

# MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Lauren Kim

## **UB Person Number:**

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### Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
   You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that  $w \in \text{Span}(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  linearly independent? Justify your answer.

$$\begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix} = \overrightarrow{W}$$

$$\begin{bmatrix} 1 & -1 & 1 & | & -2 & | & -1 & 1 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | & -2 & | &$$

$$b-6=0$$
  
if  $b=6$  ithen  $\overrightarrow{w} \in \text{Spen}(\overrightarrow{V}_1, \overrightarrow{V}_2, \overrightarrow{V}_3)$ 

$$X_1 - X_2 + X_3 = -2$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
2 & -3 & 0 & 6
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 1 & -2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

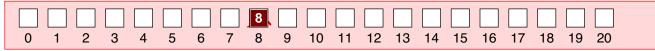
$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

to be linearly independent, every column after now reduction must be a

the set is not linearly independent.





## 2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute  $A^{-1}$ .

Compute A..

$$\begin{pmatrix}
1 - 1 & 2 & | & 1 & 0 & 0 \\
1 & 0 & 1 & | & 0 & 1 & 0 \\
0 & 2 & -1 & | & 0 & 0 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
1 - 1 & 2 & | & 1 & 0 & 0 \\
-R_1 + R_2 & | & 1 & 0 & 0 \\
0 & -1 & -1 & 1 & 0 \\
0 & 1 & -1 & -1 & 1 & 0 \\
0 & 0 & 1/2 & | & 0 & 0 & 1/2
\end{pmatrix}$$

$$\begin{pmatrix}
1 - 1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & -1 & 1 & 0 \\
0 & 0 & 1/2 & | & 1 - 1 & 1/2
\end{pmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & -2 & 3 & -1 \\
1 & 0 & 1 & | & 1 & -1 & 1 \\
0 & 2 & -1 & | & 2 & -2 & 1
\end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$(A^{T})' = (A^{-1})^{T}$$

$$AA^{-1} = I$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A^{7} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} \chi_{1} & \chi_{2} & \chi_{3} \\ y_{1} & y_{2} & y_{3} \\ 2_{1} & 2_{2} & 2_{3} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} \chi_{1} & \chi_{2} & \chi_{3} \\ -\chi_{1} & 4y_{1} = 1 \\ 2\chi_{1} & 4y_{1} - 2\chi_{1} = 3 \end{pmatrix}$$

$$(AT)^{-1} = \begin{pmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{pmatrix}$$

A = 
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
 the transpose of A).

X 1 + 4 - 2 - 2 - 1

- X 1 + 2 - 2 - 3

\[
\begin{array}{c|cccc} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 2 & 1 & -1 & 1 \end{array}
\]

\[
\begin{array}{c|cccc} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 2 & 1 & -1 & 1 \\ \end{bmatrix}
\end{array}
\[
\begin{array}{c|cccc} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ \end{bmatrix}
\end{array}
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\begin{array}{c|cccc} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \\ \end{bmatrix}
\end{array}
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\begin{array}{c|cccc} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \\ \end{bmatrix}
\end{array}
\[
\begin{array}{c|cccc} 1 & 0 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 1 \\ \end{bmatrix}
\end{array}
\[
\begin{array}{c|cccc} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 1 \\ \end{bmatrix}
\end{array}
\]

$$\begin{array}{c|c}
2 & -1 & 3 \\
 & -3 & 5 \\
 & 0 & 1 & 2 \\
 & 0 & 0 & 1 \\
 & 0 & 0 & 1 \\
 & 0 & 0 & 1 \\
 & 0 & 0 & 1 \\
 & 0 & 0 & 1 \\
 & 0 & 0 & 1 \\
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$$\begin{bmatrix}
1 & 1 & 0 & | & 2 \\
-1 & 0 & 2 & | & 5 \\
-1 & 0 & 2 & | & 5 \\
2 & 1 & -1 & | & 2
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 0 & | & 2 \\
-1 & 0 & 2 & | & 7 \\
0 & -1 & -1 & -2
\end{bmatrix}$$

$$\begin{bmatrix}
11 & 0 & | & 2 \\
0 & 1 & 2 & | & 7 \\
0 & 0 & 1 & 5
\end{bmatrix}$$

$$\begin{bmatrix}
2 & 2 & 5 & | & 5 \\
0 & 1 & 2 & | & 7 \\
0 & 0 & 1 & 5
\end{bmatrix}$$

$$\begin{bmatrix}
110 & | & 3 \\
-(02) & | & 4 \\
2 & | & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1103 \\
0127 \\
0-1-1-5
\end{bmatrix}$$

$$\begin{bmatrix}
1103 \\
0127 \\
0012
\end{bmatrix}$$

$$73 = 2$$
 $93 = 7 - 9 = 3$ 
 $3 = 3 = 0$ 
 $3 = 0$ 



4. (20 points) Let  $T: \mathbb{R}^2 \to \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \left(\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} \times_1 \\ \times_2 \end{bmatrix} - \begin{bmatrix} \times_1 & -2 \times_2 \\ \times_1 & -3 \times_2 \end{bmatrix} \right)$$

- a) Find the standard matrix of T.

$$A = \begin{bmatrix} 1 - 2 \\ 1 - 3 \end{bmatrix}$$

$$\begin{bmatrix}
1 - 2 \\
1 - 3
\end{bmatrix}
\vec{u} = \begin{bmatrix}
1 \\
1 \\
2
\end{bmatrix}$$

$$\vec{u} = \begin{bmatrix}
4 \\
3
\end{bmatrix}$$

$$\begin{pmatrix}
1-2 & 1 \\
1 & 1 & 10 \\
1-3 & -2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1-2 & 1 \\
0 & 3 & 9 \\
0 & -1 & -3
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1-2 & 1 \\
0 & 1 & 3 \\
0 & -1 & -3
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 & 1 & 3 \\
0 & -1 & 3
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 & 1 & 3 \\
0 & 0 & 3
\end{pmatrix}$$

$$u_1 - u_2 = 1$$
 $u_2 = 3$ 
 $u_1 = 4$ 



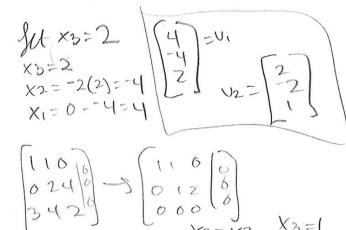
5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A: \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(v) = Av$  is one-to one or not. If  $T_A$  is not one-to-one, find two vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$ .

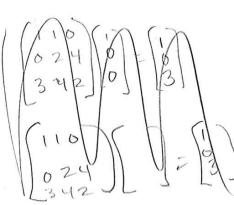
a) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ 

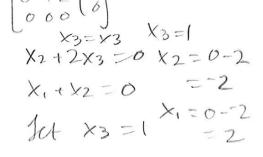
$$\mathbf{b)} \ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$

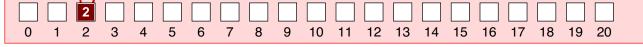
one to are if proof position in every column

not a prot column











- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True combinations of vectors in a spon are still within a spon and vice versa.

b) If u,v,w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.

True. In the set of 3,-they are all lin ind.

from each other, meaning if one vector

was taken away, the remaining two

would still be lin ind.



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

true. A matrix is a linear transform iso anything that is linearly transformed stays linearly dependent.

b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

the linear transforms will not change is Samething is within a span

