
$\begin{bmatrix} a & b \\ c & d \end{bmatrix}[v] = \lambda v$

$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} c & d \\ d & c \end{bmatrix}$

**5.** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

$\begin{bmatrix} a+c & b+d \\ b+d & a+c \end{bmatrix}$

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{v}$ is an eigenvector of $A$ corresponding to an eigenvalue $\lambda$ then $2\mathbf{v}$ is an eigenvector of $A$ corresponding to the eigenvalue $2\lambda$.

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

**b)** If $V$ is a subspace of $\mathbb{R}^2$ and $\mathbf{w}$ is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then $\mathbf{w}$ must be the zero vector.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**c)** If $A$ is a square matrix which is both symmetric and orthogonal then $A^2$ is the identity matrix.

**d)** If $A$ and $B$ are $2 \times 2$ matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

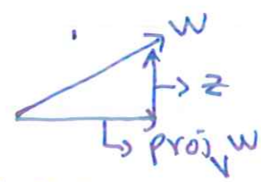
a) False The eigenvalue is the root of the characteristic polynomial which means its the solution that makes the equation $P(\lambda) = 0$ true an $n \times n$ matric can have ~~only~~ no more than $n$ roots so $2\lambda$ would not be the eigenvalue.

5

b) True projection of a vector w on subspace $V$ is unique ~~to each vector~~ ~~being projected~~ such that $z = w - \text{proj}_V w \Rightarrow w = z + \text{proj}_V w$ where $z$ is an orthogonal vector to $V$

+5

$\rightarrow$ This means the only way $\text{proj}_V w = -w$ is is vector $w$ is a zero vector.




$\rightarrow$ If $w$ is an element of vectorspace of $V$ then $\text{proj}_V w = w$ which is different from $\text{proj}_V w = -w$

c) False

2

If $A$ is symmetric then it has $n$ orthogonal eigenvector

A can then be expressed as $A = QDQ^T$

$A^2 = QD^2Q^T$ not identity matrix



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20