1. Consider the following vectors in $\mathbb{R}^4$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V.

a) Gram-Schmidt Process

$w_1: \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ $v_2: \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ $w_1 \cdot v_2 = 2+0+1+0 = 3$

$w_1 \cdot w_1 = 1+0+1+1 = 3$

$w_1 = v_1$

$w_2 = v_2 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \left(\frac{3}{3}\right) \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ w_2

$v_3: \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}$ $w_2 \cdot v_3 = 2+(-2)+0+(-3) = -3$

$w_2 \cdot w_2 = 1+0+1+1 = 3$

$w_1 \cdot v_3 = 2+1+3 = 6$

$w_3 = v_3 - \left(\frac{w_1 \cdot v_3}{w_1 \cdot w_1}\right) w_1 - \left(\frac{w_2 \cdot v_3}{w_2 \cdot w_2}\right) w_2 =$

$2-1+1 \quad 2$
$-2-0+1 \quad -1$
$-1--1+0 \quad 0$
$3-1+-1 \quad 1$

$\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \left(\frac{6}{3}\right) \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} - \left(\frac{-3}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} =$

$\begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix} \cancel{-} - \left(\frac{3}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ w_3

w_1, w_2, w_3

$\mathcal{D} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ ✓

$w_1 \cdot w_2 = 0$
$w_2 \cdot w_3 = 0$ ✓
$w_1 \cdot w_3 = 0$

b) $\text{proj}_V u$

$w_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ $w_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ $w_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ $u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$

$w_1 \cdot w_1 = 3$ $\quad u \cdot w_1 = 3$
$w_2 \cdot w_2 = 3$ $\quad u \cdot w_2 = 3$
$w_3 \cdot w_3 = 3$ $\quad u \cdot w_3 = 3$

$\text{proj}_V u = \left(\frac{u \cdot w_1}{w_1 \cdot w_1}\right) w_1 + \left(\frac{u \cdot w_2}{w_2 \cdot w_2}\right) w_2 + \left(\frac{u \cdot w_3}{w_3 \cdot w_3}\right) w_3$

$\frac{3}{3} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \left(\frac{3}{3}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \left(\frac{3}{3}\right) \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\text{proj}_V u = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20