4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

r $\lambda_1 = 3$

$$\begin{bmatrix} 1-3 & 8 & 4 \\ -2 & 11-3 & 4 \\ 2 & -8 & -1-3 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 4 & 0 \\ 2 & 8 & 4 & 0 \\ 2 & -8 & -4 & 0 \end{array}\right] \xrightarrow{\substack{-R_1+R_2 \to R_2 \\ R_1+R_3}} \left[\begin{array}{ccc|c} -2 & 8 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 4x_2 + 2x_3$
$x_2 = \text{free}$
$x_3 = \text{free}$

$$\Rightarrow \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} x_3 \Rightarrow \mathrm{Nul}(A - \lambda I_n) = \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

r $\lambda_2 = 5$

$$\begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{array}\right] \xrightarrow{\substack{-\frac{1}{2}R_1 + R_2 \to R_2 \\ \frac{1}{2}R_1 + R_3 \to R_3}} \left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -4 & -4 & 0 \end{array}\right] \xrightarrow{2R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-4R_2 + R_1 \to R_1}$$

$$\left[\begin{array}{ccc|c} -4 & 0 & -4 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\substack{-\frac{1}{4}R_1 \\ \frac{1}{2}R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 = \text{free} \end{array} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} x_3 \Rightarrow \mathrm{Nul}(A - \lambda I_n) = \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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$$A = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$$