

## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:	
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UB Person Number:	Instructions:
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## Textbooks, calculators and any other electronic devices are not permitted.

You may use one sheet of notes.

 For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

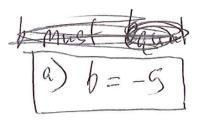
$$\begin{bmatrix}
1 & -1 & 1 & | & -2 \\
0 & 1 & 2 & | & 2 \\
2 & -3 & 0 & | & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & | & -2 \\
0 & 1 & 2 & | & 2 \\
0 & -1 & -2 & | & 4+b
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & | & -2 \\
0 & 1 & 2 & | & 2 \\
0 & -1 & -2 & | & 4+b
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 1 & 2 & | & 2 \\
0 & 0 & 0 & | & 5+b
\end{bmatrix}$$

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b) The set is linearly dependent because there is not a pivot column in every column



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute 
$$A^{-1}$$
.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
oute A<sup>-1</sup>.
$$\begin{bmatrix} 1 & -1 & 2 & | & 0 & 0 \\ 0 & | & 0 & | & 0 \\ 0 & | & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 - 2 & | & 2 & 0 & | & 0 & | & 0 & | & 0 \\ 0 - 2 & 2 & 2 - 20 & | & 0 & | & 0 & | & 0 \\ 0 & 2 - 1 & 0 & 0 & | & 0 & | & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & 0 & | & 0 & | & 0 & | & 0 & | & 0 \\ 0 & 2 & -1 & 0 & 0 & | & 0 & | & 0 & | & 0 \\ 0 & 2 & -1 & 0 & 0 & | & 0 & | & & 0 \end{bmatrix}$$

$$\begin{bmatrix}
10 & 1 & 0 & 10 \\
6 & -22 & 2 & -20 \\
0 & 0 & 1 & 2 & -2
\end{bmatrix}$$

$$\begin{bmatrix}
10 & 1 & 0 & 10 \\
6 & -72 & 2 & -20 \\
0 & 0 & 1 & 2 & -2
\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -23 - 1 \\ 1 - 1 \end{bmatrix}$$
 $\begin{bmatrix} 2 - 21 \end{bmatrix}$ 



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

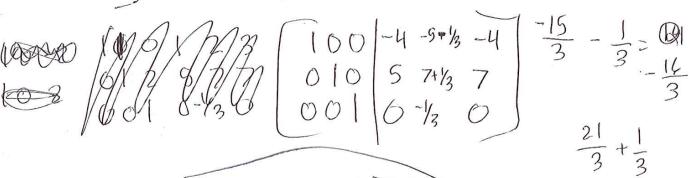
$$\vec{A} = \begin{bmatrix}
1 & 1 & 0 \\
-1 & 0 & 2 \\
2 & 1 & -1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & | & 12 & 3 \\
-1 & 0 & 2 & | & 45 & 4 \\
2 & 1 & -1 & | & 32 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & | & 12 & 3 \\
-1 & 0 & 2 & | & 45 & 4 \\
2 & 1 & -1 & | & 32 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 0 & | & 12 & 3 \\
-1 & 0 & 2 & | & 45 & 4 \\
0 & 1 & -5 & -5 & -7
\end{bmatrix}$$

$$\begin{bmatrix}
-102454 \\
012577 \\
01-59-8-7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
-102454 \\
012577 \\
00-3010
\end{bmatrix}
\rightarrow
\begin{bmatrix}
10-2-4-5-4 \\
012577 \\
06-6 0-30
\end{bmatrix}$$



$$\frac{10}{3} - \frac{1}{3} = \frac{14}{3}$$

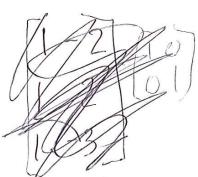


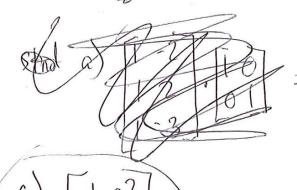
4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

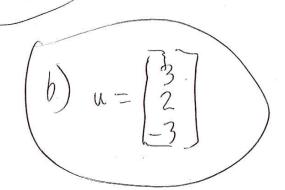
$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

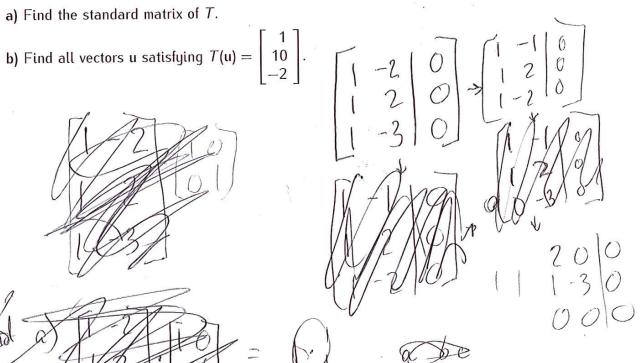
$$3 \times 2 \quad 2 \times 2$$

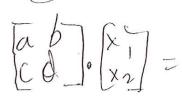
- a) Find the standard matrix of T.





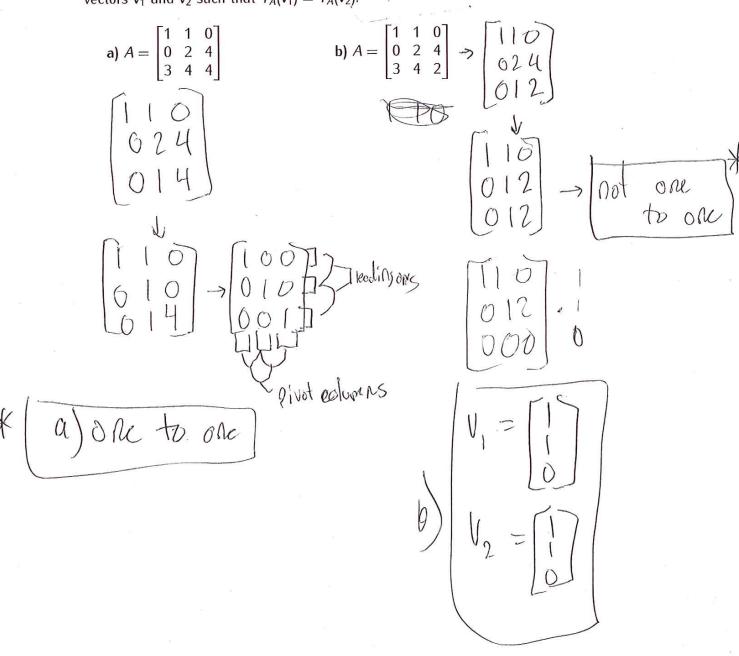








5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A : \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$ .





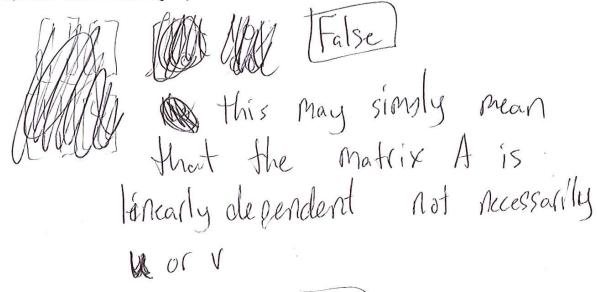
**6. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in $\mathbb{R}^3$ such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$ .
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R False Will Will Will Will Will Will Will Wil
Who spoot ou out
We could be Hojected for
we could be projected that outside the span when
Not ordelle to
b) If $u, v, w$ are vectors in $\mathbb{R}^3$ such that the set $\{u, v, w\}$ is linearly independent then the
set {u,v} must be linearly independent.
True because the you will
still have wery column as
a livot column



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $(u, v, w \in \mathbb{R}^2)$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True because if the same transformation
True because if the same transformation
is suffermed on some u that's in the
span of the some v and w; that
will be moved into that new
Span of T(V), T(W) by the
transformation T(W).