4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$\lambda_1 = 3$. $[A - 3I \mid 0]$

$$\left[\begin{array}{ccc|c} -2 & 8 & 4 & 0 \\ -2 & 8 & 4 & 0 \\ 2 & -8 & -4 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} -2 & 8 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 \end{array}\right] \div -\tfrac{1}{2} \quad \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_1 = +4x_2 + 2x_3$
$x_2 = x_2$
$x_3 = x_3$

$$x_2 \begin{bmatrix} +4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} +2 \\ 6 \\ 1 \end{bmatrix}$$

$\lambda_2 = 5$

$[A - 5I \mid 0]$

$$\left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{array}\right] -\tfrac{1}{2},\ \tfrac{1}{2} \quad \left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right] \div -\tfrac{1}{4} \quad \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right] 2 \quad \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \cdot \tfrac{1}{2} \quad \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x_1 = -x_3$
$x_2 = -x_3$
$x_3 = x_3$

$$x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20