



MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Eithne Amos

UB Person Number:

5	0	2	0	9	9	0	9
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

11

3

9

7

8

6

3

47

nan

1

2

3

4

5

6

7

TOTAL

GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$a) \quad \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{r_2 + r_1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{\begin{matrix} (-2) \cdot r_1 \\ +r_3 \end{matrix}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{bmatrix}$$

$$\xrightarrow{(-\frac{1}{3}) \cdot r_3} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b \end{bmatrix} \xrightarrow{\begin{matrix} (-1) \cdot r_2 \\ +r_3 \end{matrix}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{bmatrix}$$

$0 = b - 2$
 $b = 2$

$$b) \quad \begin{matrix} v_1 & v_2 & v_3 \\ \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \end{matrix} \xrightarrow{\text{row reduce}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\{v_1, v_2, v_3\}$ is not linearly independent because every column of $[v_1 \ v_2 \ v_3]$ is not a pivot column

hello!



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{(-1) \cdot r_1 \\ +r_2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{r_2 \leftrightarrow r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{(-2) \cdot r_2 \\ +r_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{r_3 \leftrightarrow r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{-r_3 + r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 3 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} A^T C = B (A^T)^{-1}$$

$$C = B (A^T)^{-1}$$

$$C = B (A^{-1})^T$$

$$C = B \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T .

b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$a) A = [T(e_1) \ T(e_2)] , e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} , e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

standard matrix:

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$b) T_A(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{(-1)r_1 + r_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{r_2 \cdot \frac{1}{3}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\xrightarrow{-r_1 + r_3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{2r_2 + r_1} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{r_2 + r_3} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 7$$

$$x_2 = 3$$

$$x_3 = x_3$$

$$v = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3$$



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to-one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A has pivot in every column $\rightarrow A$ is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\downarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

not one-to-one



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

true

$$\text{Span}(u, v) = \underbrace{c_1 u + c_2 v}$$

if $w + u \in \text{Span}(u, v)$, then

w must be lin. combination of u, v

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

true - each vector must be linearly independent
from each other vector for the whole set
to be linearly independent



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

false

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$.

true