

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

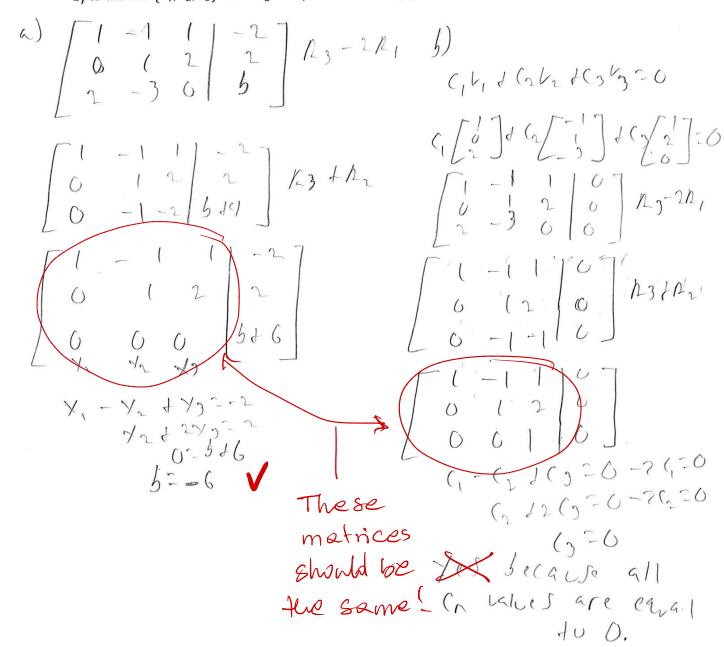
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 2 & -1 & | & 0 & 0 & | \\
0 & 2 & -1 & | & 0 & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & | & -1 & 1 & 0 \\
0 & 2 & -1 & | & 0 & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & | & -1 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 1 & | & 0 & | & 0 & |
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 0 & | & 2-2 & | & J \\
0 & 0 & | & 2-2 & | & J \\
0 & 0 & | & | & 2-2 & | & J
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 0 & | & 2-2 & | & J
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & | & 2-2 & | & J
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & | & 2-3 & | & J
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & | & 2-2 & | & J
\end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{-1} \begin{bmatrix} 2 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} A^{-1} \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 1 & 2 & 3 \\ -3 & -1 & -1 & 2 & 1 & 3 & 3 \\ 1 & 1 & 1 & 1 & 3 & 2 & 1 \end{bmatrix} + h_2 - h_1 : h_1$$

$$C = (A^{T})^{T} \cdot B$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -5 & -7 & -7 \\ -3 & +1 & -2 & | & 9 & 5 & 9 \end{bmatrix} \xrightarrow{3A_1 + A_2 : A_2}$$

$$C = \begin{bmatrix} -5 & -7 & -7 \\ 5 & 7 & -1 \end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & -5 & -5 & -7 \\
0 & -1 & -2 & -11 & -16 & -17 \\
1 & 1 & 2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & -5 & -5 & -7 \\
0 & -1 & -2 & -11 & -16 & -17 \\
1 & 1 & 2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & 0 & 0 & -5 & -5 & -7 \\
-1 & -1 & -16 & -17 & -17 \\
1 & 1 & 2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 8 & 9 & 8
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 8 & 9 & 8
\end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -5 & -7 & -7 \\ 0 & 12 & | & 11 & 16 & 17 \\ 0 & 0 & +1 & | & +3 & +7 & +9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & | & -5 & -7 & -7 \\ 0 & 10 & | & 5 & 2 & -1 \\ 0 & 0 & 1 & | & 3 & 7 & 9 \end{bmatrix}$$

$$\frac{7}{610}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

b) Find all vectors
$$\mathbf{u}$$
 satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(X_1) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

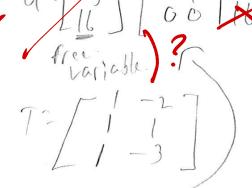
$$T(X_1) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

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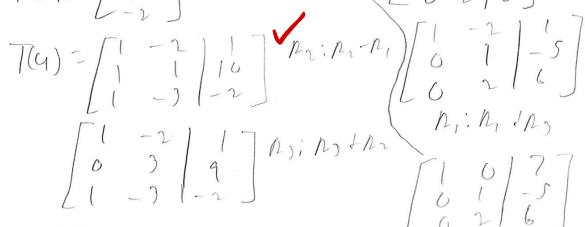
$$T(X_1) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

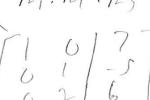
$$T(X_1) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T(4) = \begin{bmatrix} 1 & -2 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -2 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & -2/1 \\ 0 & 3/1 \end{bmatrix} h_2 : h_3 = h_3$$







5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 $(A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ $(A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} P_{1} P_{2} P_{3} P_{4} P_{5} P_$$



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

Fatse

W-[0] 4-[0] L-[0]

W+4 is a linear combination so

just he fitself may had be in span

w+2 is not ation of

w+2 is not tion of

in. combination of

a lin. on this case.

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

Trie

L. I. depends on the Lalus (1,5 m Cn

Set & U, V, L 3 & P.

(1=CL=C3=0

Set & Gets'a

(1=C2=0

Set Sets'a

Ci=C2=0

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7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent

then u, v also must be linearly dependent.

[19] 19 V heid to be

Set to be and checked

Auro ? to see if a solution

exists, i.e. (1, (2... 4)

all equal b.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True

If the Spancu, L) (antoins 9

then the Span 4 (7(L), 7(L))

contains T(4)

why?

