

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:	<u> </u>	
Kristiyan	VIG KZON GLOA	

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
							1	



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

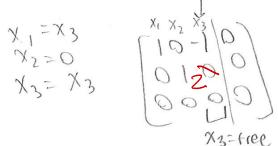
$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.

$$\begin{cases} 1 & -1 & 1 & 0 \\ 2 & -3 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 &$$

$$x_1 = x_3$$

 $x_2 = 0$
 $x_3 = x_3$



Which verns tent it is lineary dependent.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 8 & -1 \\ 2 & 4 & 1 \\ 2 & 8 & 1 \end{bmatrix}$$



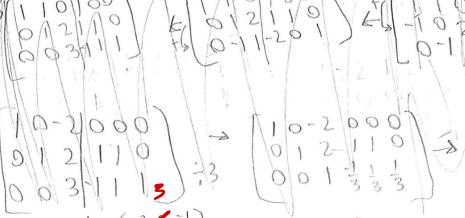
$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

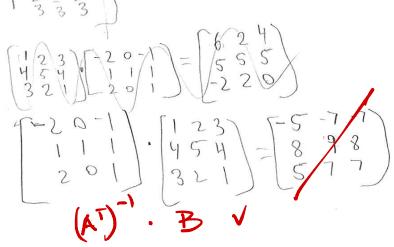
Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$



$$\left(\mathbf{A}^{\mathsf{T}}\right)^{-1} = \begin{pmatrix} -2 & \mathbf{S}^{-1} \\ 1 - 1 & 1 \\ 2 & \mathbf{S}^{-1} \end{pmatrix}$$

$$\begin{pmatrix}
 1 & 10 \\
 -1 & 02 \\
 2 & 11
 \end{pmatrix}
 \begin{pmatrix}
 -5 & -7 & -7 \\
 8 & 9 & 8 \\
 5 & 7 & 77
 \end{pmatrix}
 =$$





4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

a) Find the standard metric
$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$
.

b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(?) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \qquad T(o) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$A = \left[T(e_1) T(e_2)\right]$$

$$e_1 = \frac{1}{2} e_2 = \frac{2}{3}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}$$

$$C_{1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_{2} \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$

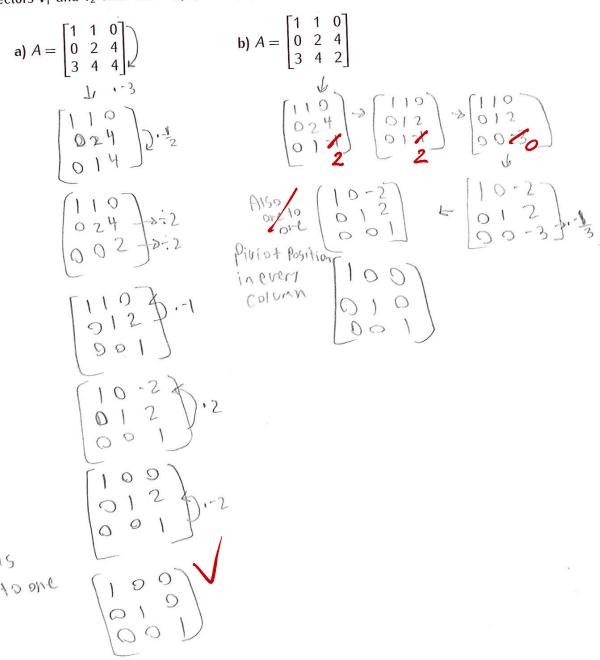


$$\begin{bmatrix}
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5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.





6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True because U is not reeded for W to be in the span IF it was given that all?

3 were in R3.

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

Fighte it would depend on What Vectors

Out of the 3.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

b) If $T:\mathbb{R}^2\to\mathbb{R}^2$ is a linear transformation and $u,v,w\in\mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

Took & why?