MTH-309T-F19-EX1-036-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Zacharias Peters

UB Person Number:

5	0	2	2	6	4	8	8
0		0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2			2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4		4	4
	5	5	5	5	5	5	5
6	6	6	6		6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8		
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $x_1v_1 + x_2v_2 + x_3v_3 = w$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\begin{array}{cccc} 2 & -3 & 0 & b \\ -2 & 2 & -2 & 4 \\ \hline 0 & -1 & -2 & b+4 \end{array} \qquad \begin{array}{cccc} 1 & 2 & 2 \\ -1 & -2 & b+4 \\ \hline 0 & 0 & b+6 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \xrightarrow{-2\text{row}(1)} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix}$$

$$\boxed{\therefore b \neq -6 \text{ for } w \in \text{Span}(v_1, v_2, v_3)}$$

b) $x_1v_1 + x_2v_2 + x_3v_3 = 0$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ \hline 1 & 0 & 3 & 0 \end{array}$$

free var $\therefore \infty$ solutions

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{-2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix} \xrightarrow{\cdot 1} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\{v_1, v_2, v_3\}$ is linearly dependent because there are infinitely many solutions when solving for the 0 vector.

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

A aug. w/ identity matrix:

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} -1$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} 2 \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} 1 \quad \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} -1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

identity matrix

A^{-1}

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

-1 0 -1 0 -1 0
1 -1 2 1 0 0
---
0 -1 1 1 -1 0

0 -2 2 2 -2 0
0 2 -1 0 0 1
---
0 0 1 2 -2 1

0 1 -1 -1 1 0
0 0 1 2 -2 1
---
0 1 0 1 -1 1

1 0 1 0 1 0
0 0 -1 -2 2 -1
---
1 0 0 -2 3 -1

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

rows $\longleftrightarrow$ columns

$$(A^T)^{-1} = (A^{-1})^T$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad (A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$A^T C = B$

$C = B(A^T)^{-1}$

$C = B(A^{-1})^T$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -2+6-3 \\ -8+15-4 \\ -6+6-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1-2+3 \\ 4-5+4 \\ 3-2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

$$v_3 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2-4+3 \\ 8-10+4 \\ 6-4+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$C = [v_1\ v_2\ v_3]$

$$\boxed{C = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $A = [T(e_1)\ \ T(e_2)]$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

b) $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$A\cdot(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] -1$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} -1$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} 1$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} 2$

$\begin{array}{ccc} x_1 & x_2 & c \end{array}$
$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = 7$

$x_2 = 3$

$$\begin{array}{ccc} -1 & 2 & -1 \\ 1 & 1 & 10 \\ \hline 0 & 3 & 9 \end{array}$$

$$\begin{array}{ccc} -1 & 2 & -1 \\ 1 & -3 & -2 \\ \hline 0 & -1 & -3 \end{array}$$

$$\begin{array}{ccc} 1 & -2 & 1 \\ 0 & 2 & 6 \\ \hline 1 & 0 & 7 \end{array}$$

$\boxed{u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} \text{ (only 1 solution)}}$

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5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ -3

$$\begin{array}{ccc} -3 & -3 & 0 \\ 3 & 4 & 4 \\ \hline 0 & 1 & 4 \end{array} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{array}{ccc} 0 & -1 & -2 \\ 0 & 1 & 4 \\ \hline 0 & 0 & 2 \end{array} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} -1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} -1$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & -1 & -2 \\ \hline 1 & 0 & -2 \end{array} \qquad \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

A has a pivot pos. in every column
$\therefore T_A(v)$ is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

A does not have a pivot pos. in every column $\therefore$
$T_A(v)$ is not one-to-one

$Nul(A) = T_A(v) = 0$

$Av = 0 \longrightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

free var.

$x_1 = 2x_3$
$x_2 = -2x_3$
$x_3 = x_3$

$x_3 = 1$: $x_1 = 2$, $x_2 = -2$, $x_3 = 1$ $\to v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$x_3 = 2$: $x_1 = 4$, $x_2 = -4$, $x_3 = 2$ $\to v_1 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

$\mathrm{Span}(u, v) = x_1 u + x_2 v$

for $w \in \mathrm{Span}(u,v)$, $w = x_1 u + x_2 v$

$w + u = x_1 u + x_2 v$

$w = (x_1 - 1)u + x_2 v$ $\therefore$ $w \in \mathrm{span}(u,v)$ because w is still a linear combination of u and v.

$\boxed{\text{True}}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

Lin. Independant: only 1 solution to $x_1 u + x_2 v + x_3 w = 0$

$0u + 0v + 0w = 0$

$\downarrow$

$0 + 0 + 0 = 0$

$\downarrow$

$0u + 0v = 0$

$\downarrow$

$0 + 0 = 0$ $\therefore$ No matter which vector is removed from the set, the set of the other two is always linearly independent.

$\boxed{\text{True}}$

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

Lin Dependent: Au, Av have ∞ solutions to $Au = 0$, $Av = 0$

$[A \mid 0] \rightarrow \infty$ solutions $\therefore$ u and v aren't necessarily linearly dependent because when any matrix is multiplied by A, the solution is a linearly dependent matrix $\therefore$ u, v are not always lin. dependent.

$\boxed{\text{False}}$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

$x_1 v + x_2 w = u \rightarrow (u \in \text{Span}(v, w))$

$\downarrow$

$T(x_1 v) + T(x_2 w) = T(u)$

$x_1 T(v) + x_2 T(w) = T(u) \rightarrow (T(u) \in \text{Span}(T(v), T(w)))$

$\boxed{\text{True}}$

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MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Michael Mu

UB Person Number:

5	0	2	6	6	9	4	8

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & 0 & 6 & b+6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$b = -6$ is the only value that would produce a solution in the span

b)
$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{bmatrix}$$

rref from A can get

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Every column doesn't have a pivot position so not linear independent

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$(A^T)^{-1} = (A^{-1})^T$$

From Problem 2: $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$$(A^T)^{-1} B = C$$

$$\begin{matrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{matrix}$$

$$(A^{-1})^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-2 \\ -1+4+3 & -2+5+2 & -3+4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 0 & -2 & -6 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

pivot position in every column, so it is one-to-one

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

every column doesn't have pivot position so not one-to-one

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - 2x_3 = 0$

$x_2 + 2x_3 = 0$

$x_3 = x_3$

if $x_3 = 1$

$x_1 - 2 = 0$

$x_1 = 2$

$x_2 + 2 = 0$

$x_2 = -2$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$T_A\left(\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}\right) = T_A\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

true

The span of (u, v) is the set of all linear combinations of u and v. If $w+u \in \text{Span}(u,v)$, that means $w+u$ is a linear combination of u and v. If we subtract a multiple of u, the result should still be obtainable from a combination of u and v since we are essentially adding $-u$ to a combination of u and v

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

true

A linearly independent set means u v and w all cannot be produced from a linear combination of the other elements in the set. This means any combination of any multiples of u and w cannot produce v. If this is the case, we still cannot produce V from any multiples of just u. The same logic applies in the opposite case of v producing u.

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

false

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$u = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ & $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are linearly independent

$Au = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ & $Av = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are linearly dependent

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

true

If u is in the Span(v, w), then it can be found through a combination of multiples of v and w, $c_1 v + c_2 w = u$

For linear transformations, $T(u) = T(c_1 v + c_2 w) = T(c_1 v) + T(c_2 w) = c_1 T(v) + c_2 T(w)$

This means that the transformation of u can be solved as a linear combination of the multiples of $T(v)$ and $T(w)$.

This means that $T(u)$ must be in the Span $(T(v), T(w))$

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Liam Carr

UB Person Number:

5	0	3	2	0	2	5	3
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a) $\begin{matrix} 1 & -1 & 1 & \cdot \\ 0 & 1 & 2 & \cdot \\ 2 & -3 & 0 & b \end{matrix}$ $(-\infty, \infty)$ - because V_1, V_2, V_3 can be multiplied by an intitie amount of different scalars

$$V_1 \begin{vmatrix} 1 \\ 0 \\ 2 \end{vmatrix} + V_2 \begin{vmatrix} -1 \\ 1 \\ -3 \end{vmatrix} + V_3 \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} = \begin{matrix} -2 \\ 2 \\ b \end{matrix}$$

(B) $V_1 \begin{vmatrix} 1 \\ 0 \\ 2 \end{vmatrix} + V_2 \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + V_3 \begin{vmatrix} 1 \\ 2 \\ 0 \end{vmatrix} \rightarrow$ the set $\{V_1, V_2, V_3\}$ is not linearly independant because V_1, V_2, and V_3 are scalar multiples of each other and therefore a vector other than $\{0\}$ can be in thier Nul space

$$\left|\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{matrix}\right.$$

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$\left|\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right| \to \left|\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right| \to \left|\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right|$$

$R_1 \times -1 + R_2$

$R_2 \times -2 + R_3$

$R_3 \times 1 + R_2$

$$\left|\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right| \to \left|\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right| \to \left|\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right|$$

$R_2 \times 1 + R_1$

$R_3 \times -2 + R_1$

$$A^{-1} = \begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$$(A^T)^{-1} = (A^{-1})^T$$

$$A^T A^T C = B \rightarrow C = B(A^T)^{-1}$$

$$C = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix} \cdot \begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} (-2+2+6), (3-2-6), (-1+2+3) \\ (-8+5+8), (12-5-8), (-4+5+4) \\ (-6+2+2), (9-2-2), (-3+2+1) \end{vmatrix}$$

$$\begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ -2 & 5 & 0 \end{vmatrix}$$

$$C = \begin{vmatrix} 6 & -5 & 4 \\ 5 & -1 & 5 \\ \cancel{-2} & 5 & 0 \\ -2 & & \end{vmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $\begin{vmatrix} 1 & 2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 1 & -3 & -2 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & -5 & -3 \end{vmatrix}$

$R_1 \times -1 + R_2 + R_2 \times -1$ $\quad R_1 \times -1 + R_3$ $\quad R_2 \times 5 + R_3$

$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & 0 & -48 \end{vmatrix} \to \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

There are no vectors (u) that satisfy

$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{vmatrix}$$

$R_1 \times -3 + R_3$ $R_2 \times \frac{1}{2}$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{vmatrix} \quad \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \checkmark$$

$R_2 \times -1$ $R_3 \times \frac{1}{2}$

a) A is one to one because it has a pivot column in every column and the Null of A is equal to zero

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

* $Nul(A) = 0$ proof

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix} \to \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix}$$

$R_1 \times -3 + R_3$ R_2

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{vmatrix} \to$$ b) $T_A(v) = Av$

it's not one to one because it doesn't have a pivot in every column and it has infinite solutions for it's Nul space

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \begin{vmatrix} 0 \\ 2 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} 2 \\ 8 \\ 10 \end{vmatrix} = \begin{vmatrix} 2 \\ 6 \\ 9 \end{vmatrix}$$

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6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

False, counter example

$w = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\quad u = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\quad v = \begin{bmatrix}0\\1\\1\end{bmatrix}$

$\begin{bmatrix}1\\0\\0\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix}$ but $\begin{bmatrix}1\\0\\0\end{bmatrix} \neq \begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}0\\1\\1\end{bmatrix}$

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

False - Counter example

True, because something in linerly independent when the vectors are not scalar multiples of each other, and taking out one vector of a set wont make it that the other 2 are now scalar multpoles of each other.

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

False

$$u = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \quad v = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad A = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$$A \cdot u = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \quad A \cdot v = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

False

True, because if $u = c_1 v + c_2 w$ then $T(u) = c_1 T(v) + c_2 T(w)$ because the transfomation is linear

MTH-309T-F19-EX1-028-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Brendan DesRosiers

UB Person Number:

5	0	1	8	6	5	8	4
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) $\text{Span}(v_1, v_2, v_3) = \left\{ \begin{matrix} \text{the set of all} \\ \text{vectors } c_1v_1 + c_2v_2 + c_3v_3 \end{matrix} \right\}$

$$x_1v_1 + x_2v_2 + x_3v_3 = w$$

$$\begin{array}{r} \\ R_3 = -2R_1 + R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

$$\begin{array}{r} \\ R_3 = +R_2 + R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b \end{array}\right]$$

$$\begin{array}{r} \\ R_1 = R_2 + R_1 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array}\right]$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{array}\right]$$

$\therefore$ b must equal 0

$$\begin{cases} x_1 = -3x_3 \\ x_2 = -2x_3 + 2 \\ x_3 = x_3 \end{cases}$$

b) the set $\{v_1, v_2, v_3\}$ is not linearly independent because every column of the matrix is not a pivot column

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$R_2 = R_1 - R_2$ $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$

$\downarrow$

$R_2 = -R_2$ $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$

$\downarrow$

$R_3 = -2R_1 + R_3$ $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$

$\downarrow$

$R_1 = R_2 + R_1$ $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$

$\downarrow$

$R_2 = R_3 + R_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$

$\downarrow$

$R_1 = -R_3 + R_1$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \Longrightarrow A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$

$A^T C = B$

Matrix Algebra Property: $(A^T)^{-1} = (A^{-1})^T$

$C = (A^T)^{-1} \cdot B$

$(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

$C = \underset{3\times 3}{\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}} \cdot \underset{3\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}}$

defined with

$C = 3\times 3$

$$C = \begin{bmatrix} -2(1)+1(4)+2(3) & -2(2)+1(5)+2(2) & -2(3)+1(4)+2(1) \\ 3(1)+(-1)(4)+(-2)(3) & 3(2)+(-1)(5)+(-2)(2) & 3(3)+(-1)(4)+(-2)(2) \\ 0(1)+1(4)+1(3) & 0(2)+1(5)+1(2) & 0(3)+1(4)+1(1) \end{bmatrix}$$

$$C = \begin{bmatrix} -2+4+6 & -4+5+4 & -6+4+2 \\ 3-4-6 & 6-5-4 & 9-4-4 \\ 4+3 & 5+2 & 4+1 \end{bmatrix}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 1 \\ 7 & 7 & 5 \end{bmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a)

$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$R_2 = -R_1 + R_2$
$R_3 = -R_1 + R_3$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right]$

$\downarrow$

$R_3 = -R_3$ $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{array}\right]$

$u = \begin{bmatrix} 2x_2 + 1 \\ -3x_2 + 9 \\ -x_2 + 3 \end{bmatrix}$

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ **b)** $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$R_3 = -3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_2 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_3 = -2R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$\downarrow$

$R_3 = -\frac{1}{4}R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right]$

$\downarrow$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

T_A is one-to-one because there is a pivot position in every column

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$R_3 = -3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 3 & 4 & 2 & 0 \end{array}\right]$

$R_2 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$R_3 = -2R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right]$

$R_1 = -R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

T_A is not one-to-one because only columns 1 and 2 have pivot positions

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\begin{cases} x_1 = 2x_3 \\ x_2 = -2x_3 \\ x_3 = x_3 \end{cases}$

• For v_1, $x_3 = 1$

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

• For v_2, $x_3 = 2$

$v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

$$u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad w = \begin{bmatrix} z_1 \\ z_1 \\ z_3 \end{bmatrix}$$

TRUE because if $w + u$ is in Span(u, v) then w will also be in Span(u, v) because they are linear combinations of one another

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$$u = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad v = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad w = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

- Since the set is linearly independent the matrix would have to be

$$\begin{bmatrix} 1 & b_1 & c_1 \\ a_2 & 1 & c_2 \\ a_3 & b_3 & 1 \end{bmatrix}$$

in order for every column to have a pivot column

set $\{u, v\}$

$$\begin{bmatrix} 1 & b_1 \\ a_2 & 1 \\ a_3 & b_3 \end{bmatrix}$$

this set is still linearly independent so the statement is TRUE

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$$A = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}, \quad u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad v = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Au and Av are LD

No matter what the vectors u, v
in $\mathbb{R}^2$ are they will have a
pivot column so statement
is FALSE

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span$(\mathbf{v}, \mathbf{w})$ then $T(\mathbf{u})$ must be in Span$(T(\mathbf{v}), T(\mathbf{w}))$.

TRUE because the properties of linear
transformations tells us that if u is
in the Span(v, w) then T(u) has
to be in the Span (T(v), T(w))

MTH-309T-F19-EX1-057-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

Emery Comstock

UB Person Number:

5	0	2	5	2	7	7	2
0	●	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	●	2	●	2	2	●
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
●	5	5	●	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	●	●	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left(\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = w\right) = \begin{array}{cccc} x_1 & x_2 & x_3 & c \\ \hline 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array} \; -2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 2+b \end{array}\right] \begin{matrix} 2 \\ 1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2+b \end{array}\right] \qquad \begin{aligned} x_1 &= -3x_3 \qquad x_3 = x_3 \\ x_2 &= 2 - 2x_3 \\ 0 &= 2+b \\ b &= -2 \end{aligned}$$

a) $\boxed{b = -2}$

b) Linearly $\boxed{\text{dependent}}$ because x_3 is a free variable in the matrix $[x_1v_1 \; x_2v_2 \; x_3v_3]$ resulting in a null space with infinite solutions

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$m = n$ ✓

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] -1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right] -2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] 1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] -1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

$A^T \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = B$ $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix}$

$c_{11} = 0.5$ $4 = 2c_{12} - c_{12}$ $4 = c_{12}$ $3 = 2c_{13} + \cancel{c_{13}} - \cancel{c_{13}}$

$\frac{3}{2} = c_{13}$

$c_1 = \begin{bmatrix} 0.5 \\ 4 \\ 1.5 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{21} \\ c_{22} \\ c_{23} \end{bmatrix}$ $2 = 2c_{21}$ $c_{21} = 1$

$5 = 1c_{22}$ $c_{22} = 5$

$2 = 2c_{23}$ $c_{23} = 1$

$c_2 = \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{31} \\ c_{32} \\ c_{33} \end{bmatrix}$ $3 = 2c_{31}$ $c_{31} = 1.5$

$4 = 1c_{32}$ $c_{32} = 4$

$1 = 2c_{33}$ $c_{33} = 0.5$

$c_3 = \begin{bmatrix} 1.5 \\ 4 \\ 0.5 \end{bmatrix}$

$$\boxed{C = \begin{bmatrix} 0.5 & 1 & 1.5 \\ 4 & 5 & 4 \\ 1.5 & 1 & 0.5 \end{bmatrix}}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

Standard Matrix of $T = \left[\, T(e_1) \;\; T(e_2) \,\right]$

$$\left[T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \right]$$

a) $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$

b) $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 11 \\ 0 & -1 & -3 \end{bmatrix} (-1)$

$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 11 \end{bmatrix} \xrightarrow{-3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

b) **No vectors** satisfy that condition

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} -3 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{bmatrix} (\tfrac{1}{2})$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} 2 -1 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} (\tfrac{1}{2})$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\text{a) One to one}}$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} -3 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix} (\tfrac{1}{2})$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} 2 -1 \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Not one to one

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] -1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{aligned} x_1 &= 2x_3 \\ x_2 &= -2x_3 \\ x_3 &= x_3 \end{aligned}$$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} x_3$$

b) $\boxed{\text{Not one to one}}$

$\boxed{v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}}$

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \text{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \text{Span}(\mathbf{u}, \mathbf{v})$.

True, vectors encompassed by a ~~elements of a~~ span are linear combinations of the vectors in that span, so ~~the~~ the linear combination of some vector and a vector in some span can only be encompassed by that span if the other vector is also a linear combination of the vectors in the span.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

~~False~~

$$C_1 U + C_2 V + C_3 W = 0$$

$$C_1 U + C_2 V = 0$$

~~False~~ ~~[illegible]~~

True, because if $\{U, V\}$ is not linearly dependent then $\{U, V, W\}$ cannot be linearly independent as multiplying W by a constant of zero would yield all solutions of $\{U, V\}$

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, Au and Av can be seen as ~~linear~~ ~~linear transformations where $T_A(u)$ ... $T_A(v)$ = Av and ...~~ ~~So if~~ multiplying by the same matrix causes a linear transformation on the solutions but will not modify the amount of solutions

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, because ~~linear transformations ensure that the transformation of the sum of 2 vectors~~

$\text{Span}(V, W) = c_1V + c_2W$

if: $U = c_1V + c_2W$

then:

$T(U) = T(c_1V + c_2W) =$

$T(U) = T(c_1V) + T(c_2W)$

$\text{Span}(T(v), T(w)) = c_1T(v) + c_2(T(w)) = T(c_1v) + T(c_2w)$

✓

MTH-309T-F19-EX1-060-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

David Flores

UB Person Number:

5	0	2	3	6	3	8	7
0	●	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	●	2	2	2	2	2
3	3	3	●	3	●	3	3
4	4	4	4	4	4	4	4
●	5	5	5	5	5	5	5
6	6	6	6	●	6	6	6
7	7	7	7	7	7	7	●
8	8	8	8	8	8	●	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) (-2) $+$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right]$$

(-1)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+2 \end{array}\right]$$

$b = -2$ or else no solution

b) Set $\{v_1, v_2, v_3\}$ is linearly dependent because if $b = -2$, and since there is no leading one (pivot position) in third column, then there is a free variable x_3 meaning infinite solutions $\Rightarrow$ linearly dependent

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$$A = \begin{array}{c} (-1) \\ + \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ (-2) \\ + \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ \\ + \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ + \\ (-2) \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\begin{array}{c} \\ + \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 2 & 4 & 5 & 4 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$$(-2)\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 2 & 1 & -1 & 3 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & -1 & -1 & 1 & -2 & -5 \end{array}\right]$$

$$-2\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 5 & 7 & 7 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$(-1)\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 5 & 0 \\ 0 & 1 & 0 & -7 & -3 & 3 \\ 0 & 0 & 1 & 6 & 5 & 2 \end{array}\right] \rightarrow C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

a) $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard matrix A:

$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$$\begin{array}{c c} & \begin{array}{ccc} u_1 & u_2 & \end{array} \\ \begin{array}{c} (-1) \\ + \\ \end{array} & \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array}\right] \end{array}$$

$$\begin{array}{c} \\ (1/3) \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array}\right]$$

$$\begin{array}{c} (-1) \\ \\ + \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{array}\right]$$

$$\begin{array}{c} \\ + \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array}\right]$$

$$\begin{array}{c} \\ (2) \\ \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

MTH-309T-F19-EX1-060-P05

5. **(20 points)** For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix} (1/2)$ b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} (1/2)$

a)

$$(-3)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix} \quad \begin{array}{c} \\ (-1) \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{array}{c} \\ \\ (1/2) \end{array}$$

$$\begin{array}{c} \\ \\ (-2) \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{c} \\ -1 \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

One-to-one

b)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix} \quad \begin{array}{c} \\ (-1) \\ \\ \end{array}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

not one-to-one

b) $T_A(v_1) = T_A(v_2)$

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

Try

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

$Av_1 = Av_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2-2+0 \\ 0-4+4 \\ 6-8+2 \end{bmatrix} = \begin{bmatrix} 4-4+0 \\ 0-8+8 \\ 12-16+4 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

6. **(10 points)** For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \mathrm{Span}(\mathbf{u}, \mathbf{v})$.

True, if a vector is in span of other vectors when modified, then that vector is in span itself.

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

True, in $\mathbb{R}^3$ if $\{u, v, w\}$ has one particular solution, then the vectors are linearly independent with one another always.

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

True, if Au, Av are linearly dependent, then

u, v are always zero vectors, which are linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v}), T(\mathbf{w})$).

True, linear transformation is in span of other

trans. if the vector of lin. trans is in span of

other vectors.