



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} = \frac{1}{|A|} \cdot A^T$$

$$\begin{aligned} |A| &= 1 \cdot [0(-1) - 2 \cdot 1] - (-1)[(1)(-1) - 1 \cdot 0] + 2[1 \cdot 2 - 0 \cdot 0] \\ &= -2 - 1 + 4 = 1 \end{aligned}$$

DETERMINANT A $\Rightarrow |A|=1$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

TO VERIFY $A \cdot A^{-1} = I$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1+1+2 & 1+0+2 & 0-2-2 \\ 1+0+2 & 1+0+1 & 0+0-1 \\ 0-2-2 & 0+0-1 & 0+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & -4 \\ 3 & 2 & -1 \\ -4 & -1 & 5 \end{bmatrix} \xrightarrow[\text{(ON SCRAP PAPER)}]{\text{ROW REDUCTION}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$



2. (10 points) Consider the following matrix:

let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{R}^3$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left[A \mid e_1 e_2 e_3 \right] = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

↓

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right]$$



$$\begin{aligned} (-1)(-2) &= 2 & 2(-1)+1 &= \\ -2(1) &= -2 & -2 \end{aligned}$$

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$(-1)(-2) = 2 - 1 = 1$$

Compute A^{-1} .

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right| \xrightarrow{\text{R}_2 = R_1 + R_2} \left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right| \xrightarrow{\text{R}_3 = -2(R_2) + R_3} \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right|$$

$$R_2 = R_1 + R_2$$

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right| \xrightarrow{R_1 = 2R_2 + R_1} \left| \begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right| \xrightarrow{R_2 = R_3 + R_2} \left| \begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right| \xrightarrow{R_1 = -R_2 + R_1} \left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right|$$

$$R_1 = -R_2 + R_1$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

?

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1+R_2 \\ \frac{1}{2}R_3}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array} \right] \xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -1 & \frac{1}{2} \end{array} \right]$$

$$\xrightarrow{2R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\substack{R_3 \leftrightarrow R_2 \\ -2R_3+R_1}} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 & -2 & 1 \end{array} \right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \quad \checkmark$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$(-1) \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} (-2+1) \\ (-1+1) \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$(1) \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\boxed{\left[\begin{array}{ccc|ccc} -2 & 3 & -1 & & & \\ 1 & -1 & 1 & & & \\ 2 & -2 & 1 & & & \end{array} \right] = A^{-1}}$$

$$\begin{array}{l} (-1+2) \\ (-1+1) \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$(-1) \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\begin{array}{l} (-2) \\ (-2) \end{array} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(-1)\cdot r_1 + r_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\
 \xrightarrow{r_2 \leftrightarrow r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(-2)\cdot r_2 + r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\
 \xrightarrow{r_3 + r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{-r_3 + r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]
 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .



2. (10 points) Consider the following matrix:

A^T or A^{-1} ?
Compute A^{-1} .

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

3×3

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$

3×3



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_2 = r_2 - r_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_1 = r_1 + r_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{r_3 = r_3 - 2r_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{r_1 = r_1 - r_3, r_2 = r_2 + r_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

A^{-1}



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -1}$$



$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -2}$$



$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot 1} \xrightarrow{-2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -2 & 1 & \\ 0 & 0 & 1 & 2 & -2 & 1 & \end{array} \right] \xrightarrow{\cdot 1}$$



$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -2 & 1 & \\ 0 & 0 & 1 & 2 & -2 & 1 & \end{array} \right]$$

$$\boxed{\begin{bmatrix} -2 & 2 & -1 \\ 1 & -2 & 1 \\ 2 & -2 & 1 \end{bmatrix}} = A^{-1}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A \cdot I = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & -1 & 2 & 1 & 0 & 0 \end{array} \right]$$

$$\underbrace{R_3 = R_3 + (-R_1)}_{\text{Row 3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 \rightarrow -R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 \rightarrow R_1 + (-R_3)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 = -R_1 + R_2 \\ R_1 = R_1 + R_2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\downarrow R_1 = R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_3 = 2R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\downarrow R_2 = R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 = R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



$$\begin{aligned}
 & -1 - 2(-1) \\
 & = -1 + 2 \\
 & = 1 \\
 & 0 - 2(-1)
 \end{aligned}$$

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 - 2R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 - R_3} = \begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \rightarrow R_2 + R_3} = \begin{bmatrix} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

am b/a/p

Compute A^{-1} .

$$\text{Augmented Matrix: } \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right]$$

Reduction

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R2} \leftarrow \text{R2} + \text{R1}} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R1} \leftarrow \text{R1} + \text{R2}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R1} \leftarrow \text{R1} - \text{R2}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R1} \leftarrow \text{R1} - 2\text{R3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R2} \leftarrow \text{R2} + \text{R1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R1} \leftarrow \text{R1} + 3\text{R2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R1} \leftarrow \text{R1} + 12\text{R3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{\text{R2} \leftarrow \text{R2} + 4\text{R3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

Check:

$$\left[\begin{array}{ccc} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right], \left[\begin{array}{ccc|c} -2 & 3 & R^{-1} \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 1 & -1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \xrightarrow{\text{Row 2} \leftarrow \text{Row 2} - \text{Row 1}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 2 & -1 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} + 2\text{Row 2}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \\ 2 & 0 & 3 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} - 2\text{Row 1}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \frac{1}{3}\text{Row 3}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

↓

$$C_1: \begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{\text{Row 2} \leftarrow \text{Row 2} - \text{Row 1}} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} + 2\text{Row 2}} \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}}$$

$\therefore A$ is invertible.

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} + \text{Row 2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} - 2\text{Row 1}}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \leftarrow C_2: \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} + 2\text{Row 1}} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 0 & 3 & 0 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \frac{1}{2}\text{Row 3}}$$

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 2 & 0 & 3 & 0 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}}$$

$$C_3: \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \text{Row 3} - 2\text{Row 1}} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix} \xrightarrow{\text{Row 3} \leftarrow \frac{1}{3}\text{Row 3}}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}}$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} + \text{Row 2}}$$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}} \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} + \text{Row 2}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xleftarrow{\text{Row 1} \leftarrow \text{Row 1} - \text{Row 2}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{l} A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow[-1 \cdot 0]{} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 1 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow[-1 \cdot 0]{} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{bmatrix} \xrightarrow[\frac{3}{2} \cdot 2]{} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ — invertible.} \end{array}$$

To find A^{-1}

$$\left[\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} \xrightarrow{\left[\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 0 \end{array} \right]} = \left[\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \end{array} \right] \xrightarrow{\left[\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]} = \left[\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \\ A^{-1} = \left[\begin{array}{ccc} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & 2 & 1 \end{array} \right] \quad \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\left[\begin{array}{ccc|cc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]} = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow[4+2, 4-1, -2+0]{} \end{array}$$

check

$$A \cdot A^{-1} = \left[\begin{array}{ccc} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right] \left[\begin{array}{ccc} -1 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & 2 & 1 \end{array} \right] = \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{array}{l} 1x-1 + -1x+2x = 0 \\ 1x+2 + -1x+x = 0 \\ 0x-2 + 2x = 0 \end{array} \quad \begin{array}{l} 1x+3 + -1x+x = 0 \\ 1x+3 + 0x-1 + x = 0 \\ 0x+3+2x-1 = 0 \end{array} \quad \begin{array}{l} 1x-1 + -1x+2x = 0 \\ 1x-1 + 0x+1 = 0 \\ 0x+1 = 0 \end{array}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(-1)} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(-2)} \\
 \downarrow \\
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{(+2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{(1)} \\
 \downarrow \\
 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{(1)} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\
 \underbrace{\qquad\qquad}_{\text{identity matrix}} \qquad\qquad \underbrace{\qquad\qquad}_{A^{-1}}
 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\text{(1)} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$



$$\text{(2)} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$



$$\text{(3)} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$



$$\text{(4)} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$



$$\text{(5)} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$



$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

check: $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \checkmark$



(ad-bc)

2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 + (-1)R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

 $R_3 + (-2)R_2$

$$\xrightarrow{-2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 + (1)R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

(ad-bc)

$$A = \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 0 & -1 & 1 \\ -1 & 2 & 1 & 1 & 2 & 1 \\ 2 & -1 & 1 & 0 & -1 & 1 \\ -1 & 2 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 & -1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} -2 & -1 & 2 & + & - & + \\ -3 & -1 & 2 & - & + & - \\ -1 & -1 & 1 & + & - & + \end{array} \right] \xrightarrow{\text{Row operations}} A^{-1} = \left[\begin{array}{ccc} -2 & 1 & 2 \\ 3 & -1 & -2 \\ 1 & 1 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A^{-1} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 4 & -2 & 1 \end{bmatrix}$$

$$\begin{aligned}
 A - D + 2G &= 1 & B - E + 2H &= 0 & C - F + 2I &= 0 \\
 A + G &= 0 & B + H &= 1 & C + I &= 0 \\
 2D - G &= 0 & 2E - H &= 0 & 2F - I &= 1 \\
 -D + G &= 1 & 2B - 2E + 4H &= 0 & 2C - 2F + 4I &= 0 \\
 2D - G &= 0 & 2E - H &= 0 & 2F - I &= 1 \\
 -2D + 2G &= 2 & -2B + 3H &= 0 & 2C + 3I &= 1 \\
 2D - G &= 0 & -2B + 2H &= 2 & 2C + 2I &= 0 \\
 G = 2 & & -H = 2 & I = 1 & \\
 A = -2 & & H = -2 & C = -1 & \\
 D = 1 & & B = 3 & F = 1 & \\
 & & E = -1 & &
 \end{aligned}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{x_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -\frac{1}{3} & \frac{4}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \xrightarrow{x_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} -1 \cdot 2 = -2 \\ 1 - -2 = 3 \end{array}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 3 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \xrightarrow{\begin{array}{l} \frac{4}{3} + \frac{5}{3} = \frac{9}{3} \\ \frac{1}{3} \cdot -1 = -\frac{1}{3} \\ -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3} \end{array}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\frac{1}{3}}$$

$$\therefore A^{-1} = \boxed{\begin{bmatrix} 0 & 3 & -\frac{1}{3} \\ \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \xrightarrow{\begin{array}{l} \frac{2}{3} \cdot -1 = -\frac{2}{3} \\ -\frac{3}{3} + -\frac{2}{3} = -\frac{1}{3} \\ -\frac{2}{3} + \frac{2}{3} = 0 \end{array}}$$

$$\downarrow \qquad \qquad \qquad \frac{3}{3} - \frac{4}{3} = -\frac{1}{3}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{3} & \frac{5}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \xrightarrow{x_2} 0 - \frac{4}{3} = -\frac{4}{3}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{aligned} & \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \\ & \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right] + \left[\begin{array}{cc|c} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{array} \right] \\ & = \left[\begin{array}{ccc} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right] = A^{-1} \\ A^{-1} \cdot A &= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right], \quad \left[\begin{array}{ccc} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right] \\ & \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{aligned}$$

We need to find a matrix S s.t.

It is equal to the Identity matrix.



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$6 - 2(-1) = 2$
 $6 - 2(1) = -2$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$\rightarrow R_1 - R_3 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$

$\rightarrow R_2 + R_3 \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$

$\therefore A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a-d+2g & b-e+2h & c-f+2i \\ a+g & b+h & c+i \\ 2d-g & 2e-h & 2f-i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a - d + 2g = 1$$

$$b - e + 2h = 0$$

$$c - f + 2i = 0$$

$$a + g = 0$$

$$b + h = 1$$

$$c + i = 0$$

$$2d - g = 0$$

$$2e - h = 0$$

$$2f - i = 1$$

$$a = -g$$

$$b = 1 - h$$

$$c = -i$$

$$d = \frac{1}{2}g$$

$$e = \frac{1}{2}h$$

$$f = \frac{1}{2} + \frac{i}{2}$$

$$-g + \frac{1}{2}g + 2g = 1$$

$$1 - h - \frac{1}{2}h + 2h = 0$$

$$-i - \frac{1}{2} - \frac{i}{2} + 2i = 0$$

$$\frac{3}{2}g = 1$$

$$\frac{1}{2}h = -1$$

$$\frac{1}{2}i = \frac{1}{2}$$

$$g = \frac{2}{3}$$

$$h = -2$$

$$i = 1$$

$$a = -\frac{2}{3}$$

$$b = 3$$

$$c = -1$$

$$d = \frac{4}{3}$$

$$e = -1$$

$$f = 1$$

$$A^{-1} = \boxed{\begin{bmatrix} -2/3 & 3/1 & -1 \\ 4/3 & -1 & 1 \\ -2/3 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right.$$

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 & -2 & 1 \end{array} \right| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right|$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right|$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right|$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 - R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 2R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2 + R_1; R_3 - 2R_1}$$

$$A^{-1} = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 - R_2; R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1; R_2 + R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$R_2 = R_1 - R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$



$$R_3 = -R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$



$$R_3 = -2R_1 + R_3 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$



$$R_1 = R_2 + R_1 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



$$R_2 = R_3 + R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



$$R_1 = -R_3 + R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \Rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{ccc} 1^{-1} & 0^{-1} & 1^{-2} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

~~Row operations~~

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & +2 & -2 & 0 \end{array} \right]$$

$$\begin{array}{l} 1^{-1} \\ 2^{-2} \\ 2^{-2} \\ -1+2 \end{array} \quad \begin{array}{l} 2^{-2} \\ 2^{-2} \\ 2 \cdot \\ -1 \cdot 2 \end{array} \quad \begin{array}{l} 100 \\ 010 \\ 001 \end{array} \quad \begin{array}{l} -2 \\ 1 \\ 2 \\ -2 \end{array} \quad \begin{array}{l} 2 \\ 1 \\ 0 \\ -2 \end{array} \quad \begin{array}{l} 2 \\ 1 \\ 0 \\ -2 \end{array}$$

$$\begin{array}{c} 1^{-1} \\ 1^{-2} \\ 0^{-2} \end{array} \quad A^{-1} = \left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{array} \right]$$

~~Final Answer~~

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$[A | I_3] = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \quad \therefore A^{-1} = \underbrace{\left[\begin{array}{ccc} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array} \right]}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{(-1) \cdot R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_1 - R_3 \\ R_2 + R_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] = [I_3 | A^{-1}]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad \begin{bmatrix} (r_1 c_1) (r_1 c_2) (r_1 c_3) \\ (r_2 c_1) (r_2 c_2) (r_2 c_3) \\ (r_3 c_1) (r_3 c_2) (r_3 c_3) \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{2R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array} \right]$$

$$\xrightarrow{R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array} \right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

~~A \times $\frac{1}{2}$~~
~~A \times $\frac{1}{2}$~~
~~A \times $\frac{1}{2}$~~

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$$a_1 - a_4 + 2a_7 = 1$$

$$-a_2 - a_4 + 2a_8 = 1$$

$$a_1 + a_7 = 0$$

$$1 + a_4 = a_7$$

$$2a_4 - a_7 = 0$$

$$1 + a_4 = 2a_4$$

$$a_4 = 1 \quad a_7 = 2 \quad a_1 = -2$$

$$a_2 - a_5 + 2a_8 = 0$$

$$2a_5 = a_8$$

$$1 - a_8 - \frac{1}{2}a_8 + 2a_8 = 0$$

$$a_2 + a_8 = 1$$

$$a_8 = 1 - a_2$$

$$\frac{1}{2}a_8 = -1$$

$$2a_5 - a_8 = 0$$

$$a_8 = -2$$

$$a_5 = -1 \quad a_2 = 3$$

$$a_3 - a_6 + 2a_9 = 0$$

$$a_3 = -a_9$$

$$-a_9 - \frac{1}{2}(1 + a_9) + 2a_9 = 0$$

$$a_3 + a_9 = 0$$

$$a_6 = \frac{1}{2}(1 + a_9)$$

$$-\frac{1}{2} + \frac{1}{2}a_9 = 0$$

$$2a_6 - a_9 = 1$$

$$a_9 = 1$$

$$a_3 = -1$$

$$a_6 = \frac{1}{2}(1 + 1) = 1$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} . $A|I$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{(2)}-\text{(1)}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{(3)}-2\text{(2)}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{\text{(1)}+\text{(2)}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{\text{(2)}+\text{(3)}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \quad I|A^{-1}$$

check

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}} \quad \text{Ans}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \quad R_2 = R_2 - R_1 \quad \rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_1 = R_1 + R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \quad R_3 = R_3 - 2R_2 \quad \rightarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$R_2 = R_2 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$R_1 = R_1 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A \cdot A^{-1} = I$$

$$\begin{array}{ccc} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array}$$

$$(-1)\cancel{6} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

$$(-2)\cancel{6} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\cancel{2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array} \right]$$

$$\cancel{2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

A aug. w/ identity matrix:

$$\begin{array}{r} \begin{array}{r} 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ \hline 0 & -1 & 1 & 1 & -1 & 0 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 6 & -2 & 2 & 2 & -2 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 2 & -2 & 1 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ \hline 0 & 1 & 0 & 1 & -1 & 1 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{r} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & -4 & 4 & -2 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \end{array} \end{array}$$

swap

$$\begin{array}{r} \begin{array}{r} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 2 & 2 & -1 \\ \hline 1 & 0 & 0 & -2 & 3 & -1 \end{array} \end{array}$$

$$\left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{D-1}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{D2}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{S_1} \left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{-1}$$

Normal form

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$\xrightarrow{\text{identity matrix}} \xleftarrow{A^{-1}}$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R2} \cdot \frac{1}{2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array} \right] \xrightarrow{\text{R3} - \text{R2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] + \frac{1}{2}\text{R3}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{R1} + \text{R2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{R1} - 2\text{R3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \quad A^{-1} = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{(c-1)} \left(\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{(c2)}$$

$$\left(\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right) \xrightarrow{(c-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right) \xrightarrow{(c-1)}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right) \xrightarrow{(c-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right) \xrightarrow{(c1)}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \Rightarrow \text{row reduction} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 0 \end{array} \right]$$

pivot in each row and column so one-to-one and onto

A is invertible

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R}_2 \cdot 2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R}_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{R}_1 - R_2} \left[\begin{array}{ccc|ccc} -1 & -1 & -1 & -2 & 0 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\text{R}_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\text{R}_3 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{\text{R}_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\text{R}_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A \cdot A^{-1} = I \quad \text{where } I \text{ is } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \leftarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 + R_2} \\
 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] \xleftarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] \xleftarrow{R_3 - 2R_2} \\
 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right] \\
 A^{-1} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}
 \end{array}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

~~$$A^{-1} = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}$$~~

$$w_1 = \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\cdot -1} \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\cdot -1} \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}}$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 2 & -1 & | & 0 & 1 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \end{array} \right] \xrightarrow{\cdot -1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 2 & -1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot 1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 2 & -1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot \frac{1}{2}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot 2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & | & 1 & -2 & 0 \\ 0 & 1 & 0 & | & 0 & -1 & 1 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot 1}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 0 & | & 2 & -3 & 1 \\ 0 & 1 & 0 & | & 1 & -1 & 1 \\ 0 & 0 & 1 & | & 0 & 1 & 0 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} 2 & -3 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left| \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right.$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right. \quad R_2 = -R_1 + R_2$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right. \quad R_3 = R_2 + R_3$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right.$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right. \quad R_2 = R_3 + R_2$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right. \quad R_1 = -R_3 + R_1$$

$$\left| \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right.$$

$$\boxed{A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:



$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\det(A) = 1(0-2) - (-1)(-1-0) + 2(2-0)$$

$$= -2 - 1 + 4 = 4 - 3 = 1 \rightarrow A^{-1} \text{ is possible}$$

$$\left| \begin{array}{ccc} a_{11} = \begin{vmatrix} 0 & -1 \\ 2 & -1 \end{vmatrix} = -2 & a_{12} = \begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} = -1 & a_{13} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 \\ a_{21} = \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} = 4 - 1 = 3 & \cancel{a_{22}} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1 & a_{23} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2 \\ a_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1 & \cancel{a_{32}} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1 & a_{33} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 0 - (-1) = 1 \end{array} \right|$$

Now, $\text{adj}(A) = \begin{bmatrix} -2 & -1 & 2 \\ 3 & -1 & 2 \\ -1 & -1 & 1 \end{bmatrix} \times \quad \quad \quad$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \quad a_{11} = \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = -2 \quad a_{12} = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1 - 4 = -3 \quad a_{13} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = -1 \quad a_{21} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1 \quad a_{22} = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1 \quad a_{23} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1 \quad a_{31} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2 \quad a_{32} = \begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} = 2 \quad a_{33} = \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = -(-1) = 1$$

$$\therefore \text{adj}(A) = \begin{bmatrix} -2 & -3 & -1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj}(A)}{\det(A)}, \frac{1}{1} \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

(Quick check: $\begin{bmatrix} (1-1)2 & \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} -2+1+4 \\ -3+4 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$)



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{x_2 + x_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-2R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 2 & -2 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{Final row operation}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ -2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left| \begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -1 & 0 & 1 \end{array} \right| \xrightarrow{\text{R}_1 \times -1 + \text{R}_2} \left| \begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 1 \end{array} \right| \xrightarrow{\text{R}_2 \times 2 + \text{R}_3} \left| \begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 \end{array} \right|$$

$$R_1 \times -1 + R_2$$

$$R_2 \times 2 + R_3$$

$$\left| \begin{array}{ccc|cc} 1 & -1 & 2 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 \end{array} \right| \xrightarrow{\text{R}_3 \times 1 + \text{R}_2} \left| \begin{array}{ccc|cc} 1 & -1 & 0 & -3 & 4 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 \end{array} \right| \xrightarrow{\text{R}_2 \times 1 + \text{R}_1} \left| \begin{array}{ccc|cc} 1 & 0 & 0 & -2 & 3 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 \end{array} \right|$$

$$R_3 \times 2 + R_1$$

$$R_2 \times 1 + R_1$$

$$(A^{-1} = \begin{vmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{vmatrix})$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{①} + \text{③}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{②} \cdot 2} \left[\begin{array}{ccc|ccc} 2 & 2 & 2 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\boxed{\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{①} + \text{③}, \text{④}} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{④} - \text{①} + 2\text{②}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]}$$

$$\boxed{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 1 & 2 & 0 & 0 & 1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{②} + (-1)\text{①}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 & -1 & 1 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{③} + (-1)\text{②}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 & -1 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array} \right]}$$

$$\boxed{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array} \right] \xrightarrow{\text{③} \cdot (-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & 0 \\ 0 & 1 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array} \right]}$$

$$A^{-1} = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 2 & -1 & 0 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & -1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot -1}$$



$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow -R_1 + R_3} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} \end{array} \right] \xrightarrow{R_1 \rightarrow R_2 + R_1} \xrightarrow{R_3 \rightarrow -R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 1 & -1 & 0 & 1 & 0 \end{array} \right]$$

$$R_3 \rightarrow -\frac{2}{1} \cdot R_3 \downarrow$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{1}{2}R_3 + R_2} \xrightarrow{R_1 \rightarrow \frac{3}{2}R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\ \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \end{array}$$

$$A^{-1} = \begin{bmatrix} -3 & 4 & -2 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\xrightarrow{\substack{R_1 \leftarrow R_1 - R_2 \\ +R_3}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow -R_2 \\ +R_3}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 \leftarrow R_2 + R_1 \\ R_3 \leftarrow R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_1 \leftarrow R_1 - R_2 \\ R_2 \leftarrow R_2 - R_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftarrow R_1 - R_2 \\ R_3 \leftarrow R_3 - 2R_1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftarrow R_1 - R_2 \\ R_2 \leftarrow R_2 - R_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

\nearrow Identity Matrix

$$\left[A \mid I \right] \xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{\text{Row Operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array} \right]$$

\nearrow Identity Matrix

$\therefore A$ is

invertible and

$$A^{-1} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & -1 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot -2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot 1}$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\cdot 1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \downarrow$$

$$\xrightarrow{(-1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \downarrow$$

$$\xrightarrow{\left(\frac{-1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \downarrow$$

$$\xrightarrow{\left(\frac{1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -1 & 1 & 0 \end{array} \right] \downarrow$$

$$\xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -1 & 1 & 0 \end{array} \right] \xrightarrow{(1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -5 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{(1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$m=n \checkmark \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3/2} \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A \cdot A^{-1} = I$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_1^1 & A_1^2 & A_1^3 \\ A_2^1 & A_2^2 & A_2^3 \\ A_3^1 & A_3^2 & A_3^3 \end{bmatrix}$$

$$\begin{cases} A_1^1 - A_4^1 + 2A_7^1 = 1 \\ A_1^2 + A_7^2 = 0 \\ 2A_4^1 - A_7^1 = 0 \end{cases}$$

$$2A_1^1 + 3A_7^1 = 1$$

$$A_1^2 + A_7^2 = 0$$

$$\begin{cases} A_7^1 = 1 \\ A_7^2 = -\frac{1}{2} \end{cases}$$

$$A_1^1 + 1 = 0$$

$$A_1^1 = -1$$

$$2A_4^1 - 1 = 0$$

$$A_4^1 = \frac{1}{2}$$

$$\begin{cases} A_2^1 - A_5^1 + 2A_8^1 = 0 \\ A_2^2 + A_8^2 = 1 \\ 2A_5^1 - A_8^1 = 0 \end{cases}$$

$$2A_2^1 + 3A_8^1 = 0$$

$$A_2^2 + A_8^2 = 1$$

$$\begin{cases} A_8^1 = -2 \\ A_8^2 = 1 \end{cases}$$

$$A_2^1 - 2 = 1$$

$$A_2^2 = 3$$

$$2A_5^1 + 2 = 0$$

$$A_5^1 = -1$$

$$\begin{cases} A_3^1 - A_6^1 + 2A_9^1 = 0 \\ A_3^2 + A_9^2 = 0 \\ 2A_6^1 - A_9^1 = 1 \end{cases}$$

$$2A_3^1 + 3A_9^1 = 1$$

$$A_3^2 + A_9^2 = 0$$

$$\begin{cases} A_9^1 = 1 \\ A_9^2 = 0 \end{cases}$$

$$\begin{cases} A_3^1 + 1 = 0 \\ A_3^2 = -1 \end{cases}$$

$$2A_6^1 - 1 = 1$$

$$A_6^1 = 1$$

$$A^{-1} = \begin{bmatrix} -1 & 3 & -1 \\ \frac{1}{2} & -1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A \cdot A^{-1} = I_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] R_2 \rightarrow R_2 - R_1 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] R_1 \rightarrow R_2 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right] R_3 \rightarrow R_3 - 2R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & 2 & -2 & 1 \end{array} \right] R_3 \rightarrow \frac{1}{3}R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] R_2 \rightarrow R_2 + R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right] R_1 \rightarrow R_1 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{3} & \frac{5}{3} & -\frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{3} & \frac{5}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{aligned}
 A &= \xrightarrow{(-1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\
 &\xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\
 &\xrightarrow{+C_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\
 &\xrightarrow{+C_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\
 &\xrightarrow{(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\
 &\xrightarrow{+C_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \rightarrow A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}
 \end{aligned}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & -1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R}_1 \leftrightarrow \text{R}_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{R}_3 - 2\text{R}_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & -2 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 3 & -2 & 0 & 1 \end{array} \right] \xrightarrow{\text{R}_3 \rightarrow \frac{1}{3}\text{R}_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 & \frac{1}{3} \end{array} \right] \xrightarrow{\text{R}_1 + \text{R}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 & \frac{1}{3} \end{array} \right]$$

Add R2 to R1

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 & \frac{1}{3} \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 - 2R_2} \\
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 2 & -2 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2 + R_1} \\
 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 2 & -2 \\ 0 & 1 & 0 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & -1 \\ 0 & 1 & 0 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{\text{swap } R_1 \text{ and } R_2} \\
 A^{-1} = \left[\begin{array}{ccc} -1 & 2 & -1 \\ 0 & 3 & -1 \\ 2 & -2 & 1 \end{array} \right]
 \end{array}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \Rightarrow & \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \Rightarrow & \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \times -2 + 0 \\ \Rightarrow & \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\ \Rightarrow & \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \\ \therefore A^{-1} = & \left[\begin{array}{ccc} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array} \right] \end{aligned}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{l}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(2)(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array} \right] \xrightarrow{R_1=R_1+R_2} \\
 \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array} \right] \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array} \right] \xrightarrow{R_2=R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 & -2 & -1 \end{array} \right]
 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & -1 \\ 2 & -2 & -1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\uparrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$

$$(R1 \times -1) + R3 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 2 & -2 & 1 \end{array} \right]$$

$$(R3 \times -1) + R2 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array} \right]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(R2 \times -1) + R3 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array} \right]$$

$$\begin{aligned} -2 - 1 + 4 &= 1 \\ 3 + 1 - 4 &= 0 \\ -1 - 1 + 2 &= 0 \\ -2 + 0 + 2 &= 0 \end{aligned}$$

$$R2 + R1 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array} \right] | -1$$

$$R3 \times -2 + R1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$(R2 \times -2) \quad \frac{4-1}{4-1} = 3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\begin{aligned} -2 + 3 - 1 &= 0 \\ 1 - 1 + 1 &= 1 \\ 2 - 2 + 1 &= 1 \end{aligned}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\text{Augmented } \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3-2R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\downarrow R_1+R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\text{So, } A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{array} \right] \cdot \left[\begin{array}{ccc} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array} \right] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\boxed{A^{-1} = \left[\begin{array}{ccc} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{array} \right]}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\text{⑥} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_2 \leftarrow \textcircled{2} - \textcircled{1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_3 = \textcircled{3} - \textcircled{2} \times 2 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 \end{array} \right] \quad \begin{array}{l} 0 = -1 \times 2 \\ = 0 + 2 \end{array}$$

$$R_2 \leftarrow \textcircled{2} + \textcircled{3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$R_1 \leftarrow \textcircled{1} - \textcircled{3} \times 2 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & 2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 3 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 3 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 2 & -2 & 4 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 & -2 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 & -1 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 2 & -2 & 1 \end{array} \right]$$

$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{cccc|cc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{cccc|cc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] = \left[\begin{array}{cccc|cc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] = \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

First, append Identity matrix, I , then find $\text{rref}(A)$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \xleftarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array} \right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_1 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -2 & 3 & -1 \\ 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{-2R_2, R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \checkmark$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 = R_2 - R_3 \\ R_1 = R_1 - R_2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \end{array} \right]$$

$$R_2 = R_2(-1)$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_3 = R_3 + 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array} \right]$$

$$R_1 = R_1 - 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array} \right]$$

$$R_1 = R_1 - 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & -3 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array} \right] \\ \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 2 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \end{array}$$

$$[A | I] \rightarrow [I | A^{-1}]$$

$$\overbrace{A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1+R_2 \\ R_2+R_1}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{2R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} -2 & 3 & -1 & & & \\ 0 & -1 & 1 & & & \\ 2 & -2 & 1 & & & \end{array} \right] = A^{-1}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{c}
 \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_2=R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1=R_2} \\
 \downarrow \\
 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_1=R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_1=R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xleftarrow[R_3=-2R_2]{} \\
 \downarrow \\
 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xleftarrow{R_1=R_3+R_2} A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}
 \end{array}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\cdot(-1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\cdot(-2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\cdot(1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\cdot(1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\cdot(-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{swap } R_1 \text{ and } R_2} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array} \right]$$

$$A^{-1} = \boxed{\begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

Handwritten steps for finding the inverse of matrix A:

- Start with the augmented matrix $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$
- Row operations:
 - $R_3 \rightarrow R_3 - 2R_1$: $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -2 & 0 & -5 & -1 & 0 & 1 \end{array} \right]$
 - $R_3 \rightarrow R_3 + 2R_2$: $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -1 & 2 & 1 \end{array} \right]$
 - $R_1 \rightarrow R_1 - R_2$: $\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -1 & 2 & 1 \end{array} \right]$
 - $R_1 \rightarrow R_1 + R_2$: $\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -1 & 2 & 1 \end{array} \right]$
 - $R_2 \rightarrow R_2 - R_1$: $\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & -3 & -1 & 2 & 1 \end{array} \right]$
- Final result: $A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$
- Annotation: "this is the answer"

Alternative method (bottom part):
 $\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + 2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array} \right]$
 $\downarrow R_1 = R_1 - R_2$
 $\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 3R_2} \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 3 & 2 & 0 & 1 \end{array} \right]$
 $\downarrow R_2 = R_3 + R_2$
 $\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ -1 & 0 & 0 & 2 & -3 & 1 \end{array} \right] \xrightarrow{-} \left[\begin{array}{ccc|ccc} 0 & -1 & 0 & -2 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ -1 & 0 & 0 & 2 & -3 & 1 \end{array} \right] \xrightarrow{-(-1)} \left[\begin{array}{ccc|ccc} 0 & -1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \\ 1 & 0 & 0 & -2 & 3 & -1 \end{array} \right] \xrightarrow{-(-1)}$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \quad \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

Compute A^{-1} .

$$\text{-1. } \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\text{-2. } \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\text{1. } \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$

$$\text{2. } \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$

$$\text{10. } \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -3 & 4 & -2 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right)$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{l} \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \\ \Rightarrow \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \Rightarrow \xrightarrow{-2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \\ \Rightarrow \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \Rightarrow \xrightarrow{-1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array} \right] \\ \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{array} \right] \Rightarrow A^{-1} = \left[\begin{array}{ccc} -2 & 2 & -1 \\ 1 & -1 & 0 \\ 0 & -2 & 1 \end{array} \right] \end{array}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 6 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \quad A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 6 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{aligned}
 A^{-1} &= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \times -1 \\
 &\quad \downarrow \\
 &= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \times -2 \\
 &= \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array} \right] \times 1 \\
 &\quad \downarrow \\
 &= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \times 1 \\
 &\quad \downarrow \\
 &= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \times 1 \\
 &\quad \downarrow \\
 &= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]
 \end{aligned}$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ -1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(2)-(1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{(3)-(2)\times 2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{(2)+(3)} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 2 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 2 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{(1)+(2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 2 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 2 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right] \xrightarrow{(1)-2(3)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

There is also another way that requires you to find a Cofactor by doing

$$\begin{bmatrix} [0 & 1] & [1 & 1] & [1 & 0] \\ [2 & -1] & [0 & -1] & [0 & 2] \\ [-1 & 2] & [2 & -1] & [] \end{bmatrix}$$

e.t.c.

Finding the cross product of each box to find a cofactor X then

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$X \text{ (this)} = \bar{A}^{-1}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{array}{l}
 \text{Step 1: } \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right] \xrightarrow[-R2]{R1} \left[\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 \end{array} \right] \xrightarrow[R2]{R3} \left[\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \\
 \text{Step 2: } \left[\begin{array}{ccc|c} 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[-R1]{R1} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[-R2]{R2} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right] \\
 \text{Step 3: } \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[R3]{R1} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[-R2]{R2} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \\
 \text{Step 4: } \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[\text{R3}]{R1} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[\text{R2}]{R2} \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \\
 \text{Step 5: } \left[\begin{array}{ccc|c} 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow[\text{R3}]{R1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]
 \end{array}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & -2 & 2 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$A^{-1} = \begin{bmatrix} -2 & 3 & -1 \\ 1 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix} \xrightarrow{\text{Row operations}} \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R2 \rightarrow R2 - R1} \begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R1 \rightarrow R1 + R2} \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 2 & -1 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R3 \rightarrow R3 - 2R2} \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & -1 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R2 \rightarrow R2 + R3} \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 1 & -1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix} \xrightarrow{R1 \rightarrow R1 - R2} \begin{bmatrix} 1 & 0 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & 0 & | & 1 & -1 & 0 \\ 0 & 0 & 1 & | & 2 & -2 & 1 \end{bmatrix}$$



2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & x_1 & y_1 & z_1 \\ 1 & 0 & 1 & x_2 & y_2 & z_2 \\ 0 & 2 & -1 & x_3 & y_3 & z_3 \end{array} \right] \xrightarrow{\text{R}_1 + R_2, R_3 \rightarrow R_3/2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & x_1 & y_1 & z_1 \\ 0 & 1 & 1 & x_2 & y_2 & z_2 \\ 0 & 1 & -\frac{1}{2} & x_3 & y_3 & z_3 \end{array} \right] \xrightarrow{\text{R}_1 - \text{R}_2, \text{R}_3 + \text{R}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & x_1 & y_1 & z_1 \\ 0 & 1 & 1 & x_2 & y_2 & z_2 \\ 0 & 0 & 0 & x_3 & y_3 & z_3 \end{array} \right]$$

$$\xrightarrow{\text{R}_1 + \text{R}_2, \text{R}_3 + \text{R}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & x_1 & y_1 & z_1 \\ 0 & 1 & 1 & x_2 & y_2 & z_2 \\ 0 & 0 & 0 & x_3 & y_3 & z_3 \end{array} \right] \xrightarrow{\text{R}_1 \rightarrow \text{R}_1 - \text{R}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & x_1 & y_1 & z_1 \\ 0 & 1 & 1 & x_2 & y_2 & z_2 \\ 0 & 0 & 0 & x_3 & y_3 & z_3 \end{array} \right]$$

$$\xrightarrow{\text{R}_2 \rightarrow \text{R}_2 - \text{R}_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & x_1 & y_1 & z_1 \\ 0 & 1 & 0 & x_2 & y_2 & z_2 \\ 0 & 0 & 0 & x_3 & y_3 & z_3 \end{array} \right] \xrightarrow{\text{R}_3 \rightarrow \text{R}_3 - 2\text{R}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & x_1 & y_1 & z_1 \\ 0 & 1 & 0 & x_2 & y_2 & z_2 \\ 0 & 0 & 1 & x_3 & y_3 & z_3 \end{array} \right]$$

$$\xrightarrow{\text{R}_2 + \text{R}_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & x_1 & y_1 & z_1 \\ 0 & 1 & 0 & x_2 & y_2 & z_2 \\ 0 & 0 & 1 & x_3 & y_3 & z_3 \end{array} \right] \xrightarrow{\text{R}_1 + (-\text{R}_2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & x_1 & y_1 & z_1 \\ 0 & 1 & 0 & x_2 & y_2 & z_2 \\ 0 & 0 & 1 & x_3 & y_3 & z_3 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{5}{3} & -\frac{5}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{4}{3} & \frac{1}{3} \end{bmatrix}$$