MTH-309T-F19-EX1-026-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	7	9	9	8	7

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

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1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right]$$

b) The set is linearly independent

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & -2 & b+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -12 & b-6 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-6}{-12} \end{matrix}$$

$c_3 = \frac{b-6}{-12}$

$c_2 = 2-2c_3$

$c_1 = -2+c_2-c_3$

$0 = \frac{(b-6)}{-12}$

$0 = b-6$

$6 = b$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 & -2 & 1 \end{array}\right] = \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

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0	1	2	3	4	5	6	7	8	9	10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$ $\begin{matrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{matrix}$

$C = BA^{T^{-1}}$

$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 2 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & -1 & -2 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 1 \end{bmatrix} = C$

$-8+15-4$ $\begin{matrix} -2 \\ 1 \\ 2 \end{matrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ -11 \\ 2 \end{bmatrix} = $

0 1 2 3 4 5 6 7 8 9 10

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$ $x_1 - x_2$

$CT(x) = T(u)$

$C_1(x) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix}$

$C_1 = 1 + 2C_2 = 7$

$C_2 = 3$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$T(V_1) = T(V_2$

$T(V_1) - T(V_2) = 0$

$T(V_1 - V_2) = 0$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix}$

$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix}$

$\left[\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}\middle|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right]$

$x_1 = -x_2$ $\quad$ $x_1 = 2$

$x_2 = -2x_3$ $\quad$ $x_2 = -2$

$x_3 = x_3$ $\quad$ $x_3 = 1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} =$

a) One-to-one

b) not, one-to-one

$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w}+\mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$$\begin{bmatrix} u_1+w_1 \\ u_2 \; w_2 \\ u_3 \; w_3 \end{bmatrix}$$ True

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True

$c_1 v + c_2 w = u$

$c_1 T(v) + c_2 T(w) = T(u)$

0 1 2 3 4 5 6 7 8 9 10