



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\text{SPAN}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \rightarrow 2R_1 - R_3 = R_3 \quad \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 + R_2 = R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2 - R_3 = R_3}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

A AFTER
ROW REDUCTION

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

$$\begin{aligned} \mathbf{v}_1 + 3\mathbf{v}_3 &= -2 \quad | \cdot (-2) & -2\mathbf{v}_1 - 6\mathbf{v}_3 &= 4 \\ \mathbf{v}_2 + 2\mathbf{v}_3 &= 2 \quad | \cdot (3) \Rightarrow & 3\mathbf{v}_2 + 6\mathbf{v}_3 &= 6 \Rightarrow -2\mathbf{v}_1 + 3\mathbf{v}_2 &= 10 \\ && & & | \downarrow \\ && & & 2\mathbf{v}_1 - 3\mathbf{v}_2 &= -10 \end{aligned}$$

$$\begin{aligned} 2\mathbf{v}_1 &= 3\mathbf{v}_2 - 10 \\ \mathbf{v}_1 &= \frac{3}{2}\mathbf{v}_2 - 5 \end{aligned}$$

CONTINUED HERE

$$\mathbf{v}_1 = \frac{3}{2}\mathbf{v}_2 - 5$$

FOR $\mathbf{v}_1 + 3\mathbf{v}_3 = -2$

$$\frac{3}{2}\mathbf{v}_2 - 5 + 3\mathbf{v}_3 = -2$$

$$\frac{3}{2}\mathbf{v}_2 + 3\mathbf{v}_3 = 3$$

$$\begin{aligned} \frac{1}{2}\mathbf{v}_2 + \mathbf{v}_3 &= 1 \\ \mathbf{v}_2 &= 2 - 2\mathbf{v}_3 \end{aligned}$$

$$\mathbf{v}_1 = \frac{3}{2} \cdot 2(1 - \mathbf{v}_3) - 5$$

$$\mathbf{v}_1 = 3 - 3\mathbf{v}_3 - 5$$

$$(\mathbf{v}_1 = -3\mathbf{v}_3 - 2)$$

$$\mathbf{v}_1 \rightarrow -3\mathbf{v}_3 - 2$$

$$\mathbf{v}_2 \rightarrow 2 - 2\mathbf{v}_3$$

$$\mathbf{v}_3 \rightarrow \mathbf{v}_3$$

• \mathbf{v}_3 HAS INFINITE MANY
OF VALUES

• SET OF $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ IS
LINEARLY DEPENDENT SINCE
 \mathbf{v}_3 IS A FREE VARIABLE.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & 0 \end{array} \right] \xrightarrow{(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & b-4 \end{array} \right] \xrightarrow{(5)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -10 & b-14 \end{array} \right] \xrightarrow{} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-14}{-10} \end{array} \right]$$

$$b = 4$$

$$Ax = b \rightarrow Av = b$$

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3 = \mathbf{w}$$

b) a set is linearly independent if it has only one solution
 Since there is a pivot position in every column of A
 Linearly independent



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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \quad \left| \begin{array}{l} R_3 = -2(R_1) + R_3 \\ R_3 + R_2 + R_3 \\ R_1 = R_2 + R_1 \end{array} \right. \quad \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} v_1 & v_2 & v_3 & \\ \hline 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \quad \left| \begin{array}{l} R_2 = R_1 - R_2 \end{array} \right. \quad \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

↓ write ~~in~~ in order for it be

b) The solution
is linear
independent.
The last row
is 0 and
because of
that there are
multiple solutions.
Also, it can't
be dependent
because every
column must
be a pivot.



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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{w} \end{array} \right] = \vec{W}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right) \xrightarrow{-2R_1+R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right) \xrightarrow{R_2+R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-6 \end{array} \right)$$

$$b-6=0$$

if $b=6$, then $\vec{w} \in \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

$$x_1 - x_2 + x_3 = -2$$

x_2

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right) \xrightarrow{\quad} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

x_3 is free

to be linearly independent, every column after row reduction must be a pivot column. (Therefore,

the set is not linearly independent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \quad (-2) \leftarrow \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 2 & -3 & 0 & | & 0 \end{bmatrix} \quad \begin{array}{l} 2-3 \\ =2+0 \end{array}$$

$$v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \quad (1) \leftarrow \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & -1 & -2 & | & 0 \end{bmatrix} \quad 2-2$$

$$\begin{bmatrix} 1 & 0 & 3 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad (1) \leftarrow \begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \quad 2 - (-1)$$

$$x_1 = x_2 - x_3 \rightarrow -2$$

$$x_2 = -2x_3 \rightarrow (2)$$

$$x_3 = \text{free} \rightarrow \begin{array}{l} b \\ (-1) \end{array}$$

(a) $b = -1$

(b) No, dependent because it has infinite solutions
 x_3 is a free variable



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a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{r_2 + r_1} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(-2) \cdot r_1} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right]$$

$$\xrightarrow{(-\frac{1}{3}) \cdot r_3} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b \end{array} \right] \xrightarrow{(-1) \cdot r_2} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array} \right] \quad \begin{array}{l} 0 = b-2 \\ b = 2 \end{array}$$

b)

$$\left[\begin{array}{ccc} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array} \right] \xrightarrow{\text{row reduce}} \left[\begin{array}{ccc} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly independent because every column of $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$ is not a pivot column

Hehe



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a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$b=6 \quad b=-6$$

a) $\left\{ \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right\} \xrightarrow{x_1v_1 + x_2v_2 + x_3v_3 = w}$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

let $x_1=0$ let $x_1=-3$
 $x_2=2$ $x_2=0$
 $x_3=0$ $x_3=1$

$$\left\{ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right\} \xrightarrow{-2} \left\{ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right\} \xrightarrow{3} \left\{ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right\}$$

$x_1 = -3x_3$ $x_3 = \text{free}$
 $x_2 = 2-2x_3$ infinite soln
 $b = -6$

$b = -6, 6$

b) $x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

since $x_3 = \text{free}$, there are infinitely many solutions

\Rightarrow set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ is linearly independent

?



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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$$\boxed{b=0}$$

RREF
&
See if
each
column
has
leading one?

$$0 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + -2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$$

$$\boxed{b=-6}$$

$$y(2) + x(-3) + 0$$

(b)

$$\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{w} \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \\ -2 & 2 & -2 & 0 \end{array} \right] \xrightarrow{-2R_1 + R_3 \rightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right] \xrightarrow{R_2 + R_3 \rightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Vectors $\mathbf{v}_1, \mathbf{v}_2$ & \mathbf{v}_3 are linearly dependant
(NOT Linearly Independent) because, \mathbf{v}_2 & \mathbf{v}_3 are free variables.

$$\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 = 0$$

$$\mathbf{v}_2 + 2\mathbf{v}_3 = 0$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

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a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\begin{aligned} a) \quad & \mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 = -2 \\ & \mathbf{v}_1 + 2\mathbf{v}_2 = 2 \\ & 2\mathbf{v}_1 - 3\mathbf{v}_2 = b \end{aligned}$$

$$\begin{aligned} & \mathbf{v}_1 = 2 - 2\mathbf{v}_2 \\ & 2 - 2\mathbf{v}_2 - \mathbf{v}_2 + \mathbf{v}_3 = -2 \\ & 3\mathbf{v}_2 + \mathbf{v}_3 = -4 \\ & \mathbf{v}_3 = -4 - 3\mathbf{v}_2 \end{aligned}$$

$$\mathbf{v}_1 = 2 - 2\mathbf{v}_2$$

$$2 - 2\mathbf{v}_2 + \mathbf{v}_3 = -2$$

$$-3\mathbf{v}_2 + \mathbf{v}_3 = -4$$

$$\mathbf{v}_3 = -4 - 3\mathbf{v}_2$$

$$\boxed{b=4}$$

$$\begin{aligned} & \Rightarrow 2 - 2\mathbf{v}_2 - \mathbf{v}_2 - 4 - 3\mathbf{v}_2 = -2 \\ & \Rightarrow \mathbf{v}_2 = 0 \\ & \mathbf{v}_1 + 2(0) = 2 \\ & \mathbf{v}_1 = 2 \end{aligned}$$

$$2\mathbf{v}_1 - 3\mathbf{v}_2 = b$$

$$2(2) - 3(0) = b$$

$$4 = b$$

$$\boxed{b=4}$$

$$b) \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 4 \end{array} \right] \xrightarrow{(-2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 0 & 8 \end{array} \right] \xrightarrow{4} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 2 & 2 \end{array} \right] \xrightarrow{(1) \cdot 3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{(2) \cdot (-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & 0 & -8 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{(1) + (2)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -10 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{(2) \cdot (-1/8)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -10 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{(1) \cdot (-1)} \left[\begin{array}{ccc|c} -1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{(1) \cdot (-1)} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{(1) \cdot 1/3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2/3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{(-1)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{array} \right]$

The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent because the set contains only 1 solution, thus being a homogeneous equation which proves that the set is linearly independent.



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a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\mathbf{w} \notin \text{cv}_1$, since 2nd row of $c\mathbf{v}_1$ is always 0.
 $\mathbf{w} \notin \text{cv}_3$, since 2nd row is same for both, but
 first row is different.

$$\mathbf{w} \in \text{cv}_2 \text{ when } c=2 \text{ and } b=-6$$

$$\begin{array}{c} \downarrow \\ \mathbf{v}_1 + 2\mathbf{v}_2 = \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = 2\mathbf{v}_2 = 2 \cdot \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}} \end{array}$$

b) Yes, since there is only one solution.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & \mathbf{v}_1 \\ 0 & 1 & 2 & \mathbf{v}_2 \\ 0 & 3 & 0 & \mathbf{v}_3 \end{array} \right] \xrightarrow{\text{Row } 3 - 3\text{Row } 2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & \mathbf{v}_1 \\ 0 & 1 & 2 & \mathbf{v}_2 \\ 0 & 0 & 1 & \mathbf{v}_3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & \mathbf{v}_1 \\ 0 & 1 & 2 & \mathbf{v}_2 \\ 0 & 5 & -2 & \mathbf{v}_3 \end{array} \right] \xrightarrow{\text{Row } 3 - 5\text{Row } 2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & \mathbf{v}_1 \\ 0 & 1 & 2 & \mathbf{v}_2 \\ 0 & 0 & -12 & \mathbf{v}_3 \end{array} \right]$$

$$\begin{cases} x_1 = c_1 \\ x_2 = c_2 \\ x_3 = c_3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & \mathbf{v}_1 \\ 0 & 1 & 2 & \mathbf{v}_2 \\ 0 & 0 & -12 & \mathbf{v}_3 \end{array} \right]$$



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$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & -4 \end{array} \right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 4 \end{array} \right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -4 \end{array} \right]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & -4 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -4 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array} \right] \xrightarrow{\text{pivot exchange}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array} \right]$ is linearly independent because each part of the matrix has a pivot point



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 2 & b+4 \end{array} \right]$$

$$C_1\mathbf{v}_1 + C_2\mathbf{v}_2 + C_3\mathbf{v}_3 = \mathbf{w} \Rightarrow \begin{cases} C_1=0 \\ C_2=2 \\ C_3=0 \end{cases} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\therefore \boxed{b = -6}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + (-2R_1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

not linearly independent because every column of the matrix is not a pivot column.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\text{a)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{R_3 = R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

$$R_4 = R_1 + R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \quad \underline{\underline{b = -6}} \rightarrow \text{For all other values of } b \text{ there will not be a solution.}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{R_3 = R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$R_1 = R_1 + R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

\downarrow free variable

~~Since~~ Since, x_3 is a free variable, the set $\{v_1, v_2, v_3\}$

is NOT linearly independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) as $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \therefore \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{w}$

$$-4 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\therefore b = (-8)$$

b) $c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\textcircled{1} \quad c_1 - c_2 + c_3 = 0 \quad \therefore c_1 = c_2 - c_3$$

$$\textcircled{2} \quad c_2 + 2c_3 = 0 \quad \therefore c_3 = -\frac{c_2}{2} \quad \therefore c_3 = 0 \quad (\because c_2 = 0)$$

$$\textcircled{3} \quad 2c_1 - 3c_2 = 0 \quad \therefore c_1 = \frac{3c_2}{2} \quad \therefore c_1 = 0 \quad (\because c_2 = 0)$$

$$\therefore c_2 - c_3 = \frac{3c_2}{2} \quad \therefore 2c_2 + 2\left(-\frac{c_2}{2}\right) = 3c_2 \quad \therefore 3c_2 = 3c_2 \quad \therefore c_2 = 0$$

$$\therefore 2c_2 - 2c_3 = 3c_2 \quad \therefore 2c_2 + 2\left(-\frac{c_2}{2}\right) = 3c_2 \quad \therefore c_2 = 0$$

as, $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ so, we can say the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

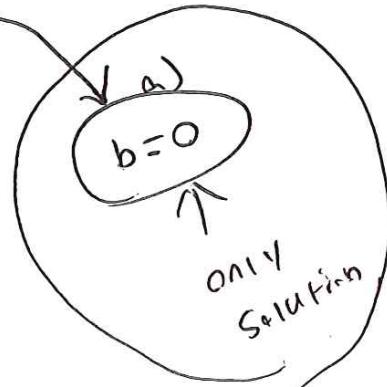


1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.



$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -4 & 0 \end{bmatrix}$$

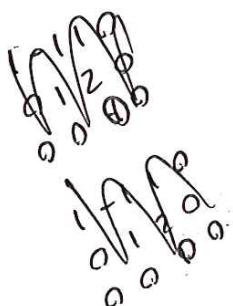
$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & -2 & b-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{bmatrix}$$

for $\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$ in span



$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

reduced matrix

b) No
linearly dependent

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a). $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$

When $b = -6$, it has infinite solutions.

$$x_1 + 3x_3 = 0$$

$$x_2 + 2x_3 = 2$$

b) $\left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & -5 & -2 \end{array} \right] \xrightarrow{R_3 + 5R_1} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -2 \end{array} \right] \xrightarrow{R_3 + 5R_2} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & -5 & -2 \end{array} \right] \xrightarrow{R_3 \downarrow} \left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \xleftarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \xleftarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$

\uparrow
 \uparrow
 \uparrow

$\left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$ is linearly independent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\text{a). } \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\cdot 2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{\cdot (-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \xrightarrow{\text{free}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

when $b+6=0$
 $b=-6$, so $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

matrix it has infinite solution, with free variable x_3 .

b) no, for set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ after row reduced,

we got $\left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$ the third column is not a pivot column.

which means it is not a linearly independent set.

$$\cancel{x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = 0} \quad x_3.$$

$$\text{For } \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \quad \cancel{x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = 0}.$$

because it has free variable $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

have infinite solutions

so not linear indep



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(a) $\chi_1 \mathbf{v}_1 + \chi_2 \mathbf{v}_2 + \chi_3 \mathbf{v}_3 = \mathbf{w}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow[-2]{\downarrow+} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow[(1)]{\downarrow+} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

$\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ as long as

$b = -6$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow[-2]{\downarrow+} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow[(1)]{\downarrow+} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Since the homogeneous equation

$\chi_1 \mathbf{v}_1 + \chi_2 \mathbf{v}_2 + \chi_3 \mathbf{v}_3 = 0$ has infinitely

many solutions due to a free

variable, the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

is NOT linearly independent.

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

↑
free

Hello!



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

a) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{R}_2 \cdot (-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{\text{R}_1 + \text{R}_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{\text{R}_3 + \text{R}_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \implies b = 6$$

b) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent because

x_3 is a free variable meaning there are infinitely many solutions.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 + (-2)R_1} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{array} \right] \xrightarrow{R_1 + (1)R_3} \left[\begin{array}{cccc} 0 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4b+2 \end{array} \right]$$

$$0 = 4b+2$$

$$4b = -2$$

$$b = -\frac{2}{4}$$

$$b = -\frac{1}{2}$$

b)

No,

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 + (-2)R_1} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4b+2 \end{array} \right]$$

$$b = -\frac{1}{2}$$

$$a_1\mathbf{v}_1 + \dots + a_n\mathbf{v}_n \neq 0$$

$$x_1 - x_2 = 1$$

$$x_2 = 2$$

$$x_1 = 3$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

$$\xrightarrow{\frac{1}{2} \times R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 2 & -3 & 0 & b \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\xrightarrow{-1 \times R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 1 & -\frac{3}{2} & 0 & \frac{b}{2} \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -\frac{1}{2} & 0 & \frac{b}{2} \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\xrightarrow{x-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -\frac{1}{2} & -1 & \frac{b}{2} + 2 \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\xrightarrow{r1 - r2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & -2(\frac{b}{2} + 2) = -b - 4 \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 0 & -b - 4 \\ 0 & 0 & 0 & b+6 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 1 & 0 & -b - 4 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

?

(b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 2 & -3 & 0 & b \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 1 & -\frac{3}{2} & 0 & b \\ 0 & 1 & 2 & 2 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -\frac{1}{2} & -1 & b \\ 0 & 1 & 2 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & b \\ 0 & 1 & 2 & 2 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} y_1 = -y_2 + y_3 \\ y_2 = y_3 \\ y_3 = y_3 \end{array}$$

No the set is not linear Independent

Some scalar multiple of \mathbf{v}_1 add to scalar multiple of \mathbf{v}_2 will produce \mathbf{v}_3 .

Since $b+6$ is in last column
 $b+6$ has to be equal to 0
 which $b = -b$ to make zero
 otherwise $b+6$ will be 1. which
 is undefined. So $b = -b$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$. $b = \text{any number but } 4$

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array} \right| \xrightarrow{x_2} \left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & b-4 \end{array} \right| \xrightarrow{x_5} \left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -10 & 10(b-4) \end{array} \right| \xrightarrow{x_1} \left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -(b-4) \end{array} \right| \xrightarrow{-b+4}$$

The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent because each vector is not a scalar multiple of each other.

$-b+4 = 0$
 $-b = -4$
 $b = 4$
 $\therefore \text{If } b=4, \text{ then } \mathbf{w} \notin \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ -2 & -3 & 0 & b \end{array} \right] \xrightarrow{(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & 2 & -4+b \end{array} \right] \xrightarrow{(5)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 12 & -4+b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & -2 \\ 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 12 & -4+b \end{array} \right] \xleftarrow{\text{divide by 12}} \left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & -2 \\ 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{-4+b}{12} \end{array} \right]$$

$\text{Span}(b) = \mathbb{R}^3$ since there be a solution to any real number

b) $\begin{cases} x_1 - x_2 + x_3 = -2 \\ x_2 + 2x_3 = 2 \\ x_3 = \frac{6+b}{12} \end{cases}$

$$\left\{ \begin{array}{l} x_1 = -2 - x_2 + x_3 \\ x_2 = 2 - 2x_3 \\ x_3 = \frac{6+b}{12} \end{array} \right.$$

ob b

Since the equation has a solution any value be given
not linearly independent set
it is a linearly independent set

$$\begin{aligned} -3(6-6) \\ -18+36 \\ \hline 8 \\ 16-12=4 \end{aligned}$$

$$2 - 2\left(\frac{6-6}{8}\right)$$

$$2 - \frac{12+6}{8}$$

$$2 - 2\left(\frac{6-6}{8}\right)$$

$$2 - \frac{12+2b}{8}$$



$$\begin{aligned} \cdot \frac{-1}{8}(6-6) = \frac{-6+6}{8} \\ \frac{6+b}{8} \end{aligned}$$

1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & 6 \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & -2 & b+4 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-6}{8} \end{array} \right] \xrightarrow{R_1 - 3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-18+3b}{8} \\ 0 & 1 & 0 & 2 - \frac{12+2b}{8} \\ 0 & 0 & 1 & \frac{b-6}{8} \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-6}{8} \end{array} \right]$$

$$x_1 = \frac{-18+3b}{8}$$

$$x_2 = \frac{4-2b}{8}$$

$$x_3 = \frac{b-6}{8}$$

Since b is in the numerator of all solutions \therefore if $b \in \mathbb{R}$ then $w \in \text{Span}(v_1, v_2, v_3)$

b) The set $\{v_1, v_2, v_3\}$ is linearly independent because there is a pivot column in every column as shown in part A where we reduced the matrix in row echelon form.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$. $A\vec{x} = \vec{b}$

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $[\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ | \ \mathbf{w}]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\cdot(-2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b \end{array} \right] \xrightarrow{\cdot(1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 0 & b \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{array} \right] \xrightarrow{\cdot(-2)} \left[\begin{array}{ccc|c} 0 & 3 & -6 & b \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{array} \right] \xrightarrow{\cdot(-3)}$$

$$\text{col}(A) = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\} \quad b = -6$$

b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{\cdot(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{\cdot(1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \cdot(6)$

$\therefore \text{the } \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ is } \boxed{\text{Not linearly independent}}$

x_3 is a free variable so there are infinite solutions, this means that they are linearly dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array} \right)$$

b) The set is linearly independent

$$\left(\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & -2 & b+4 \end{array} \right)$$

$$\left(\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -12 & b-6 \end{array} \right)$$

$$\left(\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-6}{-12} \end{array} \right)$$

$$C_3 = \frac{b-6}{-12}$$

$$C_2 = 2 - 2C_3$$

$$C_1 = -2 + C_2 - C_3$$

$$0 = \frac{(b-6)}{-12}$$

$$0 = b-6$$

$$b = b$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -3 & b \end{array} \right] \xrightarrow{R_3 + 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -1 & b+4 \end{array} \right]$

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = 0$$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -3 & 0 \end{array} \right] \xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -1 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$c_1 - c_2 + c_3 = 0 \Rightarrow c_1 = 0$$

$$c_2 + 2c_3 = 0 \Rightarrow c_2 = 0$$

$$c_3 = 0$$

Yes because all
c_n values are equal
to 0.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\text{Span}(v_1, v_2, v_3) = \left\{ \begin{array}{l} \text{the set of all} \\ \text{vectors } c_1v_1 + c_2v_2 + c_3v_3 \end{array} \right\}$

$$R_3 \leftarrow -2R_1 + R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \quad x_1v_1 + x_2v_2 + x_3v_3 = w$$

$$R_2 \leftarrow R_1 + R_2 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 3 & 2 \\ 0 & -1 & -2 & b \end{array} \right]$$

b) the set $\{v_1, v_2, v_3\}$ is not linearly independent because every column of the matrix is not a pivot column

$$R_1 \leftarrow R_2 + R_1 \quad \left[\begin{array}{ccc|c} 0 & -1 & 1 & -2 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & b \end{array} \right]$$

$\therefore b$ must equal 0

$$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 + 2 \\ x_3 = x_3 \end{cases}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

The value of b is -6

because $2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$$A_2 = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 6 \end{bmatrix}$$

It is independent because

$$\text{Null}(A) \cup \{0\}$$

Not linearly independent

$x_1 = 0$ (I think it's nearly independent)
 $x_2 = -2$ because
 $x_3 = 0$ set to 0

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(a) \mathbf{w} is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$

$$\mathbf{w} = x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3$$

$$= x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 - x_2 + x_3 \\ 0 + x_2 + 2x_3 \\ 2x_1 - 3x_2 + 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\Rightarrow \text{augmented matrix} \quad \left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & -2 \\ 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2x_1 - 3x_2 + 0 & & & \end{array} \right] \xrightarrow{\text{Row reduction}} \left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Sol $\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 + 2 \\ x_3 = x_3 \end{cases} \quad \therefore b = 2x_1 - 3x_2 = -6x_3 + 6x_3 - 6 = -6$ $b = -6$

(b) $\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & 0 \end{array} \right]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\Rightarrow x_3$ is a free variable, so $x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3 = 0$ has

infinitely many solutions.

$\Rightarrow \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

$$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) If $\mathbf{w} \in \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ then $\mathbf{w} = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3$

$$\mathbf{w} = c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathbf{w} = -2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} + \begin{bmatrix} b \\ 2b \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} b-4 \\ 2b+2 \\ -10 \end{bmatrix} \quad \begin{array}{l} b-4=0 \Rightarrow b=4 \\ 2b+2=0 \Rightarrow b=-1 \end{array}$$

b)

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \xrightarrow{2R_1-R_3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3+R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-3R_3+R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Yes $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent because every column of the row reduced matrix is a pivot column.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$2v_2 \rightarrow b = -6$$

~~$v_1 + v_2 + v_3 = 2$~~

$$x_1 - x_2 + x_3 = -2$$

$$x_2 + 2x_3 = 2$$

~~$2v_2 + v_3 + v_1 = 2$~~

$$b = 2x_1 - 3x_2$$

$b = -6 \text{ only}$

$$x_1 - (2 - 2x_3) + x_3 = -2$$

$$x_1 + 3x_3 = 0$$

$$b = 2(-3x_3) - 3(2 - 2x_3)$$

$$b = -6x_3 - 6 + 6x_3$$

$b = -6$

~~b) $\{v_1, v_2, v_3\}$ is linearly independent~~

~~because~~

~~$v_1 \neq c_1 v_2$ for any $c_1 \in \mathbb{R}$; etc.~~

~~$v_2 \neq c_2 v_3$ for any $c_2 \in \mathbb{R}$;~~

~~$v_3 \neq c_3 v_1$ for any $c_3 \in \mathbb{R}$.~~

b) $\{v_1, v_2, v_3\}$ is not linearly independent

because $3v_1 = v_3 - 2v_2$.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3 = \mathbf{w}$

Aug mat

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(3)-2(1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{(3)+2(2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$\boxed{b \neq -6}$$

b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b+6 \end{array} \right] \xrightarrow{\text{row red}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$

$x_1 \quad x_2 \quad x_3$

$x_3 = \text{free}$
 $x_1 = -3x_3$
 $x_2 = -2x_3$

Equation $x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3 = \mathbf{0}$ has infinitely many solution

So $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} v_1 & v_2 & v_3 & \\ \hline 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{array} \right]$$

$$R_1 = R_1 + R_2 \quad \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{array} \right] \quad R_3 = R_3 + R_2 \quad \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & (6+b) \end{array} \right]$$

a. $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ when $b = -6$

b. $\left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc} 1 & -1 \\ 0 & 1 \\ 0 & 0 \end{array} \right]$

The set is not linearly independent, because the v_3 column is a free variable, so there is not a leading one in each column



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) ~~$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{pmatrix} \text{ No } X$~~

$$\begin{pmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & b-4 \\ 0 & 1 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 4-b \\ 0 & 1 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 4-b \\ 0 & 0 & 0 & b-2 \end{pmatrix}$$

\therefore the last row is 0.

$$\therefore b-2=0$$

$$\therefore b=2.$$

$\therefore b=2$ such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

b) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly dependant because every column is not a pivot column



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{array}{r} 2 -3 0 b \\ -2 2 -2 4 \\ \hline 0 -1 -2 b+4 \\ \hline 1 2 2 -b+4 \\ \hline -1 -2 b+4 \\ \hline 0 0 b+6 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{-2\text{ row}(1)} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{2\cdot 1} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$\therefore b \neq -6 \text{ for } \mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

b) $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = 0$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

free var. $\therefore \infty$ solutions

$$\begin{array}{r} 1 -1 1 0 \\ 0 1 2 0 \\ \hline 1 0 3 0 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 3 & 0 & 0 \end{array} \right] \xrightarrow{-2} \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{2\cdot 1} \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent because there are infinitely many solutions when solving for the vector.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & 0 \end{array} \right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 0 & 2b \end{array} \right] \xrightarrow{R_3 - 3 \cdot R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -6 & 2b-6 \end{array} \right] \xrightarrow{-\frac{1}{6}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array} \right] \xrightarrow{R_1 + R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array} \right] \xrightarrow{-3 \cdot R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -b+3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -\frac{1}{3}b+1 \end{array} \right] \xrightarrow{R_2 - 2 \cdot R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -b+3 \\ 0 & 1 & 0 & \frac{2}{3}b \\ 0 & 0 & 1 & -\frac{1}{3}b \end{array} \right] \xrightarrow{\begin{array}{l} x_1 = -b+3 \\ x_2 = \frac{2}{3}b \\ x_3 = -\frac{1}{3}b \end{array}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_3 + 3 \cdot R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} x_1 = x_2 - x_3 \\ x_2 = -3k_3 \\ x_3 = k_3 \end{array}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

It is not linearly independent bc there are infinite solutions



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(1)-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{(2)+2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$\therefore \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \xrightarrow{(1)} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$

For $w \in \text{Span}(v_1, v_2, v_3)$, must have a sol'n. $\therefore b+6=0$

$$\boxed{b=-6}$$

- b) $\{v_1, v_2, v_3\}$ is not linearly independent
 because all columns of $\{v_1, v_2, v_3\}$ are not pivot columns.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

| a)
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{row reduce}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2b+4 \end{array} \right]$$

$2b+4$ has to equal 0 or no solution

$$\begin{aligned} 2b+8=0 \\ \rightarrow \\ \frac{2b+8}{2}=0 \\ b=-4 \end{aligned}$$

- | b) No since there is not a pivot column in every column when reduced
 also there is a free variable w/ x_3 so infinite solutions and dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{+ \cdot (1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 1 & -2 & -1 & 2+b \end{array} \right] \xrightarrow{+ \cdot (-1)}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & -2-b \end{array} \right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & -b \\ 0 & -1 & -2 & -2-b \end{array} \right] \xrightarrow{\cdot (-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & -2-b \\ 0 & 0 & 0 & -b \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & b \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array} \right]$$

b) The set $\{v_1, v_2, v_3\}$ is not lin. independent; it has more than one, trivial solution.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a. Span is all linear combinations of $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$

$$\begin{aligned} c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} &= \begin{bmatrix} c_1 - c_2 + c_3 \\ c_2 \\ 2c_1 - 3c_2 \end{bmatrix} \\ c_1 - c_2 + c_3 &= -2 \\ c_2 &= 2 \\ \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \end{bmatrix} &\xrightarrow{+(-1)} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{bmatrix} \\ &\xrightarrow{\text{Row } 1 \rightarrow R_1 - R_2} \begin{bmatrix} -3c_3 - (2 - 2c_3) + c_3 \\ 2 - 2c_3 + 2c_3 \\ 2(-3c_3) - 3(2 - 2c_3) \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} \\ &\begin{array}{l} c_1 = -3c_3 \\ c_2 = 2 - 2c_3 \\ c_3 = c_3 \\ b = -6 \end{array} \end{aligned}$$

$\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ if $b = -6$

$$\begin{aligned} b. \quad x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 &= 0 \\ x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

the equation has one solution therefore the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent

$$\begin{aligned} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] &\xrightarrow{\text{Row } 1 \rightarrow R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 0 & 0 \end{array} \right] \\ &\xrightarrow{\text{Row } 3 \rightarrow R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array} \right] \\ &\xleftarrow{\text{Row } 3 \rightarrow R_3 / 2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \end{aligned}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$-3 \leq b \leq 2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{R}_3 - 2\text{R}_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+2 \end{array} \right] \xrightarrow{\text{R}_3 + \text{R}_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array} \right]$$

$b=0$

$$\text{b)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{\text{R}_3 - 2\text{R}_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{\text{R}_3 + \text{R}_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 &= x_3 \\ x_2 &= 0 \\ x_3 &= x_3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$x_3 = \text{free}$
so infinite solutions

Which means that it is
linearly dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{Row Reduction}}$$

~~$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{Row Reduction}}$$~~

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{array} \right] \xrightarrow{\text{Row Reduction}}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6b \end{array} \right] \xrightarrow{\text{Row Reduction}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6b \end{array} \right]$$

x_3 is
free

$$b=0$$

- a) The only value of b that allows w to be in $\text{Span}(v_1, v_2, v_3)$ is 0 because if it isn't zero there is no solution to the augmented matrix after row reduction.
- b) The set $\{v_1, v_2, v_3\}$ is not linearly independent because x_3 is a free variable meaning it has infinitely many solutions.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) -6 ?

$$\left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & 6 \end{array} \right| \xrightarrow{-2} \left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & 12 \end{array} \right| \xrightarrow{-5} \left| \begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 60 \end{array} \right| \xrightarrow{\text{D}_1}$$

$$\left| \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 60 \end{array} \right|$$

free variable

b) No because the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ when row reduced in reduced row echelon form doesn't have a pivot column for every column and a pivot position in every row.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3 = \mathbf{w}$$

$$\text{RREF: } \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

$$R_3 \rightarrow R_3 - 2R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$R_3 \rightarrow R_3 + R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+4 \end{array} \right]$$

$$\text{RREF: } \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$R_3 \rightarrow R_3 + R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$R_3 \rightarrow R_2 + R_3 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+4 \end{array} \right] \quad (1)$$

~~Since every column~~ can not be a pivot column.

~~Set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly independent~~

NOW, there exists soln only if b is not a pivot column.

~~so, soln exists only when, $b \neq -4$~~

Since, $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$,

Here, soln exists only when, $b+4 \neq 1$

$$\Rightarrow b \neq -1 - 4$$

$$\Rightarrow b \neq -3$$

$\therefore b \in \mathbb{R}, \text{ except } b = -3$

$$\text{From (1)} \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+4 \end{array} \right]$$

Also, soln exist, if $b+4 = 0$
 $\therefore b = -4$.

For, $b = -4$, there is infinite soln.

See next page

$$\therefore b = \mathbb{R} \setminus \{-3\}$$

Ans:



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\text{a)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow[\text{row reduce}]{} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 + \frac{b}{3} \end{array} \right]$$

$\therefore w \in \text{Span}(v_1, v_2, v_3)$

$$\therefore 2 + \frac{b}{3} = 0$$

$$\therefore b = -6$$

$$\text{b)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow[\text{row reduce}]{} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 + \frac{b}{3} \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

there is no pivot position in last column.

Thus, the set is Linearly dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(A) $\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{vmatrix} = (-1)(-12) = 12 \neq 0$ - because $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ can be multiplied by an infinite amount of different scalars

$$\mathbf{v}_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mathbf{v}_2 \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + \mathbf{v}_3 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ b \end{pmatrix}$$

(B) $\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{vmatrix} \rightarrow$ the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly independent because $\mathbf{v}_1, \mathbf{v}_2$, and \mathbf{v}_3 are scalar multiples of each other and therefore a vector other than \mathbf{v}_3 can be in their null space



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

c) $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{②} + \text{①}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{①}(-2) + \text{③}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right] \xrightarrow{\text{②} \times 3} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

Free variable

$b = -6$

b) $x_1 = -3x_3$

$x_2 = 2 - 2x_3$

$x_3 = \text{free}$

The set $\{v_1, v_2, v_3\}$ is not linearly independent. This is because x_3 is a free variable, therefore the set has infinite solutions (and is linearly dependent.)



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$b \neq 0$$

$$b > 0$$

$$b < 0$$

b)

$$\begin{array}{ccc|c} & 1 & -1 & 1 \\ & 0 & 1 & 2 \\ & 2 & -3 & 0 \end{array}$$

\mathbf{b} is linearly independent
as the reduced form
has exactly one solution
when $A = 0$

$$\begin{array}{ccc|c} & 1 & -1 & 1 \\ & 0 & 1 & 2 \\ & 0 & 1 & 0 \end{array} \quad \begin{array}{ccc|c} & 1 & -1 & 1 \\ & 0 & 1 & 2 \\ & 0 & 0 & 2 \end{array} \quad \begin{array}{ccc|c} & 1 & 0 & 1 \\ & 0 & 1 & 0 \\ & 0 & 0 & 2 \end{array}$$

$$\begin{array}{ccc|c} & 1 & 6 & 3 \\ & 0 & 1 & 2 \\ & 0 & 0 & 2 \end{array} \quad \begin{array}{ccc|c} & 1 & 0 & 3 \\ & 0 & 1 & 0 \\ & 0 & 0 & 2 \end{array} \quad \begin{array}{ccc|c} & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & 0 & 0 & 2 \end{array}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\text{a)} \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{+1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$\mathbf{w} \in \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ if and only if $b+6=0 \quad b=-6$.

b) Set is linearly independent if and only if

$x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3 = 0$ has only trivial solution ($x_1 = x_2 = x_3 = 0$)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{+1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$



$$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases} \quad \text{infinite solutions } x_1 \neq x_3 \neq x_2,$$

so set is not linearly independent!



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 \rightarrow -2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$R_1 \rightarrow R_1 + R_2$$

$$R_3 \rightarrow R_3 + R_2$$

$b = -6$ If b equals anything other than -6 , there will be a pivot position in the last column leading to no solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$b) \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow 2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 + R_2}$$

The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly dependent because

$$x_1 \mathbf{v}_1 + x_2 \mathbf{v}_2 + x_3 \mathbf{v}_3 = 0$$

does not have only the

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

trivial solution. x_3 is a free variable giving infinite solutions making the set linearly dependent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{array}{c}
 \left[\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{cccc} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right] \\
 \left[\begin{array}{cccc} 1 & -1.5 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{cccc} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1/2 & 1 & 0 \end{array} \right] \\
 \left[\begin{array}{cccc} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3/2 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{cccc} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \\
 \left[\begin{array}{cccc} 1 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]
 \end{array}$$

So: $v_3 - v_1 = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$2v_2 = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$

So: $\{-2, -4\}$

the set $\{v_1, v_2, v_3\}$ is linearly independent because there is only one solution to $x_1v_1 + x_2v_2 + x_3v_3 = 0$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3) \right] \Rightarrow \mathbf{w} = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3$$

$$\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\xrightarrow{R1 \cdot -2 + R3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b-4 \end{array} \right] \xrightarrow{R2 + R1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-4 \end{array} \right] \xrightarrow{R4 - 1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$\xrightarrow{R2 \leftarrow 1 + R3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -b-4 \end{array} \right] \xrightarrow{R2 + R1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -b-4 \end{array} \right] \xrightarrow{\text{If } b = -6} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} -b-6 &= 0 \\ -6 &= b \end{aligned}$$

$$\begin{aligned} c_1 - c_2 + c_3 &= -2 \\ c_2 + 2c_3 &= 2 \end{aligned}$$

$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is not linearly independent
 because c_3 is a free variable.
 Infinite # of slns if $b = -6$.

No slns if $b \neq -6$.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$a) \begin{array}{l} \text{① } \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{\cdot 3} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{array} \right] \\ \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \quad \therefore \text{for all values of } b \text{ greater than } -5, \text{ } u \in \text{Span}(v_1, v_2, v_3) \end{array}$$

b) The $\{v_1, v_2, v_3\}$ is linearly independent because no vector in that set is a multiple of another.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -5 & -2 & 4+b \end{bmatrix} \xrightarrow{5} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 8 & 14+b \end{bmatrix} \xrightarrow{8=14+b} \boxed{b = -6}$$

$$\begin{array}{l} \text{b)} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 8 \end{bmatrix} \xrightarrow{\frac{1}{8}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

b) The set is linearly independent b/c
the homogenous vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = 0$
will only have one solution.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $-2\mathbf{v}_1 + 4\mathbf{v}_2 - \mathbf{v}_3 = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$$-2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 4 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -16 \end{bmatrix}$$

~~4 - 12~~

b) Aug. matrix $\xrightarrow{\text{Row } 1 \leftrightarrow \text{Row } 2} \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Row } 2 \rightarrow \text{Row } 2 - 2\text{Row } 1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Row } 4 \rightarrow \text{Row } 4 - 2\text{Row } 2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \therefore \text{The set } \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} \text{ is lin. independent.}$
 b/c there is only one solution to the homogeneous matrix

$$\xrightarrow{\text{Row } 2 \leftrightarrow \text{Row } 3} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 - 2\text{Row } 1} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 - (\text{Row } 1 + \text{Row } 2)} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{Row } 1 \rightarrow \text{Row } 1 - \text{Row } 2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & -3 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{\text{Row } 1 \rightarrow \text{Row } 1 - 2\text{Row } 2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 / 3} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{Row } 1 \rightarrow \text{Row } 1 - \text{Row } 3} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{\text{Row } 1 \rightarrow \text{Row } 1 + \text{Row } 2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{\text{Row } 1 \rightarrow \text{Row } 1 / (-1)} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left(\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{w} \right) \Leftrightarrow \begin{bmatrix} x_1 & x_2 & x_3 & c \\ 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 6 \end{bmatrix} \xrightarrow{\text{Row operations}} \dots \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 26 \end{bmatrix} \xrightarrow{x_1 = -3x_3} \begin{array}{l} x_1 = -3x_3 \\ x_2 = 2 - 2x_3 \\ 0 = 26 \end{array} \xrightarrow{26 \neq 0} \text{No solution}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 6 \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 26 \end{bmatrix} \xrightarrow{x_1 = -3x_3} \begin{array}{l} x_1 = -3x_3 \\ x_2 = 2 - 2x_3 \\ 0 = 26 \end{array} \xrightarrow{26 \neq 0} \text{No solution}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 26 \end{bmatrix} \xrightarrow{x_1 = -3x_3} \begin{array}{l} x_1 = -3x_3 \\ x_2 = 2 - 2x_3 \\ 0 = 26 \end{array}$$

a) $b = \boxed{-2}$

b) Linearly dependent because x_3 is a free variable in the matrix $\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$ resulting in a null space with infinite solutions



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) w in span if $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

$$\begin{array}{l} x_1 - x_2 + x_3 = -2 \\ x_2 + 2x_3 = 2 \\ 2x_1 - 3x_2 = b \end{array} \rightarrow \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix}$$

a) $b = -6$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

any integer can be reduced to a leading 1 except when $b = -6$

b) $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_3 = 0$$

$$x_2 + 2x_3 = 0$$

x_3 is a free variable \therefore the equation has infinitely many solutions \therefore the set is linearly dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

A) $c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3 = \mathbf{w}$ $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} = A$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

Aug.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] R_3 \rightarrow 2R_1 - R_3 \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{array} \right] R_1 \rightarrow R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{array} \right] R_3 \rightarrow R_3 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & -6-b \end{array} \right]$$

for \mathbf{w} to be in $\text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ b must equal -6 .

B) The solution to $Ax = \mathbf{0}$ is A is $\begin{bmatrix} x_1 & x_2 & x_3 & | & 0 \\ 1 & 0 & 3 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

$$x_1 = -3x_3$$

The presence of a free variable suggests infinitely many solutions to $Ax = \mathbf{0}$

$$x_2 = -2x_3$$

since the homogeneous

$$x_3 = x_3$$

equation does not have one trivial solution it is linearly dependent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\begin{array}{l}
 \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -1 & 0 & b \end{array} \right) \\
 \xrightarrow{\text{(1)} \cdot (-2)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 0 & b+4 \end{array} \right) \\
 \xrightarrow{\text{(3)} \cdot (-1)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+2 \end{array} \right)
 \end{array}$$

$b = -2$ or else no solution

- b) Set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent because if $b = -2$, and since there is no leading one (pivot position) in third column, then there is a free variable x_3 meaning infinite solutions \Rightarrow linearly dependent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3 = \mathbf{w}$$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & 0 & 6 & b+6 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

b)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

ref from A can get

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Every column doesn't have a pivot position so not linear independent

$b = -6$ is the only value that would produce a solution in the span



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 1 & -3 & -3 & b \end{array} \right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 2 & 2 \\ 1 & -3 & -3 & b \end{array} \right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -3 & 0 \end{array} \right] \xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -1 & 0 \end{array} \right] \xrightarrow{R_3 \cdot (-1)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & b \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & b \end{array} \right] \xrightarrow{R_2 \cdot (-1)} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -2 \\ 1 & 0 & 0 & b \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 1 & 0 & 0 & b \end{array} \right] \xrightarrow{R_3 + 2R_2} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 1 & 0 & 0 & b-4 \end{array} \right] \xrightarrow{R_3 \cdot \frac{1}{2}} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & \frac{b-4}{2} \end{array} \right]$$

b) They are linearly independent.

If we create an augmented matrix of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and augment with 0 instead of \mathbf{w} , we see that the matrix still reduces to the identity matrix, meaning the only solution to that equation is the trivial solution.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \quad b = -6$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(+1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right] \xrightarrow{(1)(-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & b+4 \end{array} \right] \xrightarrow{(-1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right] \quad 0 = b + 6 \quad \underline{b = -6}$$

b) Yes.

Not every column in the matrix is a pivot column.

Therefore, independent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

$$\mathbf{w} = C_1 \mathbf{v}_1 + C_2 \mathbf{v}_2 + C_3 \mathbf{v}_3$$

$$C_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 & 0 \\ 0 & -3 & 0 & b & 6 \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & \frac{b}{3} \end{bmatrix}$$

$\frac{b}{3} = 2 \Rightarrow b = 6$ so that the matrix is consistent and has solution that makes $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

b)

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$$x_1 + 3x_3 = 0 \quad x_1 = -3x_3$$

$$x_2 + 2x_3 = 0 \quad x_2 = -2x_3$$

$$x_2 + 2x_3 = 0 \quad x_2 = -2x_3 \quad x_3 = x_3$$

$$x = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} x_3$$

so that there are other solutions other than $x_1 = x_2 = x_3 = 0$

$\therefore \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} \xrightarrow{b=-6} w = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} \quad \boxed{b = -6}$$

$$v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} -1 \\ 0 \\ -6 \end{bmatrix}$$

$$-2v_1 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$b) \quad \left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array} \right] \xrightarrow{(2)} \left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{array} \right] \xrightarrow{R_3 \leftrightarrow R_3} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

yes, linearly independent
 because every col. is a pivot column



J

1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$\rightarrow \mathbf{w}$ can be written as a

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$. linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$
- b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(a) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(R_1 \times -2) + R_3}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \xrightarrow{R_2 + R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right]$$

b = -6 for
 $\mathbf{w} \in \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

(otherwise get no solution)
due to pivot position in
column of constants

(b) $c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

\Leftrightarrow this set is linearly independent if homo. eqn. only has trivial solution \therefore

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{(R_1 \times -2) + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_2 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

↓

free variable!
not every column of
the matrix is a pivot
therefore the set

$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$
are not linearly
independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $V = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}$ reduced $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

~~(check for rank)~~
 ~~$x_2 + 2x_3 = 2$~~
 ~~$2x_1 - 3x_2 + x_3 = 0$~~
 $b=0$

b. $V = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} R(V) = 2.$

the number of ~~row~~ v_1, v_2, v_3 is 3.

$$R(V) = 2 < 3$$

so, the set $\{v_1, v_2, v_3\}$ is linearly independent.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$V = \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{-2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{+} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \xrightarrow[\text{free}]{} \begin{array}{l} x_1 = -3x_3 \\ x_2 = 2 - x_3 \\ x_3 = x_3 \end{array} \quad \boxed{b = -6}$$

$$b) \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} x_1 = -3x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{array}$$

$x_3 \geq 1$

Yes linear independent

$$-3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} - 2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$a. b_1 \mathbf{v}_1 + b_2 \mathbf{v}_2 + b_3 \mathbf{v}_3 = \mathbf{w}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \left| \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right.$$

$$(3) - (2) \times 1$$

$$\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \left| \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{array} \right.$$

$$(3) + (2)$$

$$\left| \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array} \right.$$

$$b-2=0$$

$$\boxed{b=2}$$

$$b. \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} (1) \\ (2) \\ (3) \end{array} \left| \begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right.$$

$$(1) \times (2) - (2)$$

$$\left| \begin{array}{ccc|c} 2 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right.$$

$$2x_1 - 3x_2 = 0$$

$$x_2 - 2x_3 = 0$$

$$x_1 = \frac{3}{2}x_2$$

$$x_2 = x_2$$

$$x_3 = \frac{1}{2}x_2$$

$$= \begin{bmatrix} \frac{3}{2} \\ 1 \\ -\frac{1}{2} \end{bmatrix} x_2 \rightarrow \text{Infinity solutions}$$

Set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ not linear independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 5+b \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 5+b \end{array} \right]$$

↑ free var

~~Must be zero~~

a) $b = -5$

b) The set is linearly dependent because there is not a pivot column in every column



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

(a)

$$\begin{aligned} C_1\mathbf{v}_1 + C_2\mathbf{v}_2 + C_3\mathbf{v}_3 &= C_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} C_1 \\ 0 \\ 2C_1 \end{bmatrix} + \begin{bmatrix} -C_2 \\ C_2 \\ -3C_2 \end{bmatrix} + \begin{bmatrix} C_3 \\ 2C_3 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} C_1 - C_2 + C_3 \\ C_2 + 2C_3 \\ 2C_1 - 3C_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \end{aligned}$$

$C_1 - C_2 + C_3 = -2$
 $C_2 + 2C_3 = 2$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

$C_1 + 3C_3 = 0$
 $C_2 + 2C_3 = 2$

$$\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix} \Rightarrow 2C_1 - 3C_2 = -6C_3 - 3(2 - 2C_3) = -6C_3 - 6 + 6C_3 = -6$$

$b = -6$

(b)

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

no. it is dependent

↓
infinite



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $w = 2v_2, \boxed{b=6}$
 $w = v_1 + v_3, \boxed{b=2}$
 $w = 3v_1 + 2v_2, \boxed{b=0}$

b) The set is not linearly independent because there is a linear combination of vectors v_1 and v_2 which give v_3 .

$$3v_1 + 2v_2 = v_3$$

$$3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = v_3$$



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & -2 & b+4 \end{array} \right]$$

$$\xrightarrow{R_3 = R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$$

$$b+6=0 \Rightarrow b=-6$$

b) $x_1 v_1 + x_2 v_2 + x_3 v_3 = 0$

aug matrix

$$\left[\begin{array}{ccc|c} v_1 & v_2 & v_3 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right]$$

Reducing rows

$$R_3 = R_3 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right]$$

$$\xrightarrow{R_3 = R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

free variable means
infinitely many solutions

the set $\{v_1, v_2, v_3\}$ is not
linearly independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) ~~$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{w}$~~ ~~$b = 2$~~

~~$\mathbf{v}_1 + \mathbf{v}_3 = \mathbf{w}$~~

$$2\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}, b = -6$$

$$\boxed{b = -6}$$

$$-3\mathbf{v}_1 + \mathbf{v}_3 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}, b = -6$$

b) The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, because the vectors
 cannot linearly combine to cancel one another

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

cannot cancel all terms

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

no scalar multiple



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_2 + R_3} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{array} \right] \xrightarrow{R_2 + R_3} \left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+2 \end{array} \right]$$

$\mathbf{w} \in \text{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ as long as $b=2$ if $b \neq 2$ then
 no solution

the set of vectors is linearly dependent
 because even if you had $b=2$ then there
 would still be a free variable

if $\mathbf{w} \notin \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ then $\mathbf{w} = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + c_3 \mathbf{v}_3$
 there is a nontrivial answer to this equation



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $C_1\mathbf{v}_1 + C_2\mathbf{v}_2 + C_3\mathbf{v}_3 = \mathbf{w}$

$$C_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + C_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

$$\begin{aligned} C_1 - C_2 + C_3 &= -2 \\ C_2 + 2C_3 &= 2 \\ 2C_1 + C_2 &= b \end{aligned}$$

$$\begin{aligned} b &= 2C_1 + C_2 \\ C_1 &= -3 \\ C_2 &= 0 \\ C_3 &= 1 \end{aligned} \quad \begin{aligned} C_1 &= -\frac{3}{2} \\ C_2 &= 1 \\ C_3 &= \frac{1}{2} \end{aligned}$$

$$b = -5, -2$$

b) $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 2 & 6 \\ 2 & -3 & 0 & 6 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & -1 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 0 \end{array} \right]$

No, there must be a pivot column in every column and have only one solution



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $\left[\begin{array}{ccc|c} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{w} \end{array} \right]$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{(-2)} \downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{(1)} \downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right] \xrightarrow{2 \cdot (1)} \downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array} \right] \xrightarrow{\sim} \begin{aligned} 0x_1 + 0x_2 + 0x_3 &= 6+b \\ 0 &= 6+b \end{aligned}$$

free $6+b=0$ for it to have a solution

$$\boxed{b = -6}$$

b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent because x_3 is a free variable indicating that the system has infinitely many solutions, making it linearly dependent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & 0 & b+4 \end{array} \right] \xrightarrow{R_1 = R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & -b-6 \\ 0 & 0 & 2 & b+6 \\ 0 & -1 & 0 & b+4 \end{array} \right] \xrightarrow{R_1 = R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{b}{2} \\ 0 & -1 & 0 & b+6 \\ 0 & 0 & 2 & b+6 \end{array} \right] \cdot (-1) \cdot \left(\frac{1}{2} \right)$$

All values of b
 are $c_1(2) + c_2(-3) + c_3(0)$
 that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

$$\begin{aligned} x_1 &= 0 \\ x_2 &= -b-4 \\ x_3 &= b+6 \end{aligned}$$

b) Yes, the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent because after row reduction every column is a pivot column.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 = \mathbf{w}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a)

~~-2.~~ $\xrightarrow{\text{Augmented matrix}}$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$

for $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

$b+6=0$

$b=-6$

1. $\xrightarrow{\text{ }} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$

1. $\xrightarrow{\text{ }} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$

1) $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array} \right]$

b) $\left[\begin{array}{c} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{array} \right] \xrightarrow{\text{reduce row echelon form}}$

row echelon form is not linearly independent because it's reduce has more than one free variable so the null space meaning that it is linearly dependent (it has infinitely many solutions), here only one trivial solution to the null space which would be the $\mathbf{0}$ vector.

$$\left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

free variable



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\xrightarrow{-2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right) \xrightarrow{\cdot 1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array} \right)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 4+b \end{array} \right] \quad b \Rightarrow -4$$

b) From part a \rightarrow

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

No, the set $\{v_1, v_2, v_3\}$ is not linearly independent because the solution would be a trivial solution if it were linearly independent. In this case, it leads to infinitely many solutions.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+b \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+b \end{array} \right]$$

$w \in \text{Span}(v_1, v_2, v_3)$ for $b = -6$

b)

$$\left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array} \right] \xrightarrow[\text{reduction}]{} \left[\begin{array}{ccc} x_1 & x_2 & x_3 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_3 = 3x_1 + 2x_2$$

The set $\{v_1, v_2, v_3\}$ is not linearly independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{-2}$$

↓

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4b \end{array} \right] \xrightarrow{*1}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+4b \end{array} \right]$$

⋮

$\therefore w \in \text{Span}(v_1, v_2, v_3)$

$$2+4b=0$$

$$\cancel{b} = \frac{1}{2}$$

$$b = -\frac{1}{2}$$

(b)

$\{v_1, v_2, v_3\}$ is not linearly independent,
 because there is not every column of matrix
 is pivot column.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

c) $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ means that $(\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3) = \mathbf{w}$ has solution

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{(1) \times 2 - (3)}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & -4-b \end{bmatrix} \xrightarrow{(2) - (3)}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

Since 6+b is in constant column,

$$6+b=0$$

$$\therefore b = -6$$

b) $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{(1) \times 2 - (3)}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \xrightarrow{(2) - (3)}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since x_3 is free variable,
this homogenous equation has
many solutions.

This means that the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$
is (not) linearly independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & b+4 \\ 0 & 1 & 2 \\ 0 & -1 & b-6 \end{array} \right]$$

$$\begin{aligned} x_1 - x_2 + x_3 &= -2 \\ x_2 + 2x_3 &= 2 \\ 2x_1 + 3x_2 &= b \end{aligned}$$

$$-12x_3 = b-6$$

$$x_2 = 2 + \frac{b-6}{12}$$

$$x_1 = -2 + \left(2 + \frac{b-6}{12}\right) + \frac{b-6}{12}$$

$$x_1 = 3\left(\frac{b-6}{12}\right)$$

$$b = ?$$

$$\left\{ \begin{array}{l} x_1 = 3\left(\frac{b-6}{12}\right) \\ x_2 = 2 + 2\left(\frac{b-6}{12}\right) \\ x_3 = -\frac{b-6}{12} \end{array} \right.$$

(b) to check for linear independence

$$\left[\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & 0 \end{array} \right] \xrightarrow{\text{row}} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{array} \right] \xrightarrow{\text{red}} \left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

Doesn't reduce to identity

so

Linerly Dependent

Also, No vectors Appear to be able to be represented by any other in the set.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\text{augmented matrix: } \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{\text{R2} \leftrightarrow \text{R3}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{\text{R3} + \text{R2}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{array} \right] \xrightarrow{\text{R2} \rightarrow \text{R2} - 2\text{R1}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array} \right] \xrightarrow{\text{R2} \rightarrow \text{R2} - 2\text{R1}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

$$x_1 = -3x_3$$

$$x_2 = 2 - 2x_3$$

$$x_3 = 0 = b+6$$

$$\underline{b = -6}$$

b) Yes, it is linearly independent because v_1, v_2, v_3 are not multiples of each other. There is no way to get anything but 0 in second row of v_1 and therefore we can't get such multiple of v_1 so that we obtain v_2, v_3 .



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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & 0 & b \end{array} \right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 12 & 0 & b+2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+\frac{b}{2} \end{array} \right]$$

$$2 + \frac{b}{2} = 0$$

$$\frac{b}{2} = -2$$

$$b = -4$$

The set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent because $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are not scalar multiples of each other.



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

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- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
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a)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \right] \xrightarrow{R3 \rightarrow R3 - 2R1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-4 \end{array} \right] \xrightarrow{\substack{R3 \rightarrow \\ R3 + R2}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b-2 \end{array} \right]$$

when $b \neq 2$

b) The set isn't linearly independent because since the span is made up of linear combinations of the vectors all the vectors in the span are linearly dependent



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- a) Find all values of b such that $\mathbf{w} \in \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.
 b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_2 \times -2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_1 + (-R_3) \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right] \xrightarrow{\text{restore } R_3} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \text{free}$$

b) It is linearly dependent since the reduced form contains a free variable meaning it has infinitely many solutions.

a) $b = 2$