

MTH 309T LINEAR ALGEBRA

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5 ① ① ② ③ ④ ⑥ ⑦ ⑧ ⑨	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	3456	4567	3 4 5 6 7	⑥ ⑦		७ 1 0 1 0 0 0 0 0 0 0 0 0 0		 Textbooks, calculators and any other electronic devices are not permitted You may use one sheet of notes. For full credit solve each problem fully, showing all relevant work. 			
1		2		3		4	5		6	7	TOTAL	GRADE

20	10	5	20	20	6	2			83	B+
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .







3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A)

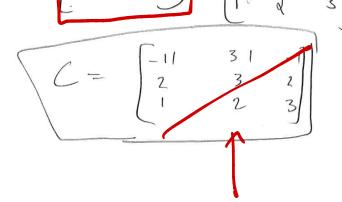
me matrix as in Problem 2, and let
$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{T}C = B \text{ (where } A^{T} \text{ is the transpose of } A \text{)}.$$

$$AT = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{pmatrix} \qquad (AT)^{-1} = \begin{pmatrix} -2 & 1 & 2 \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

$$AT C = B \Rightarrow (AT)^{-1} (AT) C = B AT)^{-1} \Rightarrow C = B(AT)^{-1}$$

$$C = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 & 2 \\ -3 & -1 & -2 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} -1 & 31 & -1 \\ 2 & 3 & 2 \\ 1 & 3 & 3 \end{pmatrix}$$



metria multiple too.

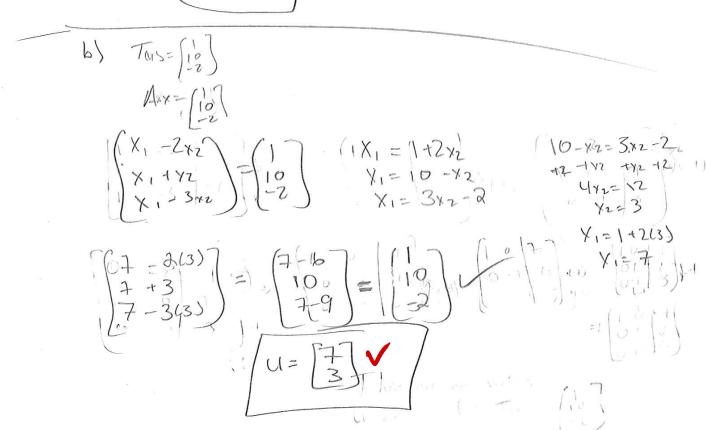


4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)
$$\{T(e_i) \mid T(e_i)\} = [T([i]) \mid T([i])] = \begin{bmatrix} 1-210 \\ 1+0 \\ 1-310 \end{bmatrix} = \begin{bmatrix} 1-210 \\ 1-310$$





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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d) $\begin{bmatrix} 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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6)
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 0 & 0 \\ 0 & 1 & 2 & 0 \\$$

MTH-309T-F19-EX1-038-P06



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in Span(u, v)$ then $w \in Span(u, v)$.

Polse. If u,u,w are vectors in

? The Soto was E Span, wmust be a linear combination of u+v not u+u.

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

True of Spar (1) the If

[UIVING is linearly independent, only harve

trivial solution as the solution to homogeneous

equations ..., CIU + CIV = 0 must also

only have the trivial solution as the

arow.



- **7.** (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

The For Au and Ar to be Then's dependent, u and v must have infinitely many solutions. ? Some combination of use other than the trivial solution well result in the Overtor.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True. If u is in Spen (u,w), then it is a linear combination of v& wo. Herefore its

transformation must be in the spen as well & why?