



DO NOT WRITE OUTSIDE THE MARKED AREA

# MTH 309Y LINEAR ALGEBRA

## EXAM 3

December 11, 2018

Name: \_\_\_\_\_

Person Number: \_\_\_\_\_

- Textbooks and electronic devices (calculators, cellphones etc.) are not permitted.
- You may use one sheet of notes.
- For full credit explain your answers fully, showing all work.
- Each problem is worth 20 points.

1	
2	
3	
4	
5	
Total:	



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1. Consider the following vectors in  $\mathbb{R}^4$ :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$  is a basis of some subspace  $V$  of  $\mathbb{R}^4$ .

- a) Find an orthogonal basis  $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$  of the subspace  $V$ .
- b) Compute the vector  $\text{proj}_V \mathbf{u}$ , the orthogonal projection of  $\mathbf{u}$  on  $V$ .



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2. Find the equation  $f(x) = ax + b$  of the least square line for the points  $(1, 0)$ ,  $(-1, 2)$ ,  $(2, 1)$ .



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3. Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of  $\lambda$  given below determine if it is an eigenvalue of  $A$ .

a)  $\lambda = 0$

b)  $\lambda = -1$

c)  $\lambda = -2$



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4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of  $A$  are  $\lambda_1 = 3$  and  $\lambda_2 = 5$  diagonalize this matrix; that is, find a diagonal matrix  $D$  and an invertible matrix  $P$  such that

$$A = PDP^{-1}$$

Note: you do not need to compute  $P^{-1}$ .



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5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $A$  is a  $2 \times 2$  matrix and  $\mathbf{v}$  is an eigenvector of  $A$  corresponding to an eigenvalue  $\lambda$  then  $2\mathbf{v}$  is an eigenvector of  $A$  corresponding to the eigenvalue  $2\lambda$ .

b) If  $V$  is a subspace of  $\mathbb{R}^2$  and  $\mathbf{w}$  is a vector such that  $\text{proj}_V \mathbf{w} = -\mathbf{w}$  then  $\mathbf{w}$  must be the zero vector.

c) If  $A$  is a square matrix which is both symmetric and orthogonal then  $A^2$  is the identity matrix.

d) If  $A$  and  $B$  are  $2 \times 2$  matrices which are both orthogonally diagonalizable, then the matrix  $A + B$  is also orthogonally diagonalizable.