

1. Consider the following vectors in \mathbb{R}^4 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of \mathbb{R}^4 .

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V .

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V .

~~$$v_1 \cdot v_2 = (1)(2) + (0)(1) + (-1)(-1) + 0 = 2 + 0 + 1 + 0 = 3$$~~

a.) Use Gram-Schmidt: $\langle w_1, w_1 \rangle = 3$

$$w_1 = v_1$$

$$\langle w_1, v_2 \rangle = (1)(2) + 0 + 1 + 0 = 3$$

$$w_2 = v_2 - \frac{\langle w_1, v_2 \rangle}{\langle w_1, w_1 \rangle} w_1 = v_2 - w_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = w_2$$

$$w_3 = v_3 - \frac{\langle w_1, v_3 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle w_2, v_3 \rangle}{\langle w_2, w_2 \rangle} w_2 = v_3 - 2w_1 + w_2 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\langle w_1, v_3 \rangle = 2 + 0 + 1 + 3 = 6$$

$$\langle w_2, v_3 \rangle = 2 - 2 + 0 - 3 = -3$$

$$\langle w_2, w_2 \rangle = 3$$

~~$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} \right\}$$~~

$$\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$b.) \text{proj}_V u = \frac{\langle u, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 + \frac{\langle u, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2 + \frac{\langle u, w_3 \rangle}{\langle w_3, w_3 \rangle} w_3$$

$$= w_1 + w_2 + w_3 = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \text{proj}_V u$$

$$\langle u, w_1 \rangle = 3 + 0 - 3 + 3 = 3$$

$$\langle u, w_2 \rangle = 3 + 3 + 0 - 3 = 3$$

$$\langle u, w_3 \rangle = 3 - 3 + 3 + 0 = 3$$

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