1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\text{SPAN}(V_1\ V_2\ V_3) = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \qquad W = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{2 \cdot R1 - R3 = R3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R1 + R2 = R1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R2 - R3 = R3} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$[2 \cdot 1 - 2]$ $[2 \cdot -1 - (-3)]$ $2 \cdot 1 - 0$

CONTINUED HERE

A AFTER ROW REDUCTION

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} V_1 + 3V_3 = -2 \quad / \cdot (-2) \\ V_2 + 2V_3 = 2 \quad / \cdot (3) \end{matrix} \Rightarrow \begin{matrix} -2V_1 - 6V_3 = 4 \\ 3V_2 + 6V_3 = 6 \end{matrix} \Rightarrow -2V_1 + 3V_2 = 10$$

$$\Downarrow$$

$$2V_1 - 3V_2 = -10$$

$$2V_1 = 3V_2 - 10$$

$$\boxed{V_1 = \frac{3}{2}V_2 - 5}$$

CONTINUED HERE

$V_1 = \frac{3}{2}V_2 - 5$

FOR $V_1 + 3V_3 = -2$

$\frac{3}{2}V_2 - 5 + 3V_3 = -2$

$\frac{3}{2}V_2 + 3V_3 = 3$

$\frac{1}{2}V_2 + V_3 = 1$

$\boxed{V_2 = 2 - 2V_3}$

$V_1 = \frac{3}{2} \cdot 2(1 - V_3) - 5$

$V_1 = 3 - 3V_3 - 5$

$\boxed{V_1 = -3V_3 - 2}$

$V_1 \to -3V_3 - 2$

$V_2 \to 2 - 2V_3$

$V_3 \to V_3$

- b HAS INFINITE MANY OF VALUES
- SET OF $\{V_1, V_2, V_3\}$ IS LINEARLY DEPENDENT SINCE V_3 IS A FREE VARIABLE.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

MTH-309T-F19-EX1-002-P01

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right] \cdot(-2) \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & b-4 \end{array}\right] (-5) \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -10 & b-14 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-14}{-10} \end{array}\right]$$

$b = 4$

$Ax = b \rightarrow Av = b$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

b) a set is linearly independent if it has only one solution since there is a pivot position in every column it is linearly independent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \quad R_3 = -2(R_1) + R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{bmatrix} \quad R_3 + R_2 + R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix} \quad R_1 = R_2 + R_1$$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix} \quad R_2 = R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

b) The solution is linear independent. The last row is 0 and because of that there are multiple solutions. Also, it can't be dependent because every column must be a pivot.

b) before ~~...~~ in order for it be

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$[\, v_1 \quad v_2 \quad v_3 \,] = \vec{w}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-6 \end{array}\right]$$

$b-6=0$

if $b=6$, then $\vec{w} \in \text{span}(\vec{v_1}, \vec{v_2}, \vec{v_3})$

$x_1 - x_2 + x_3 = -2$

x_2

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3 is free

to be linearly independent, every column after row reduction must be a pivot column. Therefore, the set is not linearly independent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\underline{w} \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$V_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ $\quad (-2) \cdot \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right]$ $\quad 2-3$ $\quad =2+6$

$V_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$

$V_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ $\quad (1) \cdot \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right]$ $\quad 2-2$

$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow (1) \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\quad 2-(-1)$

$x_1 = -x_3 \rightarrow -2$

$x_2 = -2x_3 \rightarrow (2)$

$x_3 = \text{free} \rightarrow b$

(-1).

(a) $\boxed{b=-1}$

(b) No, dependent because it has infinite solutions x_3 is a free variable

0 1 2 3 4 5 6 7 8 9 10 11 ~~12~~ 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)
$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{r2+r1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow[+r3]{(-2)\cdot r1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{bmatrix}.$$

$$\xrightarrow{(-\frac{1}{3})\cdot r3} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b \end{bmatrix} \xrightarrow[+r3]{(-1)\cdot r2} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{bmatrix}$$

$0 = b - 2$

$b = 2$

b)
$$\begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \end{array} \xrightarrow{\text{row reduce}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\{v_1, v_2, v_3\}$ is not linearly independent because every column of $[v_1\ v_2\ v_3]$ is <u>not</u> a pivot column

Hello!

0 1 2 3 4 5 6 7 8 9 10 ~~11~~ 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$b=6 \quad b=-6$

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

let $x_1 = 0$, $x_2 = 2$, $x_3 = 0$

let $x_1 = -3$, $x_2 = 0$, $x_3 = 1$

$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{array}\right] \rightarrow \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right]$

$x_1 = -3x_3$, $x_3 = \text{free}$

$x_2 = 2 - 2x_3$ infinite soln

$b = -6$

$\boxed{b = -6, 6}$

b) $x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

since x_3 = free, there are infinitely many solutions

$\Rightarrow$ set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ is linearly independent

?

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$. c_1, c_2, c_3

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

RREF & See if each column has leading one?

a) $c_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$\boxed{b \neq 0}$

$0\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$\boxed{b = -6}$

$y(2) + x(-3) + 0$

(b)
$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \leftarrow -2R_1 + R_3 \to R_3$$
$-2 \quad +2 \quad -2 \quad 0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \leftarrow R_2 + R_3 \to R_3$$
$+0 \quad +1 \quad +2 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\leftarrow$ Vectors V_1, V_2 & V_3 are Linearly dependant (NOT Linearly Independent) because, V_2 & V_3 are free variables.

$V_1 - V_2 + V_3 = 0$

$V_2 + 2V_3 = 0$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☒	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1 -1 1 -2

MTH-309T-F19-EX1-009-P01

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $V_1 - V_2 + V_3 = -2$

$V_1 + 2V_2 = 2$

$2V_1 - 3V_2 = b$

~~$V_1 = 2 - 2V_2$~~

~~$2 - 2V_2 - V_2 + V_3 = -2$~~

~~$-3V_2 + V_3 = -4$~~

~~$V_3 = -4 - 3U_2$~~

~~$2 - 2V_2 - V_2 - 4 - 3U_2 = -2$~~

~~$2 - 3V_2 - 4 - 3U_2 = -2$~~

~~$-3V_2 - 4 - 3V_2 = -4$~~

~~$V_2 = 0$~~

~~$V_1 = 2$~~

~~$b = 4$~~

$V_1 = 2 - 2V_2$

$2 - 2V_2 + V_3 = -2$

$-3V_2 + V_3 = -4$

$V_3 = -4 - 3V_2$

$= 2 - 2V_2 - V_2 - 4 - 3V_2 = -2$

$\Rightarrow \underline{V_2 = 0}$

$V_1 + 2(0) = 2$

$\underline{V_1 = 2}$

$2V_1 - 3V_2 = b$

$2(2) - 3(0) = b$

$4 = b$

$\boxed{b = 4}$

b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 4 \end{array}\right] (-2) = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 0 & 8 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & 0 & 8 \\ 0 & 1 & 2 & 2 \end{array}\right] (1)$

$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 3 & 2 \end{array}\right] \cdot \frac{1}{3} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 1 & 2/3 \end{array}\right] \cdot (-1) \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 2/3 \end{array}\right] (1) \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2/3 \end{array}\right]$

$\Downarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2/3 \end{array}\right] (-1) \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2/3 \end{array}\right]$

The set $\{V_1, V_2, V_3\}$ is linearly independent because the the set contains only 1 solution, thus being a homogeneous equation which proves that the set is linearly independent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \operatorname{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $w \notin cv_1$ since 2nd row of cv_1 is always 0.
$w \notin cv_3$ since 2nd row is same for both, but first row is different.
$w \in cv_2$ when $c = 2$ and $b = -6$ ✓

$$w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = 2v_2 = 2 \cdot \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$$

$\boxed{b = -6}$

b) Yes, since there is only 1 solution!

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & c_1 \\ 0 & 1 & 2 & c_2 \\ 2 & 3 & 0 & c_3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & c_1 \\ 0 & 1 & 2 & c_2 \\ 0 & 5 & -2 & c_3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 3 & c_1 \\ 0 & 1 & 2 & c_2 \\ 0 & 0 & -12 & c_3 \end{array}\right] \to$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & c_1 \\ 0 & 1 & 0 & c_2 \\ 0 & 0 & 1 & c_3 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= c_1 \\ x_2 &= c_2 \\ x_3 &= c_3 \end{aligned}}$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13 ✗	14	15	16	17	18	19	20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \begin{array}{l} v_1 \\ v_2 \\ v_3 \end{array}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

b) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \cdot -2$

$\downarrow$

$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 0 \end{bmatrix} \cdot 1$

$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{array}{l} \cdot \frac{1}{2} \\ \cdot 1 \\ \cdot 1/2 \end{array}$

$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot 1$

$\downarrow$

$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is linearlly independant because each part of the matrix has a pivot point}}$

$$\begin{matrix} -2 & -2 \\ 0 & 1 \\ -4 & -3 \end{matrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 \to R_3 + (-2R_1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right]$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = w \Rightarrow \begin{matrix} c_1 = 0 \\ c_2 = 2 \\ c_3 = 0 \end{matrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\therefore \boxed{b = -6}$$

b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + (-2R_1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

not linearly independent because every column of the matrix is not a pivot column.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_1} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

$$R_1 = R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

$b = -6$ → For all other values of b there will not be a solution.

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_1} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 = R_1 + R_2 \rightarrow \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

↳ Free variable

~~Since col.~~ Since, X_3 is a free variable, the set $\{V_1, V_2, V_3\}$ is NOT linearly independent

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) as $W \in \text{span}(V_1, V_2, V_3)$ $\therefore V_1 + V_2 + V_3 = W$

$$-4 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$\therefore b = (-8)$.

b) $c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

① $c_1 - c_2 + c_3 = 0$ $\quad \therefore c_1 = c_2 - c_3$ $\quad \therefore c_3 = 0$ $[\because c_2 = 0$

② $c_2 + 2c_3 = 0$ $\quad \therefore c_3 = -\frac{c_2}{2}$

③ $2c_1 - 3c_2 = 0$ $\quad \therefore c_1 = \frac{3c_2}{2}$ $\quad \therefore c_1 = 0$ $[\because c_1 = 0$

$\therefore c_2 - c_3 = \frac{3c_2}{2}$

$\therefore 2c_2 - 2c_3 = 3c_2$ $\quad \therefore 2c_2 + 2\left(\frac{c_2}{2}\right) = 3c_2$ $\quad \therefore 3c_2 = 3c_2$

$\therefore c_2 = 0$

as, $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ so, we can say the set $\{V_1, V_2, V_3\}$ is linearly independent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $b=0$

only solution

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-2 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{matrix}$$

for $\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$ in span

$$\begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -4 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

~~reduces~~ b)

$$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

reduced matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) No

$\boxed{\text{linearly dependent}}$

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a). $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{\cdot -2} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \xrightarrow{+} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$

When $b = -6$, it has infinite solutions.

$x_1 + 3x_3 = 0$

$x_2 + 2x_3 = 2$

b) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{\times -2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -5 & -2 \end{bmatrix} \xrightarrow{+} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & -5 & -2 \end{bmatrix} \times 5$

$\rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} + \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \times -\frac{1}{2} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

It's linearly independent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free}} & b+6 \end{bmatrix}$

$(-3)+(2)$ $\quad 0+N\cdot 2$ $\quad b+(-2)(2)$

when $b+6=0$
$b=-6$. so $w \in \mathrm{Span}(v_1, v_2, v_3)$

~~matrix~~ it has infinite solution. with free variable x_3.

b). no, for set $\{v_1\ v_2\ v_3\}$ after row reduced

we got $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ the third column is not a pivot column.

which ~~meath~~ means it is not a linearly independent set.

~~$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \ 0 \ x_3$~~.

for $v_1\ v_2\ v_3$. $\quad x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \to 0.$

because it has free variable.

have infinite solution

so not linear indep

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \operatorname{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] (-2) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] (1) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right] (1)$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right]$$

$w \in \operatorname{Span}(v_1, v_2, v_3)$ as long as $\underline{b = -6}$.

(b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] (-2) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] (1) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] (1)$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free

Since the homogeneous equation $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$ has infinitely many solutions due to a free variable, the set $\{v_1, v_2, v_3\}$ is <u>NOT</u> linearly independent.

Hello!

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathrm{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathrm{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathrm{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathrm{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathrm{w} \in \mathrm{Span}(\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3)$.

b) Is the set $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{\cdot(-2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right] \Longrightarrow \quad b = 6$$

b) The set $\{v_1, v_2, v_3\}$ is <u>linearly dependent</u> because x_3 is a free variable meaning there are infinitely many solutions.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}.$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{R_3+(-2)R_1} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{bmatrix} \xrightarrow[R_3+(1)R_2]{R_1+(1)R_2} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4b+2 \end{bmatrix}$$

$0 = 4b+2$

$4b = -2$

$b = -\frac{2}{4}$

$\boxed{b = -1/2}$

b) No,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{R_3+(-2)R_1} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$a_1 v_1 + \ldots + a_n v_n \neq 0$

$x_1 - x_2 = 1$

$x_2 = 2$

$x_1 = 3$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$\frac{1}{2}\times$ $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 2 & -3 & 0 & b \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$-1\times$ b $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 1 & -\frac{3}{2} & 0 & \frac{b}{2} \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -\frac{1}{2} & -1 & \frac{b}{2}+2 \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$\times -2$ $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -\frac{1}{2} & -1 & \frac{b}{2}+2 \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$\times -1$ $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & -2(\frac{b}{2}+2) = -b-4 \\ 0 & 1 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & -b-4 \\ 0 & 0 & 0 & b+6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 1 & 0 & -b-4 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

Since $b+6$ is in last colum $b+6$ has to be equal to 0 which $b=-6$ to make zero otherwise $b+6$ will be 1. which is undefind. So $\boxed{b=-6}$

?

(b)

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & -\frac{1}{2} & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$X_1 = -X_2 + X_3$
$X_2 = X_3$
$X_3 = X_3$

No the set is not linear Independent Some Scalar mutiple of v_1 add to Scalar multiple of v_2 will product V_3.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$. b = any number but 4

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right] \quad 3-2 \quad \times 2$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & b-4 \end{array}\right] \times 5$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -10 & 10(b-4) \end{array}\right] \times -\frac{1}{10}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \underbrace{-(b-4)}_{-b+4} \end{array}\right]$$

$-b+4=0$

$-b=-4$

$b=4$

$\therefore$ If $b=4$, then $W \notin \text{Span}(V_1, V_2, V_3)$

The set $\{V_1, V_2, V_3\}$ is linearly independent because each vector is not a scalar multiple of each other

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & 2 & -4+b \end{array}\right] \; 5$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 12 & 6+b \end{array}\right] \; \tfrac{1}{12}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{6+b}{12} \end{array}\right]$$

Span(b) = $\mathbb{R}^3$ since it'd be a solution to any real number

b)

$$\left.\begin{array}{l} x_1 - x_2 + x_3 = -2 \\ x_2 + 2x_3 = 2 \\ x_3 = \frac{6+b}{12} \end{array}\right\} \quad \begin{array}{l} x_1 = -2 - x_2 + x_3 \\ x_2 = 2 - 2x_3 \\ x_3 = \frac{6+b}{12} \end{array}$$

Since the equation has a solution any value of b given
it is ~~a~~ not a linearly independent set

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

$-3(6-b)$

$\frac{-18+3b}{8}$

$16-12=4$

$2-2\left(\frac{6-b}{8}\right)$

$2-\frac{12+b}{8}$

$2-2\left(\frac{6-b}{8}\right)$

$2-\frac{12+2b}{8}$

$\frac{-1}{8}\, b-6 = \frac{-b+6}{8}$

$\frac{6+b}{8}$

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & -2 & b+4 \end{array}\right] \begin{array}{l}\xrightarrow{R_1+R_2} \\ \\ \xrightarrow{R_1-5R_2}\end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -8 & b-6 \end{array}\right] \xrightarrow{R_3 \cdot \frac{-1}{8}}$

$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{6-b}{8} \end{array}\right] \begin{array}{l}\xrightarrow{R_1-3R_3} \\ \xrightarrow{R_2-2R_3}\end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-18+3b}{8} \\ 0 & 1 & 0 & 2-\frac{12+2b}{8} \\ 0 & 0 & 1 & \frac{6-b}{8} \end{array}\right] \rightarrow \therefore$

$x_1 = \frac{-18+3b}{8}$

$x_2 = \frac{4+2b}{8}$

$x_3 = \frac{6-b}{8}$

Since b is in the numerator of all solutions $\therefore$ if $b \in \mathbb{R}$ then $w \in \text{Span}(v_1, v_2, v_3)$

b) The set $\{v_1, v_2, v_3\}$ is linearly independent because there is a pivot column in every column as shown in part A where we reduced the matrix in row echelon form.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

3

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$. $A\vec{x} = b$

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $[v_1 \ v_2 \ v_3 \mid w]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & -3 & 0 & b \\ 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \end{array}\right] \cdot(1)$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & b \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{array}\right] \cdot(-2) \to \left[\begin{array}{ccc|c} 0 & -3 & -6 & b \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{array}\right] \cdot(3)$$

$\mathrm{Col}(A) = \mathrm{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ $\boxed{b = -6}$

b) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \cdot(-2) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \cdot(1) \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot(-1)$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{\text{Not linearly independent}}$

x_3 is a free variable so there are infinite solutions. this means that they are linearly dependent.

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right]$$

b) The set is linearly independent

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & -2 & b+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -12 & b-6 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-6}{-12} \end{matrix}$$

$c_3 = \frac{b-6}{-12}$

$c_2 = 2-2c_3$

$c_1 = -2+c_2-c_3$

$0 = \frac{(b-6)}{-12}$

$0 = b-6$

$6 = b$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] R_3 + R_2$$

$$\begin{array}{cccc} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right] \\ x_1 \quad x_2 \quad x_3 \end{array}$$

$x_1 - x_2 + x_3 = -2$
$x_2 + 2x_3 = 2$
$0 = b + 6$
$b = -6$

b)
$c_1v_1 + c_2v_2 + c_3v_3 = 0$

$$c_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right] R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 - c_2 + c_3 = 0 \rightarrow c_1 = 0$
$c_2 + 2c_3 = 0 \rightarrow c_2 = 0$
$c_3 = 0$

Yes because all c_n values are equal to 0.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\text{Span}(v_1, v_2, v_3) = \left\{ \begin{matrix} \text{the set of all} \\ \text{vectors } c_1v_1 + c_2v_2 + c_3v_3 \end{matrix} \right\}$

$x_1v_1 + x_2v_2 + x_3v_3 = w$

$R_3 = -2R_1 + R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$R_3 = +R_2 + R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b \end{array}\right]$

$R_1 = R_2 + R_1 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array}\right]$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array}\right]$$

$\therefore$ b must equal 0

$$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 + 2 \\ x_3 = x_3 \end{cases}$$

b) the set $\{v_1, v_2, v_3\}$ is not linearly independent because every column of the matrix is not a pivot column

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

The value of b is -6

because $2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$

It is independent because Null(A) is {0}

2 -
-3 - 2

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -5 & -2 & 0 \end{matrix}$

~~Yes it is linearly independent?~~

-5+5
-2+10

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 8 & 0 \end{matrix}$

$x_1 = 0$
$x_2 = -2$
$x_3 = 0$

~~It is linearly independent because x_2 can be ... when set to 0~~

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$ $\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & \cancel{0} \\ 0 & 0 & 1 & 0 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a) W is a linear combination of V_1, V_2, V_3

$W = X_1V_1 + X_2V_2 + X_3V_3$

$= X_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + X_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + X_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} X_1 - X_2 + X_3 \\ 0 + X_2 + 2X_3 \\ 2X_1 - 3X_2 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$\Rightarrow$ augmented matrix $\begin{array}{ccc|c} X_1 & X_2 & X_3 & \\ \end{array}$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \end{array}\right] \xrightarrow{\text{Row Reduction}} \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{array}\right]$

X_3 : free variable

Sol $\begin{cases} X_1 = -3X_3 \\ X_2 = -2X_3 + 2 \\ X_3 = X_3 \end{cases}$

$\therefore b = 2X_1 - 3X_2$
$= -6X_3 + 6X_3 - 6$
$= -6$

$\boxed{b = -6}$

(b) $[\,V_1 \;\; V_2 \;\; V_3 \mid 0\,]$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \xrightarrow{R_3 + R_2} \begin{array}{ccc} X_1 & X_2 & X_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

free variable

$\Rightarrow$ X_3 is a free variable, so $X_1V_1 + X_2V_2 + X_3V_3 = 0$ has infinetely many solutions.

$\begin{cases} X_1 = -3X_3 \\ X_2 = -2X_3 \\ X_3 = X_3 \end{cases}$

$\Rightarrow \{V_1, V_2, V_3\}$ is linearly dependent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) if w span (v_1, v_2, v_3) th $w = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$w = c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$w = -2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$w = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} + \begin{bmatrix} b \\ 2b \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} b-4 \\ 2b+2 \\ -10 \end{bmatrix}$$

$b - 4 = 0 \Rightarrow b = 4$

$2b + 2 = 0 \quad b = -1$

b)
$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{2R_1 - R_3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 + R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 - R_3} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-3R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-2R_3 + R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

yes $\{v_1, v_2, v_3\}$ is linearly independent because every colum of the ~~rm~~ reduced matrix is a pivot column.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$x_1 - x_2 + x_3 = -2$

$x_2 + 2x_3 = 2$

$2v_2 \to b = -6$

$b = 2x_1 - 3x_2$

$\boxed{b = -6 \text{ only}}$

$x_1 - (2 - 2x_3) + x_3 = -2$

$x_1 + 3x_3 = 0$

$b = 2(-3x_3) - 3(2 - 2x_3)$

$b = -6x_3 - 6 + 6x_3$

$\boxed{b = -6}$

~~b) $\{v_1, v_2, v_3\}$ is linearly independent because~~

~~$v_1 \neq c_1 v_2$ for any $c_1 \in \mathbb{R}$;~~

~~$v_2 \neq c_2 v_3$ for any $c_2 \in \mathbb{R}$;~~

~~$v_3 \neq c_3 v_1$ for any $c_3 \in \mathbb{R}$.~~

$\boxed{\text{b) } \{v_1, v_2, v_3\} \text{ is not linearly independent because } 3v_1 = v_3 - 2v_2.}$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $C_1 V_1 + C_2 V_2 + C_3 V_3 = W$

Aug mat

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{③-2①} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] \xrightarrow{③+②} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$\boxed{b \neq -6}$

b) $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{\text{row red}} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x_3 = \text{free}$
$x_1 = -3x_3$
$x_2 = -2x_3$

Equation $x_1 V_1 + x_2 V_2 + x_3 V_3 = 0$ has infinitely many solution

So $\boxed{\{V_1, V_2, V_3\} \text{ is linearly dependent}}$

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12 ☐ 13 ☐ 14 ☐ 15 ☐ 16 ☐ 17 ☐ 18 ☐ 19 ☐ 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_1} \begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{array}$$

$$R_1 = R_1 + R_2 \quad \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{bmatrix} \quad R_3 = R_3 + R_2 \quad \left|\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & (6+b) \end{array}\right|$$

a. $w \in \mathrm{Span}(v_1, v_2, v_3)$ when $b = -6$

b. $\left|\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array}\right| \longrightarrow \quad 1 \quad -1$

The set is not linearly independent, because the V_3 column is a free ~~fa~~ variable, so there is not a leading one in each column

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$(-2)\begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -3 & 0 & b \\ 0 & 1 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -1 & -2 & b-4 \\ 0 & 1 & 2 & 2 \end{bmatrix}$

$2\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 4-b \\ 0 & 1 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 4-b \\ 0 & 0 & 0 & b-2 \end{bmatrix}$

$\because$ the last row is 0.

$\therefore b-2=0$.

$\therefore b=2$.

$\therefore$ $b=2$ such that $w \in \mathrm{Span}(v_1,v_2,v_3)$

b) the set $\{v_1, v_2, v_3\}$ is not linearly dependant because every column is not a pivot column

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{array}{cccc} 2 & -3 & 0 & b \\ -2 & 2 & -2 & 4 \\ \hline 0 & -1 & -2 & b+4 \end{array} \qquad \begin{array}{ccc} 1 & 2 & 2 \\ -1 & -2 & b+4 \\ \hline 0 & 0 & b+6 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{-2\,\text{row}(1)} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

$\boxed{\therefore\ b \neq -6 \text{ for } w \in \mathrm{Span}(v_1, v_2, v_3)}$

b) $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ \hline 1 & 0 & 3 & 0 \end{array}$$

free var $\therefore \infty$ solutions

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\{v_1, v_2, v_3\}$ is linearly dependent because there are infinitely many solutions when solving for the 0 vector.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right] R3 \cdot R2 \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 0 & 2b \end{array}\right] R3 - 3 \cdot R2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -6 & 2b-6 \end{array}\right] \cdot -\frac{1}{6} \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array}\right] R1 + R2$$

$3(-\frac{1}{3}b+1)$

$-b+3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array}\right] -3 \cdot R3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -b+3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array}\right] R2 - 2 \cdot R3$$

$$\begin{array}{cccc} 1 & 0 & 0 & -b+3 \\ 0 & 1 & 0 & \frac{2}{3}b \\ 0 & 0 & 1 & -\frac{1}{3}b \end{array}$$

$x_1 = -b+3$

$x_2 = \frac{2}{3}b$

$x_3 = -\frac{1}{3}b$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] R3 + 3 \cdot R2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & 0 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

-2

$x_1 = x_2 - x_3$

$x_2 = -3x_3$

$x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$

It is not linearly independent bc there are infinite solutions

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1v_1 + x_2v_2 + x_3v_3 = w$

$x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] (\circ -2) \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] (\circ 1)$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right] (\circ 1) \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$

For $w \in \mathrm{Span}(v_1, v_2, v_3)$, must have a sol'n. $\therefore$ $b+6=0$

$\boxed{b=-6}$

b) $\{v_1, v_2, v_3\}$ is not linearly independent because all columns of $[v_1 \; v_2 \; v_3]$ are not pivot columns.

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

1a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xRightarrow{\text{row reduce}} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2(b+4) \end{array}\right]$

$2b+4$ has to equal 0 or no solution

$2b+8=0$

$-8 \quad -8$

$\frac{2b}{2} = \frac{-8}{2}$

$\boxed{b=-4}$

1b) No since there is not a pivot column in every column when reduced

also there is a free variable w/ x_3 so infinite solutions and dependent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot(1) \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 1 & -2 & -1 & 2+b \end{array}\right] \cdot(-1)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & -2-b \end{array}\right] \cdot 1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & -b \\ 0 & -1 & -2 & -2-b \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & -2 & -2-b \\ 0 & 0 & 0 & -b \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & b \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array}\right]$$

b) The set $\{v_1, v_2, v_3\}$ is <u>not</u> lin. independent; it has more than one, trivial solution.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a. Span is all linear combinations of $c_1v_1+c_2v_2+c_3v_3$

$$c_1\begin{bmatrix}1\\0\\2\end{bmatrix}+c_2\begin{bmatrix}-1\\1\\-3\end{bmatrix}+c_3\begin{bmatrix}1\\2\\0\end{bmatrix}=\begin{bmatrix}c_1-c_2+c_3\\c_2+2c_3\\2c_1-3c_2\end{bmatrix}$$

$c_1-c_2+c_3=-2$
$c_2+2c_3=2$

$$\left[\begin{array}{ccc|c}1&-1&1&-2\\0&1&2&2\end{array}\right] \xrightarrow{+(1)} \left[\begin{array}{ccc|c}1&0&3&0\\0&1&2&2\end{array}\right]$$

free

$c_1=-3c_3$
$c_2=2-2c_3$
$c_3=c_3$

$$\begin{bmatrix}-3c_3-(2-2c_3)+c_3\\2-2c_3+2c_3\\2(-3c_3)-3(2-2c_3)\end{bmatrix}=\begin{bmatrix}-2\\2\\-6\end{bmatrix}$$

$b=-6$

$w \in Span(v_1,v_2,v_3)$ if $b=-6$

b. $x_1v_1+x_2v_2+x_3v_3=0$

$$x_1\begin{bmatrix}1\\0\\2\end{bmatrix}+x_2\begin{bmatrix}-1\\1\\-3\end{bmatrix}+x_3\begin{bmatrix}1\\2\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&-1&1&0\\0&1&2&0\\2&-3&0&0\end{array}\right] \to \left[\begin{array}{ccc|c}1&-1&1&0\\0&1&2&0\\0&-1&0&0\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&3&0\\0&1&2&0\\0&0&2&0\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&3&0\\0&1&0&0\\0&0&2&0\end{array}\right] \to \left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

the equation has one solution therefore the set $\{v_1,v_2,v_3\}$ is linearly independent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$-3 \le b \le 2$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{+(\cdot -2)} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-2 \end{array}\right] +$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & b \end{array}$$
$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array}\right]$$

$b = 0$

b) $$\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \cdot -2 \rightarrow \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] + \rightarrow \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot -1$$

$x_1 = x_3$
$x_2 = 0$
$x_3 = x_3$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 =$ free
So infinite Solutions
Which means that it is linearly dependent.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$b=0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot -2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6b \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{x_3 \text{ is free}} & 6b \end{array}\right]$$

a) The only value of b that allows w to be in Span(v_1, v_2, v_3) is 0 because if it isnt zero there is no solution to the augmented matrix after row reduction.

b) The set $\{v_1, v_2, v_3\}$ is not linearly independant because x_3 is a free variable meaning it has infinitely many solutions.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) -6 ?

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & 6 \end{array}\right] \; -2 \qquad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & -12 \end{array}\right] \; -5 \qquad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 60 \end{array}\right] \; \cdot 1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 60 \end{array}\right]$$

free variable

b) No Because the set $\{V_1, V_2, V_3\}$ when ~~row redu~~ in reduced row echelon form dosent have a pivot column for every column and a pivot position in every row.

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = w$$

(b)

RREF :- $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$

$\xrightarrow{R_3 \to R_2 + R_3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \boxed{0} \end{bmatrix}$

~~So every~~ Since every column, can not be a pivot column,

$\boxed{\text{Set } \{v_1, v_2, v_3\} \text{ is not linearly independent}}$

(a)

RREF: $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$

$\xrightarrow{R_3 \to R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & \boxed{b+4} \end{array}\right]$ — (1)

Now, there exists sol^n only if is not a pivot column.

~~So, sol^n exists only when, b+~~

Since, $w \in \mathrm{Span}(v_1, v_2, v_3)$,

Here, sol^n exists only when, $b+4 \neq 1$

$\Rightarrow b \neq 1-4$

$\Rightarrow b \neq -3$

$\therefore b \in \mathbb{R}$, except $b = -3$.

Ans :-

~~From (1),~~ ~~$R_2 \to R_1 + R_2$~~ $\left[1 \;\; -1 \;\; 1 \;\; -2 \right.$

$R_2 \to$

Also, sol^n exist, if $b+4=0$

$\therefore b = -4$.

For, $b=-4$, there is infinite sol^n.

$\therefore$ ~~$b = \{b \in \mathbb{R}\}$~~

~~See next Page~~

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow[\text{reduce}]{\text{row}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+\frac{b}{3} \end{array}\right]$

$\because w \in \mathrm{span}(v_1, v_2, v_3)$

$\therefore 2+\frac{b}{3} = 0$

$\therefore b = -6$

b) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow[\text{reduce}]{\text{row}} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

there is no pivot position in last column.

Thus, the set is Linearly dependent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a) $(-\infty, \infty)$ – because V_1, V_2, V_3 can be multiplied by an infinite amount of different scalars

$$V_1 \begin{vmatrix} 1 \\ 0 \\ 2 \end{vmatrix} + V_2 \begin{vmatrix} -1 \\ 1 \\ -3 \end{vmatrix} + V_3 \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} = \begin{matrix} -2 \\ 2 \\ b \end{matrix}$$

(B) $V_1 \begin{vmatrix} 1 \\ 0 \\ 2 \end{vmatrix} + V_2 \begin{vmatrix} -1 \\ 1 \\ -3 \end{vmatrix} + V_3 \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix}$ → the set $\{V_1, V_2, V_3\}$ is not linearly independant because V_1, V_2, and V_3 are scalar multiples of each other and therefore a vector other than $\{0\}$ can be in thier Nul space

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

!=0

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$c_1v_1 + c_2v_2 + c_3v_3 = w$

a) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} V_1 + \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} V_2 + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{②+①} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{①(-2)+③} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{array}\right] \xrightarrow{②(3)+③} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free variable}} & b+6 \end{array}\right]$$

$\boxed{b = -6}$

b) $x_1 = -3x_3$

$x_2 = 2 - 2x_3$

$x_3 =$ free

The set $\{V_1, V_2, V_3\}$ is not linearly independent. This is because x_3 is a free variable, therefore the set has infinite solutions (and is linearly dependent.)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$b \neq 0$

$b > 0$

$b < 0$

b)

$$\begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{matrix}$$

b is linearly independent as the reduced form has exactly one solution when A = 0

$$\begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{matrix} \qquad \begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix} \qquad \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix} \qquad \begin{matrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix} \qquad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot -2 \rightarrow \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] \cdot 1 \rightarrow \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right] \cdot 1 \rightarrow \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$

$w \in \mathrm{span}(v_1, v_2, v_3)$ if and only if $b+6=0$ $\quad b=-6$

b) set is linearly independent if and only if

$x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$ has only trivial solution $(x_1 = x_2 = x_3 = 0)$

$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \cdot -2 \rightarrow \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \cdot 1 \rightarrow \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot 1 \rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases}$ infinite solutions $x_1 \neq x_3 \neq x_2$

so set is <u>not</u> linearly independent!

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 \to -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$R_1 \to R_1 + R_2$
$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$b = -6$ If b equals anything other than -6, there will be a pivot posinon in the last column leading to no solutions

b)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \Rightarrow -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right]$$

$R_3 \Rightarrow R_2 + R_3$
$R_1 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The set $\{v_1, v_2, v_3\}$ is not linearly dependent because $x_1v_1 + x_2v_2 + x_3v_3 = 0$ does not have only the trivial solution. x_3 is a free variable giving infinite solutions making the set linearly dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1.5 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1/2 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

b: $v_3 - v_1 = \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$

$2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$b: \{-2, -6\}$

the set $\{v_1, v_2, v_3\}$ is linearly independent because there is only one solution to $x_1v_1 + x_2v_2 + x_3v_3 = 0$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$[w \in Span(v_1, v_2, v_3)] \Rightarrow w = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$R1 \cdot -2 + R3$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & w \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] R4 \cdot -1$$

$R2 \cdot -1 + R3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -b-4 \end{array}\right] \xrightarrow{R2+R1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -b-\cancel{4-2} \\ & & & -6 \end{array}\right]$$

If $b = -6$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-b - 6 = 0$

$-6 = b$

$c_1 - c_2 + c_3 = -2$

$c_2 + 2c_3 = 2$

$\{v_1, v_2, v_3\}$ is not linearly independent because c_3 is a free variable.

Infinite # of sltns if $b = -6$.

No sltns if $b \neq -6$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\cdot 1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \to \cdot -2 \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \to \cdot 3 \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right]$ $\therefore$ for all values of b greater than -5, w $\in \text{Span}(v_1, v_2, v_3)$

b) The $\{v_1, v_2, v_3\}$ is linearly independent because no vector in that set is a multiple of another.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \cdot -2 \rightarrow \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & -2 & 4+b \end{bmatrix} \cdot 5 \rightarrow \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 8 & 14+b \end{bmatrix}$$

$8 = 14 + b$

a) $\boxed{b = -6}$

b) $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 8 \end{bmatrix} \cdot \tfrac{1}{8} \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot -2 \rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot -1 \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot 1$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) The set is linearly independent b/c the homogenous vector equation $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$ will only have one solution.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $-2v_1 + 4v_2 - v_3 = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$$-2\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -16 \end{bmatrix}$$

$-4 \; -12$

b) aug. matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \to (-2)\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & -3 & 0 & 0 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$$\to (-1)\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right](1) \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]\left(\tfrac{1}{3}\right)$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right](-1) \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ The set $\{v_1, v_2, v_3\}$ is lin. independent b/c there is only one solution to the homogeneous matrix

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left(\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = w \right) = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \end{array} -2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & \end{array}\right] \begin{matrix} 2 \\ 1 \end{matrix} \quad \cancel{\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 12 \end{array}\right]} \quad \cancel{\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 12 \end{array}\right]} \quad \cancel{\begin{matrix} x_1 = -3x_3 \\ x_2 = 2 - 2x_3 \\ x_3 = x_3 \end{matrix}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+b \end{array}\right] \quad \begin{matrix} x_1 = -3x_3 \\ x_2 = 2 - 2x_3 \\ 0 = 2 + b \\ b = -2 \end{matrix} \quad x_3 = x_3$$

a) $\boxed{b = -2}$

b) Linearly $\boxed{\text{dependent}}$ because x_3 is a free variable in the matrix $[x_1 v_1 \ x_2 v_2 \ x_3 v_3]$ resulting in a null space with infinite solutions

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) w in span if $\underset{x_1}{c_1}v_1 + \underset{x_2}{c_2}v_2 + \underset{x_3}{c_3}v_3 = w$

$x_1 - x_2 + x_3 = -2$
$x_2 + 2x_3 = 2$
$2x_1 - 3x_2 = b$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

any integer can be reduced to a leading 1 except when $b=-6$

a) $b = -6$

b) $x_1v_1 + x_2v_2 + x_3v_3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 3x_3 = 0$
$x_2 + 2x_3 = 0$
x_3 is a free variable $\therefore$ the equation has infinitely many solutions $\therefore$ the set is linearly dependent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} = A$$

A) $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

Aug.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 \to 2R_1 - R_3} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{bmatrix} \xrightarrow{R_1 \to R_2 + R_1}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -6-b \end{bmatrix}$$

For w to be in the span(v_1, v_2, v_3) b must equal -6.

B) The solution to $Ax = 0$ is

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_1 = -3x_3$

$x_2 = -2x_3$

$x_3 = x_3$

The presence of a free variable suggests infinitely many solutions to $Ax = 0$. Since the homogenous equation does not have one trivial solution it is Linearly Dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) (-2) $+$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

(-1)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+2 \end{array}\right]$$

$b = -2$ or else no solution

b) Set $\{v_1, v_2, v_3\}$ is linearly dependent because if $b = -2$, and since there is no leading one (pivot position) in third column, then there is a free variable x_3 meaning infinite solutions $\Rightarrow$ linearly dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & 0 & 6 & b+6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$b = -6$ is the only value that would produce a solution in the span

b) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}$

rref from A can get

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Every column doesn't have a pivot position so not linear independent

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$\mathrm{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathrm{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathrm{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathrm{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathrm{w} \in \mathrm{Span}(\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3)$.

b) Is the set $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ linearly independent? Justify your answer.

a)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot 1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot -1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 1 & -3 & -3 & b \end{array}\right] \cdot 1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 1 & 0 & 5 & 2 \\ 1 & -3 & -3 & b \end{array}\right] \cdot -1 \quad \left[\begin{array}{ccc|c} 0 & 0 & -2 & -2 \\ 1 & 0 & 5 & 2 \\ 1 & -3 & -3 & b \end{array}\right] \cdot \frac{-1}{2} \quad \left[\begin{array}{ccc|c} 1 & -3 & -3 & b \\ 1 & 0 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \cdot -1$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -3 & b \\ 0 & 3 & 8 & 2-b \\ 0 & 0 & 1 & 1 \end{array}\right] \cdot 1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 3 & 8 & 2-b \\ 0 & 0 & 1 & 1 \end{array}\right] \cdot -5, \ \cdot -8 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 3 & 0 & -6-b \\ 0 & 0 & 1 & 1 \end{array}\right] \cdot \frac{1}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & \frac{-6-b}{3} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

b) They are linearly independent.
If we create an augmented matrix of v_1, v_2, v_3 and augment with 0 instead of w, we see that the matrix still reduces to the identity matrix, meaning the only solution to that equation is the trivial solution.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \quad b=-6$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$[-2 \;\; 2 \;\; -2 \;\; +4]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot(-2) \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right] \cdot(1) \; [0 \;\; 1 \;\; 2 \;\; 2]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right] \qquad 0 = b+6 \qquad \underline{b=-6}$$

b) Yes.

Not every column in the matrix is a pivot column.

Therefore, independent.

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $W \in Span(V_1, V_2, V_3)$

$W = C_1V_1 + C_2V_2 + C_3V_3$

$C_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & \frac{b}{-3} \end{bmatrix}$

$\frac{b}{-3} = 2 \quad b = -6$ so that the matrix is consistant and has solution that makes $W \in Span(V_1, V_2, V_3)$

b) $\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$
$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

$X_1 + 3X_3 = 0 \quad X_1 = -3X_3$
$X_2 + 2X_3 = 0 \quad X_2 = -2X_3$
$X_2 + 2X_3 = 0 \quad \cancel{X_2 = -2X_3} \; X_3 = X_3$

$X = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} X_3$

so that there are other ~~soth~~ solutions other that $X_1 = X_2 = X_3 = 0$

$\therefore \{V_1, V_2, V_3\}$ is linearly dependent.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} \xrightarrow{b=-6} w = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} \qquad \boxed{b=-6}$$

$$v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad -3v_1 = \begin{bmatrix} -3 \\ 0 \\ -6 \end{bmatrix}$$

$$-2v_1 = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix}$$

b) $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{(-2)} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix} \xrightarrow{R_1 = R_2 + R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 4 & -2 \end{bmatrix} \xrightarrow{R_3 = R_2 + R_3} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

yes, linearly independent
because every col. is a pivot column

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$. → w can be written as a linear combination of v_1, v_2, v_3

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] (R1 \times -2) + R3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] R2 + R3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right] R2 + R1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right]$$

$\boxed{b = -6 \text{ for } w \in \mathrm{span}(v_1, v_2, v_3)}$

(otherwise get no solution) due to pivot position in column of constants

(b)

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

↳ this set in linearly independent if homo. eqn. only has trivial solution ∴

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] (R1 \times -2) + R3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R2 + R3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free variable! not every column of the matrix is a pivot therefore the set $\{v_1, v_2, v_3\}$ are not linearly independent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $V = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{\text{reduced}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} \cancel{x_1 - x_2 + x_3 = -2} \\ \cancel{x_2 + 2x_3 = 2} \\ \cancel{2x_1 - 3x_2 = b} \end{cases}$ $b = 0$

b. $V = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $R(V) = 2.$

the number of V_1, V_2, V_3 is 3.

$R(V) = 2 < 3$

So. the set $\{V_1, V_2, V_3\}$ linearly independent.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$\mathrm{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathrm{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathrm{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathrm{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathrm{w} \in \mathrm{Span}(\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3)$.

b) Is the set $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ linearly independent? Justify your answer.

$$V = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \xrightarrow{+} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free}} & 6+b \end{array}\right] = \begin{array}{l} x_1 = -3x_3 \\ x_2 = 2 - x_3 \\ x_3 = x_3 \end{array} \quad \boxed{b = -6}$$

b) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{array}$$

$x_3 = 1$

$\boxed{\text{Yes linear independent}}$

$$-3\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - 2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a. $b_1 V_1 + b_2 V_2 + b_3 V_3 = W$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{array}{c} ① \\ ② \\ ③ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

③ - ②×1

$$\begin{array}{c} ① \\ ② \\ ③ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{array}\right]$$

③+②

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array}\right]$$

$b-2=0$

$\boxed{b=2}$

b. $$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{c} ① \\ ② \\ ③ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

①×② - ②

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$2x_1 - 3x_2 = 0$

$x_2 - 2x_3 = 0$

$x_1 = \frac{3}{2}x_2$

$x_2 = x_2$

$x_3 = -\frac{1}{2}x_2$

$2x_1 = 3x_2$

$x_1 = \frac{3}{2}x_2$

$x_2 + 2x_3 = 0$

$2x_3 = -x_2$

$x_3 = -\frac{1}{2}x_2$

$= \begin{bmatrix} \frac{3}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} x_2$ → Infinity solutions

Set $\{v_1, v_2\, v_3\}$ not linear independent

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 5+b \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free var}} & 5+b \end{array}\right]$$

~~b must equal~~

$\boxed{\text{a) } b = -5}$

b) The set is $\boxed{\text{linearly dependent}}$ because there is not a pivot column in every column

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)

$$C_1V_1 + C_2V_2 + C_3V_3 = C_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 \\ 0 \\ 2C_1 \end{bmatrix} + \begin{bmatrix} -C_2 \\ C_2 \\ -3C_2 \end{bmatrix} + \begin{bmatrix} C_3 \\ 2C_3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 - C_2 + C_3 \\ C_2 + 2C_3 \\ 2C_1 - 3C_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$C_1 - C_2 + C_3 = -2$

$C_2 + 2C_3 = 2$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \end{bmatrix} = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{bmatrix} \end{array}$$

$C_1 + 3C_3 = 0$

$C_2 + 2C_3 = 2$

$$\begin{bmatrix} -3C_3 \\ 2-2C_3 \\ C_3 \end{bmatrix} \Rightarrow 2C_1 - 3C_2 = -6C_3 - 3(2-2C_3)$$

$$= -6C_3 - 6 + 6C_3 = -6$$

$b = -6$

(b).

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & \end{matrix} \\ \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

infinite

no. it is dependent

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) If: $W = 2V_2$, $\boxed{b = -6}$

$W = V_1 + V_3$, $\boxed{b = 2}$

$W = 3V_1 + 2V_2$, $\boxed{b = 0}$

b) The set is not linearly independent because there is a linear combination of vectors V_1 and V_2 which give V_3.

$$3V_1 + 2V_2 = V_3$$

$$3\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = V_3$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$

$R_3 = R_3 - 2R_1$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$

$R_3 = R_2 + R_3$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$

$b+6=0$

$\boxed{b=-6}$

b) $x_1v_1 + x_2v_2 + x_3v_3 = 0$

aug matrix

$[\,v_1 \ \ v_2 \ \ v_3 \,|\, 0\,]$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right]$

Reducing rows

$R_3 = R_3 - 2R_1$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right]$

$R_3 = R_3 + R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

free variable means infinitly many solutions

the set $\{v_1, v_2, v_3\}$ is not linearly indipendent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\cancel{V_1 + V_3 = w \mid b = 2}$

$\cancel{-V_2 + V_3 = V}$

$2V_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}, b = -6$

$-3V_1 + V_3 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}, b = -6$

$\boxed{b = -6}$

b) The set $\{V_1, V_2, V_3\}$ is linearly independent, because the vectors cannot linearly combine to cancel one another

cannot cancel all terms

$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$

cannot cancel all terms

no scaler multiple

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$\mathrm{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathrm{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathrm{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathrm{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathrm{w} \in \mathrm{Span}(\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3)$.

b) Is the set $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{\overset{=R_3}{-2R_1+R_3}} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{bmatrix} \xrightarrow{R_2+R_3} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{bmatrix}$$

$w \in span(v_1, v_2 v_3)$ aslong as $b=2$ if $b \neq 2$ then nosoultion

the set of vectors is linearly dependent because even if you had $b=2$ then there would still be a free variable

if $w \in \{v_1 v_2 v_3\}$ then $w = c_1 v_1 + c_2 v_2 + c_3 v_3$ antlere is a nontrivial answer to this equation

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$C_1V_1 + C_2V_2 + C_3V_3 = W$$

$$C_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$C_1 - C_2 + C_3 = -2$

$0 \quad C_2 + 2C_3 = 2$

$2C_1 + C_2 + \quad = b$

$b = 2C_1 + C_2$

$C_1 = -3$, $C_2 = 0$, $C_3 = 1$

$C_1 = -\frac{3}{2}$, $C_2 = 1$, $C_3 = \frac{1}{2}$

$b = -5, -2$

b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 2 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right]$$

No, there must be a pivot column in every column and have only one solution

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \operatorname{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} V_1 & V_2 & V_3 & w \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \cdot(-2)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \cdot(1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \cdot(1)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right] \rightarrow 0x_1 + 0x_2 + 0x_3 = 6+b$$

$$0 = 6+b$$

free

$6+b=0$ for it to have a solution

$$\boxed{b = -6}$$

b) $\{v_1, v_2, v_3\}$ is linearly dependent because x_3 is a free variable indicating that the system has infinitely many solutions, making it linearly dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 0 & b+4 \end{array}\right] +$

$\begin{bmatrix} 1 & -1 & -2 & -2 \\ 0 & 0 & 2 & b+6 \\ 0 & -1 & 0 & b+4 \end{bmatrix} + \xrightarrow{R_1 = R_1 - R_3} \begin{bmatrix} 1 & 0 & -2 & -b-6 \\ 0 & 0 & 2 & b+6 \\ 0 & -1 & 0 & b+4 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 & -b-6 \\ 0 & -1 & 0 & b+4 \\ 0 & 0 & 2 & b+6 \end{bmatrix} \xrightarrow{R_1 = R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & b+4 \\ 0 & 0 & 2 & b+6 \end{array}\right] \begin{array}{l} \\ \cdot(-1) \\ \cdot(\frac{1}{2}) \end{array}$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -b-4 \\ 0 & 0 & 1 & \frac{b+6}{2} \end{bmatrix}$

a) All values of b are $c_1(2) + c_2(-3) + c_3(0)$ that $w \in \text{Span}(v_1, v_2, v_3)$

b) yes, the set $\{v_1, v_2, v_3\}$ is linearly independent because after row reduction every column is a pivot column.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

MTH-309T-F19-EX1-079-P01

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

$b_1 v_1 + b_2 v_2 + b_3 v_3 = w$

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) Augmented matrix

$$-2 \cdot \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

for $w \in \text{Span}(v_1, v_2, v_3)$

$b + 6 = 0$

$b = -6$

$$1 \cdot \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$$1 \cdot \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

b)

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

reduce row echelont form

null space

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

free variable

b) $[v_1, v_2, v_3]$ is not linearly independent because it's reduce row echolon form has a free variable so the null space has more then one trivial solution (it has infinitly many solutions), meaning that it is linearly dependent, linearly independent vectors would have only one trivial solution to the null space which would be the 0 vector.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\cdot -2 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \Rightarrow \cdot 1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4+b \end{array}\right] \qquad b \Rightarrow -4$$

b) From part a $\rightarrow$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

No, the set $\{v_1, v_2, v_3\}$ is not linearly independent because the solution would be a trivial solution if it were linearly independent. In this case, it leads to infinitely many solutions.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$w \in \mathrm{Span}(v_1, v_2, v_3)$ for $b = -6$

b)
$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow[\text{reduction}]{\text{row}} \begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = 3x_1 + 2x_2$

The set $\{v_1, v_2, v_3\}$ is not linearly independent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \operatorname{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \times -2$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{bmatrix} \times 1$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+4b \end{bmatrix}$$

$$\Updownarrow$$

$w \in \operatorname{Span}(v_1, v_2, v_3)$

$2 + 4b = 0$

$\cancel{b = \frac{1}{2}}$

$b = -\frac{1}{2}$

(b) $\{v_1, v_2, v_3\}$ is not linearly independent, because there is not every column of matrix is pivot column.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $w \in \text{Span}(v_1, v_2, v_3)$ means that $c_1v_1 + c_2v_2 + c_3v_3 = w$ has solution

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} ① \times 2 - ③$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{bmatrix} ② - ③$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

Since $6+b$ is in constant column.

$\therefore$ $6+b=0$

$b=-6$

$\therefore$ $b = -6$

b) $x_1v_1 + x_2v_2 + x_3v_3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} ① \times 2 - ③$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} ② - ③$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since x_3 is free variable, this homogenous equation has many solutions.

This means that the set $\{v_1, v_2, v_3\}$ is not linearly independent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right]$$

$$\begin{array}{ccc|c} 0 & 5 & -2 & b+4 \\ 0 & 0 & -12 & b-6 \end{array}$$

$x_1 - x_2 + x_3 = -2$

$x_2 + 2x_3 = 2$

$\cancel{2x_1 + 3x_2 = b}$

$-12x_3 = b-6$

$x_3 = -\frac{b-6}{12}$

$x_1 = 3\left(\frac{b-6}{12}\right) \quad x_2 = 2 + 2\left(\frac{b-6}{12}\right)$

$x_2 = 2 + \frac{b-6}{6}$

$x_1 = -2 + \left(2 + \frac{b-6}{6}\right) + \frac{b-6}{12}$

or

$x_1 = 3\left(\frac{b-6}{12}\right)$

$b = \mathbb{R}?$ $\boxed{6}$

(b) to check for linear independance

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow[\text{red}]{\text{row}} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3/2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Doesn't reduce to identity

so

$\boxed{\text{Liniarly Dependent}}$

Also, No vectors Appear to be able to be representa by any other in the set.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\begin{array}{l} +R2 \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] = \begin{array}{l} \\ \\ -2R1 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] = \begin{array}{l} \\ \\ +3R2 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & 6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$x1 = -3x3$

$x2 = 2 - 2x3$ $\qquad \underline{b = -6}$

$x3 = 0 = b+6$

b) Yes, it is linearly independent because $v1, v2, v3$ are not multiples of each other. There is no way to get anything but 0 in second row of $v1$ and therefore we can't get such multiple of $v1$ so that we obtain $v2, v3$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & -2 \\ 0 & 1/2 & 1 & \frac{-2-b}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 2+\frac{b}{2} \end{array}\right]$$

$2 + \frac{b}{2} = 0$

$\frac{b}{2} = -2$

$b = -4$

The set $\{v_1, v_2, v_3\}$ are linearly independent because v_1, v_2, v_3 are not scalar multiples of each other.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)
$$\begin{array}{cccc} 2 & -2 & 2 & -4 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R3 \to R3 - 2R1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{array}\right] \xrightarrow{R3 \to R3 + R2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array}\right]$$

when $b = 2$

b) The set isn't linearly independent because since the span is made up of linear combinations of the vectors all the vectors in the span are linearly dependent

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R\times -2} \left[\begin{array}{ccc|c} -2 & 2 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_2+R_1} \begin{bmatrix} -2 & 2 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$R_1+(-R_3) \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \xrightarrow{\text{restore } R_3} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{array}{c} \quad\quad\quad \text{free} \\ \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) It is linearly dependent since the reduced form contains a free variable meaning it has infinetely many solutions.

a) $b = 2$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20