

3. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$\begin{matrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} & \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \\ A & b \end{matrix}$$

$$A^T A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A^T A \begin{bmatrix} a \\ b \end{bmatrix} = A^T b$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array} \right] \cdot \left(\frac{1}{6} \right) \rightarrow \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 2 & 3 & 3 \end{array} \right] \downarrow (-2) \rightarrow$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 7/3 & 3 \end{array} \right] \cdot (3/7) \rightarrow \left[\begin{array}{cc|c} 1 & 1/3 & 0 \\ 0 & 1 & 9/7 \end{array} \right] \uparrow (-1/3) \rightarrow$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & -3/7 \\ 0 & 1 & 9/7 \end{array} \right]$$

$$a = -3/7$$

$$b = 9/7$$

$$\boxed{f(x) = -\frac{3}{7}x + \frac{9}{7}}$$