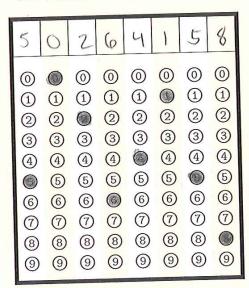


MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

| Name: | | |
|---------|----------|--|
| Michael | Leishear | |

UB Person Number:



Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | TOTAL | GRADE |
|---|---|---|---|---|---|----|-------|-------|
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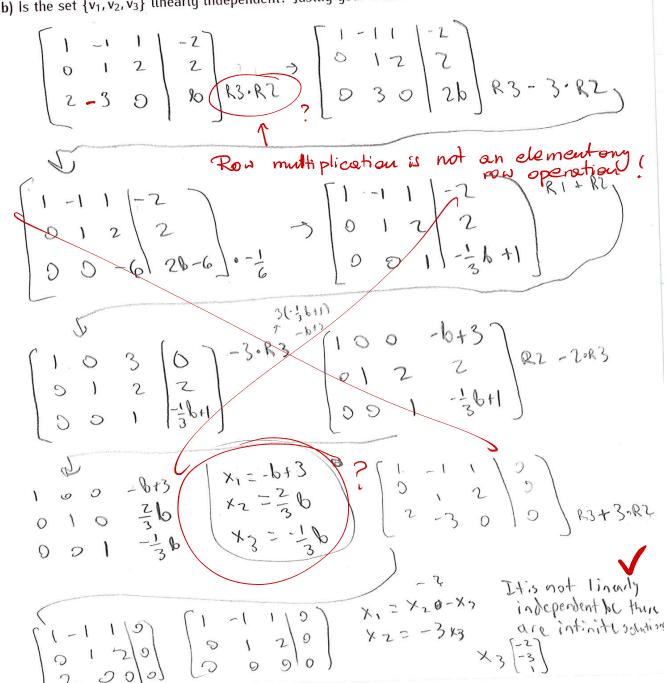
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1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 \\ 0 & 2 & -1 & | & 0 & 0 \\ 0 & 2 & -1 & | & 0 & 0 \end{bmatrix} \stackrel{R7 - \frac{1}{2}}{}$$

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$$\begin{bmatrix}
1 & -1 & 2 & | &$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \left[\begin{array}{rrr} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{array} \right]$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 23 \\ 454 \\ 3 & 21 \end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 1 & 0 & 0 \\
0 & 1 & 2 & 1 & 1 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 0 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 4 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 2 & 1 & 1 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 1 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 \\
0 & 0 & 1 & 1 & 2 & 1 & 2 & 1
\end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

tors u satisfying
$$T(u) = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$
.

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad T(e_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$f_{an} hard matrix \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix}
1 & -2 & | & 1 \\
0 & | & 1 & | & 10 \\
0 & -3 & | & -2
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -2 & | & 1 \\
0 & | & 20/3 \\
0 & -3 & | & -2
\end{bmatrix}$$

$$\begin{bmatrix}
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0 & -3 & | & -2
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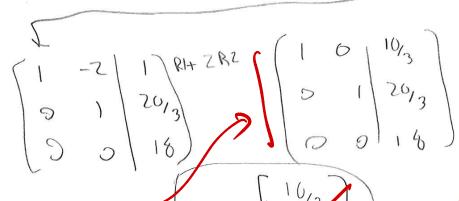
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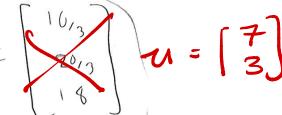
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0 & -3 & | & -2 & | \\
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This mould _____ mean: no solutions





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to-one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -4 & 0 \end{pmatrix}$$
, $-\frac{1}{4}$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

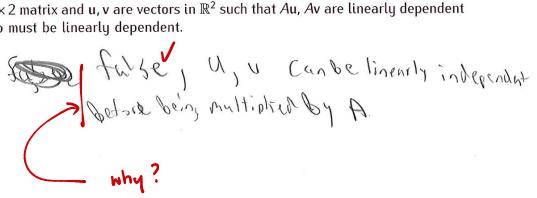
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b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

Folse, Ell, UB Could have a different!
Solution with nothing trivial



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

