$\begin{bmatrix} a & b \\ c & d \end{bmatrix}[v] = \lambda v$ $\quad$ $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} c & d \\ d & c \end{bmatrix}$

$\begin{bmatrix} a+c & b+d \\ b+d & a+c \end{bmatrix}$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and $\mathbf{w}$ is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then $\mathbf{w}$ must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

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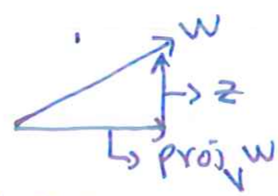
a) False The eigenvalue is the root of the characteristic polynomial which means its the solution that makes the equation $P(\lambda) = 0$ true an $n \times n$ matric can have no more than n roots so 2λ would not be the eigen value.

+5

b) True projection of a vector w on subspace V is unique ~~to each vector~~ ~~being projected~~ such that $z = w - \text{proj}_V w \Rightarrow w = z + \text{proj}_V w$ where z is an orthogonal vector to V

→ This means the only way $\text{proj}_V w = -w$ is if vector w is a zero vector.



→ If w is an element of vectorspace of V then $\text{proj}_V w = w$ which is different from $\text{proj}_V w = -w$

2

c) False If A is symmetric then it has n orthogonal eigenvector A can then be expressed as $A = QDQ^T$

$A^2 = QD^2Q^T$ not identity matrix