MTH 309T LINEAR ALGEBRA
EXAM 1
October 3, 2019

Name:

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UB Person Number:

5 0 1 7 1 8 3 3

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.

- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

10	10	10	20	20	4	6	--	--	80	B
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\underline{w} \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$(-2) \cdot \left(\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \right) \quad \begin{array}{l} -2 \\ 2 \\ b \end{array} \quad \begin{array}{l} 2 \cdot 2 - 3 \\ = 2 + b \end{array}$$

$$V_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad (1) \cdot \left(\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 0 \end{bmatrix} \right) \qquad 2 - 2$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{(1)} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad 2 - (-1)$$

$$X_1 = x - X_3 \rightarrow -2$$
$$X_2 = -2X_3 \rightarrow (2)$$
$$X_3 = \text{free} \rightarrow b$$

(a) $\boxed{b = -1}$ $\qquad b = -6 \qquad (-1).$

(b) No, dependent because it has infinite solutions X_3 is a free variable ✓

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$(-1) \subsetneq \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix}$

$\begin{array}{l} -2+1 \\ 1-1:-1 \end{array}$

$(-2) \subsetneq \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix}$

$(1)\cdot \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$

$\begin{array}{l} -1+2 \\ -1+1 \end{array}$

$(1) \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$

$(-1)\cdot \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 1 & -1 & 1 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$

-2

$\begin{bmatrix} 1 & 0 & 0 & | & -2 & 3 & -1 \\ 0 & 1 & 0 & | & 1 & -1 & 1 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$

$\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}$ ✔

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$(A^T)^{-1} A^T C = (A^T)^{-1} B$

$C = (A^{-1})^T \cdot B$ ✓

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$3 \times 3 \qquad\qquad 3 \times 3$

$2 + 6$
$-2 + 4 + 6 =$
$\overset{1+4}{-4 + 5 + 4}$
$-6 + 4 + 2$
$-2 + 2$

$3 + 4 - 6$
$-1 + 6$

$6 - 5 - 4$
$1 + 4$

$9 - 4 - 2$
$5 - 2$

$$\begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$ ✓

$-1 + 4 + 3$
$3 + 3$

$-2 + 5 + 2$
$-3 + 4 + 1 \cdot 1$

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ✗
0 1 2 3 4 5 6 7 8 9 10

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$7 - 6 = 1$
$7 + 3 = 10$
$7 - 9 = -2$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

ⓐ $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ ✓

ⓑ $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

ⓑ $T(u) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ ✓

$(-1) \curvearrowright \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$

$\begin{matrix} -1+10 \\ (-1) \end{matrix} \curvearrowright \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$\begin{matrix} (-3) \\ \tfrac{2-3}{} \end{matrix} \curvearrowright \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{bmatrix} \nrightarrow (2) \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$-1+2$

$-9+$

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = $
$(-3) \bigcirc \left[\begin{array}{ccc|c} 1 & 1 & 6 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$-3 \bigcirc \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$(-1) \bigcirc \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$

$(-1) \bigcirc \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$-2+2 = $

$-2+4 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ ✓

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ ✓

a) ONE-TO-ONE

b) NOT ONE-TO-ONE

$X_1 = -X_2$

$X_2 = -2X_3$

$X_3 = $ free

$V_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$V_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$ ✓

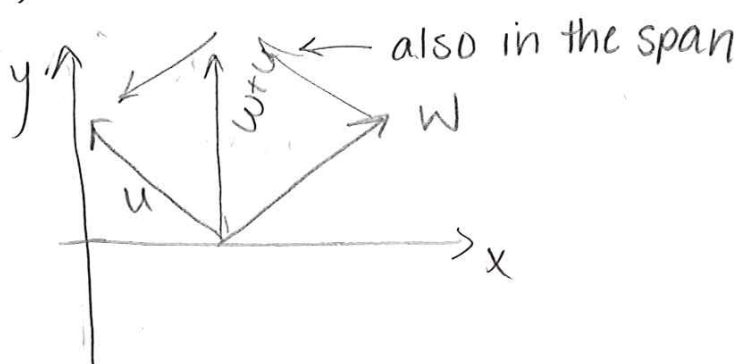| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 ✗ |

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in $\mathbb{R}^3$ such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True:

if $x = w$, then $x_0 = w + u)$?

also in the span

graphical interpretation

?

b) If u, v, w are vectors in $\mathbb{R}^3$ such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

These are still lin. indep

False

u v w

$$A = \begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 2 & 4 & | & 0 \\ 3 & 4 & 4 & | & 0 \end{bmatrix}$$ but

$$A = \begin{bmatrix} 1 & 1 & | & 9 \\ 0 & 2 & | & 3 \\ 3 & 4 & | & 5 \end{bmatrix}$$

is linear independent

[after row reduction. each value of u,v,w corresponds to a unique solution]

is inconsistent after row reduction because last row is

$$\begin{bmatrix} x & y & | & z \\ a & b & | & c \\ 0 & 0 & | & \# \end{bmatrix}$$

No solution; linearly dependent

0 1 2 3 ☒ 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in $\mathbb{R}^2$ such that Au, Av are linearly dependent then u, v also must be linearly dependent.

$$\text{''} \quad \overset{A}{\underset{2}{\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}}} \overset{u}{\begin{bmatrix} 5 \\ 10 \end{bmatrix}}$$

FALSE,

just because Au, Av are linearly dependent does not mean that u, v will also be linearly dependent

$$(-2) \, G \, \overset{A}{\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}} \overset{v}{\begin{bmatrix} 5 \\ 7 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

b) If $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then $T(u)$ must be in Span$(T(v), T(w))$.

True

Col(A) = set of values for T_A
so if u is in span (v, w) for matrix A then it will also be in the span of the transformation for A.

?