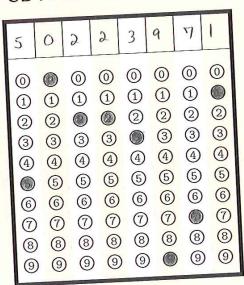


MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:			
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UB Person Number:



Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1 2 3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.
- a) $[v, v, v_3 | \omega]$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & f3 & 0 & | & 6 \\ 1 & -1 & 1 & | & -2 & | \\ 0 & 1 & 2 & | & 2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 - 3 & 0 & | & 5 \\ 1 & 0 & 3 & | & 0 \\ 0 & 1 & 2 & | & 2 \end{bmatrix} \xrightarrow{(-a)} \rightarrow \begin{bmatrix} 0 - 3 - 6 & | & 5 \\ 4 & 0 & 3 & | & 0 \\ 0 & 1 & 2 & | & 2 \end{bmatrix} \xrightarrow{(-3)}$$

$$Col(A) = span \left\{ \begin{bmatrix} c \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right\} \begin{bmatrix} b = -6 \end{bmatrix}$$

X3 is a free variable so there are infinite soldiens, this means that they are linearly dependent.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}, \begin{bmatrix} a & b & C \\ \partial & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a - O + 2g & b - e + 2h & C - f + 2i \\ a + g & b + h & C + i \\ 20 - g & 2e - h & 2f - i \end{bmatrix} \begin{bmatrix} 1 & O \\ 0 & 1 \\ 0 & O \end{bmatrix}$$

$$f = \frac{1}{a} + \frac{i}{a}$$

$$-(-\frac{1}{\lambda}-\frac{i}{\lambda}+\lambda i)=0$$

$$\frac{3}{2}g = 1$$

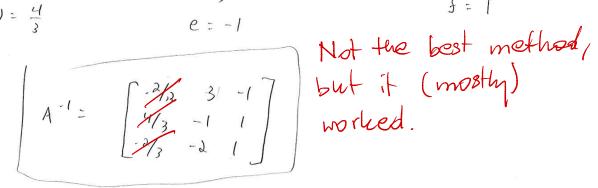
$$c - \frac{2}{c}$$

$$\frac{1}{\lambda} 4 = -1$$

$$\frac{1}{\lambda}i = \frac{1}{\lambda}$$

$$9 = \frac{2}{3}$$

$$a = -\frac{\lambda}{3}$$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Simpler: C=(AT).B

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{T:} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & \lambda \\ 1 & 1 & -1 \end{bmatrix}$$

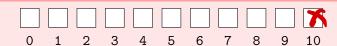
A=[10] AT:[10] and A was computed in problem 2.

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & \lambda \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{20} & C_{33} \\ C_{31} & C_{12} & C_{33} \end{bmatrix} - \begin{bmatrix} 1 & \lambda & 3 \\ 4 & 5 & 4 \\ 3 & \lambda & 1 \end{bmatrix}$$

$$(3) = \frac{5}{\lambda} + \frac{1}{\lambda} C_{31}$$

$$\frac{1}{2}(2) - \frac{1}{2} = 2$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$





4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)
$$\begin{bmatrix} c & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - \partial x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$ax_1 + (x_2 = x_1 - \lambda x_2$$

$$C \times_{1} + \partial x_{2} = X_{1} + X_{2}$$

 $e \times_{1} + f \times_{2} = X_{1} - 3 \times_{2}$

$$A = \begin{bmatrix} 1 & -\lambda \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

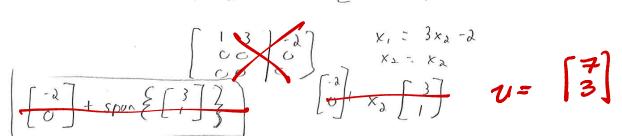
$$\begin{bmatrix} 1-2 \\ 1-3 \end{bmatrix} \cdot 0 = \begin{bmatrix} 10 \\ -2 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1-2 \\ 1-3 \end{bmatrix} \cdot 0 = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1-2 \\ 1-3 \end{bmatrix} \cdot 0 = \begin{bmatrix} 10 \\ 1-3 \end{bmatrix} \cdot 0$$

$$\begin{bmatrix} 1-3 \\ 1-3 \end{bmatrix} \cdot 0 = \begin{bmatrix} 10 \\ 1-3$$

$$-9 \qquad \left[\begin{array}{c|c} 0 & | & 3 \\ 1 & | & 0 \\ 1 & 3 \end{array}\right] \xrightarrow{7} \left[\begin{array}{c|c} 1 & -3 & | & -\lambda \\ 0 & | & 3 \end{array}\right] \xrightarrow{7} \left[\begin{array}{c|c} 0 & | & 3 \\ 0 & | & 3 \end{array}\right]$$





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to-one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 2 & 4 & 0 \\
 3 & 4 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 63 \\
 63
 \end{bmatrix}$$

$$\begin{bmatrix} 10 - 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 2 \times 3$$

$$\times_3 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$Nul(A) = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{cases} -2 \\ 0 \end{bmatrix}$$

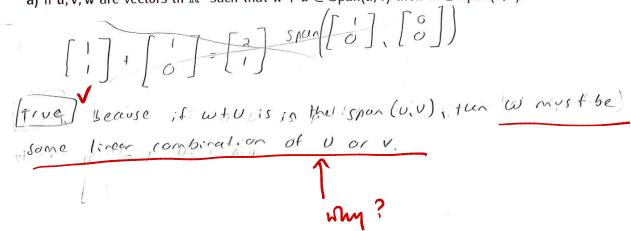
$$V_2 = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$= \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\} \left\{ \begin{array}{c} 0 \\ -2 \end{array} \right\}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u,v,w are vectors in \mathbb{R}^3 such that $w+u\in Span(u,v)$ then $w\in Span(u,v)$.



b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

True, if the whole sot is linearly in dependent, then
its parts must hold the relation.



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

True, thors formations are a linear operator so their original relations half frue to the outlones of a tensloretien.

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