

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Matthew Cho

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
		ï	4					

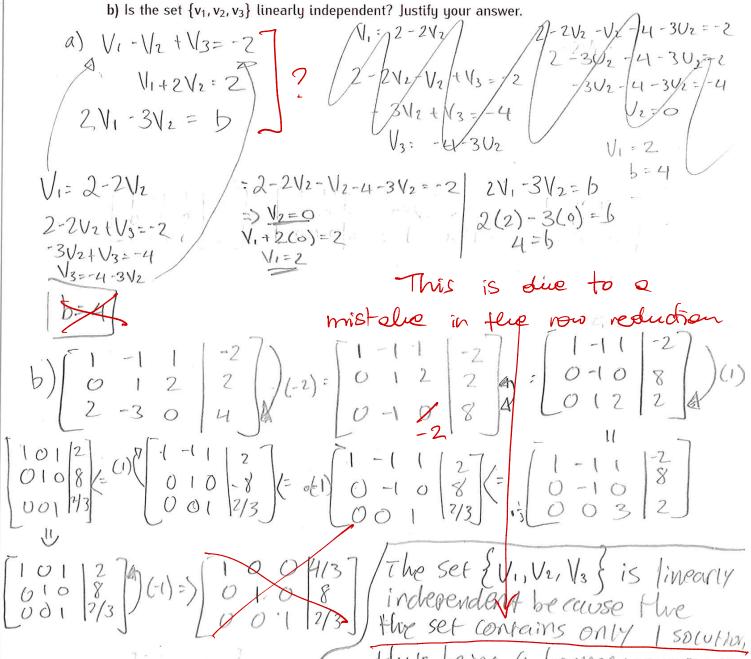
Thus being a homogeneous eavation which proves that the set is linearly independent



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

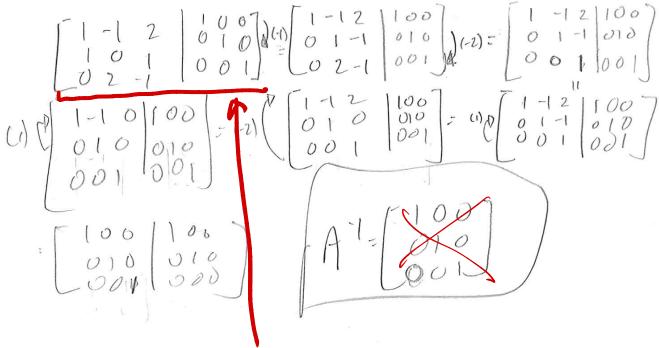




2. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .





3. (10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

A: [1-12]

 $(A^{\tau})^{-1} = (A^{-1})^{\tau} B^{\tau}$

 $C: \left\{ \begin{array}{c} C_{2}(A^{T}) \cdot B \\ 0 & 0 \end{array} \right\} \cdot \left[\begin{array}{c} 123 \\ 4154 \\ 321 \end{array} \right]$

The motion inverse is mong, but even dis responding shis; the metric multiplies wong too.



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- **b)** Find all vectors **u** satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$T(u+v) = Tu+tv$$

$$T(u) = \begin{cases} u_1 + u_2 \\ u_1 + u_2 \\ u_1 + u_2 \end{cases}$$

$$T(u+v) = \begin{cases} u_1 + v_1 - 2u_2 - 2v_2 \\ u_1 + v_1 + u_2 + v_2 \\ u_1 + v_1 - 3u_1 - 3v_2 \end{cases}$$

$$T(v) = \begin{cases} v_1 - 2v_2 \\ v_1 + v_2 \\ v_1 - 3v_2 \end{cases}$$

$$= \begin{cases} (u_1 + v_1) - 2 (u_2 + v_2) \\ (u_1 + v_1) + (u_2 + v_2) \\ (u_1 + v_1) - 3 (u_2 + v_2) \end{cases}$$

$$S + and and matrix$$

$$A \left(T(e_1) T(e_2)\right) = \begin{cases} 1 - 2 \\ 1 \\ 1 - 3 \end{cases}$$

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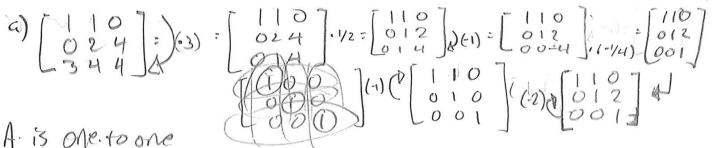
$$A \left(T(e_1) T(e_2)\right) = \begin{cases} 1$$



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A \colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to-one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$
 b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$$



A. is one to one

because every column has a pivot position and the marrix is a homogeneous earer ton

$$b) \begin{pmatrix} 0 & 2 & 4 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{pmatrix} b \begin{pmatrix} 0 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} b \begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} b \begin{pmatrix} 0 & 1 & 2 \\$$

A's one wone because every column has a frust position and the matrix is a homogeneous earestion.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.

Vi=[3], Vi=[3]; W:[3], W:[6] $V_1 + 0v_2 = 1$ $0v_1 + 0v_2 = 1$ $0v_1 + 0v_2 = 0$ $v_1 + 0v_2 = 4$ $v_2 = 8$ $v_3 = 1$ $v_4 = 1$ v

> b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

The peause in order for a set to be Imparty independent they Must have | socution CHOMO geneous caration) 50 that means that U,U, W all only have I squition which thems This could be explained more clearly...



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.

b) If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u,v,w\in \mathbb{R}^2$ are vectors such that u is

in Span(v, w) then T(u) must be in Span(T(v), T(w)).

This is T(u) = T(u) = T(u).