

3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$

b) $\lambda = -1$

c) $\lambda = -2$

$$A - 0I = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & | & 0 \\ 1 & 1 & 0 & | & 0 \\ 4 & 2 & 2 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & | & 0 \\ 1 & 1 & 0 & | & 0 \\ 0 & -2 & -6 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & | & 0 \\ 0 & 0 & 6 & | & 0 \\ 0 & -2 & -6 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 2 & | & 0 \\ 0 & 0 & 6 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow \text{not an eigenvalue}$$

$$A + I = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 0 \\ 4 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & | & 0 \\ 1 & 2 & 0 & | & 0 \\ 4 & 2 & 3 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ 0 & -2 & -5 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 4 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ 0 & 0 & -9 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \rightarrow x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow \text{not an eigenvalue}$$

$$A - 2I = \begin{bmatrix} -2 & 1 & 2 \\ 1 & -1 & 0 \\ 4 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 2 & | & 0 \\ 1 & -1 & 0 & | & 0 \\ 4 & 2 & 0 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 2 & | & 0 \\ 1 & -1 & 0 & | & 0 \\ 0 & 6 & 8 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 2 & | & 0 \\ 0 & 6 & 8 & | & 0 \\ 0 & 6 & 8 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 2 & | & 0 \\ 0 & 6 & 8 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix} \rightarrow (-\lambda)(1-\lambda)(2-\lambda) + 0 + 2 - 8(1-\lambda) - 0 - (2-\lambda) \\ = (-\lambda + \lambda^2)(2-\lambda) + 2 - 8 + 8\lambda - 2 + \lambda \\ = (-2\lambda + 3\lambda^2 - \lambda^3) - 8 + 9\lambda \\ = -\lambda^3 + 3\lambda^2 + 7\lambda - 8 = 0$$

$$-(0)^3 + 3(0)^2 + 7(0) - 8 = -8 \neq 0 \leftarrow \text{not an eigenvalue}$$

$$-(-1)^3 + 3(-1)^2 + 7(-1) - 8 = -11 \neq 0 \leftarrow \text{not an eigenvalue}$$

$$-(-2)^3 + 3(-2)^2 + 7(-2) - 8 = -2 \neq 0 \leftarrow \text{not an eigenvalue}$$

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