

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of  $A$  are  $\lambda_1 = 3$  and  $\lambda_2 = 5$  diagonalize this matrix; that is, find a diagonal matrix  $D$  and an invertible matrix  $P$  such that

$$A = PDP^{-1}$$

Note: you do not need to compute  $P^{-1}$ .

$$\text{Null}(A - \lambda_1 I) = \begin{bmatrix} -2 & 8 & 4 & 0 \\ -2 & 8 & 4 & 0 \\ 2 & -8 & -4 & 0 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} -2 & 8 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-\frac{1}{2}} \begin{bmatrix} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 = 4x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Null}(A - \lambda_2 I) = \begin{bmatrix} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{bmatrix} \xrightarrow{-\frac{1}{2}} \begin{bmatrix} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{bmatrix} \xrightarrow{\begin{smallmatrix} x_2 \rightarrow 1 \\ x_3 \rightarrow 1 \end{smallmatrix}} \begin{bmatrix} 1 & -2 & -1 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{bmatrix} \xrightarrow{\begin{smallmatrix} x_2 \rightarrow 1 \\ x_3 \rightarrow 1 \end{smallmatrix}} \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -4 & -4 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}} \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & -4 & 0 \end{bmatrix} \xrightarrow{\begin{smallmatrix} x_2 \rightarrow 1 \\ x_3 \rightarrow 1 \end{smallmatrix}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$A = PDP^{-1}$$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$$

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