



# MTH 309T LINEAR ALGEBRA

## EXAM 1

October 3, 2019

Name:

Judy Mei

UB Person Number:

5	0	2	3	7	0	3	2
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

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TOTAL

GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of  $b$  such that  $w \in \text{Span}(v_1, v_2, v_3)$ .

b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 + R_1} \left[ \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 + (-R_2)} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{restore } R_3} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{matrix} \text{free} \\ \uparrow \\ R_1 \end{matrix}$$

b) It is linearly dependent since the reduced form contains a free variable meaning it has infinitely many solutions.

$$a) b = 2$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} -(+1) \\ 1=1=0 \\ 0+0=0 \end{array}$$

$$\begin{array}{l} x_1 - x_2 + 2x_3 \\ x_1 + 0x_2 + x_3 \\ 0x_1 + 2x_2 + 2x_3 \end{array} \begin{array}{l} 1 \\ 0 \\ 0 \end{array} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{R_2 + (-R_1)} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{array}{l} 0 \\ 1 \\ 1 \end{array}$$

$$\begin{array}{l} R_1 + R_2 \\ R_3 + (-2R_2) \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{R_3 + (-2R_2)} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -3 & 2 \end{bmatrix} \begin{array}{l} 0 \\ 0 \\ -1/3 \end{array} \xrightarrow{\times 1/3} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -2/3 \end{bmatrix} \begin{array}{l} 0 \\ 0 \\ -1/3 \end{array}$$

$$\xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -5/3 \\ 0 & 0 & 1 & -2/3 \end{bmatrix} \begin{array}{l} -R_1 + (-R_3) \\ -5/3 \\ -2/3 \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2/3 \\ 0 & 1 & 0 & -5/3 \\ 0 & 0 & 1 & -2/3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ -5/3 & -5/3 & 1/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$$



3. (10 points) Let  $A$  be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad Ax = B$$

Find a matrix  $C$  such that  $A^T C = B$  (where  $A^T$  is the transpose of  $A$ ).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1) \ T(e_2)]$  where  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$   $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)



5. (20 points) For each matrix  $A$  given below determine if the matrix transformation  $T_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  given by  $T_A(v) = Av$  is one-to-one or not. If  $T_A$  is not one-to-one, find two vectors  $v_1$  and  $v_2$  such that  $T_A(v_1) = T_A(v_2)$ .

a)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

b)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a)



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. In order to be linearly independent, each column is a pivot column. So if you do set  $\{u, v\}$  they will both still have a pivot column in each column.





7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $A$  is a  $2 \times 2$  matrix and  $u, v$  are vectors in  $\mathbb{R}^2$  such that  $Au, Av$  are linearly dependent then  $u, v$  also must be linearly dependent.

False,

b) If  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that  $u$  is in  $\text{Span}(v, w)$  then  $T(u)$  must be in  $\text{Span}(T(v), T(w))$ .

True,