4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$T(V) = T \cdot V$

$V = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad T(V) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$ ✓ $\quad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$T(u) = T \cdot u$ $\quad T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ $\quad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\underset{3\times 2}{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}} \cdot u = \underset{3\times 1}{\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}}$

$2\times 1 \rightarrow u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$u_1 - 2u_2 = 1$

$u_1 + u_2 = 10$

$u_1 - 3u_2 = -2$

$\begin{cases} u_1 - 2u_2 = 1 \\ -u_1 - u_2 = -10 \end{cases}$

$-3u_2 = -9$

$\boxed{u_2 = 3}$

$u_1 + 3 = 10$ $\quad \boxed{u_1 = 7}$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

SCRAP PAPER

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow[\text{REDUCTION}]{\text{ROW}} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} Te_1 \end{bmatrix} \begin{bmatrix} Te_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} c = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 20 \\ 4 \end{bmatrix}$$

b) $T(u) = c_1 x_1 + c_2 x_2$

$u = c_1 e_1 + c_2 e_2$

$T(u) = T(c_1 e_1) + T(c_2 e_2)$

$= c_1 T(e_1) + c_2 T(e_2)$

$= \begin{bmatrix} T(e_1) + T(e_2) \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \cdot U$

$$\left[\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right]$$

$$\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \cdot U$$

$U = \text{Span}\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot (u) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix} \qquad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] = \begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{matrix}$$

$R_2 = \cancel{R_2 - R_1}\ -R_1 + R_2$

$R_3 = -R_1 + R_3$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}\ {}^{1/3} = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix} = R_2 + R_3 = \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix} =$$

$$R_1 = 2(R_2) + R_1 = \left.\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}\right| \begin{matrix} 1 \\ 3 \\ 0 \end{matrix}$$

$$\begin{matrix} x_1 = \\ x_2 = \end{matrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$Tx = Ax$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{matrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$. $= A\vec{u}$ $\quad \vec{u}$ is 2×1

$$= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\vec{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \qquad \boxed{\vec{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$u_1 - u_2 = 1$

$u_2 = 3$

$u_1 = 4$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$7 - 6 = 1$
$7 + 3 = 10$
$7 - 9 = -2$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a) $\boxed{T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

(b) $\boxed{T(u) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

(b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$(-1)\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$

$-1+10$

$$(-1)\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{array}\right]$$

$$(-3)\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{array}\right] (2) \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$2 \cdot 3$

$-1+2$

$-9+$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = [T(e_1) \;\; T(e_2)]$, $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

standard matrix:

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $T_A(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{(-1)r1+r2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{r2 \cdot \frac{1}{3}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$

$\xrightarrow{-r1+r3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{2r2+r1} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{r2+r3} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = 7$
$x_2 = 3$
$x_3 = x_3$

$v = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$ $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ $u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 0 & -1 & -3 \end{array}\right] \cdot 1/4$$

$$\to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \to \begin{array}{ccc} x_1 & x_2 & \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 - 2x_2 = 1$ $x_1 = 1 + 2x_2 = 1 + 2(3) = 7$

$x_2 = 3$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$ or any multiple of $u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

check $T\left(\begin{bmatrix} 7 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 7 - 6 \\ 7 + 3 \\ 7 - 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ ✓

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

2 rows to 3 columns?

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

Standard

(a) $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{bmatrix} = \boxed{\begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

$x_1 = 1 + 2x_2$ $\Rightarrow 1 + 2(3) = \boxed{7}$ $x_1 = 7$

$1 + 2x_2 + x_2 = 10$ $\Rightarrow 1 + 3x_2 = 10 \Rightarrow 3x_2 = 9 = \boxed{x_2 = 3}$

$1 + 2x_2 - 3x_2 = -2 \Rightarrow 1 - x_2 = -2$

$\boxed{x_2 = 3}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$T(u+v) = Tu + Tv$

$$T(u) = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} \qquad T(u+v) = \begin{bmatrix} u_1 + v_1 - 2u_2 - 2v_2 \\ u_1 + v_1 + u_2 + v_2 \\ u_1 + v_1 - 3u_2 - 3v_2 \end{bmatrix} =$$

A)

$$T(v) = \begin{bmatrix} v_1 - 2v_2 \\ v_1 + v_2 \\ v_1 - 3v_2 \end{bmatrix} \qquad = \begin{bmatrix} (u_1+v_1) - 2(u_2+v_2) \\ (u_1+v_1) + (u_2+v_2) \\ (u_1+v_1) - 3(u_2+v_2) \end{bmatrix}$$

standard matrix

$$A(T(e_1)\ T(e_2)] =$$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

Standard Matrix

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \Rightarrow A: \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 1 & 1 & 0 & 10 \\ 1 & 3 & 0 & -2 \end{array}\right] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 3 \\ -17 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 1 & 1 & 0 & 10 \\ 1 & 3 & 0 & -2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 3 & 0 & 9 \\ 0 & 5 & 0 & -2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 5 & 0 & -2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & -17 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = 3\times 2$ since $3\times 2 \cdot 2\times 1$

$$\begin{bmatrix} C_1 & C_4 \\ C_2 & C_5 \\ C_3 & C_6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$C_1x_1 + C_4x_2 = 1x_1 - 2x_2$ $\quad C_1 = 1 \quad C_4 = -2$

$C_2x_1 + C_5x_2 = 1x_1 + 1x_2$ $\quad C_2 = 1 \quad C_5 = 1$

$C_3x_1 + C_6x_2 = 1x_1 - 3x_2$ $\quad C_3 = 1 \quad C_6 = -3$

Standerd Matrix: $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$\boxed{T(u) = \text{Span}\left(\begin{bmatrix} 7 \\ 3 \end{bmatrix}\right)}$

$x_1 = 7$

$x_2 = 3$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$ $\cdot -1$ $\cdot -1$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right]$ $\cdot \frac{1}{3}$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right]$ $\cdot 1$

$\downarrow$

$$\begin{array}{ccc} x_1 & x_2 & \\ \end{array}$$
$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x_1 = 2x_2 + 1 \\ x_2 = 3 \end{cases}$

$\downarrow$

$\begin{bmatrix} x_1 = 7 \\ x_2 = 3 \end{bmatrix}$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$ Standard Matrix of $\boxed{T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

$T(e_2) = \begin{bmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b) $\left[\begin{array}{c|c} x_1 - 2x_2 & 1 \\ x + x_2 & 10 \\ x_1 - 3x_2 & -2 \end{array}\right] \Rightarrow \begin{array}{ccc} x_1 & x_2 & \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{R_2 \to R_2 + (-R_3)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-R_1)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 0 & -1 & -3 \end{array}\right] \xrightarrow{R_2 \to R_2 + (4R_3)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & -3 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$x_1 - 2x_2 = 1$

$x_1 + x_2 = 10$ We ge $x_1 = 7$, $x_2 = 3$

$x_1 - 3x_2 = -2$

(b) $u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \rightarrow \begin{array}{l} 7a + 3b = 1 \\ 7c + 3d = 10 \\ 7e + 3f = -2 \end{array}$$

3×2 $\quad 2 \times 1$

Standard Matrix of $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

(a)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

$\frac{2}{3}$ 3

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard matrix of T $\therefore$ $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) as linear transformation,

$Au = T(u)$

$$\therefore \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

(1) $u_1 - 2u_2 = 1$

(2) $u_1 + u_2 = 10$

(3) $u_1 - 3u_2 = -2$

$\therefore u_1 = 1 + 2u_2$ $\quad \therefore u_1 = 1 + 2(3) = 1 + 6 = 7$

$\therefore 1 + 2u_2 + u_2 = 10$ $[\because u_1 = 1 + 2u_2]$ $\therefore 3u_2 = 9$ $\therefore u_2 = 3$

$\therefore u_1 - 3(3) = -2$ $[\because u_2 = 3]$

$\therefore u_1 - 9 = -2$ $\quad \therefore u_1 = 7$

$$\therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\begin{matrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{matrix}$

b) $x_1 = 7$
$x_2 = 3$

$\begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$

$\begin{matrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a). $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -3 \end{bmatrix}_{x-1} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 21 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{bmatrix}_{x-1}$

$-4 \quad -1 \quad 0 \quad x-\frac{1}{2}$

$\begin{matrix} 2 & 1 & 0 \\ 1 & 1 & 0 & x-2 \\ 0 & 1 & 3 \end{matrix}$

$\begin{matrix} 0 & +1 & 0 & x-1 \\ 1 & 1 & 0 \\ 0 & 1 & 3 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -3 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a. $T_A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b. $\begin{array}{c} \begin{matrix} -1 & 2 & -1 \end{matrix} \\ \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & -3 \end{bmatrix} \div 3$

$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \Leftarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & -3 \end{bmatrix}$

free

$\begin{cases} x_1 = -7x_3 \\ x_2 = -3x_3 \\ x_3 = x_3 \end{cases}$ $x = \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$ $u \in \text{Span}\begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T \colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a) $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

(b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ $\quad Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] (-1) \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{array}\right] (\tfrac{1}{4}$$

$$\to (-1) \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] (-1) \to (2) \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

$u_1 = 7$

$u_2 = 3$

$$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} \qquad T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \qquad \text{Standard matrix of } T = A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $A \cdot u = T(u)$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} u \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{\cdot(-1)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{\cdot(-1)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -1 & -3 \\ 0 & 3 & 9 \end{array}\right] \xrightarrow{\cdot(-1)} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{array}\right] \cdot 2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{array}\right] \xrightarrow{\cdot(-3)} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned} \qquad u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Check: $T\left(\begin{bmatrix} 7 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 7 - 2(3) \\ 7 + 3 \\ 7 - 3(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ ✓

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2\cdot 0 \\ 1+0 \\ 1-3\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2\cdot 1 \\ 0+1 \\ 0-3\cdot 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard matrix of T is equal

$T\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \times -1$ $\quad \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \times 2$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \times -1$ $\quad \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \times \frac{1}{3}$ $\quad x_1 = 7$
$x_2 = 3$
$x_3 = x_3$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \times -1$ So when u equal $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ will satisfaing $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \times -1$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T. $\longrightarrow T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\left.\begin{array}{l} e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \end{array}\right\} T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = T(\vec{u})$

$$\begin{cases} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{cases}$$

$\downarrow$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \times 1$$

$-2 \times 1 = -2$

$1 - -2 = 3$

$1 \times 1 = 1$

$10 - 1 = 9$

$\downarrow$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \times 1$$

$-2 \times 1 = -2$

$-3 + 2 = -1$

$1 \times 1 = 1$

$-2 + 1 = -3$

$\downarrow$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \times \frac{1}{3} \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \times -1 \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -6 \end{array}\right] \Rightarrow \text{Inconsistent system}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$A = [T(x_1), T(x_2)]$

$$T\left(\begin{bmatrix} x_1 \\ x_1 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_1 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

Standard Matrix $= \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$x_1 - 2x_2 = 1$

$x_1 + x_2 = 10$

$x_1 - 3x_2 = -2$

$$\Rightarrow \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & \end{array} \\ \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \end{array} \Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 9 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 9 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} -19 \\ 11 \\ -29 \end{bmatrix} \checkmark$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$, $\therefore e_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $e_2 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\therefore$ Std Matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $A(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \begin{array}{l} \\ \to R_2 - R_1 \to \\ \to R_3 - R_1 \to \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \to \frac{R_3}{3} \to$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \begin{array}{l} \to R_1 + 2R_2 \to \\ \\ \to R_3 + R_2 \to \end{array} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \to$ $x_1 = 7$, $x_2 = 3$, $x_3 = x_3$,

Since $\mathbb{R}^2 \to \mathbb{R}^3$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$

$a x_1 + c x_2 = x_1 - 2x_2$

$c x_1 + d x_2 = x_1 + x_2$

$e x_1 + f x_2 = x_1 - 3x_2$

$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $Au = b$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ $\quad \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \cdot (-1) \to$

$\to \left[\begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \cdot (-1) \to \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 1 & 3 \\ 0 & 4 & 12 \end{array}\right] \cdot \frac{1}{4} = \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\quad x_1 = 3x_2 - 2$, $\quad x_2 = x_2$

$\boxed{\begin{bmatrix} -2 \\ 0 \end{bmatrix} + \text{span}\left\{\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right\}}$ $\quad \begin{bmatrix} -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \quad x_1 - x_2$$

$CT(x) = T(u)$

$C_1(x) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix}$

$C_1 = 1 + 2C_2 = 7$

$C_2 = 3$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \qquad \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix}$$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^2$

$u = \begin{bmatrix} 7 \\ -5 \\ 16 \end{bmatrix}$ $\quad \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -5 \\ 0 & 0 & 16 \end{array}\right]$

free variable

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{array}{c} \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \\ \begin{matrix} x_1 & x_2 \end{matrix} \end{array}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T(u) = \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \quad R_2 : R_2 - R_1$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \quad R_3 : R_3 + R_2$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array}\right] \quad R_3 : R_3 - R_1$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 1 \\ 0 & 2 & 6 \end{array}\right] \quad R_2 : R_2 - R_3$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -5 \\ 0 & 2 & 6 \end{array}\right]$$

$R_1 : R_1 + R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -5 \\ 0 & 2 & 6 \end{array}\right]$$

$R_3 : R_3 - 2R_2$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = [\,T(e_1) \quad T(e_2)\,]$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$R_2 = -R_1 + R_2$

$R_3 = -R_1 + R_3$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$

$\downarrow$

$R_3 = -R_3$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{array}\right]$$

$$u = \begin{bmatrix} 2x_2 + 1 \\ -3x_2 + 9 \\ -x_2 + 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors u satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$\begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \end{bmatrix}$ [3

2x1 2x1

2x3 3x1 = 2x1

2x3 [

3x2 2x1

A) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

3-1

7-3

7-

B)

$V = C_p \cancel{\begin{bmatrix} 3 \\ 7 \end{bmatrix}} \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

Anything multiple of this vector.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find $\underbrace{\text{the standard matrix of } T}_{}$. A

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) A is a 3X2 matrix.

$A = [T(e_1) \;\; T(e_2)]$, $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ is the standard matrix of T.

(b) $T(u) = T_A(u) = A \cdot u = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$, $u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

augmented matrix

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & \end{matrix} \\ \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \end{array} \xrightarrow{\text{row reduction}} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

Sol

$$\begin{cases} x_1 = 7 \\ x_2 = 3 \end{cases}$$

$\therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T. im defining this as A.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$

$T(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

standard matrix of T: $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} * \overset{u}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left.\begin{aligned} 1x_1 - 2x_2 &= 1 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 \end{aligned}\right\}$$

$x_1 = 1 + 2x_2$

$1 + 2x_2 + x_2 = 10 \Rightarrow 1 + 3x_2 = 10$

$3x_2 = 9$

$x_2 = 3$

↓ backsb

$x_1 - 2(3) = 1$

$x_1 = 7$

$7 - 3(3) = -2$

$7 - 9 = -2$ ✓

so $u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

or any multiple of it.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

std. matrix of $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

~~$x_1 - 2x_2 = 1$~~

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & 3 & -2 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 5 & -3 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 5 & -3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ \boxed{0} & 0 & -18 \end{array}\right]$$

$\boxed{\therefore \text{There are no vectors } u \text{ satisfying } T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a) $$A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \\ a_{31}x_1 + a_{32}x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$a_{11} = 1 \quad a_{12} = -2 \quad a_{21} = 1 \quad a_{22} = 1 \quad a_{31} = 1 \quad a_{32} = -3$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

(b) $$T(u) = Au = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{\text{row red}} \begin{array}{cc} u_1 & u_2 & \\ \end{array}\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$u_1 = 7$
$u_2 = 3$

$$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1 0

0 1

0 0

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a. $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 & -2x_2 \\ x_1 & +x_2 \\ x_1 & -3x_2 \end{bmatrix}$ $\quad x_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\qquad \begin{vmatrix} T(e_1) & T(e_2) \\ 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$ $\qquad \begin{vmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

$A = \begin{vmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

$\begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix}$ $\qquad \begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{matrix}$ $\qquad \begin{matrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{matrix}$

$\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{matrix}$ $\qquad \begin{matrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$ $\qquad \begin{matrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{matrix}$ $\qquad \begin{matrix} x_1 = 7 \\ x_2 = 3 \end{matrix}$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\therefore$ The Standard Matrix of T.

$T : c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $\therefore T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\therefore c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\begin{bmatrix} c_1 - 2c_2 \\ c_1 + c_2 \\ c_1 - 3c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$. $\therefore T(u) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = [T(e_1) \;\; T(e_2)]$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$A\cdot(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] -1$

$\begin{array}{ccc} -1 & 2 & -1 \\ 1 & 1 & 10 \\ \hline 0 & 3 & 9 \end{array}$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} -1$

$\begin{array}{ccc} -1 & 2 & -1 \\ 1 & -3 & -2 \\ \hline 0 & -1 & -3 \end{array}$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} 1$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} 2$

$\begin{array}{ccc} 1 & -2 & 1 \\ 0 & 2 & 6 \\ \hline 1 & 0 & 7 \end{array}$

$\begin{array}{ccc} x_1 & x_2 & c \end{array}$
$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\boxed{u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \text{ (only 1 solution)}}$

$x_1 = 7$

$x_2 = 3$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$e1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad T(e_2) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Standard matrix $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 0 & -3 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -3 & -2 \end{array}\right] \xrightarrow{R2 \cdot -\frac{1}{3}R3} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 20/3 \\ 0 & -3 & -2 \end{array}\right] R3+3R2$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 20/3 \\ 0 & 0 & 18 \end{array}\right] R1+2R2 \quad \left[\begin{array}{cc|c} 1 & 0 & 10/3 \\ 0 & 1 & 20/3 \\ 0 & 0 & 18 \end{array}\right]$$

$$\boxed{u = \begin{bmatrix} 10/3 \\ 20/3 \\ 18 \end{bmatrix}}$$

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $[T(e_1) \;\; T(e_2)] = [T(\begin{bmatrix}1\\0\end{bmatrix}) \;\; T(\begin{bmatrix}0\\1\end{bmatrix})] = \begin{bmatrix} 1-2(0) & 0-2(1) \\ 1+0 & 0+1 \\ 1-3(0) & 0-3(1) \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$Ax = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$x_1 = 1 + 2x_2$
$x_1 = 10 - x_2$
$x_1 = 3x_2 - 2$

$10 - x_2 = 3x_2 - 2$
$+2 \; +x_2 \quad +x_2 \; +2$
$4x_2 = 12$
$x_2 = 3$

$x_1 = 1 + 2(3)$
$x_1 = 7$

$\begin{bmatrix} 7 - 2(3) \\ 7 + 3 \\ 7 - 3(3) \end{bmatrix} = \begin{bmatrix} 7-6 \\ 10 \\ 7-9 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ ✓

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$\boxed{\text{standard matrix of } T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $T(u) = \begin{matrix} 1 \\ 10 \\ -2 \end{matrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \Rightarrow \text{row reduction} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = 7$

$x_2 = 3$

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

b) $\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

a) $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b) $$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \overset{+\ \cdot(-1)}{\Longrightarrow} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -3 \end{array}\right] \cdot(-1) \Longrightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{array}\right]$$

$$\Longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{array}\right] \Longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 1 & 1 & 10 \end{array}\right] + \cdot -1 \Longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right]$$

$$u = \text{Span}\left(\begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}\right)$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

standard matrix of $T = A$

a. A is 3×2 matrix where $A \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$

$$\begin{bmatrix} \circ & \circ \\ \circ & \circ \\ \circ & \circ \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b. Find $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \leftarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right]$$

$x_1 = 7 \quad x_2 = 3$

$$u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$e_1 = \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \quad e_2 = \begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$

$T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \qquad T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{array}{cc|c} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \cdot(-1) \to \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -2 \end{bmatrix} \cdot -1$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \xleftarrow{+} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -2 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -2 \end{bmatrix} \div 3$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$A = [T(e_1) \ T(e_2)]$ $\quad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b)

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \cdot -1$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -4 & -12 \end{array}\right] \cdot -1$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -4 & -12 \end{array}\right] \cdot 2$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & -14 \end{array}\right] \cdot 1/3$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -14 \end{array}\right] \cdot 2$

$\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & -14 \end{array}\right]$

$0 \neq -14$

$\therefore$ There are no solutions for u that ~~that~~ satisfies $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

4. **(20 points)** Let $T\colon \overset{n}{\mathbb{R}^2} \to \overset{m}{\mathbb{R}^3}$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $m \times n$ matrix

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \qquad \begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{bmatrix}$$

free

b)

$x_1 - 2x_2$

$x_1 + x_2$

$x_1 - 3x_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 1 & 1 & 0 & 10 \\ 1 & -3 & 0 & -2 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a) $\cancel{T(e_1)}$ $[e_1] = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\therefore T([e_1]) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\qquad$ $\therefore T([e_2]) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

So, Standard matrix, $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$ Ans:-

(b) From definition, $T(u) = A \cdot u = A \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, as $T: \mathbb{R}^2 \to \mathbb{R}^3$

Now, $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$A \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

Since, $T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\Downarrow$

$$\begin{cases} u_1 - 2u_2 = 1 & \text{--- (1)} \\ u_1 + u_2 = 10 & \text{--- (2)} \\ u_1 - 3u_2 = -2 & \text{--- (3)} \end{cases}$$

from (2), $u_1 = 10 - u_2$

$\therefore$ " (1), $10 - u_2 - 2u_2 = 1$

$\Rightarrow 10 - 3u_2 = 1$

$\Rightarrow 10 - 1 = 3u_2$

$\therefore u_2 = \frac{9}{3} = 3$, $u_1 = 10 - 3 = 7$

$\therefore \boxed{\vec{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} = 7\hat{u}_1 + 3\hat{u}_2}$

Ans:-

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

the standard matrix : $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array} \Rightarrow v = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{vmatrix} 1 & 2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 1 & -3 & -2 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & -5 & -3 \end{vmatrix}$

$R_1 \times -1 + R_2 + R_2 \times -1$ $\quad R_1 \times -1 + R_3$ $\quad R_2 \times 5 + R_3$

$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & 0 & -48 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

There are no vectors (u) that satisfy

$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(u) = T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -2 & -8 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 0 & -9 \end{array}\right]$

→ No vector satisfies T(u).

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$A = [T(e_1), T(e_2)]$$

where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ = standard matrix of T

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b)

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{\cdot -1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{\cdot -1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{\cdot \frac{1}{3}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & 0 \end{bmatrix} \cdot \frac{1}{3}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\cdot 2} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{cases} x_1 = 7 \\ x_2 = 3 \end{cases} \qquad u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

chk $\begin{bmatrix} 7-2(3) \\ 7+3 \\ 7-3(3) \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard Matrix of T:

$$A = [T(e_1)\ T(e_2)] = \left[T\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\right)\ T\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\right)\right] = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)
$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \begin{array}{l} R_2 \to -R_1 + R_2 \\ R_3 \to -R_1 + R_3 \end{array} \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] R_2 \to \frac{R_2}{3}$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \begin{array}{l} R_3 \to R_2 + R_3 \\ R_1 \to 2R_2 + R_1 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$u = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

there is a unique solution

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+(0) \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A = [T(e_1)\ T(e_2)]$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard Matrix:

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $T(\vec{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$A\vec{u} = b$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \vec{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

R1 $\cdot -1$ + R2

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{R1\cdot -1 + R3} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] = \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \; R3 \cdot -1 \; = \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \Rightarrow \begin{array}{l} u_1 - 2u_2 = 1 \\ u_2 = 3 \end{array}$$

$u_1 - 2(3) = 1$

$u_1 - 6 = 1$

$u_1 = 7$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = [\,T(e_1),\ T(e_2)\,] \quad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{bmatrix} = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)

$$\cdot -1 \quad \begin{array}{cc} x_1 & x_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \cdot -1 \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$$

$$\to \begin{array}{c} \cdot \frac{1}{3} \\ \cdot -1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \to \cdot -1 \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= 2x_2 \\ x_2 &= x_2 \end{aligned}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \qquad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

a) Find the standard matrix of T.

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2(1) \\ 1 \\ -3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

a) $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $Tu = Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ $\qquad A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \begin{matrix} \cdot -1 \\ \cdot -1 \end{matrix} \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 8 \\ 0 & -1 & -4 \end{array}\right] \longrightarrow$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -1 & -4 \\ 0 & 3 & 8 \end{array}\right] \cdot -1 \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 3 & 8 \end{array}\right] \cdot -3 \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{array}\right]$$

$\boxed{\text{no solutions for } u}$

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$x_1 = 2x_2 + 1$

a) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 - 0 \\ 1 + 0 \\ 1 - 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 - 2 \\ 0 + 1 \\ 0 - 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$A = \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $[$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

Standard Matrix of $T = \begin{bmatrix} T(e_1) & T(e_2) & [redacted] \end{bmatrix}$

$$\left[T\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right] = \begin{bmatrix} 1-2(0) \\ 1+\cancel{0} \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

a) $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \begin{matrix} \\ 2-1 \\ +1 \end{matrix} \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 11 \\ 0 & -1 & -3 \end{bmatrix} \begin{matrix} \\ \\ (-1) \end{matrix}$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 11 \end{bmatrix} 2-3 \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

b) $\boxed{\text{No vectors}}$ satisfy that condition

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T. T is 3x2 $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$-1\left(\begin{aligned} u_1 - 2u_2 &= 1 \\ u_1 + u_2 &= 10 \\ u_1 - 3u_2 &= -2 \end{aligned}\right.$$

$-u_2 = -3$

$u_2 = 3 \quad u_1 = 7$

$u = \left(\begin{bmatrix} 3 \\ 7 \end{bmatrix}\right)$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

A) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\boxed{\text{Standard matrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

B) $x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$ $R_2 \to R_1 - R_2$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & -9 \\ 1 & -3 & -2 \end{bmatrix}$ $R_3 \to R_1 - R_3$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & -9 \\ 0 & 1 & 3 \end{bmatrix}$ $R_2 \to 3R_3 + R_2$ $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ $R_1 \to 2R_3 + R_1$

$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ $R_3 \longleftrightarrow R_2$ $\begin{array}{cc|c} x_1 & x_2 & \\ \hline \end{array}$ $\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 = 7$
$x_2 = 3$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard matrix A:

$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$$\begin{array}{cc} & \begin{array}{cc|c} u_1 & u_2 & \end{array} \\ \begin{array}{c} (-1) \\ + \\ \\ \end{array} & \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \end{array}$$

$$\begin{array}{c} \\ (1/3) \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$$

$$\begin{array}{c} (-1) \\ + \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{array}\right]$$

$$\begin{array}{c} \\ + \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right]$$

$$\begin{array}{c} \\ (2) \\ \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & -2 & -6 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $[\,T(e_1) \quad T(e_2)\,]$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & - & 0 \\ 1 & + & 0 \\ 1 & - & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\text{Standard Matrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & - & 2 \\ 0 & + & 1 \\ 0 & - & 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

b) $x_1 - 2x_2 = 1$

$x_1 + x_2 = 10$

$x_1 - 3x_2 = -2$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) the standard matrix of T

is $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 10 \\ 1 & -2 & 1 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 10 \\ 0 & 3 & 9 \\ 0 & 4 & 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 10 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

$u_1 = 7$

$u_2 = 3$

$u_3 = u_3$

$$u = \begin{bmatrix} 7 \\ 3 \\ u_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_3.$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $T(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{-1} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{-1} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 0 & -1 & -3 \end{array}\right] \xrightarrow{-2} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 4 & 12 \end{array}\right] \Big\}\ \begin{matrix} x_1 = 7 \\ x_2 = 3 \end{matrix}$

$T\left(\begin{bmatrix} 7 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 7-6 \\ 7+3 \\ 7-9 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$Ax = T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T. $\longrightarrow \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$ $\Rightarrow$ standard matrix of $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$(R1 \times -1) + R2$ $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$(R1 \times -1) + R3$ $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] /3$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] /-1$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$ $(R3 \times 2) + R1$

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$ $(R2 \times -1) + R3$

$\longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 = 7$
$x_2 = 3$

$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{matrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{matrix}$$

$$\boxed{T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b) $$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$x_1 = 7$

$x_2 = 3$

$$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a. $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b. $\begin{array}{c} ① \\ ② \\ ③ \end{array}\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$ ② - ①

$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}.$

③ - ①

$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$

7.

③ x 2 - ①

$= \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & 0 \end{array}\right]$

$3x_2 = 9$

$x_2 = 3$

$x_1 - 2x_2 = 1$

$x_1 - 6 = 1$

$x_1 = 7$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

3x2 2x2

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

a) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $u = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = (T(e_1), T(e_2))$ $\quad T(e_1) = T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\quad T(e_2) = T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-2\\1\\-3\end{bmatrix}$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $T(u) = Au = \begin{bmatrix}1\\10\\-2\end{bmatrix}$ $\quad \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & \end{array} \\ \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x_2 = 3$

$x_1 = 7$ $\quad \begin{bmatrix}7\\3\end{bmatrix}$

$$u = \begin{bmatrix}7\\3\end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. (20 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\boxed{T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{aligned} x_1 - x_2 &= 1 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x_1 &= 11 \rightarrow x_1 = 4.5 \text{ or } 9/2 \\ 9/2 - 3x_2 &= -2 \\ -3x_2 &= 13/2 \\ x_2 &= -13/6 \end{aligned}$$

But $9/2 - 13/6 \neq 10$ $\therefore$ $\boxed{\text{There are no vectors, } u\text{, which satisfy } T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$$[3 \times 2][2 \times 1] = [3 \times 1]$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

3×2 $\quad 2 \times 1$ $\quad 3 \times 1$

$\text{rref}\begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{pmatrix}$ would solve it

$$\boxed{U = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

$c_1 - 2c_2 = 1$

$c_1 + c_2 = 10$

$c_1 - 3c_2 = -2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

rref

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -7 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

$c_2 = 3$

$c_2 = 7$

$c_1 - 2c_2 = 1$

$c_1 = +2(7) + 1$

$c_1 = 15$

15-14

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad \boxed{T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b) $1 = x_1 - 2x_2$

$10 = x_1 + x_2$

$-2 = x_1 - 3x_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -3 & -9 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x_1 = 7 \\ x_2 = 3 \end{matrix} \quad \boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

(a) $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

(b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow{-R_1+R_3} = \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{-R_1+R_2} = \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 1 \end{array}\right]$

=

$\boxed{\begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned}}$

$7 - 2(3) = 1$

$7 + 3 = 10$

$7 - (3)(3) = -2$

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\overset{A}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}\right) = \overset{A}{\begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$ V

$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$u_1 - 2u_2 = 1$
$u_1 + u_2 = 10$
$u_1 - 3u_2 = -2$

$u = \begin{bmatrix} \; \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = [T(e_1) \;\; T(e_2)]$

$T(e_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad T(e_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$Au = T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \cdot(-1)$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right] \cdot(-1)$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \cdot(1/3)$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \cdot(1)$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] (2)$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \longrightarrow \boxed{\begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}}$$

$$\boxed{u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $u = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $v = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(u) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix}$ $T(v) = \begin{bmatrix} b_1 - 2b_2 \\ b_1 + b_2 \\ b_1 - 3b_2 \end{bmatrix}$

$T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T(e_2) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

standard matrix of $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $R_3 = R_3 - R_1$ $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$+2R_2$ $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & -3 \end{array}\right]$

$\downarrow$

$\begin{bmatrix} 3 & 0 & 21 \\ 1 & 1 & 10 \\ 0 & 1 & -3 \end{bmatrix} * \left(\frac{1}{3}\right) \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 1 & 1 & 10 \\ 0 & 1 & -3 \end{array}\right]$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & -3 \end{bmatrix}$

$x_1 = 7$
$x_2 = x_2$

all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ are: $\begin{bmatrix} 7 \\ x_2 \end{bmatrix}$ where x_2 is free

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{matrix} 0 - 2 \\ 0 + 1 \\ 0 - 3 \end{matrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$3 \times 2 \quad 2 \times 1$

$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \\ A_5 & A_6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \\ A_5 & A_6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

$A_1 = 1$ $\quad A_2 = -2$

$A_3 = 1$ $\quad A_4 = 1$

$A_5 = 1$ $\quad A_6 = 3$

standard matrix of T is $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$

b) $-1 \cdot \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & 3 & -2 \end{array}\right]$ $\quad \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & 0 & -11 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 & -11 \\ 0 & 1 & 3 \\ 0 & 0 & 13 \end{bmatrix}$

no Solutions

$1 \cdot \begin{bmatrix} 0 & -3 & -9 \\ 1 & 1 & 10 \\ 1 & 3 & -2 \end{bmatrix}$ $\quad -1 \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 7 \\ 1 & 0 & -11 \end{bmatrix}$

$-\frac{1}{3} \begin{bmatrix} 0 & -3 & -9 \\ 1 & 1 & 10 \\ 1 & 0 & -11 \end{bmatrix}$ $\quad \begin{bmatrix} 0 & 1 & 3 \\ 0 & 0 & 18 \\ 1 & 0 & -11 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) \Rightarrow T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) \Rightarrow T\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard Matrix of T:

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $AV = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$

$(1/3)\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}$ $\begin{bmatrix} 7 \\ 3 \end{bmatrix} \Rightarrow u$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard matrix of $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 = 1 + 2x_2$

$x_2 = 3$

$\begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$

$x_1 = 7$
$x_2 = 3$

$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & -1 & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\mathrm{Nul}(A) = \{0\}$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

all vectors satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$: $\left\{\begin{bmatrix} 7 \\ 3 \end{bmatrix}\right\}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$0 - 2 = -2$

$0 - 3 = -3$

(a) $T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

standard matrix of T : $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

(b) $u = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$T(u) = T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$a_1 - 2a_2 = 1$

$a_1 + a_2 = 10$

$3a_2 = 9$ $a_2 = 3$

$a_1 = 7$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Let T be 3x2 matrix

$$T = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \\ C_5 & C_6 \end{bmatrix}$$

$$T\left(\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}\right) = \begin{bmatrix} X_1 - 2X_2 \\ X_1 + X_2 \\ X_1 - 3X_2 \end{bmatrix}$$

$$C_1X_1 + C_2X_2 = X_1 - 2X_2$$
$$C_3X_1 + C_4X_2 = X_1 + X_2$$
$$C_5X_1 + C_6X_2 = X_1 - 3X_2$$

$\therefore C_1 = 1, C_2 = -2, C_3 = 1, C_4 = 1, C_5 = 1, C_6 = -3$

$$\therefore T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

Let u be vector $\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

aug. matrix

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{(1)-(3)} \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(2)-(1)} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(2)\div 3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(2)-(3)} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{(1)+(2)\times 2} \begin{array}{c} \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\therefore \begin{cases} C_1 = 7 \\ C_2 = 3 \end{cases} \qquad \therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $$\begin{bmatrix} x_1-2x_2 \\ x_1+x_2 \\ x_1-3x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b) $$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$ SO $x_1-2x_2=1$, $x_1+x_2=10$, $x_1-3x_2=-2$

$x_1=1+2x_2$

$x_1=1+6=7$

or
$$\begin{matrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -4 & -12 \end{matrix}$$

SO $-4x_2=-12$

$x_2=3$

$x_1=7$

SO $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] = [$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

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Standard Matrix $T = \begin{bmatrix} 1 & -2 \\ 1 & 2 \\ 1 & -3 \end{bmatrix}$

$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 2 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 9 \\ 0 & 1 & 3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 4 & 9 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & -3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right]$

$x_1 - 2x_2 = 1$

$x_1 + x_2 = 10$

$x_1 - 3x_2 = -2$

$-3x_2 = -9$

$x_2 = 3$

$x_1 + 3 = 10$

$x_1 = 7$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

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a) Find the standard matrix of T.

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a) $A = [T(e_1) \quad T(e_2)]$ where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}}$

$T(e_1) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20