4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} Te_1 \end{bmatrix} \begin{bmatrix} Te_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} c = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 20 \\ 4 \end{bmatrix}$$

b) $T(u) = c_1 x_1 + c_2 x_2$

$u = c_1 e_1 + c_2 e_2$

$T(u) = T(c_1 e_1) + T(c_2 e_2)$

$= c_1 T(e_1) + c_2 T(e_2)$

$= \begin{bmatrix} T(e_1) T(e_2) \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \circ U$

$$\begin{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \cdot U$$

$U = \text{Span}\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot (u) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix} \qquad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & \end{array} \\ \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \end{array} = \begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{matrix}$$

$R_2 = \cancel{R_? - R_1}\ -R_1 + R_2$

$R_3 = -R_1 + R_3$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}\ \tfrac{1}{3} = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix} = R_2 + R_3 \qquad = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix} =$$

$$R_1 = 2(R_2) + R_1 = \left.\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}\right| \begin{matrix} 1 \\ 3 \\ 0 \end{matrix}$$

$$\begin{matrix} x_1 = \\ x_2 = \end{matrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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$\frac{9}{3}$ 3

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard matrix of T :- $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) as linear transformation,

$Au = T(u)$

$$\therefore \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$\therefore u_1 = 1 + 2u_2$ $\quad \therefore u_1 = 1 + 2(3) = 1 + 6 = 7$

(1) $u_1 - 2u_2 = 1$ $\quad \therefore 1 + 2u_2 + u_2 = 10$ $[\because u_1 = 1 + 2u_2]$ $\therefore 3u_2 = 9$ $\therefore u_2 = 3$

(2) $u_1 + u_2 = 10$

(3) $u_1 - 3u_2 = -2$ $\quad \therefore u_1 - 3(3) = -2$ $[\because u_2 = 3]$

$\therefore u_1 - 9 = -2$ $\quad \therefore u_1 = 7$

$$\therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a. $T_A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix} \div 3$

$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$

$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & -3 \end{bmatrix}$

free

$\begin{cases} x_1 = -7x_3 \\ x_2 = -3x_3 \\ x_3 = x_3 \end{cases}$ $x = \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$ $u \in \text{Span}\begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1 0
0 1
0 0

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a. $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 & -2x_2 \\ x_1 & +x_2 \\ x_1 & -3x_2 \end{bmatrix}$ $x_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{vmatrix} T(e_1) & T(e_2) \\ 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$ $\begin{vmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

$A = \begin{vmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

$\begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix}$ $\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{matrix}$ $\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix}$ $\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$ $\begin{matrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$ $x_1 = 7$
$x_2 = 3$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

a) $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{+\ \cdot(-1)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -3 \end{array}\right] \cdot(-1) \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 1 & 1 & 10 \end{array}\right] \xrightarrow{+\ \cdot -1} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right]$

$$u = \text{Span}\left(\begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}\right)$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(u) = T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -2 & -8 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 0 & -9 \end{array}\right]$

$\to$ No vector satisfies T(u).

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$A = [T(e_1), T(e_2)]$

a) Find the standard matrix of T.

where $e_1 = \begin{smallmatrix} 1 \\ 0 \end{smallmatrix}$ $e_2 = \begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b)

$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

3x2 2x2

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

a) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $u = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1-2x_2\\x_1+x_2\\x_1-3x_2\end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix}1\\10\\-2\end{bmatrix}$.

a) $T\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1-2(0)\\1+0\\1-3(0)\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\qquad T\begin{bmatrix}0\\1\end{bmatrix} = \begin{matrix}0-2\\0+1\\0-3\end{matrix} = \begin{bmatrix}-2\\1\\3\end{bmatrix}$

$3\times 2 \quad 2\times 1$

$\begin{bmatrix}A_1 & A_2\\A_3 & A_4\\A_5 & A_6\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\qquad \begin{bmatrix}A_1 & A_2\\A_3 & A_4\\A_5 & A_6\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-2\\1\\3\end{bmatrix}$

$A_1 = 1$ $\qquad A_2 = -2$

$A_3 = 1$ $\qquad A_4 = 1$

$A_5 = 1$ $\qquad A_6 = 3$

standard matrix of T is $\begin{bmatrix}1 & -2\\1 & 1\\1 & 3\end{bmatrix}$

b) $-1\cdot\begin{bmatrix}1 & -2 & 1\\1 & 1 & 10\\1 & 3 & -2\end{bmatrix}$

$1\cdot\begin{bmatrix}0 & -3 & -9\\1 & 1 & 10\\1 & 3 & -2\end{bmatrix}$

$-\tfrac{1}{3}\begin{bmatrix}0 & -3 & -9\\1 & 1 & 10\\1 & 0 & -11\end{bmatrix}$

$-1\begin{bmatrix}0 & 1 & 3\\1 & 1 & 10\\1 & 0 & -11\end{bmatrix}$

$-1\begin{bmatrix}0 & 1 & 3\\1 & 0 & 7\\1 & 0 & -11\end{bmatrix}$

$\begin{bmatrix}0 & 1 & 3\\0 & 0 & 18\\1 & 0 & -11\end{bmatrix}$

$\begin{bmatrix}1 & 0 & -11\\0 & 1 & 3\\0 & 0 & 13\end{bmatrix}$

no solutions

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ +1 & -3 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] = \Big[$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20