1. Consider the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

$0+0+{}^-3$

$\frac{2+(-2)+0+(-3)}{1+1+0+1}$

$(-1)\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

The set $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ on V.

a) $w_1 = v_1 \quad \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}}_{w_1}$

$w_2 = v_2 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1$

$\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{2+0+1+0}{1+0+1+1} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ $\quad \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}}_{w_2}$

$w_3 = \overset{v_3}{\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}} - \left(\frac{w_1 \cdot v_3}{w_1 \cdot w_1}\right) w_1 - w_2 \left(\frac{w_2 \cdot v_3}{w_2 \cdot w_2}\right)$

$\frac{2+0+1+3}{3} \; \frac{6}{3} = 2 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 2 \end{bmatrix}$

$w_3 = \begin{bmatrix} \cancel{2} \\ -2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} \cancel{2} \\ 0 \\ -2 \\ 2 \end{bmatrix} - \begin{bmatrix} \cancel{-1} \\ \cancel{-1} \\ 0 \\ 1 \end{bmatrix}$

$w_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

$$\boxed{\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}}$$ ✓

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b) $\text{proj}_V u = \left(\frac{u \cdot v_1}{v_1 \cdot v_1}\right) v_1 + \left(\frac{u \cdot v_2}{v_2 \cdot v_2}\right) v_2 + \left(\frac{u \cdot v_3}{v_3 \cdot v_3}\right) v_3$

$-\frac{6}{3} + {}^-\frac{1}{3}$

$1 \left(\frac{3+0+-3+3}{1+0+1+1}\right) \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \frac{6}{6}\,1\;\frac{6+3+(-3)+0}{4+1+1+0} \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} + \frac{6}{18}\,\frac{1}{3}\;\frac{6+(-6)+(-3)+9}{4+4+1+9} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}$ $\quad \begin{matrix} 2/3 \\ -2/3 \\ -1/3 \\ 1 \end{matrix}$

$\frac{9}{3} + \frac{2}{3}$

$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2/3 \\ -2/3 \\ -1/3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 11/3 \\ 1/3 \\ -7/3 \\ 2 \end{bmatrix}}$