5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and **w** is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then **w** must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

a) false: The +5

f $\lambda = 2$ eigenvalue would

d x_2 in $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$? stay the same, as

s a free root,

om $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ v and 2v wouldn't

ould be eigenvectors be linearly independent

taining $\lambda \to v = 2v \not\Rightarrow 2\lambda$

b) True; This has to

+1 be true because This would be the only way Proj_V would equal $-w$. no other combo

? would produce this besides the trivial solution

c)

+1 True; The Identity matrix, which is both A and A^2 in this case, is the only matrix that could be both symmetric and orthogonal

d) ~~true~~ false; This is not always the case. When

+1 matrixies are added, properties are not always preserved, such as diagonizability:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ 2 & 5 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

diagonizable diagonizable not diagonizable

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20