

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

Name:		
Jessica Z	Luritis	

UB Person Number:

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
								1.63

							0	nan
1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

a) Find all values of b such that $w \in Span(v_1, v_2, v_3)$. Vinear combination of v_1, v_2, v_3 linearly independent? Institution

b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

(a)
$$\begin{bmatrix} 1 & -1 & 1 & | & -2 \\ 0 & 1 & 2 & | & 2 \\ 2 & -3 & 0 & | & b \end{bmatrix}$$
 (B) $\begin{bmatrix} 1 & -1 & 1 & | & -2 \\ 2 & 2 & | & 2 \\ 2 & -3 & 0 & | & b \end{bmatrix}$ Lp this set in linear independent if home eqn. only has the solution in Solution in

$$c_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

LD this set in linearly independent is homo. ean only has trivial

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$
 R2+R3

not every column of the motion is a pivot therefore the set are not linearly



 $\sqrt{2}$. (10 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

Compute A^{-1} .

Compute A-1.

$$\begin{cases}
1 & -1 & 2 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & -1 & 0 & 0 & 1
\end{cases}$$

$$\begin{pmatrix}
R_{1}x-1 \end{pmatrix} + R_{3} \begin{pmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
0 & 2 & -1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0
\end{cases}$$

$$\begin{pmatrix}
R_{3}x-1 \end{pmatrix} + R_{2} \begin{pmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
0 & 2 & -1 & 0 & 0 & 1 \\
0 & 1 & -1 & -1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
R_{2}x-1 \end{pmatrix} + R_{3} \begin{pmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
0 & 1 & -1 & -1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
R_{2}x-1 \end{pmatrix} + R_{3} \begin{pmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 1 \\
0 & 1 & -1 & 2 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 1 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
R_{2}x-1 \end{pmatrix} + R_{3} \begin{pmatrix}
1 & -1 & 2 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 1 \\
0 & 0 & 1 & 2 & -1 & 1 \\
0 & 0 & 1 & 2 & -2 & 1
\end{pmatrix}$$

$$\begin{pmatrix}
1 & 0 & 2 & 2 & -1 & 1 \\
0 & 1 & 0 & 1 & -1 & 1 \\
0 & 0 & 1 & 2 & -2 & 1
\end{pmatrix}$$

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\end{pmatrix}$$

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1 & 0 & 2 & 2 & -1 & 1 \\
0 & 1 & 0 & 1 & 2 & -2 & 1
\end{pmatrix}$$



(10 points) Let A be the same matrix as in Problem 2, and let

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that $A^TC = B$ (where A^T is the transpose of A).

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 5 & 0 \\ -7 & -3 & 3 \\ 6 & 5 & 2 \end{bmatrix}$$

$$0 | 1 | 2 | 3$$

 $2 | 5 | 7 | (R.1x-2) + R.3$
 $-1 | 3 | 2 |$

$$\begin{bmatrix} 1 & 1 & 0 & | & 1 & 2 & 3 \\ 0 & 1 & 2 & | & 5 & 7 & 7 \\ 0 & 1 & -1 & | & 1 & -2 & -5 \end{bmatrix}$$
 R2+R3=R3

$$\begin{bmatrix} 1 & 1 & 0 & | & 12 & 3 \\ 0 & 12 & | & 57 & 7 \\ 0 & 0 & 1 & | & 65 & 2 \end{bmatrix} R \cdot 2x - 1 + R \cdot 1$$

$$(6x-2)+5$$
 $(2x-2)+7$
 $-12+5$ $-4+7$
 -7 $(5x-2)+7$ 3
 $-10+7$



a) Find the standard matrix of
$$T$$
.

b) Find all vectors \mathbf{u} satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$.

$$\begin{bmatrix} x_1 - 3x_2 \\ 1 & 1 \end{bmatrix}$$

of $T([x_2])$



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

b)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix} (RIX-3) + R3$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix} (R3X-1) + R7$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} (R3X-1) + R7$$

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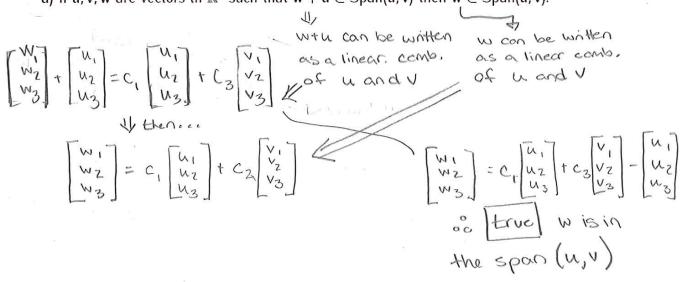
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} (R3X-1) + R7$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} (R3X-1) + R7$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 & 2 \\ 0 &$$



- 6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in \text{Span}(u, v)$ then $w \in \text{Span}(u, v)$.



b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

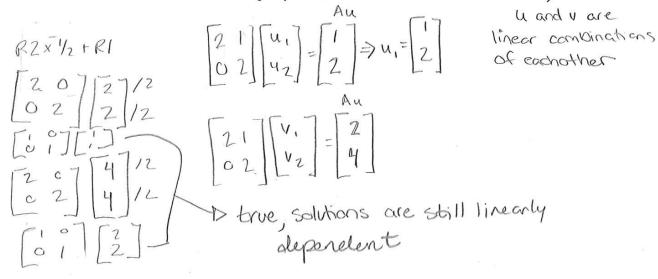
True if u,u, and w are already linearly independent and are not linear combinations of each other, the the set {u,u} must also already be linearly independent

{[i] [i] } > linearly independent

{[i] [i] } > still linearly independent



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a <u>linear transformation</u> and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).