

MTH 309T LINEAR ALGEBRA EXAM 1

Name: Charles Balle	2/				
UB Person Number: 5 0 2 7 0 3 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 6 6 6 6 6 6 6 6 6 6 6 6 6 7 7 7 7 7 7 8 8 8 8 8 8 9 9 9 9 9 9	 Instructions: Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes. For full credit solve each problem fully, showing all relevant work. 				
	5 6 7 TOTAL GRADE				

20	10	10	18	8	6	6	2	10	88	A-
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- 4 V1 + Q V2 + B3 V3 = W

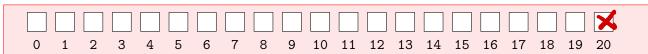
b) Is the set
$$\{v_1, v_2, v_3\}$$
 linearly independent? Justify your answer.

Augmented motify

 $\begin{cases}
1 & -1 & 1 & -2 \\
0 & 1 & 2 & 2 \\
2 & -3 & 0 & b
\end{cases}$
 $\begin{cases}
507 & w \in b \\
6+6=0
\end{cases}$

Sor
$$w \in Span(V_1, V_2)^3$$

 $b+6=0$
 $b=-6$





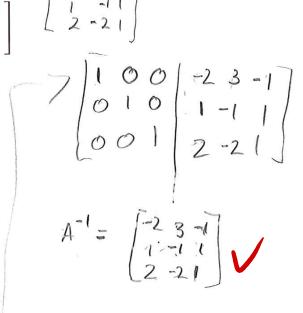
2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 \cdot 1 \\ 1 & -1 \cdot 1 \\ 2 & -2 \cdot 1 \end{bmatrix}$$

Compute A^{-1} .

$$-2.6601-121000$$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

 \widehat{C} such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 7 \\ 2 & 1 & -1 \end{bmatrix} \qquad (A^{T})^{-1} = \begin{bmatrix} -2 & 12 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{T})^{-1} = \begin{bmatrix} -2 & 12 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = (A_L)_{-1} B$$

$$C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$$C = (AT)^{-1} B = (A^{-1})^{-1} B$$

$$C = (AT)^{-1} B = (A^{-1})^{-1} B$$

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$$O = (AT)^{-1} B = (AT)^{-1} B$$

$$C = (A^T)^{-1} B = (A^{-1})^T B$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of
$$T$$
.

b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

C) $T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -$



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$
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c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$
c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0$

Solution is one to one as every row well column has a princt polosition or leading ore



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True 1 be cause is w + U is

Is in spen (), w) then U-v=w

which is a so in span (u,u)?

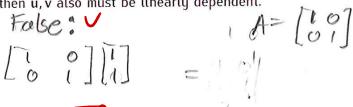
b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ myst be linearly independent.

True. is Eu, VIW3 are linearly independent than EUIV3 is linearly independent because GUIGNTIGWED would have only one trivial solution, meaning that Gutgutesme CIUTBAW or CIUTBAW or CIUTBAW it have recombination to sum to 0 wither than multipling by 0, is they did than Eu, VIW3 would have a nother solution to GU +C2V +C3W =0 which would make 1: En UIV, W3 linearly dependent. Since GUTGV they are linearly independent



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



This matrix will not give a counterexample.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True: if u is in the spen (v, w)

then a combination GV + GW = U, so

T (E, V + C, W) = T(a) as T(a, V + C, W)

can be split into (IT (V) + C, T(W) = T(W)

which means that there is a linear combiner.

T(C) and T(G) that equal T(a)