4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

basis of $(A-3I) = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2,\ \frac{1}{2}R_3]{\frac{1}{2}R_1} \begin{bmatrix} -1 & 4 & 2 \\ -1 & 4 & 2 \\ 1 & -4 & -2 \end{bmatrix} \xrightarrow{R_3+R_1} \begin{bmatrix} -1 & 4 & 2 \\ -1 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{R_2-R_1} \begin{bmatrix} -1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1R_1} \begin{bmatrix} 1 & -4 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow x_1 - 4x_2 - 2x_3 = 0 \rightarrow x_1 = 4x_2 + 2x_3$

$x_2 = x_2$

$x_3 = x_3$

$\rightarrow x = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad \rightarrow \text{basis} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

basis of $(A-5I) = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \xrightarrow[\frac{1}{2}R_2,\ \frac{1}{2}R_3]{\frac{1}{4}R_1} \begin{bmatrix} -1 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & -4 & -3 \end{bmatrix} \xrightarrow[R_3+R_1]{R_2-R_1} \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_3} \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\xrightarrow[R_1-2R_2]{R_3-R_2} \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{-1R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \\ x_3 = x_3 \end{matrix} \rightarrow \begin{matrix} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 = x_3 \end{matrix}$

$\rightarrow x = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad \rightarrow \text{basis} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

so $\boxed{P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} + D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}}$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20