

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Daviel Maas

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

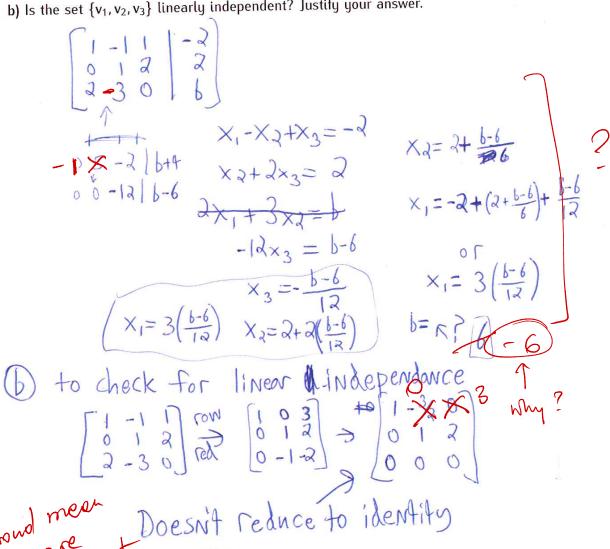
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1,v_2,v_3\}$ linearly independent? Justify your answer.



This man are all Doesn't reduce to identify
that the pendent So
Liniarly Dependent

2 Also, No vectors Appear to be able to be represented
by only other in the set.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

There is also aNother Way that requires you to find a Cofactor by doing

$$\begin{array}{ccccc}
 & & & & & & & & \\
 & + & - & + & & & & \\
 & + & - & + & & & & \\
 & + & - & + & & & & \\
 & + & - & + & & & & \\
\end{array}$$

$$\times \left(\begin{array}{c} + \\ + \\ \end{array} \right) = A$$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \left[\begin{array}{rrr} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{array} \right]$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

Find a matrix C such that
$$A'C = B$$
 (where A' is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

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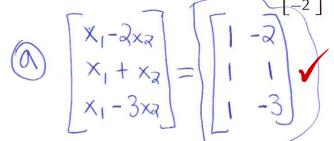
$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4$$

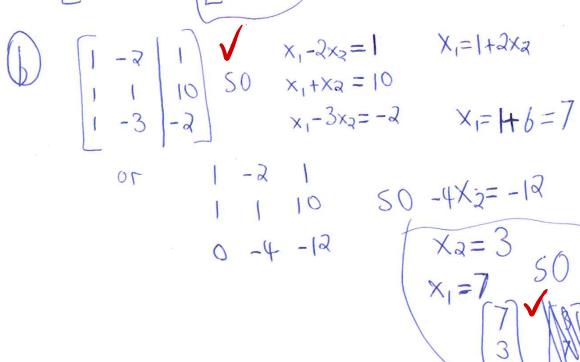


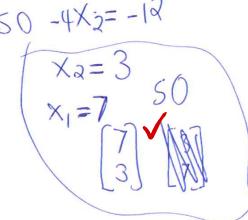
4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$.





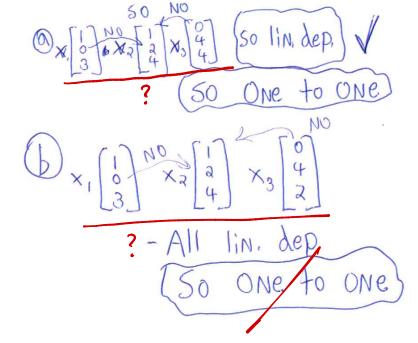




5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

one to one if all Colla) are lin, dep.



Linear independence does not always mean that one vector is a multiple of another — this works only for linear independence of two vectors (not three).



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u,v,w are vectors in \mathbb{R}^3 such that $w+u\in Span(u,v)$ then $w\in Span(u,v)$.

$$N = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

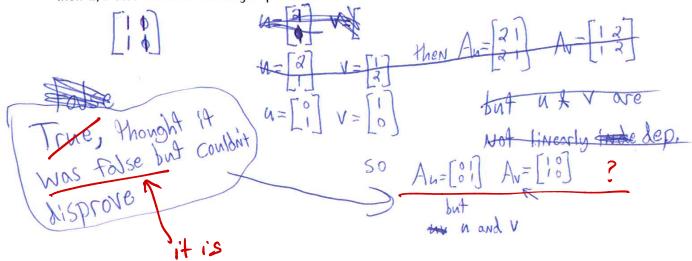
must be linearly independent.

$$a + v = w$$
 $a + v = w$
 $a + w = w$

Tepresent the other.



- **7.** (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

