4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

For $\lambda = 3$, $A - \lambda I = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$Nul(A-3I) = span\left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ = eigenspace of A corresponding to λ_1

$A - 5I = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 2 \\ -1 & 3 & 2 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 \\ -2 & 4 & 2 \\ -1 & 4 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & -3 & 2 \\ 0 & -2 & -2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$Nul(A-5I) = Span\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ $P = \begin{bmatrix} 4 & 2 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ 18/20

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