MTH-309T-F19-EX1-029-P00

MTH 309T LINEAR ALGEBRA

EXAM 1

October 3, 2019

Name:

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UB Person Number:

5	0	2	5	8	5	3	9
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9

Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

1	2	3	4	5	6	7	TOTAL	GRADE
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1. **(20 points)** Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

The value of b is -6

because $2v_2 = \begin{bmatrix} -2 \\ 2 \\ -6 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & -3 & 0 & 0 \end{bmatrix}$

It is independent because Null(A) is {0}

2-
-3-2

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -5 & -2 & 0 \end{matrix}$

~~Yes it is linearly independent?~~

-5+5
-2+10

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 8 & 0 \end{matrix}$

$x_1 = 0$
$x_2 = -2$
$x_3 = 0$

~~It is linearly independent because x_2 can be when set to 0~~

$\begin{matrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$ $\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & \\ 0 & 0 & 1 & 0 \end{matrix}$

$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

2. **(10 points)** Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1}.

$1-1 \quad 0--1 \quad 1-2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{cccccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & +2 & -2 & 0 \end{array}$$

$$\begin{array}{cccccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array}$$

-2

$2-2 \quad 2-2$

$-1+{+2} \quad 2 \cdot \quad -1-2$

$1-1$

$1-2$

$0-2$

$$A^{-1} = \begin{bmatrix} -2, 3, -1 \\ 1, -1, 1 \\ 2, -2, 1 \end{bmatrix}$$

0 1 2 3 4 5 6 7 8 9 10

3. **(10 points)** Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^T C = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 0 & 2 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix} \quad x_1 = 0,\ x_2 = 3,\ x_3 = 7$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$B = [$

$x_1 = x_2\ 1 - x_2$

$x_2 = 5 - x_3$

$x_3 = 7$

$5 - 7 = -2$

$1 + 2 = 1$

1
-2
7

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

0 2
4 1
2 0

$3\times3 \quad 3\times3 = 3\times3$

3 -2 0 1
-3 0 2 3
6 -2 -1 = 3

3
0
1

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

1 1 0 2
0 1 2 7
0 -1 1 -2

1 1 0 2 $x_1 = 2$
0 1 2 7 $x_2 = 5$
0 0 3 5 x_3

$$x_1\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_3\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad x_1\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$x_1\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} x_2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

2 0 2
-2 6 = 4
4 -2 -2

2
-3
6

7
-2
9

1 1
-1 6 = 5
2 3 5

4. **(20 points)** Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.

b) Find all vectors $\mathbf{u}$ satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

A) $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$\begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \end{bmatrix}$ $\begin{bmatrix} 3 \end{bmatrix}$

2×1 2×1

2×3 3×1 = 2×1

2×3

3×2 2×1

3 - 1

7-3

7-

B) $v = c_p \begin{bmatrix} 3 \\ 7 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

Anything multiple of this vector.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

5. (20 points) For each matrix A given below determine if the matrix transformation $T_A\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathrm{v}) = A\mathrm{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors v_1 and v_2 such that $T_A(\mathrm{v}_1) = T_A(\mathrm{v}_2)$.

a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 4 & 4 \end{bmatrix}$$

4-3 $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$

4-2
$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

This is ~~onto~~ one to one because there is a pivot in every ~~row~~ col.

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{matrix}$$

$x_1 = -x_2$
$x_2 = -2x_3$
$x_3 = \text{free}$

2
-2
1

-4
2

This isn't one to one cuse there isn't a pivot in every col.

$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ $v_2 \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{w} + \mathbf{u} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$ then $\mathbf{w} \in \operatorname{Span}(\mathbf{u}, \mathbf{v})$.

$U = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$

$W = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \operatorname{span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$

The statement is true

False

b) If $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are vectors in $\mathbb{R}^3$ such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent then the set $\{\mathbf{u}, \mathbf{v}\}$ must be linearly independent.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$

~~This is false because just because~~

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This is false beause just becuse

$u + v + w = 0$

doesn't men

$u + v = 0$

0 1 2 3 4 5 6 7 8 9 10

7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and $\mathbf{u}, \mathbf{v}$ are vectors in $\mathbb{R}^2$ such that $A\mathbf{u}$, $A\mathbf{v}$ are linearly dependent then $\mathbf{u}, \mathbf{v}$ also must be linearly dependent.

$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

True because if you have inf solution and then transform it is still the same.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ are vectors such that $\mathbf{u}$ is in Span($\mathbf{v}, \mathbf{w}$) then $T(\mathbf{u})$ must be in Span($T(\mathbf{v})$, $T(\mathbf{w})$). False

~~False~~

True

$u = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ Span$\left(\begin{bmatrix} 7 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$

$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$ $\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix}\right)$

2×1 1×1

2×1

2+2

True because of the example of a liner transformation of [2]

0 1 2 3 4 5 6 7 8 9 10