1 2

$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ~~xxx~~ $\begin{matrix} x_1 = x_2 \\ \lambda_2 = x_2 \end{matrix} \to \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \to x_2 = 0$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

a) false ~~true~~, ~~because multiplying~~ $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ $\lambda_1 = 1$ then $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \to v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

but

+5 so false $\lambda_2 = 2$ then $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\begin{matrix} x_2 = 0 \\ 0 = 0 \end{matrix}$ no v_2

b) True, because ~~xxx~~ for the proj to equal the original vector,

+3 the vector must already fall on the projected plan

with $0 = -0$ as the only vector that could ~~x~~ possibly be

equal to it's negative proj

c) ~~false~~, because $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ is a square, symmetric, & orthogonal

+1 matrix but $A^2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ which is not the Identity matrix

D) True, because the resulting matrix will still be

+5 symmetrical which means that it will also be

orthogonally diagonizable

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20