

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1} .

$$\begin{aligned} \text{basis of } (A - 3I) &= \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \xrightarrow{\substack{\frac{1}{2}R_1 \\ \frac{1}{2}R_2 \\ \frac{1}{2}R_3}} \begin{bmatrix} -1 & 4 & 2 \\ -1 & 4 & 2 \\ 1 & -4 & -2 \end{bmatrix} \xrightarrow{R_3+R_1} \begin{bmatrix} -1 & 4 & 2 \\ -1 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix} \\ &\xrightarrow{R_2-R_1} \begin{bmatrix} -1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1R_1} \begin{bmatrix} 1 & -4 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{aligned} x_1 - 4x_2 - 2x_3 &= 0 \rightarrow x_1 = 4x_2 + 2x_3 \\ 0x_2 &= 0 \\ 0x_3 &= 0 \end{aligned} \\ &\rightarrow x = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \rightarrow \text{basis} = \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\} \end{aligned}$$

$$\begin{aligned} \text{basis of } (A - 5I) &= \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \xrightarrow{\substack{\frac{1}{4}R_1 \\ \frac{1}{2}R_2 \\ \frac{1}{2}R_3}} \begin{bmatrix} -1 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & -4 & -3 \end{bmatrix} \xrightarrow{\substack{R_2-R_1 \\ R_3+R_1}} \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{-1R_1} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \\ &\xrightarrow{\substack{R_3-R_2 \\ R_1-2R_2}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{aligned} x_1 + x_3 &= 0 \rightarrow x_1 = -x_3 \\ x_2 + x_3 &= 0 \rightarrow x_2 = -x_3 \\ 0x_3 &= 0 \rightarrow x_3 = x_3 \end{aligned} \\ &\rightarrow x = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \rightarrow \text{basis} = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\} \end{aligned}$$

$$50 \quad P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad \& \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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