# MTH 309Y LINEAR ALGEBRA

## EXAM 3

December 11, 2018

Name: ______________________________

Person Number: ______________________________

- Textbooks and electronic devices (calculators, cellphones etc.) are not permitted.
- You may use one sheet of notes.
- For full credit explain your answers fully, showing all work.
- Each problem is worth 20 points.

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| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total: | |

DO NOT WRITE OUTSIDE THE MARKED AREA

**1.** Consider the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace $V$ of $\mathbb{R}^4$.

**a)** Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace $V$.

**b)** Compute the vector $\mathrm{proj}_V \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ on $V$.

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**2.** Find the equation $f(x) = ax + b$ of the least square line for the points $(1,0)$, $(-1,2)$, $(2,1)$.

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**3.** Consider the following matrix $A$:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of $\lambda$ given below determine if it is an eigenvalue of $A$.

**a)** $\lambda = 0$ **b)** $\lambda = -1$ **c)** $\lambda = -2$

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**4.** Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of $A$ are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix $D$ and an invertible matrix $P$ such that

$$A = PDP^{-1}$$

Note: you do not need to compute $P^{-1}$.

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**5.** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{v}$ is an eigenvector of $A$ corresponding to an eigenvalue $\lambda$ then $2\mathbf{v}$ is an eigenvector of $A$ corresponding to the eigenvalue $2\lambda$.

**b)** If $V$ is a subspace of $\mathbb{R}^2$ and $\mathbf{w}$ is a vector such that $\mathrm{proj}_V \mathbf{w} = -\mathbf{w}$ then $\mathbf{w}$ must be the zero vector.

**c)** If $A$ is a square matrix which is both symmetric and orthogonal then $A^2$ is the identity matrix.

**d)** If $A$ and $B$ are $2 \times 2$ matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.