



MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	9	0	3	2	6
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

10

10

10

20

20

4

2

2

0

78

B

1

2

3

4

5

6

7

PIAZZA

HILL

TOTAL

GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
 b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

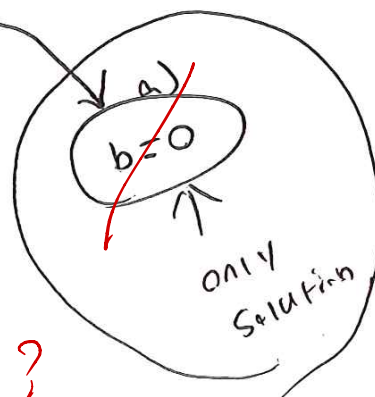
$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b \end{bmatrix}$$

No solution for $\begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$ in span



?



reduces matrix

linearly dependent

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) No linearly dependent

why?



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Reduction

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Check:

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$



12
34
56

135
246

3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

~~Matrix A~~

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{T^{-1}} \cdot B = C \quad \checkmark$$

$$A^{T^{-1}} = \begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} -2+4+6 &= -7 \\ 3-4-6 &= -7 \\ -1+4+3 &= 6 \end{aligned}$$

$$\begin{bmatrix} -2 & 1 & 2 \\ 3 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 0 \\ 2 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

$$\begin{aligned} -4+5+4 &= 5 \\ 6-5-4 &= -3 \\ -2+5+2 &= 5 \end{aligned}$$

$$C = \begin{bmatrix} 8 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix} \quad \checkmark$$

$$\begin{aligned} -6+4+2 &= 0 \\ 9-4-2 &= 3 \\ -3+4+1 &= 2 \end{aligned}$$



4. (20 points) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation given by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T .

b) Find all vectors u satisfying $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.



a) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$ ✓



$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

b) $x_1 = 7$
 $x_2 = 3$
 $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$ ✓





5. (20 points) For each matrix A given below determine if the matrix transformation $T_A: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T_A(v) = Av$ is one-to-one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(v_1) = T_A(v_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 4 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 0 \text{ Pivot} \\ 0 & 1 & 0 \text{ Pivot} \\ 0 & 0 & 1 \text{ Pivot} \end{array}$$

one to one

~~one to one~~

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{array}$$

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}$$

Not Pivot

Not one-to-one

$$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$$

$$\begin{array}{ccc} 0 & & \\ 0 & & \\ 0 & & \end{array}$$

$$\begin{array}{l} x_1 = 2x_3 \\ x_2 = -2x_3 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} u & v & w \end{matrix}$$

$$w + u \in \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

True, ✓
if double plus
u kept it the
Span of u, v ,
the w by itself
should be in the
Span.
?

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

True, ✓

because if, $\{u, v, w\}$
was linearly independent, then
making w zero didn't change it
to be dependent, so removing
 w entirely changes nothing.
?
Actually the set $\{u, v, 0\}$
would be linearly dependent

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

True, the ^{linear} transformation must
be one to one and onto, so
the span would be kept intact. | ?