Preface

I first learned Serre's definition of intersection multiplicity from Mel Hochster, back when I was an undergraduate. I was immediately intrigued by this surprising connection between homological algebra and geometry. As it has always been for me when learning mathematics, I wanted to know how I could have guessed this definition for myself—what are the underlying principles that *tell us* to go looking in homological algebra for a definition of multiplicity. This question has been in the back of my mind for most of my mathematical life. It took me a long time to accept that the answers to such questions are not often readily available; one has to instead make do with vague hints and partial explanations. I still believe, though, that the answers exist *somewhere*—and that it is the ultimate job of mathematicians to uncover them. So perhaps it is better said this way: those questions often don't have simple answers *yet*.

During my first year of graduate school I tried to puzzle out for myself the secrets behind Serre's definition. Thanks to the Gillet-Soulé paper [GS] I was led to K-theory, and similar hints of topology seemed to be operating in work of Roberts [R]. Coincidentally, MIT had a very active community of graduate students in topology, and I soon joined their ranks. Although there were other factors, it is not far from the truth to say that I became a topologist in order to understand Serre's definition.

In Winter quarter of 2012 I taught a course on this material at the University of Oregon. The graduate students taking the course converted my lectures into LaTeX, and then afterwards I both heavily revised and added to the resulting document. The present notes are the end result of this process. I am very grateful to the attending graduate students for the work they put into typesetting the lectures. These students were: Jeremiah Bartz, Christin Bibby, Safia Chettih, Emilio Gardella, Christopher Hardy, Liz Henning, Justin Hilburn, Zhanwen Huang, Tyler Kloefkorn, Joseph Loubert, Sylvia Naples, Min Ro, Patrick Schultz, Michael Sun, and Deb Vicinsky.