

3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$

b) $\lambda = -1$

c) $\lambda = -2$

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix} = -\lambda[(1-\lambda)(2-\lambda)] - 1[2-\lambda] + 2[4-(4-\lambda)]$$

The roots are $\lambda = 0, 3, 5/4$
 therefore $\lambda = 0$ is an eigenvalue while
 $\lambda = -1$ and $\lambda = -2$
 are not

$$\begin{aligned} & 1 - 2\lambda - \lambda + \lambda^2 \\ & -\lambda[\lambda^2 - 3\lambda + 1] \\ & -\lambda^3 + 3\lambda^2 - \lambda - 2 + 4 + 8 - 8\lambda \\ & (-\lambda^3 + 3\lambda^2 - 8\lambda + 10) \end{aligned}$$

$$\lambda^2(-\lambda + 3) \quad 2(4\lambda + 5) \quad -4\lambda = -\frac{5}{4}$$

$$\lambda = 0 \quad \lambda = 3 \quad \lambda = 5/4$$

$$-\lambda^3 + 3\lambda^2 - \lambda + 2 + 4 + 8 - 16\lambda$$

$$-\lambda^3 + 3\lambda^2 - 16\lambda + 14$$

$$\lambda^2(-\lambda + 3) \quad +7 \quad -6$$

$$\lambda = 0 \quad \lambda = 3$$

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$$\lambda = 0 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

$$\lambda = -1 = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 0 \\ 4 & 2 & 3 \end{bmatrix}$$

$$\lambda = -2 = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 3 & 0 \\ 4 & 2 & 4 \end{bmatrix}$$