

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
							4	

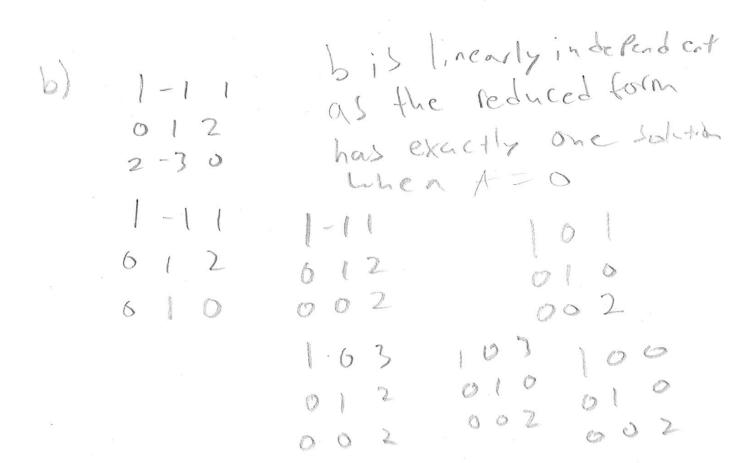
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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.





2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

$$\begin{vmatrix}
1 - 12100 \\
1 0 1010 \\
0 2 - 1001
\end{vmatrix}$$

$$\begin{vmatrix}
1 - 12100 \\
6 - 11110
\end{vmatrix}$$

$$\begin{vmatrix}
6 2 - 1001
\end{vmatrix}$$

$$\begin{vmatrix}
1 - 12100 \\
0 - 11110
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 1 & 1 & 0 \\
0 - 1 & 1 & 1 & 0
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 1 & 1 & 0 \\
0 - 1 & 1 & 1 & 0
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 1 & 1 & 0 \\
0 - 1 & 1 & 1 & 0
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 1 & 1 & 0 \\
0 - 1 & 1 & 1 & 1
\end{vmatrix}$$

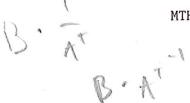
$$\begin{vmatrix}
0 - 1 & 0 & 1 & 2 & 1 \\
0 - 1 & 0 & 1 & 1
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 0 & 1 & 2 & 1 \\
0 - 1 & 0 & 1 & 1
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 0 & 1 & 1 \\
0 - 1 & 0 & 1
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 0 & 1 & 1 \\
0 - 1 & 0 & 1
\end{vmatrix}$$

$$\begin{vmatrix}
0 - 1 & 0 & 1 & 1 \\
0 - 1 & 0 & 1
\end{vmatrix}$$





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$T.$$

- a) Find the standard matrix of T.
- b) Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$. where $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$T(e_1) = T(G) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

 $T(e_2) = T(G) = \begin{bmatrix} -2 \\ 1 & 3 \end{bmatrix}$

b)
$$T(u) = \begin{bmatrix} 1 & 0 \\ -2 \end{bmatrix}$$

a)



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$





- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u,v,w\}$ is linearly independent then the set $\{u,v\}$ must be linearly independent.

$$X_1V_1 + X_2V_2 + X_3W = 0$$
 $X_3V = 3$ $X_3V = -6$



- **7.** (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

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A=[60] v=-[6] V=(1)

Av, Av are dereding

but V, V are indereded

b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).

 $U = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad V = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad V = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $T(u) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad T(u) = \begin{bmatrix} -2 \\ -1 \end{bmatrix} \left(T(u) = \begin{bmatrix} -3 \\ -1 \end{bmatrix}\right)$ $U \in Spen(v, w)$ $T(u) \notin Spen(T(u), T(w))$