

2. Find the equation  $f(x) = ax + b$  of the least square line for the points  $(1, 0)$ ,  $(-1, 2)$ ,  $(2, 1)$ .

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$3 \times 2 \quad \quad 2 \times 3$

~~$$A^T A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$$~~

$$A^T A x = A^T b$$

$$b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$2 \times 3 \quad 3 \times 1$

$$0 + (-2) + 2$$

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

~~$$\begin{array}{l} \frac{1}{2} \left( \begin{bmatrix} 2 & 0 & 3 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 3 & -1 & 5 & | & 1 \end{bmatrix} \xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & -1 & 5/2 & | & 1 \end{bmatrix} \xrightarrow{R_3 \times 2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & -2 & 5 & | & 2 \end{bmatrix} \xrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & 0 & 3 & | & 4 \end{bmatrix} \xrightarrow{R_3 \div 3} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 \times 1/2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & 1/2 & | & 5/3 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 \times 2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & 1 & | & 10/3 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 - 2R_3} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 2 & -1 & | & 2 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 \times 1/2} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_1 - 3/2 R_3} \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 + 1/2 R_3} \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 5/3 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \end{array}$$~~

~~$$\begin{array}{l} \begin{bmatrix} 1 & 0 & 3/2 & | & 0 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_1 - 3/2 R_3} \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & -1/2 & | & 1 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \xrightarrow{R_2 + 1/2 R_3} \begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 5/3 \\ 0 & 0 & 1 & | & 4/3 \end{bmatrix} \end{array}$$~~

$x_1 = -2$   
 $x_2 = 5/3$   
 $x_3 = 4/3$

$$\frac{4}{10} \quad \frac{2}{9} \quad \frac{3}{2}$$

$$\left(\frac{3}{2}, 0\right) \quad \left(\frac{7}{9}, 2\right) \quad \left(-\frac{4}{9}, 1\right)$$

$$a = \frac{1 - 0}{\frac{4}{18} - \frac{36}{18}} = \frac{1}{-\frac{44}{18}}$$

$$b = 0$$

$$f(x) = 0.4x + 0$$

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