$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then $2v$ is an eigenvector of A corresponding to the eigenvalue 2λ.

+5 ✓ False, while you can multiply eigenvectors to scale them they still correspond to the original eigenvalue, λ.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V w = -w$ then w must be the zero vector. True, it is impossible for the projection to return -w on a non-zero vector. +1 2

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix. True, properties of orthonormal vectors $v_i \cdot v_j = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$ +1 2.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

+5 d) True, If A and B are orthogonally diagonalizable then they have to be symmetric matrices

A+B will still be symmetric as adding B top of diagonal will cancel out with Bs bottom of diagonal.

Since (A+B) is symmetric it is orthogonally diagonalizable

a)

$\det\left(\begin{bmatrix} 3-\lambda & 2 \\ 1 & 8-\lambda \end{bmatrix}\right)$

$(3-\lambda)(8-\lambda) - 2$

$24 - 11\lambda + \lambda^2$

$-8 \quad -3$

$(\lambda - 8)(\lambda - 3) = 0$

$\lambda = 8 \quad \lambda = 3$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\frac{-2}{2}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + \frac{0}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

$1/9 + 4/9 + 4/9 = 1$ $\quad 2/9 + 2/9 - 4/9 = 0$ $\quad 2/9 - 4/9 + 2/9 = 0$

$2/9 + 2/9 - 4/9 = 0$ $\quad 4/9 + 1/9 + 4/9 = 1$ $\quad 4/9 - 2/9 - 2/9 = 0$

$2/9 - 4/9 + 2/9 = 0$ $\quad 4/9 - 2/9 - 2/9 = 0$ $\quad 4/9 + 4/9 + 1/9 = 1$