



MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	5	8	5	3	9
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1 2 3 4 5 6 7 TOTAL GRADE

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14

10

3

15

20

3

2

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10

74

B-

1

2

3

4

5

6

7

PIAZZA

HILL

TOTAL

GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

The value of b is -6

because $2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

How do you know that this is the only value which works?

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$\begin{matrix} 2- \\ 3-2 \end{matrix}$

$$1 - 1 \ 1 \ 0$$

$$0 \ 1 \ 2 \ 0$$

$$0 \ -5 \ -2 \ 0$$

$-5+5$

$-2+10$

$$1 - 1 \ 1 \ 0$$

$$0 \ 1 \ 2 \ 0$$

$$0 \ 0 \ 0 \ 0$$

$$1 - 1 \ 1 \ 0$$

$$0 \ 1 \ 2 \ 0$$

$$0 \ 0 \ 1 \ 0$$

Is it linearly independent?

$$x_1 = 0$$

$$x_2 = -2$$

$$x_3 = 0$$

It is linearly independent because x_2 is not 0.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} 1-1 & 0-1 & 1-2 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} 1-1 & 0-1 & 1-2 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -2 & 0 \end{bmatrix} \quad \begin{matrix} 1-1 & 0-1 & 1-2 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1-1 \\ 1-2 \\ 0-2 \end{matrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix} \begin{matrix} x_1 = 0 \\ x_2 = 3 \\ x_3 = 7 \end{matrix}$$

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ x_2 &= 5 - x_3 \\ x_3 &= 7 \\ 5 - 7 &= -2 \\ 1 + 2 &= 1 \end{aligned}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 3 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$3 \times 3 \quad 3 \times 3 = 3 \times 3$

$$\begin{bmatrix} 3 & -2 & 0 & 1 \\ -3 & 0 & 2 & 0 \\ 6 & -2 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \begin{matrix} x_1 = 0 \\ x_2 = 5 \\ x_3 = 7 \end{matrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ -2 & 6 & 4 \\ 4 & -2 & -2 \end{bmatrix}$$

$$C = ?$$



4. (20 points) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T .

b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

A) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ ✓

B)

$v = c_1 \begin{bmatrix} 3 \\ 7 \end{bmatrix} + c_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

~~Any multiple of
this vector.~~

Where did this
come from?



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to-one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$$

4-3 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

4-2 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

This is ^{one-to-one} ~~not~~ because
there is
a pivot in every
given col.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -x_2 \\ x_2 &= -2x_3 \\ x_3 &= \text{free} \end{aligned}$$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

This isn't one to one
one case there isn't
a pivot in every col.

$$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

$u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
 $\begin{bmatrix} 3 \\ 3 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$
 The statement is true \leftarrow $\begin{bmatrix} 3 \\ 3 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$ \rightarrow False

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$ This is false because just because
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ This is ~~false~~ because just because
 $u+v+w=0$
 does not mean
 $u+v=0$?



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

~~$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$~~ ~~$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$~~ $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

~~$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$~~ ~~$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$~~ $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

~~True~~ because if you have
inf solution and then transform it is still
the same. | ?

b) If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in $\text{Span}(v, w)$ then $T(u)$ must be in $\text{Span}(T(v), T(w))$. *True*

~~True~~ $u = \begin{bmatrix} 8 \\ 2 \end{bmatrix} \text{Span}\left(\begin{bmatrix} 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$

what is
T here ?

$\rightarrow \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix}$ | ?

2×1 1×1

2×1

2×2 \uparrow

True ~~True~~ because

of the example

of a linear transformation of $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

An example does not show
that this is always true.