$$c_{32} = -3 + -2 + -1 = -6$$

$$c_{33} = 6 + 4 + 1 = 11$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad B_{11} = [1 \ 1 \ 0] \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 1 \Rightarrow 2c_1$$

$$A^T C = B$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{T-1} \cdot A^T C = A^{T-1} \cdot B$$

$$C = A^{T-1} \cdot B$$

$$A^{T-1} = \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array} \right]$$

$$A^{T-1} \cdot B =$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+0-3 \\ 0+0+6 \\ 0+4-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+0-2 \\ 0+0+4 \\ 0+5-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+0-1 \\ 0+0+2 \\ 0+4-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A^{T-1} \cdot B = \boxed{\begin{bmatrix} -2 & 0 & 2 \\ 4 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix}} \quad C$$

$$\begin{array}{l} R_2 \rightarrow -2R_1 + R_2 \\ R_3 \rightarrow R_1 + R_3 \end{array} \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array} \right]$$

$$R_2 \rightarrow -R_2$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{array}{l} R_1 \rightarrow -R_2 + R_1 \\ R_3 \rightarrow -R_2 + R_3 \end{array} \left[\begin{array}{ccc|cc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow -R_3 + R_2 \\ R_1 \rightarrow R_3 + R_1 \end{array} \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right]$$

$$A^{T-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ -2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^{-1})^T A^T C = B (A^{-1})^T$$

$$I C = B (A^{-1})^T$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} (-2+6-3) & (1-2+3) & (-2-4+3) \\ (-8+15-4) & (4-5+4) & (8-10+4) \\ (-6+6-1) & (3-2+1) & (6-4+1) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

A

$$(A^T A^T)C = A^T B$$

$$I C = A^T B$$

$$C = A^T B$$

$$\begin{array}{c|ccc} \boxed{1} & \boxed{-1} & \boxed{2} & \\ \boxed{1} & \boxed{0} & \boxed{1} & \\ \boxed{0} & \boxed{2} & \boxed{-1} & \end{array} \times \begin{array}{c|cc} \boxed{1} & \boxed{2} & \boxed{3} \\ \boxed{4} & \boxed{5} & \boxed{4} \\ \boxed{3} & \boxed{2} & \boxed{1} \end{array} = \boxed{\begin{bmatrix} 3 & 1 & 1 \\ 4 & 4 & 4 \\ 5 & 8 & 7 \end{bmatrix}} = C$$

$$\begin{aligned} 1(1) - 1(4) + 2(3) \\ = 1 - 4 + 6 \\ = 3 \end{aligned}$$

$$\begin{aligned} 1(1) + 0 + 1(3) = 4 \\ 1(2) + 0 + 1(2) = 4 \\ 1(3) + 0 + 1(1) = 4 \end{aligned}$$

$$1(2) - 1(5) + 2(2)$$

$$\begin{aligned} 0 + 2(4) - 1(3) = 8 - 3 = 5 \\ 0 + 2(5) - 1(2) = 10 - 2 = 8 \\ 0 + 2(4) - 1(1) = 8 - 1 = 7 \end{aligned}$$

$$= 2 - 5 + 4$$

$$= 1$$

$$1(3) - 1(4) + 2(1)$$

$$= 3 - 4 + 2$$

$$= 1$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = B \cdot \frac{1}{A^T} = B \cdot A^{T^{-1}}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad \Rightarrow \quad \left(\begin{array}{ccc|ccc} 6 & 0 & -2 & -6 & 0 & b \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array} \right)$$

$$A^{T^{-1}} = \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right) \quad \Rightarrow \quad \left(\begin{array}{ccc|ccc} 6 & 0 & 0 & -3 & -3 & 3 \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array} \right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right) \quad \Rightarrow \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 5/2 & -1/2 & -3/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array} \right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array} \right) \quad \xrightarrow{\text{Identity Matrix}} \quad A^{T^{-1}}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right) \quad C = B \cdot A^{T^{-1}} = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}_{3 \times 3} \cdot \underbrace{\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}_{3 \times 3}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & -1 & 1 & 1 \end{array} \right) \quad C = \boxed{\begin{bmatrix} -\frac{1}{2} & 5 & -\frac{3}{2} \\ -2 & -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 & \frac{1}{2} \end{bmatrix}}$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 2 & 6 & 2 & 2 & 0 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array} \right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 5 & -1 & -3 \\ 0 & 0 & 6 & -3 & 3 & 3 \end{array} \right)$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} \cdot (A^{-1})^T$$

$$C = (A^{-1})^T B$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^T)^{-1} \cdot \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+4+6 \\ 3-4-6 \\ -1+4+3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

$$(A^T)^{-1} \cdot \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -4+5+4 \\ 6-5-4 \\ -2+5+2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$$

$$(A^T)^{-1} \cdot \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+4+2 \\ 9-4+2 \\ -3+4+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{array}{c} 3 \times 3 \\ \left[\begin{array}{ccc} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{array} \right] \cdot \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 1 \\ 4 \\ 3 \end{array} \right] \end{array}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = B \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix}$$

$$c_{11} = 0.5 \quad 4 = 2c_{12} - c_{11} \quad 4 = c_{12} \quad 3 = 2c_{13} + c_{12} - c_{11}$$

$$c_1 = \begin{bmatrix} 0.5 \\ 4 \\ 1.5 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} c_{21} \\ c_{22} \\ c_{23} \end{bmatrix} \quad 2 = 2c_{21} \quad c_{21} = 1$$

$$c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad 5 = 1c_{22} \quad c_{22} = 5$$

$$c_3 = \begin{bmatrix} 1.5 \\ 4 \\ 0.5 \end{bmatrix} \quad 2 = 2c_{23} \quad c_{23} = 1$$

$$C = \begin{bmatrix} 0.5 & 1 & 1.5 \\ 4 & 5 & 4 \\ 1.5 & 1 & 0.5 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{array}{l} \begin{aligned} c_1 + c_4 &= 1 \\ -c_1 + 2c_7 &= 4 \\ 2c_1 + c_4 + c_7 &= 3 \end{aligned} \end{array} \quad \begin{array}{l} \begin{aligned} c_2 + c_5 &= 2 \\ -c_2 + 2c_8 &= 5 \\ 2c_2 + c_5 + c_8 &= 2 \end{aligned} \end{array} \quad \begin{array}{l} \begin{aligned} c_3 + c_6 &= 3 \\ -c_3 + 2c_9 &= 4 \\ 2c_3 + c_6 + c_9 &= 1 \end{aligned} \end{array}$$

$$\begin{array}{lll} c_1 + c_7 = 2 & c_2 + c_8 = 0 & c_3 + c_9 = -2 \\ -c_1 + 2c_7 = 4 & -c_2 + 2c_8 = 5 & -c_3 + 2c_9 = 4 \\ 3c_7 = 6 & 3c_8 = 5 & 3c_9 = 2 \\ c_7 = 2 & c_8 = \frac{5}{3} & c_9 = \frac{2}{3} \\ -c_1 + 4 = 4 & -c_2 + \frac{10}{3} = \frac{15}{3} & -c_3 + \frac{4}{3} = \frac{12}{3} \\ (c_1 = 0) & (c_2 = -\frac{5}{3}) & (c_3 = -\frac{8}{3}) \\ 4(0) + c_4 = 1 & \frac{5}{3} + c_5 = \frac{6}{3} & -\frac{8}{3} + c_6 = \frac{3}{3} \\ (c_4 = 1) & (c_5 = \frac{11}{3}) & c_6 = \frac{11}{3} \end{array}$$

$$C = \begin{bmatrix} 0 & -\frac{5}{3} & -\frac{8}{3} \\ 1 & \frac{11}{3} & \frac{11}{3} \\ 2 & \frac{5}{3} & \frac{2}{3} \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} (A^T)^{-1} A^T C &= B(A^T)^{-1} \\ C &= B(A^T)^{-1} \end{aligned}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Inverse of A^T

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \xrightarrow{\text{R}_2 \leftrightarrow R_1} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 2 & 1 & -1 \end{bmatrix} \xrightarrow{\text{R}_2 + R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix} \xrightarrow{\text{R}_3 - 2R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{R}_3 \rightarrow R_3 - R_1}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{\text{R}_2 \rightarrow R_3 + R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{\text{R}_1 \rightarrow R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{\text{R}_3 \rightarrow R_3 + R_1}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{\text{R}_1 \rightarrow R_1 + R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{\text{R}_3 \rightarrow R_2 + R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \xleftarrow{\text{R}_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R}_2 \rightarrow R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Inverse } A^T} \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{R}_3 \rightarrow R_3 - R_1 - R_2}$$

$$(A^T)^{-1} \cdot B = C$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0+4+6 & 0+5+4 & 0+4+2 \\ 1-4-6 & 2-5+1 & 3-4-2 \\ 0+4+3 & 0+5+2 & 0+4+1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 10 & 9 & 6 \\ -9 & 7 & -9 \\ 7 & 7 & 5 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow{+1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right]$$

$$\xrightarrow{(2)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right]$$

$$\xrightarrow{(3)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array} \right]$$

$$\xrightarrow{+2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$\xrightarrow{(-1)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \rightarrow C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} = (A^{-1})^T$$

From Problem 2: $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$$(A^T)^{-1} B = C$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{\text{R1}+R2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{\text{R2}-2\text{R1}} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 2 & 0 & -2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 4 & 5 & 4 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xrightarrow{\text{R1}-\text{R2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 4 & 5 & 4 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xrightarrow{\text{R2}-2\text{R1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -8 & -5 & 0 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -8 & -5 & 0 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ -8 & -5 & 0 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} A & Q & X \\ B & R & Y \\ C & S & Z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} A_1 + B_1 + C_1 &= 1 \\ A_1 + B_2 + C_2 &= 4 \\ A_2 + B_1 + C_1 &= 3 \end{aligned}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ 7 & -3 & 3 \\ 0 & 5 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} X & Y & Z & \\ \hline 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{array} \right] \xrightarrow{(1)(1)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{array} \right]$$

$$\begin{aligned} Q(1) + R(1) + S(1) &= 2 \\ Q(1) + R(2) + S(2) &= 5 \\ Q(2) + R(1) + S(1) &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{array} \right] \xrightarrow{(1)(1)}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -5 \end{array} \right] \xrightarrow{(1)(1)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -2 \end{array} \right]$$

$$\begin{aligned} Q(1) + R(1) &= 1 \\ Q(1) + R(2) &= 5 \\ Q(2) + R(1) &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 2 & 1 & -1 & 3 \end{array} \right] \xrightarrow{(1)(2)}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{(1)(2)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$\begin{aligned} Q(1) + R(1) &= 1 \\ Q(1) + R(2) &= 5 \\ Q(2) + R(1) &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \end{array} \right] \xrightarrow{(1)(1)}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{(1)(1)} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$\begin{aligned} Q(1) + R(1) &= 1 \\ Q(1) + R(2) &= 5 \\ Q(2) + R(1) &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 8 \end{array} \right] \xrightarrow{(1)(2)}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{x} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array} \right] \xrightarrow{R=3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array} \right] \xrightarrow{S=5} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array} \right]$$

$$\begin{aligned} Q(1) + R(1) &= 1 \\ Q(1) + R(2) &= 5 \\ Q(2) + R(1) &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array} \right] \xrightarrow{A=8} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{array} \right]$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^T C = B$$

$$\text{let } C = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_4 & C_5 & C_6 \\ C_7 & C_8 & C_9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 2 \\ -1 & 0 & 2 & 5 \\ 2 & 1 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & -1 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 1 & 1 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -2 & -4 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B$$

$$C = (A^T)^{-1} B$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2+4+6}{3-4-6} & \frac{-4+5+4}{6-5-4} & \frac{-6+4+2}{9-4-2} \\ \frac{3-4-6}{2-5-2} & \frac{1-4-3}{2-5-2} & \frac{3-4-1}{3-4-1} \end{bmatrix}$$

$$C = \boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 13 \\ 6 & 5 & 12 \end{bmatrix}}$$

check $A^T C = B$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 13 \\ 6 & 5 & 12 \end{bmatrix} = \begin{bmatrix} 8-7+0 & 0+3+0 & 4 \\ -5+0+10 & 10-3-5 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ -1 & 0 & 2 & | & 4 & 5 & 4 \\ 2 & 1 & -1 & | & 3 & 2 & 1 \end{bmatrix} \quad R_1 + R_2$$

this matrix
is solution

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 2 & 1 & -1 & | & 3 & 2 & 1 \end{bmatrix} \quad (R_1 \times -2) + R_3$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & -1 & -1 & | & 1 & -2 & -5 \end{bmatrix} \quad R_2 + R_3 = R_3$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix} \quad R_2 \times -1 + R_1$$

$$2 \times 2 \xrightarrow{+4} \\ 4 \times -4 = 0$$

$$(6 \times 2) - 4 \\ 12 - 4$$

$$8 \\ (5 \times 2) \xrightarrow{+5} \\ 10 - 5 \\ 5$$

$$(2 \times 2) + 7$$

$$-4 + 7$$

$$(6 \times 2) + 5$$

$$-12 + 5$$

$$-7$$

$$(5 \times 2) + 7$$

$$-10 + 7$$

$$-3$$

$$\begin{bmatrix} 1 & 0 & -2 & | & -4 & -5 & -4 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix} \quad R_3 \times 2 + R_1$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 8 & 5 & 0 \\ 0 & 1 & 0 & | & -7 & -3 & 3 \\ 0 & 0 & 1 & | & 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A). E.

$$\begin{aligned} A^T C C^{-1} &= B C^{-1} \\ A^T E &= B C^{-1} \\ B^{-1} A^T &= B^{-1} B C^{-1} \\ B^{-1} A^T &= E C^{-1} \\ C^{-1} &= B^{-1} A^T \\ \Rightarrow C &= \begin{bmatrix} 1 & 0 & 0 \\ -4 & 10 & \\ 7 & -4 & 3 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} C^{-1} &= B^{-1} A^T \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -4 & 10 & \\ 7 & -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \\ 10 & 2 & \\ 2 & -1 & \end{bmatrix} A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \end{aligned}$$

$$= 1 \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix} - \begin{bmatrix} 0 \\ 10 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 17 \end{bmatrix}$$

$$= 1 \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix} + 0 + \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 10 \end{bmatrix}$$

$$= 0 + 2 \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 7 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -5 & -4 & 2 \\ 17 & 10 & -7 \end{bmatrix}$$

$$\text{Q.E.D.} \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -5 & -4 & 2 & 0 & 1 & 0 \\ 17 & 10 & -7 & 0 & 0 & 1 \end{array} \right] \quad \text{E.}$$

$$C = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = B \cdot (A^T)^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 2 & -1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$

$$\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 10 & 5-14=-9 \\ 4 & 5 & 5 & 5 & 9 & 9 & \\ 3 & 3 & 2 & 7 & 7 & 1 & \end{array}$$

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3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{array}{l} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right]$$

$$R_2 = \textcircled{2} + \textcircled{1}$$

$$R_1 = R_1 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & 5 & 6 \\ 0 & 1 & 0 & 19 & -3 & 3 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array} \right]$$

$$R_3 = \textcircled{3} - \textcircled{1} \times 2 \quad | \cdot 5$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 2 & -2 & -5 \end{array} \right]$$

$$C = \begin{bmatrix} 10 & 5 & 0 \\ -9 & -3 & 3 \\ 7 & 5 & 2 \end{bmatrix}$$

$$R_3 = \textcircled{3} + \textcircled{2}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array} \right]$$

$$R_2 = R_2 - 2R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -9 & -3 & 3 \\ 0 & 0 & 1 & 7 & 5 & 2 \end{array} \right]$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 2 \\ -1 & 0 & 2 & 4 & 5 \\ 2 & 1 & -1 & 3 & 2 \end{array} \right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|cc} 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 \\ 0 & 1 & -3 & -5 & -8 \end{array} \right)$$

$$\left[\begin{array}{cccc} -1 & 0 & 2 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & -5 & -8 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{cccc} -1 & 0 & 2 & 4 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -3 & 10 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{ccccc} 1 & 0 & -2 & -4 & -5 & -4 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -9 + \frac{1}{3} & -4 \\ 0 & 1 & 0 & 5 & 7 + \frac{1}{3} & 7 \\ 0 & 0 & 1 & 6 & -\frac{1}{3} & 0 \end{array} \right] \quad \frac{-15}{3} - \frac{1}{3} = \textcircled{11} \quad -\frac{14}{3}$$

$$C = \begin{bmatrix} -4 & -\frac{16}{3} & -4 \\ 5 & \frac{22}{3} & 7 \\ 0 & -\frac{1}{3} & 0 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c} \left[\begin{array}{ccccc} 1 & -1 & 2 & 1 & 2 \\ -1 & 0 & 1 & 1 & 2 \\ 2 & 1 & -1 & 1 & 2 \end{array} \right] \\ \xrightarrow{\text{Row operations}} \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 & 7 \\ 0 & -1 & 1 & 1 & 2 \end{array} \right] \\ \xrightarrow{\text{Row operations}} \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 & 7 \\ 0 & 0 & 1 & 6 & 5 \end{array} \right] \\ = \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 & 7 \\ 0 & 0 & 1 & 6 & 5 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 & 7 \\ 0 & 0 & 1 & 6 & 5 \end{array} \right] = \left[\begin{array}{ccccc} 1 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -7 & -3 \\ 0 & 0 & 1 & 6 & 5 \end{array} \right] \end{array}$$

$$\begin{array}{l} \left[\begin{array}{ccccc} 1 & 0 & 0 & 8 & 5 \\ 0 & 1 & 0 & -7 & -3 \\ 0 & 0 & 1 & 6 & 5 \end{array} \right] \quad \left[\begin{array}{c} -8x_4 - 5x_5 \\ 3 + 7x_4 + 3x_5 \\ 2 - 6x_4 - 5x_5 \\ x_4 \\ x_5 \end{array} \right] \\ \quad \begin{aligned} x_3 + 6x_4 + 5x_5 &= 2 \\ x_2 - 7x_4 - 3x_5 &= 3 \\ x_1 + 8x_4 + 5x_5 &= 0 \end{aligned} \end{array}$$

$$(A^T)^{-1} A^T C = (A^T)^{-1} B \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^T)^{-1} B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad \begin{array}{l} x_1 + x_2 + 0x_3 = 1 \\ -x_1 + 0x_2 + 2x_3 = 4 \\ 2x_1 + x_2 - x_3 = 3 \end{array} \quad \begin{array}{l} -x_1 + x_3 = -2 \\ -x_1 + 2x_3 = 4 \\ x_3 = 6 \end{array} \quad \begin{array}{l} \therefore x_1 = 8 \\ \text{and } x_2 = -7 \end{array}$$

$$C_1 = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad \begin{array}{l} x_1 + x_2 + 0x_3 = 2 \\ -x_1 + 0x_2 + 2x_3 = 5 \\ 2x_1 + x_2 - x_3 = 2 \end{array} \quad \begin{array}{l} -x_1 + x_3 = 0 \\ -x_1 + 2x_3 = 5 \end{array} \quad \begin{array}{l} x_1 = x_3 = 5 \\ \therefore x_2 = -3 \end{array}$$

$$C_2 = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \quad \begin{array}{l} x_1 + x_2 + 0x_3 = 3 \\ -x_1 + 0x_2 + 2x_3 = 4 \\ 2x_1 + x_2 - x_3 = 1 \end{array} \quad \begin{array}{l} -x_1 + x_3 = 2 \\ -x_1 + 2x_3 = 4 \end{array} \quad \begin{array}{l} x_3 = 2 \\ \text{and } x_1 = 4 \\ \therefore x_2 = -1 \end{array}$$

$$C_3 = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$\therefore C = \begin{bmatrix} 8 & 5 & 4 \\ -7 & -3 & -1 \\ 6 & 5 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

3×3

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

B
 3×3

$$C = B A^{-1}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 3 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

C is a 3×3 matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

A^T

C

B

$$C = \begin{bmatrix} -2+2+6 & 3+3+6 \\ -8+5+8 & 12+5+8 \\ -6+2+2 & 9+3+2 \end{bmatrix} \quad \boxed{0} \quad \boxed{0} \quad \boxed{0}$$

$$C = \begin{bmatrix} 6 & 12 & 0 \\ 5 & 25 & 0 \\ -2 & 14 & 0 \end{bmatrix}$$

for Matrix division

$$\text{1) } a/b = a \cdot b^{-1}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$C = (A^T)^{-1} B \quad (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ -3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} (-2+4+6) & (1+10+12) & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}^{+4+5+4}$$

$$C = \boxed{\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad Q \times 3$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2+4+3 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} & c_1 & c_2 & c_3 \\ \begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix} & \left| \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{matrix} \right. \end{array} \right] \xrightarrow{R_3 - 2R_2 + R_3} \left[\begin{array}{ccc|ccc} & 1 & 1 & 0 & 1 & 2 & 3 \\ & -1 & 0 & 2 & 4 & 5 & 4 \\ & 0 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|ccc} & 1 & 1 & 0 & 1 & 2 & 3 \\ & 0 & 1 & 2 & 4 & 5 & 4 \\ & 0 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} & 1 & 1 & 0 & 1 & 2 & 3 \\ & 0 & 0 & 2 & 3 & 4 & 3 \\ & 0 & 1 & -1 & 2 & 1 & 2 \end{array} \right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} & 1 & 1 & 0 & 1 & 2 & 3 \\ & 0 & 0 & 2 & 3 & 4 & 3 \\ & 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \xrightarrow{R_3 \leftarrow R_3 \cdot 2} \left[\begin{array}{ccc|ccc} & 1 & 1 & 0 & 1 & 2 & 3 \\ & 0 & 0 & 1 & -1 & 1 & 1 \\ & 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -1 & 3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xrightarrow{(1) \leftrightarrow (2)} \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xleftarrow{(2) \leftrightarrow (3)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xrightarrow{(2) \leftrightarrow (3)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \xrightarrow{(2) \leftrightarrow (3)} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \xrightarrow{(2) \leftarrow R_2 - 5R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \xrightarrow{(2) \leftarrow R_2 / 2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \xrightarrow{(1) \leftrightarrow (2)} \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -1 & 3 & 3 \\ 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \xrightarrow{(1) \leftarrow R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 2 & 6 \\ 0 & 1 & 0 & -1 & 3 & 3 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 2 & 6 \\ 0 & 1 & 0 & -1 & 3 & 3 \\ 0 & 0 & 1 & 11 & 12 & 9 \end{array} \right] \quad C = \begin{bmatrix} 0 & 2 & 6 \\ -1 & 3 & 3 \\ 11 & 12 & 9 \end{bmatrix}$$



$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T C = B \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \rightarrow x_1 = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \rightarrow x_2 = \begin{bmatrix} 13 \\ -11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{31} \\ x_{32} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \rightarrow x_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 13 & 0 \\ -7 & -11 & 3 \\ 6 & 9 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^{-1})^T \cdot A^T C = B$$

~~$$(A^T)^{-1} A^T C = (A^T)^{-1} B$$~~

$$C = (A^{-1})^T B$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

3×3

$(3) \times 3 = 3 \times 3$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 4 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+8+6=12 \\ 3-4-6=-7 \\ -1+4+3=6 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 5 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -4+10+4=10 \\ 6-5-4=-3 \\ -2+5+2=5 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + 4 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6+8+2=4 \\ 9-4-2=3 \\ -3+4+1=2 \end{bmatrix}$$

$$C = \begin{bmatrix} 12 & 10 & 4 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad (A^T)^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^T C = B$$

$$C = (A^T)^{-1} B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$$-2 \left(\begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{bmatrix} \right)$$

$$2 \left(\begin{bmatrix} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{bmatrix} \right) \rightarrow \begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{aligned} A^T C &= B \\ A^{T^{-1}} A^T C &= A^{T^{-1}} B \\ I C &= A^{T^{-1}} B \\ C &= A^{T^{-1}} B \end{aligned}$$

$C = 3 \times 3$ matrix

$$\begin{bmatrix} -a & 1 & a \\ 3 & -1 & -a \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find inverse of A^T

$$\begin{bmatrix} \frac{8}{-7} & \frac{5}{-3} & \frac{0}{3} \\ \frac{-7}{6} & \frac{-3}{5} & \frac{3}{2} \end{bmatrix} = C$$

$$\xrightarrow{1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \xrightarrow{2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{1} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right] \xrightarrow{2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 2 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\text{Inverse of } A^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$3 \times 3 \qquad 3 \times 3$$

$$\begin{aligned} -2 + 4 + 6 &= 8 \\ -4 + 5 + 4 &= 5 \\ -6 + 4 + 2 &= 0 \end{aligned}$$

$$3 + (-4) + (-6) = -7$$

$$6 + (-5) + (-4) = -3$$

$$9 + (-4) + (-2) = 3$$

$$-1 + 4 + 3 = 6$$

$$-2 + 5 + 2 = 5$$

$$-3 + 4 + 1 = 2$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$(A^T)^{-1} \cdot A^T C = (A^T)^{-1} \cdot B$$

$$C = (A^T)^{-1} \cdot B$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$C = (A^{-1})^T \cdot B$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = (A^{-1})^T \cdot B = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} (-2+4+6) & (-4+5+4) & (-6+4+2) \\ (3-4-6) & (6-5-4) & (9-4-2) \\ (-1+4+3) & (-2+5+2) & (-3+4+1) \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot B^{-1} \quad \left[\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 0 \\ 4 & 5 & 4 & 0 & 1 \\ 3 & 2 & 1 & 0 & 0 \end{array} \right]$$

=



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^T C = B$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \begin{array}{l} (1+2) \\ (2-3) \\ (0 \times 2 - 3) \end{array}$$

$$\downarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 1 & 1 & -1 & 2 & 5 \end{array} \right] \begin{array}{l} (1+2) \\ (2-3) \\ (0 \times 2 - 3) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \begin{array}{l} (1+2) \\ (2-3) \times 2 \end{array}$$

$$\downarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \begin{array}{l} (1-2) \\ (0-2) \end{array}$$

$$\downarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\text{so } \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = B A^{-1 T}$$

$$A^{-1} = \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -5 & 24 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\left(\begin{array}{l} (-5+6-3) \\ (-20+5-4) \\ (-15+6-1) \end{array} \right) \left(\begin{array}{l} (2-2+3) \\ (8-5+4) \\ (6-2+1) \end{array} \right) \left(\begin{array}{l} (4-4+3) \\ (16-10+4) \\ (12-4+1) \end{array} \right)$$

$$C = \boxed{\begin{bmatrix} -2 & 3 & 3 \\ -9 & 7 & 10 \\ -10 & 5 & 9 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & 0 & -5 & 24 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|cc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & -4 & -1 & -2 & 0 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right]$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\left| \begin{array}{ccc} 2 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & 1 & 0 \end{array} \right|$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$(A^{-1})^T C = B$$

$$(A^{-1})^T A^T C = (A^{-1})^T B$$

$$C = (A^{-1})^T B$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 5 & 4 \\ 0 & -1 & 0 \\ -3 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ -10 \\ -10 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array} \right]$$

$$\downarrow R_3 \rightarrow R_3 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 8 & 2 \end{array} \right] \xleftarrow{R_1 \leftrightarrow R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xleftarrow{R_2 \rightarrow R_2 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right] \xleftarrow{\text{row swap}} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array} \right]$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad Ax = B$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$