MTH-309T-F19-EX1-003-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Louis Kerner

UB Person Number:

5	0	1	9	9	6	6	8

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE
8	7	6	17	4	2	3	--	--	47	D

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \quad R_3 = -2(R_1) + R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{bmatrix} \quad R_3 + R_2 + R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix} \quad R_1 = R_2 + R_1 \quad \checkmark$$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix} \quad R_2 = R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

b) The solution is linear independent. The last row is 0 and because of that there are multiple solutions. Also, it can't be dependent because every column must be a pivot.

This would mean that vectors are lin. dependent

b) ~~cannot~~ in order for it be

Answer for a) ?

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

$(-1)(-2) = 2$ $\quad$ $2(-1)+1 =$

$-2(1) = -2$ $\quad$ -2

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$(-1)(-2) = 2 - 1 = 1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \checkmark \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \checkmark$$

$R_2 = (-1)R_1 + R_2$ $\quad$ $R_3 = -2(R_2) + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & \cancel{0}\ {-2} & 1 \end{array}\right] \quad R_1 = 2R_2 + R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = -R_2 + R_1$

$$\boxed{A^{-1} = \begin{matrix} 0 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{matrix}}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T: \begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix} \cdot C = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{matrix}$$

$(AB)^T = B^T A^T$

$2 \times 4 = 8$

$A^T \cdot C = B$

$\frac{B}{A^T} = C = \cancel{B \cdot (A^T)^{-1}}$

$(A^T)^{-1} \cdot B$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = 0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 1\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$-1 + 2 + 3 = 4$

$-4 + 5 + 4 = 5$

$-3 + 2 + 1 = 0$

$$0\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad 0\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \quad 1\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 4 & 3 \\ 1 & 5 & 4 \\ 1 & 0 & 1 \end{bmatrix} = C$$

0 1 2 3 4 5 6 7 8 9 10

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot (u) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix} \qquad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \checkmark$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \checkmark = \begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{matrix}$$

$R_2 = \cancel{R_2 - R_1}$ $-R_1 + R_2$

$R_3 = -R_1 + R_3$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}\ \tfrac{1}{3} = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix} = R_2 + R_3 = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix} =$$

$$R_1 = 2(R_2) + R_1 = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{matrix} x_1 = \\ x_2 = \end{matrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☒	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{bmatrix}$ $R_2 = R_2 \cdot \frac{1}{2}$

$R_3 = -3R_1 + R_3$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{matrix}$

$R_3 = -3R_1 + R_3$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & & \end{matrix}$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 4 & 0 \end{matrix}$ $R_3 = R_3 \times -1$

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{matrix}$ $R_3 = R_2 - R_3$

b) $\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{matrix}$ $R_3 = R_2 + R_3$

a) $\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$ $R_3 = R_2 + R_1$

Onto Pivot column

$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$

One to one

one-to-one or not? (Also, row reduction is wrong)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☒	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

It is ~~true~~ false. No matter how much you ~~can~~ change it the rule only applies for multiplication

?

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False. linearly independant means: that there ~~is only~~ must be ~~one~~ ~~infinite solutions~~. We do not know which vector has a free variable. so we dont know 100 percent of the time.

?

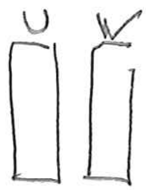
Exampe
Free variable
u, v, w

u and v can be dependant

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.



~~True~~, if they have a unique solution, it ~~doesnt matter~~ doesn't matter whether they are ~~seperated or not~~ represented in a matrix or not

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, ✓ Span$(v_1, \dots v_p)$ = set of all linear combination $c_1V_1 + c_2V_2$.

why? → Thus it holds ~~every~~ than T(u) must be in Span(T(v), T(w))

0	1	2	3	4	5	6	7	8	9	10
☐	☐	☐	☒	☐	☐	☐	☐	☐	☐	☐