4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$$Null(A-\lambda_1 I) = \left[\begin{array}{ccc|c} -2 & 8 & 4 & 0 \\ -2 & 8 & 4 & 0 \\ 2 & -8 & -4 & 0 \end{array}\right] \rightarrow -\frac{1}{2}\left[\begin{array}{ccc|c} -2 & 8 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = \begin{bmatrix} x_1 = 4x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$Null(A-\lambda_2 I) = -\frac{1}{4}\left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ +2 & -8 & -6 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right] \tfrac{1}{2} \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$A = PDP^{-1}$$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \qquad D = \cancel{\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}}$$

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