where $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ is eigenvector corresponding to λ

3. Consider the following matrix A:

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix} \qquad A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}.$$

$v_2 + v_3 = \lambda v_1$
$v_1 + v_2 = \lambda v_2$
$4v_1 + 2v_2 + 2v_3 = \lambda v_3$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$ NO b) $\lambda = -1$ NO c) $\lambda = -2$ Yes

$$-\lambda I) = \det\left(\begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix}\right) = 0 = -1\left[2-\lambda-4\right] + (1-\lambda)\left[-2\lambda+\lambda^2-8\right] - 0$$

$$2+\lambda+(-2\lambda+\lambda^2-8+2\lambda^2-\lambda^3+8\lambda) = 0$$

$$-\lambda^3+3\lambda^2+7\lambda-6=0$$

$$\lambda^3-3\lambda^2-7\lambda+6=0 \quad ✓$$

a) $0^3-3(0)^2-7(0)+6=0$ $6 \neq 0 \rightarrow$ Not an eigenvalue

b) $(-1)^3-3(-1)^2-7(-1)+6=0$ ✓

$-1-3+7+6=0$ ✓

$9 \neq 0 \rightarrow$ Not an eigenvalue

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c) $(-2)^3-3(-2)^2-7(-2)+6=0$ ✓

$-8-12+14+6=0$

$0=0$ ✓ $\rightarrow$ IS an eigenvalue