MTH-309T-F19-EX1-051-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

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Name:

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UB Person Number:

5	0	2	6	0	8	1	3

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE
20	5	5	20	19	4	7	2	10	90	A-

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 \to -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$R_1 \to R_1 + R_2$
$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

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$b = -6$ If b equals anything other than -6, there will be a pivot position in the last column leading to no solutions

b) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_3 \Rightarrow -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right]$$

$R_3 \Rightarrow R_2 + R_3$
$R_1 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The set $\{v_1, v_2, v_3\}$ is not linearly dependent because $x_1v_1 + x_2v_2 + x_3v_3 = 0$ does not have only the trivial solution. x_3 is a free variable giving infinite solutions making the set linearly dependent

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] \quad R2 \leftrightarrow R3$$

$$\downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R3 \to -R1+R3 \quad \to \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & \cancel{0}\ {-1} & 1 & 0 \end{array}\right]$$

$$R2 = \frac{R2}{2} \quad \downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \cancel{0}\ {-1} & 0 & -\frac{1}{2} \end{array}\right] \xleftarrow[R3 \to -R2+R3]{R1 \to R2+R1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -1 & \cancel{0}\ {-1} & 1 & \cancel{0} \end{array}\right]$$

$$R3 \to \frac{-2}{1} \cdot R3 \quad \downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R1 \to -\frac{3}{2}R3+R1]{R2 \to \frac{1}{2}R3+R2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^{T-1} \cdot A^T C = A^{T-1} \cdot B$$

$$C = A^{T-1} \cdot B \quad ✓$$

$$A^{T-1} = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R2 \to -2R1+R2$
$R3 \to R1+R3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R2 \to -R2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \end{array}\right]$$

$R1 \to -R2+R1$
$R3 \to -R2+R3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$R2 \to -R3+R2$
$R1 \to R3+R1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \cancel{1} & \cancel{0} & \cancel{-1} \\ 0 & 1 & 0 & \cancel{0} & \cancel{0} & \cancel{2} \\ 0 & 0 & 1 & \cancel{0} & \cancel{1} & \cancel{-1} \end{array}\right]$$

$$A^{T-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$A^{T-1} \cdot B =$

$$\cancel{\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+0-3 \\ 0+0+6 \\ 0+4-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+0-2 \\ 0+0+4 \\ 0+5-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+0-1 \\ 0+0+2 \\ 0+4-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$$A^{T-1} B = \begin{bmatrix} -2 & 0 & 2 \\ 6 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix} = C$$

Simpler: $(A^T)^{-1} = (A^{-1})^T$

Then use problem 2.

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0	1	2	3	4	5	6	7	8	9	10

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) Standard Matrix of T:

$$A = [T(e_1)\ T(e_2)] = \left[T\left(\begin{pmatrix}1\\0\end{pmatrix}\right) T\left(\begin{pmatrix}0\\1\end{pmatrix}\right)\right] = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \checkmark$$

b)
$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \xrightarrow[R3 \to -R1+R3]{R2 \to -R1+R2} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right] R2 \to \frac{R2}{3}$$

$$\downarrow$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right] \begin{array}{l} R3 \to R2+R3 \\ R1 \to 2R2+R1 \end{array}$$

$$\downarrow$$

$$u = \begin{bmatrix} \cancel{5}\ 7 \\ 3 \\ 0 \end{bmatrix} \qquad \left[\begin{array}{cc|c} 1 & 0 & \cancel{5}\ 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

there is a unique solution

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$ $R3 \to -3R1 + R3$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$ $R2 \to \frac{R2}{2}$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$ $R1 \to -R2 + R1$, $R3 \to -R2 + R3$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$ $R3 \to \frac{R3}{2}$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $R2 \to -2R3 + R2$, $R1 \to 2R3 + R1$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $\leftarrow$ A IS one to one w/ a pivot pos, in every column

✓

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$ $R3 \to -3R1 + R3$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$ $R1 = -\frac{1}{2}R2 + R1$, $R3 = -\frac{1}{2}R2 + R3$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 2x_3$

$2x_2 = -4x_3 \Rightarrow x_2 = -2x_3$

$x_3 = x_3$

$\mathrm{Nul}(A) = \mathrm{Span}\left\{ \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}} \right\}$

T(A) is not one to one because there is not a pivot position in every column

$V_1 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ $V_2 = \begin{bmatrix} 2 \\ \cancel{5}\ 3 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right) = 0$

$T\left(\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}\right) = 0$

$TA(V_1) = TA(V_2)$

$TA\left(\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}\right) = TA\left(\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}\right)$

$\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$ is in Nul(A)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

$w + u \in \text{Span}(u,v) \quad w \in \text{Span}(u,v)$

$w = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$w + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$w = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$w + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$w = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$

$\left[\begin{array}{cc|c} 1 & 4 & 2 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{array}\right] \xrightarrow[R_2 \to -2R_1 + R_2]{R_3 \to -3R_1 + R_3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & -6 & -3 \end{array}\right] \xrightarrow{R_3 = -2R_2 + R_3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2 / -3} \left[\begin{array}{cc|c} 1 & 4 & 2 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{array}\right]$

$\boxed{\text{True}}$ ✓ because $w + u$ linear combination of u (w) to be in span therefore it will be in span(u,v)

?

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b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix}$

$\boxed{\text{True}}$ ✓ because if set $\{u, v, w\}$ is all linearly independent from each other, then u and v has to be linearly independent

why?

0	1	2	3	4	5	6	7	8	9	10
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$A\begin{bmatrix}-3\\1\end{bmatrix} = \cancel{\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}-4+2\\-6+4\end{bmatrix} = \begin{bmatrix}-2\\-2\end{bmatrix}}$

$A\cdot\begin{bmatrix}1\\0\end{bmatrix} = \cancel{\begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1+0\\2+0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}}$ } dependent

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}$

$A = \left[\begin{array}{cc|c}1&2&0\\2&4&0\end{array}\right]$ $R_2 \to -2R_1+R_2$

$\begin{bmatrix}1&2&0\\0&0&0\end{bmatrix}$

$\cancel{v = \begin{bmatrix}1\\1\end{bmatrix} \quad u = \begin{bmatrix}2\\2\end{bmatrix}}$

$\cancel{A\cdot\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}-2+2\\-4+4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}}$

$\cancel{A\cdot\begin{bmatrix}-4\\2\end{bmatrix} = \begin{bmatrix}1&2\\2&4\end{bmatrix}\cdot\begin{bmatrix}-4\\2\end{bmatrix} = \begin{bmatrix}-4+4\\-8+8\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}}$

$x_1 = -2x_2$

$x_2 = x_2$

$\mathrm{Nul}(A) = \mathrm{Span}\left\{\begin{bmatrix}-2\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\right\}$

False ✓ $v = \begin{bmatrix}1\\0\end{bmatrix}$ $u = \begin{bmatrix}0\\1\end{bmatrix}$

$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

$A\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1&1\\1&1\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

2 examples $\begin{bmatrix}-3\\2\end{bmatrix}$, ✓

$A(v)$, $A(u)$ are linearly dependent, but v, u is linearly independent

b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v})$, $T(\mathbf{w})$).

$u = \begin{bmatrix}1\\1\end{bmatrix}$ $v = \begin{bmatrix}1\\0\end{bmatrix}$ $w = \begin{bmatrix}0\\1\end{bmatrix}$ $u \in \mathrm{Span}(v, w)$

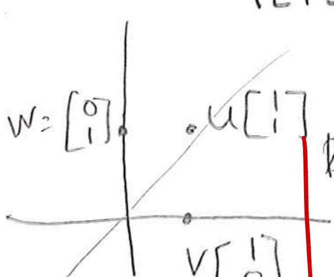
$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}-5\\-5\end{bmatrix}$

$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \to \begin{bmatrix}-5\\0\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) \to \begin{bmatrix}0\\-5\end{bmatrix}$

$\boxed{\text{True}}$ because with any reflection, T(u) will be a linear combination of T(v) and T(w) if u is in Span(v, w)

?



Reflect over x-axis	reflect over line y=x	reflect over y-axis
$T(u) = \begin{bmatrix}1\\-1\end{bmatrix}$	$T(u) = \begin{bmatrix}1\\1\end{bmatrix}$	$T(u) = \begin{bmatrix}-1\\1\end{bmatrix}$
$T(w) = \begin{bmatrix}0\\-1\end{bmatrix}$	$T(w) = \begin{bmatrix}1\\0\end{bmatrix}$	$T(w) = \begin{bmatrix}0\\1\end{bmatrix}$
$T(v) = \begin{bmatrix}1\\0\end{bmatrix}$	$T(v) = \begin{bmatrix}0\\1\end{bmatrix}$	$T(v) = \begin{bmatrix}-1\\0\end{bmatrix}$

An example does not show that this is always true...

0 1 2 3 4 5 6 7 8 9 10