

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019											
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7 7		7 7	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	1 2 3 4 5 6 7 8 9	 Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes. For full credit solve each problem fully, showing all relevant work. 						
1	2	3	4	5	6	7	TOTAL	GRADE			

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1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

the set of vectors is wrearly dependent because even if you had be often there would still be a fire variable V

if w E { v, va v } then w = c, v, + c2 va + c3 vg
anthere is a nontrivial answer to this equation



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

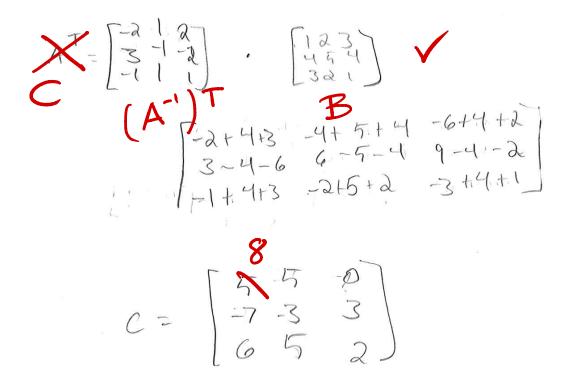
$$\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = A^{-1}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$
 $Q \chi \chi \tilde{g}$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

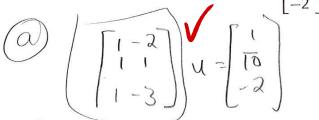




4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$. $V_1 \neq V_2$

vectors
$$\mathbf{v}_1$$
 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$. $V_1 \neq V_2$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$
b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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$$\begin{bmatrix} 1 & 0$$

$$V_{1} = \begin{bmatrix} 4 \\ -4 \end{bmatrix} \quad V_{2} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$2$$



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True if uis in the span as well then wtuEspar(uv)

Because

W = -u + v? $V = u_1 v_2 = v_3 v_4 = v_4 v_5 = v_4 v_5 = v_5 = v_5 v_6 = v_5 v_6 = v_6 v_6 =$

b) If u,v,w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

false

UV are dependent

dependent dependent



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

Took Becouse A Stay the same so inorder
to Keep Au, Av dependent a V must be dependent

Not necessarily

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True if TiBa traslation done to all

Vectors then the vectors will change
in the Sume magnitude

Translation is not a

linear transformation.