1. Consider the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace V.

b) Compute the vector $\mathrm{proj}_V \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ on V.

$$w_1 = v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$w_2 = v_2 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{(1\cdot 2 + 0\cdot 1 + -1\cdot -1 + 1\cdot 0)}{(1\cdot 1 + 0\cdot 0 + -1\cdot -1 + 1\cdot 1)} w_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - 1\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \checkmark$$

$$w_3 = v_3 - \left(\frac{w_1 \cdot v_3}{w_1 \cdot w_1}\right) w_1 - \left(\frac{w_2 \cdot v_3}{w_2 \cdot w_2}\right) w_2 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{(2+0+1+3)}{(1+0+1+1)}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \frac{(2-2+0-3)}{(1+1+0+1)}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \checkmark$$

$$\boxed{\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}} \checkmark$$

$$\mathrm{proj}_V u = \left(\frac{u \cdot w_1}{w_1 \cdot w_1}\right) w_1 + \left(\frac{u \cdot w_2}{w_2 \cdot w_2}\right) w_2 + \left(\frac{u \cdot w_3}{w_3 \cdot w_3}\right) w_3 = \frac{(3+0-3+3)}{(1+0+1+1)} w_1 + \frac{(3+3+0\cancel{+3})^{-3}}{(1+1+0+1)} w_2 + \frac{(3-3+3+0)}{(1+1+1+0)} w_3$$

$$= w_1 + \cancel{3} w_2 + w_3 = \begin{bmatrix} 1+3+1 \\ 0+3-1 \\ -1+0+1 \\ 1+3+0 \end{bmatrix} = \boxed{\begin{bmatrix} 5 \\ 2 \\ 0 \\ 4 \end{bmatrix}}$$

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