**1. (20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & 3 & 0 & b \end{array}\right] \xrightarrow{\cdot(-2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 5 & 0 & b-4 \end{array}\right] \xrightarrow{(-5)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -10 & b-14 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & \frac{b-14}{-10} \end{array}\right]$$

$b = 4$

$Ax = b \rightarrow Av = b$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = w$

b) a set is linearly independent if it has only one solution since there is a pivot position in every column it is linearly independent

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**a)** Find all values of $b$ such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \quad R_3 = -2(R_1) + R_3$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \quad R_3 + R_2 + R_3$$

$$\left[\begin{array}{cccc} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{array}\right] \quad R_1 = R_2 + R_1$$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix} \quad R_2 = R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 6+b \end{bmatrix}$$

b) The solution is linear independent. The last row is 0 and because of that there are multiple solutions. Also it cant be dependent because every column must be a pivot.

in order for it be

1. (20 points) Consider the following vectors in $\mathbb{R}^3$:

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a) Find all values of $b$ such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) as $w \in$ span $(v_1, v_2, v_3)$ $\quad \therefore v_1 + v_2 + v_3 = w$

$$-4 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

$\therefore b = (-8)$.

b) $c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

① $c_1 - c_2 + c_3 = 0 \quad \therefore c_1 = c_2 - c_3 \quad \therefore c_3 = 0 \; [\because c_2 = 0$

② $c_2 + 2c_3 = 0 \quad \therefore c_3 = -\frac{c_2}{2}$

③ $2c_1 - 3c_2 = 0 \quad \therefore c_1 = \frac{3c_2}{2} \quad \therefore c_1 = 0 \; [\because c_1 = 0$

$\therefore c_2 - c_3 = \frac{3c_2}{2} \quad \therefore 2c_2 + 2\left(\frac{c_2}{2}\right) = 3c_2 \quad \therefore 3c_2 = 3c_2$

$\therefore 2c_2 - 2c_3 = 3c_2 \quad \therefore c_2 = 0$

as, $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ so, we can say the set $\{v_1, v_2, v_3\}$ is linearcly independent.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**1. (20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of $b$ such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

b) Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a). $\begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free}} & b+6 \end{bmatrix}$

$(-3)(-2)$ $\quad 0+(-1)\cdot 2 \quad$ $b+(-2)(2)$

when $b+6=0$
$b=-6$. so $w \in \mathrm{Span}(v_1, v_2, v_3)$

~~matrix~~ it has infinite solution. with free variable $x_3$.

b). no, for set $\{v_1\ v_2\ v_3\}$ after row reduced.
we got $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ the third column is not a pivot column.
which ~~meath~~ means it is not a linearly independent set.

for $v_1\ v_2\ v_3$. $\cancel{x_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 0}$.

because it has free variable.
have infinite solution.
so not linear indep.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

**b)** Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\begin{array}{cccc} v_1 & v_2 & v_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_1} \begin{matrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{matrix}$$

$$R_1 = R_1 + R_2 \quad \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & (4+b) \end{bmatrix} \qquad R_3 = R_3 + R_2 \quad \begin{vmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & (6+b) \end{vmatrix}$$

a. $w \in \mathrm{Span}(v_1, v_2, v_3)$ when $b = -6$

b. $\begin{vmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{vmatrix} \rightarrow \quad 1 \quad -1$

The set is not linearly independent, because the $V_3$ column is a free ~~fa~~ variable, so there is not a leading one in each column

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**1.** **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

**b)** Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

a) $x_1 v_1 + x_2 v_2 + x_3 v_3 = w$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] +\cdot(1) \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 1 & -2 & -1 & 2+b \end{array}\right] +\cdot(-1)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & -2-b \end{array}\right] +\cdot 1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 0 & 0 & -b \\ 0 & -1 & -2 & -2-b \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & -1 & -2 & -2-b \\ 0 & 0 & 0 & -b \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & b \\ 0 & 1 & 2 & 2+b \\ 0 & 0 & 0 & b \end{array}\right]$$

b) The set $\{v_1, v_2, v_3\}$ is <u>not</u> lin. independent; it has more than one, trivial solution.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**1.** **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$c_1v_1 + c_2v_2 + c_3v_3 = w$

a) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} V_1 + \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} V_2 + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{②+①} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \xrightarrow{①(-2)+③} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & b \end{array}\right] \xrightarrow{②(3)+③} \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free variable}} & b+6 \end{array}\right]$$

$\boxed{b = -6}$

b) $x_1 = -3x_3$

$x_2 = 2 - 2x_3$

$x_3 = \text{free}$

The set $\{V_1, V_2, V_3\}$ is not linearly independent. This is because $x_3$ is a free variable, therefore the set has infinite solutions (and is linearly dependent.)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

**1. (20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $w \in \mathrm{Span}(v_1, v_2, v_3)$.

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$b \neq 0$

$b > 0$

$b < 0$

b) b is linearly independent as the reduced form has exactly one solution when $A = 0$

$$\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -3 & 0 \end{array}$$

$$\begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{array} \qquad \begin{array}{ccc} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{array} \qquad \begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{array} \qquad \begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{array} \qquad \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{array}$$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $\mathbf{w} \in \mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

**b)** Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly independent? Justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & 4+b \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 5+b \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & \underbrace{0}_{\text{free var}} & 5+b \end{array}\right]$$

~~Must equal~~

$\boxed{\text{a) } b = -5}$

b) The set is $\boxed{\text{linearly dependent}}$ because there is not a pivot column in every column

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |

**1. (20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $w \in \text{Span}(v_1, v_2, v_3)$.

$b_1 v_1 + b_2 v_2 + b_3 v_3 = w$

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

a) Augmented matrix

$$-2 \cdot \begin{bmatrix} 1 & -1 & 1 & | & -2 \\ 0 & 1 & 2 & | & 2 \\ 2 & -3 & 0 & | & b \end{bmatrix}$$

for $w \in \text{Span}(v_1, v_2, v_3)$

$b + 6 = 0$

$b = -6$

$$1 \cdot \begin{bmatrix} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 0 & -1 & -2 & b+4 \end{bmatrix}$$

$$1 \cdot \begin{bmatrix} 1 & -1 & 1 & | & -2 \\ 0 & 1 & 2 & | & 2 \\ 0 & 0 & 0 & | & b+6 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 0 & 3 & | & 0 \\ 0 & 1 & 2 & | & 2 \\ 0 & 0 & 0 & | & b+6 \end{bmatrix}$$

reduce row echelont form

null space

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

free variable

b) $[v_1, v_2, v_3]$ is not linearly independent because it's reduce row echelon form has a free variable so the null space has more then one trivial solution (it has insinitly many solutions), meaning that it is linearly dependent. (linearly independent vectors would have only one trivial solution to the null space which would be the 0 vector.

1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

**a)** Find all values of $b$ such that $w \in \text{Span}(v_1, v_2, v_3)$.

**b)** Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

$$\begin{array}{c} +R2 \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] = \begin{array}{c} \\ \\ -2R1 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 2 & -3 & 0 & b \end{array}\right] = \begin{array}{c} \\ \\ +3R2 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & -3 & -6 & 6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & b+6 \end{array}\right]$$

$x1 = -3x3$

$x2 = 2 - 2x3$

$x3 = 0 = b+6$

$\underline{b = -6}$

b) Yes, it is linearly independent because v1, v2, v3 are not multiples of each other. There is no way to get anything but 0 in second row of v1 and therefore we can't get such multiple of v1 so that we obtain v2, v3.

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ | ☐ |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |