

MTH 309T LINEAR ALGEBRA EXAM 1

Ţ			ı Nı			m	00	Instructions:
(a) (b) (c) (c) (c) (c) (c) (c) (c) (c) (c) (c	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0				1 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	① ② ③ ④	3 0 1 2 4 5 6 7 8 9	 Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes. For full credit solve each proble fully, showing all relevant work.

12	10	10	19	20	3	9	2	10	94	A
1	2	3	4	5	6	7	PIAZZA	HILL	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$
 Correct on swer

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
2-3 & 0 & b
\end{bmatrix}$$

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5-2 & 4+b
\end{bmatrix}$$

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5-2 & 4+b
\end{bmatrix}$$

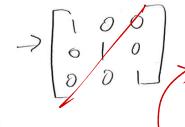
$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5+6 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5+6 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5+6 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1-1 & 1-2 \\
0 & 1 & 2 & 2 \\
0 & 5+6 & 0
\end{bmatrix}$$

 $\frac{1}{5} = \frac{1}{5} = \frac{1}$



b) The set is linearly independent b/c
the homogenous vector equation x, v, + x, v, + b, 3 v, = 0
will only how one solution.

Correct conclusion, but it comes from incorrect computations.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute
$$A^{-1}$$
.

Compute
$$A^{-1}$$
.

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
1 & 0 & 1 & | & 0 & 1 & 0 \\
0 & 2 & -1 & | & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & | & -1 & | & 0 & 0 \\
0 & 2 & -1 & | & 0 & 0 & 1
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 2 & | & 1 & 0 & 0 \\
0 & 1 & -1 & | & -1 & | & 0 \\
0 & 2 & -1 & | & 0 & 0 & 1
\end{bmatrix}$$



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^{T})^{-1} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{T})^{-1}$$
, $\begin{bmatrix} 2\\5\\2 \end{bmatrix}$ $\begin{bmatrix} -4+5+4\\6-5-4\\-2+5+2 \end{bmatrix}$ = $\begin{bmatrix} 5\\-3\\5 \end{bmatrix}$

$$(A^{T})^{-1} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 + 4 + 2 \\ 9 - 4 + 2 \\ -3 + 4 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \qquad \begin{cases} e_1 \ge \begin{bmatrix} 1 \\ 0 \end{bmatrix} & e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{cases}$$

$$\begin{bmatrix} a \end{bmatrix} \begin{bmatrix} 1 - 2 \\ 1 & 1 \\ 1 - 3 \end{bmatrix}$$

a) Find the standard matrix of
$$T$$
.

b) Find all vectors \mathbf{u} satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(e_1) = T(0)$$

$$T(e_2) = T(0)$$

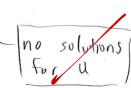
$$T(e_2) = T(0)$$

$$T(e_3) = T(0)$$

$$T(e_4) = T(0)$$

$$T(e$$

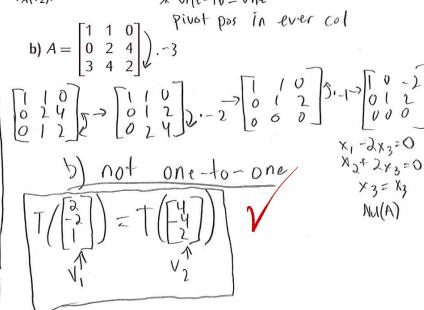
b)
$$T_{\alpha} = Au = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$
 $A = \begin{bmatrix} -2 \\ 1-3 \end{bmatrix}$





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$





6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

True Since $u \in \text{Span}(u_1v)$ for $u + w \in \text{Span}(u_1v)$ w would have to

The some combination of cu + dv where chd are

some constants.

Therefore $w \in \text{Span}(u + v)$ Why?

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent. Linearly dependent homogenous vector equation has mare than a solution. Therefore let $u = \{v, v\}$ when $\{v, v\}$ is linearly dependent of $\{v, v\}$ in this case) and $\{v, v\}$ independent of $\{v, v\}$ in this case) can only be linearly dependent. If $\{v, v\}$ is linearly independent if $\{v, v\}$ bound by linearly independent independent.



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent

Ay & Av : both = [0] which is linearly dependent

but u and v ore not linearly dependent

These vectors are actually dependent since V = 4.7.Try eg. U = [0], V = [1]instead.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u,v,w\in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

If u is in Span (v, w) it is a combination of cv+dw where c and d are some constants

so T(u) = T(cv + dw) = T(cv) + T(dw) = cT(v) + dT(w)

Which means that T(u) is a combination of the T(u) & T(u) & T(w) which is what it means to be in Span (1)

2 5 8 10 3