**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True - Linearly dependent sets have infinitely many solutions

a set of two vectors is linearly independent if and only if one vector is a scalar multiple of the other.

Since $A(v+u) = Au + Av$, if $Au$ and $Av$ are linearly dependent $u, v$ must also be linearly dependent.

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True! $T(u+v) = T(u) + T(v)$ $\quad$ $u \in \text{Span}(v, w)$

$T(cv) = cT(v)$ $\quad$ $T(u) \in \text{span}(T(v), T(w))$?

Span holds through transformations

$T \cdot S(v) = (A \cdot B)v = A(B(v))$

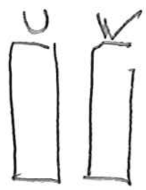
$\text{col}(A) = \text{row}(B)$

0 1 2 3 4 5 6 7 8 9 10

MTH-309T-F19-EX1-003-P07

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.




True, if they have a unique solution, it doesnt matter whether they are ~~seperated or not~~ represented in a matrix or not

**b)** If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True, Span$(v_1, \dots v_p)$ = set of all linear combination $c_1 v_1 + c_2 v_2$.

thus it holds ~~every~~ than $T(u)$ must be in Span$(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False.

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

~~False.~~

True.

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False. ~~Au~~ ~~$Au = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $Ab = [$~~

$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ∴ $Au$, $Av$ is linearly dependent.

$u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ~~A~~

$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

∴ $u, v$, is not dependent.

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False.

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ This statement is false

$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 4 \end{vmatrix}$ $\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 9 \end{vmatrix}$

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

T(u) must be in Span(T(v), T(w)), be

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False; the matrix A can cause Au, Av to become lin. dependent even if u, v are lin. independent.

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

True; let $u = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$T(u) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

$T(v) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$T(w) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

T(u) is in Span(T(v), T(w))

So, T(u) must be in Span(T(v), T(w))

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

True because Au, Av are a linear combination of A.

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

True because of the matrix property $c \cdot T = c(T)$

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Au, Av are dependent

but u, v are independent

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in $\mathrm{Span}(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in $\mathrm{Span}(T(\mathbf{v}), T(\mathbf{w}))$.

False

$u = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(u) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $T(v) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ $T(w) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$u \in \mathrm{Span}(v, w)$

$T(u) \notin \mathrm{Span}(T(v), T(w))$

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2 \times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

this may simply mean that the matrix A is linearly dependent not necessarilly u or v

**b)** If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

~~True because~~
True because if the same transformation is performed on some u thats in the span of some v and w, ~~it will~~ u will be moved into that new span of T(v), T(w) by the transformation T(u).

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False:

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} =$

**b)** If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span(v, w) then $T(\mathbf{u})$ must be in Span($T$(v), $T$(w)).

True: if $u$ is in the span $(v, w)$ then a combination $c_1 V + c_2 W = u$, so $T(c_1 v + c_2 w) = T(u)$ as $T(c_1 v + c_2 w)$ can be split into $c_1 T(v) + c_2 T(w) = T(u)$ which means that there is a linear combination $T(v)$ and $T(w)$ that equal $T(u)$

0 1 2 3 4 5 6 7 8 9 10

**7. (10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

**a)** If $A$ is a $2\times 2$ matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & \cancel{0} \\ 0 \end{bmatrix} =$$

TRUE

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Because if Au and AV yield the linearly dependent result, then u v are also dependent.

**b)** If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

~~True. Because applying the~~

FALSE

0 1 2 3 4 5 6 7 8 9 10