MTH-309T-F19-EX1-008-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

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Name:

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UB Person Number:

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1	2	3	4	5	6	7	TOTAL	GRADE
10	5	7	20	4	6	--	52	nan

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$. c_1, c_2, c_3

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

RREF & See if each column has leading one?

(a) $c_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$\boxed{b \neq 0}$

$0\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$\boxed{b = -6}$

$y(2) + x(-3) + 0$

(b)
$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \leftarrow -2R_1 + R_3 \to R_3$$
$-2 \quad +2 \quad -2 \quad 0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \leftarrow R_2 + R_3 \to R_3$$
$+0 \quad +1 \quad +2 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
$\leftarrow$ Vectors v_1, v_2 & v_3 are Linearly dependant (NOT Linearly Independent) because, v_2 & v_3 are free variables.

$v_1 - v_2 + v_3 = 0$

$v_2 + 2v_3 = 0$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

10

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3x3

A^T or A^{-1}?

Compute $\boxed{A^{-1}.}$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

3x3.

0 1 2 3 4 5 6 7 8 9 10

5

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}}_{3\times 3} \underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}}_{3\times 3} = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}_{3\times 3}$$

$A^T C = B$

~~$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ 3×2 $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $2\times$~~

~~Colums have to be same?~~

Come back!

$1x_1 - 1x_2 + 2x_3 = 1$

Hello?

7

4. **(20 points)** Let $T: \mathbb{R}^2 \xrightarrow{} \mathbb{R}^3$ be a linear transformation given by

u

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

2 rows to 3 columns?

a) Find the $\boxed{\text{standard matrix of } T.}$

b) Find all vectors u satisfying $\boxed{T(\mathbf{u})} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

Standard

(a) $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{bmatrix} = \boxed{\begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

$x_1 = 1 + 2x_2 \Rightarrow 1 + 2(3) = \boxed{7} \; x_1 = 7$

$1 + 2x_2 + x_2 = 10 \Rightarrow 1 + 3x_2 = 10 \Rightarrow 3x_2 = 9 = \boxed{x_2 = 3}$

$1 + 2x_2 - 3x_2 = -2 \Rightarrow 1 - x_2 = -2$

$\boxed{x_2 = 3}$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

20

Come back

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

→ One-to-one
pivot columns
→ Nul(A) ?

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$$

$\frac{1}{2}R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right] \leftarrow -3R_1 + R_3 \to R_3$$

$-3 \quad -3 \quad +0 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

pivot column because column without a leading 1.

NOT one-to-one.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \leftarrow \frac{1}{2}R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right] \leftarrow -3R_1 + R_3 \to R_3$$

$-3 \quad -3 \quad +0 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \leftarrow -R_2 + R_3 \to R_3$$

$+0 \quad -1 \quad -2 \quad +0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \leftarrow$$

One to one because no pivot columns?

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True because, w+u consists of w as well and when $w+u \in Span(u,v)$; w must be in Span(u,v).

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

Counterexample:

let $\{u,v,w\}$ be set $\{5,6,8\}$ respectively. Then, $\{5,6,8\}$ doesn't mean $\{5,8\}$ is also linearly independent.

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\boxed{\mathbf{u}, \mathbf{v}}$ also must be linearly dependent.

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$Au = \left[\begin{array}{cc|c} 1 & 2 & u_1 \\ 3 & 4 & u_2 \end{array}\right]$

$Av = \left[\begin{array}{cc|c} 1 & 2 & v_1 \\ 3 & 4 & v_2 \end{array}\right]$

linearly independent

True because, Since Au & Av are linearly dependent, u & v must also be.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\boxed{\mathrm{Span}(\mathbf{v}, \mathbf{w})}$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

Come back.

$u \in \mathrm{Span}(v, w)$

then

$T(u) \in \mathrm{Span}(T(v), T(w))$?!

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6	7	8	9	10