



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T(v) = T \cdot v$$

$$v = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad T(v) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \quad \checkmark \quad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T(u) = T \cdot u$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \quad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

↓

$$3 \times 2 \qquad \qquad \qquad 3 \times 1$$

$$2 \times 1 \rightarrow u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{cases} u_1 - 2u_2 = 1 \\ u_1 + u_2 = 10 \\ u_1 - 3u_2 = -2 \end{cases}$$

$$\begin{cases} u_1 - 2u_2 = 1 \\ -u_1 - u_2 = -10 \\ u_1 + 3 = 10 \end{cases} \quad \begin{array}{l} u_2 = 3 \\ u_1 = 7 \end{array}$$

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$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \xrightarrow[\text{REDUCTION}]{} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} \quad u = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

a)

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T(e_1) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_1) = T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$\{T(e_1)\} \cap \{T(e_2)\} \cap \{x_1\} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$b) T(u) = c_1 e_1 + c_2 e_2$$

$$u = c_1 e_1 + c_2 e_2$$

$$T(u) = T(c_1 e_1) + T(c_2 e_2)$$

$$= c_1 T(e_1) + c_2 T(e_2)$$

$$= \left[ T(e_1) \ T(e_2) \right] \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \cdot u$$

$$\left[ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right]$$

$$\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = A \cdot u$$

$$T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$u = \text{span} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 20 \\ 4 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot (v) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \quad T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} x_1 & x_2 & \\ \hline 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\substack{R_2 = R_2 + R_1 \\ R_3 = -R_1 + R_3}} \left[ \begin{array}{ccc} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right]$$

$$\left( \begin{array}{ccc} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right) \xrightarrow{\frac{1}{3}R_2} \left[ \begin{array}{ccc} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{R_2 + R_3} \left[ \begin{array}{ccc} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$R_1 = 2(R_2) + R_1 = \left| \begin{array}{ccc} 1 & 0 & | & 1 \\ 0 & 1 & | & 3 \\ 0 & 0 & | & 0 \end{array} \right|$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \quad \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} = A\vec{w}$

$\vec{u}$  is  $2 \times 1$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \vec{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} u_1 - u_2 &= 1 \\ u_2 &= 3 \\ u_1 &= 4 \end{aligned}$$

$$\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$\begin{aligned} 7 - 6 &= 1 \\ 7 + 3 &= 10 \\ 7 - 9 &= -2 \end{aligned}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

(a)  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

(b)  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

(b)  $T(\mathbf{u}) = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$$(-1) \left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 1 & 1 & 10 & \\ 1 & -3 & -2 & \end{array} \right)$$

$$(-1) \left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 3 & 9 & 10 \\ 1 & -3 & -2 & \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & 0 & 7 & \\ 0 & 0 & 0 & \\ 0 & 1 & 3 & \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 3 & 9 & \\ 0 & -1 & -3 & \end{array} \right) \xrightarrow{(2) \times (-1)} \left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & \\ 0 & 1 & 3 & \end{array} \right)$$

$$\xrightarrow{-1 \times 2} \quad \xrightarrow{-3 + 1}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1) \ T(e_2)]$ ,  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

standard matrix:

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $T_A(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{(-1)r_1+r_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{r_2 \cdot \frac{1}{3}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\xrightarrow{r_1+r_3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{2r_2+r_1} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{r_2+r_3} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = 7$$

$$x_2 = 3$$

$$x_3 = x_3$$

$$v = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_3$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

c)  $T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$      $T(e_1) = T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$      $T(e_2) = T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

d)  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$      $\mathbf{u} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$      $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \xrightarrow{\text{R2} \rightarrow R2 - R1} \begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 1 & -3 \end{bmatrix} \xrightarrow{\text{R3} \rightarrow R3 - R1} \begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 0 & -1 \end{bmatrix} \xrightarrow{\text{R2} \rightarrow R2 / 4} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \xrightarrow{\text{R1} \rightarrow R1 + R2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{u}$

$\xrightarrow{\text{R1} \rightarrow R1 + R3} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \xrightarrow{\text{R3} \rightarrow R3 + R1} \begin{bmatrix} 1 & x_2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$      $x_1 - 2x_2 = 1$      $x_1 = 1 + 2x_2 = 1 + 2(3) = 7$   
 $x_2 = 3$

$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$  or any multiple of  $\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

check  $T \left( \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 7 - 6 \\ 7 + 3 \\ 7 - 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \checkmark$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$\text{2 rows } T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

to 3 columns?

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

*Standard*

$$(a) T = \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \end{bmatrix}$$

$$(b) \mathbf{u} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : \begin{bmatrix} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$x_1 = 1 + 2x_2 \Rightarrow 1 + 2(3) = 7 \quad x_1 = 7$$

$$1 + 2x_2 + x_2 = 10 \Rightarrow 1 + 3x_2 = 10 \Rightarrow 3x_2 = 9 \Rightarrow x_2 = 3$$

$$1 + 2x_2 - 3x_2 = -2 \Rightarrow 1 - x_2 = -2$$

$$(x_2 = 3)$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T(\mathbf{u} + \mathbf{v}) = T\mathbf{u} + T\mathbf{v}$$

$$T(\mathbf{u}) = \begin{pmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{pmatrix}$$

$$T(\mathbf{u} + \mathbf{v}) = \begin{pmatrix} u_1 + v_1 - 2u_2 - 2v_2 \\ u_1 + v_1 + u_2 + v_2 \\ u_1 + v_1 - 3u_2 - 3v_2 \end{pmatrix}$$

$$T(\mathbf{v}) = \begin{pmatrix} v_1 - 2v_2 \\ v_1 + v_2 \\ v_1 - 3v_2 \end{pmatrix}$$

$$= \begin{pmatrix} (u_1 + v_1) - 2(u_2 + v_2) \\ (u_1 + v_1) + (u_2 + v_2) \\ (u_1 + v_1) - 3(u_2 + v_2) \end{pmatrix}$$

standard matrix

$$A(T(e_1) \ T(e_2)) =$$

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

Standard Matrix

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$$

b)

$$T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1 & -2 & 0 & | & 1 \\ 1 & 1 & 0 & | & 10 \\ 1 & 3 & 0 & | & -2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -17 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 0 & | & 1 \\ 1 & 1 & 0 & | & 10 \\ 1 & 3 & 0 & | & -2 \end{pmatrix} \xrightarrow{\text{Row operations}} \begin{pmatrix} 1 & -2 & 0 & | & 1 \\ 0 & 3 & 0 & | & 9 \\ 0 & 5 & 0 & | & -2 \end{pmatrix} \xrightarrow{\text{Row operations}} \begin{pmatrix} 1 & -2 & 0 & | & 1 \\ 0 & 1 & 0 & | & 3 \\ 0 & 5 & 0 & | & -2 \end{pmatrix} \xrightarrow{\text{Row operations}} \begin{pmatrix} 1 & -2 & 0 & | & 1 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 0 & | & 17 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = 3 \times 2$  since  $3 \times 2 = 2 \times 1$

$$\begin{bmatrix} c_1 & c_4 \\ c_2 & c_5 \\ c_3 & c_6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$c_1 x_1 + c_4 x_2 = x_1 - 2x_2$$

$$\begin{aligned} c_1 &= 1 & c_4 &= -2 \\ c_2 &= 1 & c_5 &= 1 \end{aligned}$$

$$c_2 x_1 + c_5 x_2 = x_1 + 1x_2$$

$$\begin{aligned} c_2 &= 1 & c_5 &= 1 \\ c_3 &= 1 & c_6 &= -3 \end{aligned}$$

$$c_3 x_1 + c_6 x_2 = x_1 - 3x_2$$

Standard Matrix:

$$\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$\boxed{T(\mathbf{u}) = \text{Span} \left( \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} \right)}$$

$$x_1 = 7$$

$$x_2 = 3$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$\begin{cases} x_1 = 2x_2 + 1 \\ x_2 = 3 \end{cases}$$

b)  $\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{-1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 2 & 11 \\ 0 & -4 & -3 \end{array} \right] \xrightarrow{-1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 5.5 \\ 0 & -4 & -3 \end{array} \right]$

$$\left[ \begin{array}{c} x_1 = 7 \\ x_2 = 3 \end{array} \right]$$

$\downarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 5.5 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{\cdot\frac{1}{3}} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & \frac{11}{3} \\ 0 & -1 & -1 \end{array} \right] \xrightarrow{+1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & \frac{11}{3} \\ 0 & 0 & -2 \end{array} \right]$$

$u = \boxed{\begin{bmatrix} 7 \\ 3 \end{bmatrix}}$

$\downarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & \frac{11}{3} \\ 0 & -1 & -1 \end{array} \right] \xrightarrow{+1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & \frac{11}{3} \\ 0 & 0 & 0 \end{array} \right]$$

$\downarrow$

$$\left[ \begin{array}{cc|c} x_1 & x_2 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & \frac{11}{3} \\ 0 & 0 & 0 \end{array} \right]$$



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$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(e_1) = \begin{bmatrix} 1 & -2(0) \\ 1 & +0 \\ 1 & -3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \text{Standard Matrix of } T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 0 & -2(1) \\ 0 & +1 \\ 0 & -3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

b)  $\begin{array}{|c c|} \hline x_1 & x_2 \\ \hline x_1 - 2x_2 & 1 \\ x_1 + x_2 & 10 \\ x_1 - 3x_2 & -2 \\ \hline \end{array} \Rightarrow \begin{array}{|c c|} \hline x_1 & x_2 \\ \hline 1 & -2 \\ 1 & 1 \\ 1 & -3 \\ \hline \end{array} \xrightarrow{R_2 \rightarrow R_2 + R_3} \begin{array}{|c c|} \hline 1 & -2 \\ 0 & 4 \\ 1 & -3 \\ \hline \end{array}$

$$\xrightarrow{R_3 \rightarrow R_3 + (-R_1)} \begin{array}{|c c|} \hline 1 & -2 \\ 0 & 4 \\ 0 & -1 \\ \hline \end{array} \xrightarrow{R_2 \rightarrow R_2 + 4R_3} \begin{array}{|c c|} \hline 1 & -2 \\ 0 & 0 \\ 0 & -1 \\ \hline \end{array} \xrightarrow{R_2 \leftrightarrow R_3} \begin{array}{|c c|} \hline 1 & -2 \\ 0 & -1 \\ 0 & 0 \\ \hline \end{array}$$

$$\xrightarrow{R_2 \rightarrow -R_2} \begin{array}{|c c|} \hline 1 & -2 \\ 0 & 1 \\ 0 & 0 \\ \hline \end{array} \xrightarrow{R_1 \rightarrow R_1 + 2R_2} \begin{array}{|c c|} \hline 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \hline \end{array} \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}$$

$$\boxed{\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$x_1 - 2x_2 = 1$$

$$x_1 + x_2 = 10$$

$$x_1 - 3x_2 = -2$$

We get  $x_1 = 7, x_2 = 3$

(b)  $\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \rightarrow \begin{array}{l} 7a + 3b = 1 \\ 7c + 3d = 10 \\ 7e + 3f = -2 \end{array}$$

$3x_2$        $2x_1$

Standard Matrix of  $T =$

(a)

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$



g3  
3

4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a) Standard matrix of  $T$  :-  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) as linear transformation

$$Au = T(u)$$

$$\therefore \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\textcircled{1} \quad u_1 - 2u_2 = 1$$

$$\textcircled{2} \quad u_1 + u_2 = 10$$

$$\textcircled{3} \quad u_1 - 3u_2 = -2$$

$$\begin{aligned} \therefore u_1 - 2u_2 &= 1 & \therefore u_1 = 1 + 2(3) = 1 + 6 = 7 \\ \therefore 1 + 2u_2 + u_2 &= 10 \quad [\because u_1 = 1 + 2u_2] & \therefore 3u_2 = 9 \quad \therefore u_2 = 3 \\ \therefore u_1 - 3(3) &= -2 \quad [\because u_2 = 3] & \\ \therefore u_1 - 9 &= -2 & \therefore u_1 = 7 \end{aligned}$$

$$\therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

b)  $x_1 = 7$   
 $x_2 = 3$

$$\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

a)

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a).  $T : \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$

b)

$$\begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{pmatrix} \xrightarrow{\text{R}_1 \leftrightarrow \text{R}_2} \begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -3 \end{pmatrix} \xrightarrow{\text{R}_3 + \text{R}_1} \begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\text{R}_2 - \text{R}_1} \begin{pmatrix} 2 & 0 & 21 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\text{R}_1 - 2\text{R}_2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 10 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\text{R}_1 \leftrightarrow \text{R}_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\text{R}_2 - \text{R}_1} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 10 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\text{R}_2 / 10} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a.  $T_A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & -3 \end{bmatrix}^{-1}$   
 $\begin{bmatrix} x_1 & x_2 & x_3 \\ 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$   
 free.

$$\begin{cases} x_1 = -7x_3 \\ x_2 = -3x_3 \\ x_3 = x_3 \end{cases} \quad \mathbf{x} = \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix} \quad \mathbf{u} \in \text{Span} \left[ \begin{pmatrix} -7 \\ -3 \\ 1 \end{pmatrix} \right]$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$(a) A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$(b) T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \quad A \mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \quad \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow[-1]{R_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow[-1]{R_3 - R_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & -1 \end{array} \right] \xrightarrow[-1]{R_2 - 3R_3} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 12 \\ 0 & 1 & -1 \end{array} \right] \xrightarrow[-1]{R_1 + 2R_3} \left[ \begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 0 & 12 \\ 0 & 1 & -1 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array} \right] \xleftarrow{\text{(1)} R_3 - R_2} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \xleftarrow{\text{(1)} R_1 - R_3} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \xleftarrow{\text{(1)} R_1 \leftrightarrow R_3} \left[ \begin{array}{cc|c} 0 & 1 & 3 \\ 0 & 0 & 0 \\ 1 & 0 & 7 \end{array} \right] \xleftarrow{\text{(1)} R_1 \leftrightarrow R_3} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$u_1 = 7$$

$$u_2 = 3$$

$$u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} \quad T(e_1) = T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$T(e_2) = T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} \quad \text{standard matrix of } T = A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $A \cdot u = T(u)$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{(-1)} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\downarrow \cdot (-1)} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{\downarrow \rightarrow} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -1 & -3 \\ 0 & 3 & 9 \end{array} \right] \xrightarrow{(-1)} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{array} \right] \xrightarrow{\cdot 2} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 6 \\ 0 & 3 & 9 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{array} \right] \xrightarrow{\downarrow \cdot (-3)} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \quad \begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned} \quad u = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$$

check:  $T \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 - 2(3) \\ 7 + 3 \\ 7 - 3(3) \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix} \quad \checkmark$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 - 2 \cdot 0 \\ 1 + 0 \\ 1 - 3 \cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 - 2 \cdot 1 \\ 0 + 1 \\ 0 - 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard matrix of  $T$  is equal

$$T \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{x_1 -} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 2 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{x_2 -} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{x_1 -} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 0 & -1 & -2 \end{array} \right] \xrightarrow{x_3 +} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 0 & 0 & 7 \end{array} \right]$$

So when  $\mathbf{u}$  equal  $\begin{bmatrix} 7 \\ 9 \\ 7 \end{bmatrix}$  will satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 0 & 1 & 7 \end{array} \right] \xrightarrow{x_1 -} \left[ \begin{array}{cc|c} 1 & -3 & 8 \\ 0 & 1 & 9 \\ 0 & 0 & 2 \end{array} \right]$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .  $\rightarrow T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$\left. \begin{array}{l} e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \end{array} \right\} T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = T(\bar{\mathbf{u}})$$

$$\begin{cases} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{cases}$$

$\downarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\begin{array}{l} -2x_1 = -2 \\ 1 - 2 = 3 \\ 10 - 1 = 9 \end{array}} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -1 \end{array} \right] \xrightarrow{\begin{array}{l} x_1 = 1 \\ 3x_1 = 9 \\ -1 = -1 \end{array}}$$

$\downarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -1 & -1 \end{array} \right] \xrightarrow{\begin{array}{l} -2x_1 = -2 \\ 3 + 2 = 1 \\ -1 + 1 = 0 \end{array}} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

$\downarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right] \xrightarrow{\frac{1}{3}} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{x_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{array} \right] \Rightarrow \text{Inconsistent system}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)

$$A = [T(x_1) \quad T(x_2)]$$

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

Standard Matrix =  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $\begin{aligned} x_1 - 2x_2 &= 1 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 \end{aligned}$

$$\Rightarrow \left[ \begin{array}{ccc|c} x_1 & x_2 & & \\ 1 & -2 & 1 & 1 \\ 1 & 1 & 10 & \\ 1 & -3 & -2 & \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 0 & 9 & 9 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 9 & 9 \end{array} \right]$$

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$T \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix} = \begin{bmatrix} -19 \\ 11 \\ 29 \end{bmatrix} \quad \checkmark$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}, \therefore e_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, e_2 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$\therefore$  Std Matrix  $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $A(\mathbf{u}) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -2 & -3 \end{bmatrix} \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow R_2 - R_1 \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right] \rightarrow R_3 - R_1 \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \frac{R_3}{3} \rightarrow$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \rightarrow R_1 + 2R_2 \rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \begin{aligned} x_1 &= 7 \\ x_2 &= 3 \\ x_3 &= x_3 \end{aligned} \quad \text{Since } \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ x_3 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$

$$ax_1 + bx_2 = x_1 - 2x_2$$

$$cx_1 + dx_2 = x_1 + x_2$$

$$ex_1 + fx_2 = x_1 - 3x_2$$

$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $A\mathbf{v} = \mathbf{b}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \mathbf{v} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\cdot(-1)}$$

$$\xrightarrow{\cdot(1)} \left[ \begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\cdot(-1)}$$

$$\left[ \begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 1 & 10 \\ 0 & 4 & -2 \end{array} \right] \xrightarrow{\cdot\frac{1}{4}}$$

$$\left[ \begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 1 & 10 \\ 0 & 1 & 3 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 3x_2 - 2$$

$$x_2 = x_2$$

$$\begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} + \text{span} \left\{ \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \quad \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

$$CT(x) = T(u) \quad C_1 = 1 + 2C_2 = 7$$

$$C_1(k) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$$

$$C_2 = 3$$

$$\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right| \quad \left| \begin{array}{cc|c} 1 & -2 & 7 \\ 1 & 1 & 3 \\ 1 & -3 & -2 \end{array} \right| = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{pmatrix}$$

$$u = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{pmatrix}$$

$$\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right|$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \begin{matrix} \\ \\ x_1 \\ x_2 \end{matrix}$$

$$T \begin{pmatrix} 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\text{R}_2: R_2 - R_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 1 & 0 & ? \end{array} \right] \xrightarrow{\text{R}_3: R_3 - R_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 0 & 2 & ? \end{array} \right]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^2$$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 5 \\ 16 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} ? \\ -5 \\ 16 \end{bmatrix}$$

free variable.

$$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 0 & 2 & 6 \end{array} \right] \xrightarrow{\text{R}_3: R_3 - R_2} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 8 \\ 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{R}_2: R_2 - R_1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{array} \right]$$

$R_1: R_1 + R_2$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{array} \right]$$

$R_3: R_3 - 7R_2$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(\mathbf{e}_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(\mathbf{e}_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = [T(\mathbf{e}_1) \quad T(\mathbf{e}_2)]$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\begin{array}{l} R_2 \leftarrow R_1 + R_2 \\ R_3 \leftarrow R_1 + R_3 \end{array} \quad \left| \begin{array}{ccc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right. \quad \downarrow$$

$$R_3 \leftarrow R_3 - R_2 \quad \left| \begin{array}{ccc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right.$$

$$\left| \begin{array}{ccc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{array} \right.$$

$$\mathbf{u} = \begin{bmatrix} 2x_2 + 1 \\ -3x_2 + 9 \\ -x_2 + 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

$$\begin{matrix} [2x3] & \begin{bmatrix} x_1 & x_2 \\ x_1 & x_2 \\ x_1 & x_2 \end{bmatrix} & [3] \\ \downarrow & \downarrow & \downarrow \\ 2x1 & & 2x1 \end{matrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$\begin{matrix} 2x3 & 3x1 & 2x1 \\ \downarrow & \downarrow & \downarrow \\ 2x1 & & 2x1 \end{matrix}$$

A)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$\begin{matrix} 2x3 & 3x1 \\ \downarrow & \downarrow \\ 3x2 & 2x1 \end{matrix}$$

$$\begin{matrix} 3x1 & 7x3 \\ \downarrow & \downarrow \\ 7x1 & 7x1 \end{matrix}$$

B)  $\mathbf{v} = c_1 \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$

Anything multiple of  
this vector.



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

A

"

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A$  is a  $3 \times 2$  matrix.

$$A = [T(e_1) \ T(e_2)] , \ e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ e_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$\therefore A = \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}_{\text{is the standard matrix}} \text{ of } T.$

$$(b) \ T(u) = T_A(u) = A \cdot u = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}, \ u = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

augmented matrix

$$\left[ \begin{array}{cc|c} x_1 & x_2 & \\ 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\text{row reduction}} \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} \text{sol} \\ \left\{ \begin{array}{l} x_1 = 1 \\ x_2 = 3 \end{array} \right. \end{array}$$

$$\therefore u = \underbrace{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}_{\text{sol}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ . im defining pass as A.

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$a) T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

standard matrix;  $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left. \begin{array}{l} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{array} \right\} \quad \begin{array}{l} x_1 = 1 + 2x_2 \\ 1 + 2x_2 + x_2 = 10 \Rightarrow 1 + 3x_2 = 10 \\ 3x_2 = 9 \\ x_2 = 3 \end{array}$$

$$x_1 - 2(3) = 1$$

$$x_1 = 7$$

$$7 - 9 = -2 \checkmark$$

↓  
so  $u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

or any multiple  
of it.



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

std. matrix of  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$\begin{array}{l} x_1 - 2x_2 = 1 \\ x_1 + x_2 = 10 \\ x_1 - 3x_2 = -2 \end{array} \quad \left( \begin{array}{ccc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$$

$$\downarrow \quad \left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 3 & 9 & 9 \\ 0 & 5 & -3 & -3 \end{array} \right)$$

$$\downarrow \quad \left( \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 5 & -3 & -3 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} 1 & 0 & 7 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & -18 & -18 \end{array} \right)$$

$\therefore$  There are no vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

(a)  $A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \\ a_{31}x_1 + a_{32}x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$

$$a_{11}=1 \quad a_{12}=-2 \quad a_{21}=1 \quad a_{22}=1 \quad a_{31}=1 \quad a_{32}=-3$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

(b)  $T(u) = Au = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \text{ row red } \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 11 \\ 0 & 1 & 3 \end{array} \right]$$

$$\begin{aligned} u_1 &= 1 \\ u_2 &= 3 \end{aligned}$$

$$\boxed{u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a,  $T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \times e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A = \begin{vmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$$

$$\begin{vmatrix} T(e_1) & T(e_2) \\ 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{vmatrix} \sim \begin{vmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{vmatrix} \quad \begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned}$$

$$\boxed{\mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)  $T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$

$\therefore$  The Standard Matrix of  $T$

$$T = c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} \quad \therefore T = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$$

b)  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$$

$$\begin{bmatrix} c_1 - 2c_2 \\ c_1 + c_2 \\ c_1 - 3c_2 \end{bmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix} \quad \therefore T(\mathbf{u}) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1) \ T(e_2)]$

~~Find  $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$~~

$$\Rightarrow T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

b)  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$A \cdot u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{array}{r} 1 & -2 & 1 \\ 0 & 2 & 6 \\ \hline 1 & 0 & 7 \end{array}$$

$$u = \begin{bmatrix} x_1 \\ x_2 \\ c \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 3 \end{bmatrix} \quad (\text{only 1 solution})$$

$$\begin{array}{r} -1 & 2 & -1 \\ 1 & 1 & 10 \\ \hline 0 & 3 & 9 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 6 \\ 1 & -3 & -2 \end{bmatrix}$$

$$x_1 = 7$$

$$x_2 = 3$$

$$\begin{array}{r} -1 & 2 & -1 \\ 1 & -3 & -2 \\ \hline 0 & -1 & -3 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 1 & -1 & -3 \end{bmatrix} \xrightarrow{21} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{21}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$\begin{aligned} e_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} & T(e_1) &= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} & T(e_2) &= \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} \\ e_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} & & & & \end{aligned}$$

Standard matrix  $\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 0 & -3 \end{pmatrix}$

$$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -3 & -2 \end{array} \right) \xrightarrow{R2 - \frac{1}{3}R3} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 20/3 \\ 0 & -3 & -2 \end{array} \right) \xrightarrow{R3 + 3R2}$$

$$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 20/3 \\ 0 & 0 & 18 \end{array} \right) \xrightarrow{R1 + 2R2} \left( \begin{array}{cc|c} 1 & 0 & 10/3 \\ 0 & 1 & 20/3 \\ 0 & 0 & 18 \end{array} \right)$$

$\mathbf{u} = \begin{bmatrix} 10/3 \\ 20/3 \\ 18 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)  $\left\{ T \begin{pmatrix} 1 \\ 0 \end{pmatrix}, T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} \right\}$

$$\boxed{\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}} \quad T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-2(1) \\ 1+1 \\ 1-3(1) \end{pmatrix} = \begin{pmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$$

b)  $T \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

$$A \mathbf{x} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & -3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$$

$$\begin{aligned} x_1 - 2x_2 &= 1 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 \end{aligned}$$

$$\begin{aligned} 10 - x_2 &= 3x_2 - 2 \\ 4x_2 &= 12 \\ x_2 &= 3 \end{aligned}$$

$$x_1 = 1 + 2(3)$$

$$\begin{pmatrix} 7 & 2(3) \\ 7 & +3 \\ 7 & -3(3) \end{pmatrix} = \begin{pmatrix} 7-16 \\ 10 \\ 7-9 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$$

$$\boxed{u = \begin{pmatrix} 7 \\ 10 \\ -2 \end{pmatrix}}$$

This is one solution

$$\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

c)  $\boxed{T(e_1) \quad T(e_2)}$

standard  
matrix of  $T = \boxed{\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}}$

b)  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \Rightarrow \text{row reduction} \quad \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 7$$

$$x_2 = 3$$

$$\mathbf{u} = 4 \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

b)  $\boxed{\mathbf{u} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

$$\text{b) Find all vectors } u \text{ satisfying } T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}.$$

$$\text{a) } A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$$A = \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 1 & -2 & | & 1 \\ 1 & 1 & | & 10 \\ 1 & -3 & | & -2 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & -2 & | & 1 \\ 1 & 1 & | & 10 \\ 0 & -1 & | & -3 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & -2 & | & 1 \\ 1 & 1 & | & 10 \\ 0 & 1 & | & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & | & 6 \\ 1 & 1 & | & 10 \\ 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 0 & | & 6 \\ 0 & 1 & | & 3 \\ 1 & 1 & | & 10 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 0 & | & 6 \\ 0 & 1 & | & 3 \\ 0 & 0 & | & 1 \end{bmatrix}$$

$$u = \text{Span}\left(\begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix}\right)$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

standard matrix of  $T = A$

$A$  is  $3 \times 2$  matrix where  $A \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$

$$\begin{bmatrix} * & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b. Find  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -2 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \leftarrow \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \leftarrow \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right]$$

$$x_1 = 1 \quad x_2 = 3$$

$$u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$u = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right) \xrightarrow{(-1)} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -4 & -2 \end{array} \right)$$

$$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right) \xleftarrow{\text{Add } 2\text{ times Row 2 to Row 1}} \left( \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right) \xleftarrow{\text{Divide Row 3 by 3}}$$

$$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right) \xleftarrow{\text{Add } 2\text{ times Row 3 to Row 1}} \left( \begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right)$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)

$$A = [T(\ell_1) \ T(\ell_2)] \quad \ell_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \ell_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(\ell_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(\ell_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & -14 \end{array} \right] \xrightarrow{\text{Row } 2 \rightarrow \text{Row } 2/3}$$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\text{Row } 2 \rightarrow \text{Row } 2 - \text{Row } 1}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 10 \\ 1 & -4 & -2 \end{array} \right] \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 - \text{Row } 1}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 10 \\ 0 & -4 & -12 \end{array} \right] \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 + 4 \cdot \text{Row } 2}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -14 \end{array} \right] \xrightarrow{\text{Row } 2 \rightarrow \text{Row } 2 \cdot 3}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & -14 \end{array} \right] \xrightarrow{\text{Row } 3 \rightarrow \text{Row } 3 + 14}$$

$\therefore$  There are no solutions for  $\mathbf{u}$  that satisfy  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $M \times n$  matrix

$$\mathbf{x}_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{x}_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \left[ \begin{array}{ccc} 1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{array} \right] & & \text{free} \end{array}$$

b)

$$\begin{aligned} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 1 & 1 & 0 & 10 \\ 1 & -3 & 0 & -2 \end{array} \right]$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

(a)  $T\left(\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}, e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\Rightarrow T\left(\begin{bmatrix} e_1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \Rightarrow T\left(\begin{bmatrix} e_2 \end{bmatrix}\right) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

So, Standard matrix,  $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$  Ans:-

(b) From definition,  $T(\mathbf{u}) = A \cdot \mathbf{u} = A \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ , as  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

Now,  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$   
 $A \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

Since,  $T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_2 \\ u_1 - 3u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{cases} u_1 - 2u_2 = 1 & (1) \\ u_1 + u_2 = 10 & (2) \\ u_1 - 3u_2 = -2 & (3) \end{cases}$$

From (2),  $u_1 = 10 - u_2$

$\therefore$  from (1),  $10 - u_2 - 2u_2 = 1$

$\Rightarrow 10 - 3u_2 = 1$

$\Rightarrow 10 - 1 = 3u_2$

$\therefore u_2 = \frac{9}{3} = 3, u_1 = 10 - 3 = 7$

$\therefore \vec{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix} = 7\hat{u}_1 + 3\hat{u}_2$

Ans:-



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$a) T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

the standard matrix :  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$b) \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\text{ref}} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array} \Rightarrow \mathbf{v} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)  $\begin{vmatrix} 1 & 2 \\ 1 & 1 \\ 1 & -3 \end{vmatrix}$

b)  $\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 9 \\ 0 & -4 & -3 \end{vmatrix}$

$R_1 \times 1 + R_2 + R_3 \rightarrow R_1 \times -1 + R_3 \rightarrow R_2 \times 5 + R_3$

$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 9 \\ 0 & 0 & -49 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{vmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

There are no vectors ( $\mathbf{u}$ ) that satisfy

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T(\mathbf{u}) = T \left( \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix}$

b) 
$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -2 & -8 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & 0 & -9 \end{array} \right]$$

→ No vector satisfies  $T(\mathbf{u})$ .



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$A = [T(e_1), T(e_2)]$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ . where  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

b)

$$T(u) = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$U = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)  $A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$  = standard matrix of  $T$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 + R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 8 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 / 3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\begin{array}{l} x_1=1 \\ x_2=3 \end{array}} \begin{array}{l} x_1=1 \\ x_2=3 \end{array} \quad \mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ ? \end{bmatrix}$$

$$\text{such } \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a) Standard Matrix of  $T$ :

$$A = [T(e_1) \ T(e_2)] = [T\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \ T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)] = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow -R_1 + R_2 \\ R_3 \rightarrow -R_1 + R_3}} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{R_2}{3}}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_2 + R_3 \\ R_1 \rightarrow 2R_2 + R_1}} \left[ \begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$u = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$

there is a  
unique solution



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

$$A = [T(e_1) \ T(e_2)]$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a) Standard Matrix :

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $T(\vec{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$A\vec{u} = b$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \vec{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{array}{l}
 \text{R2} \cdot -1 \\
 + \text{R2} \\
 \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right) \xrightarrow{\substack{\text{R1} \leftrightarrow \text{R2} \\ \text{R3} \cdot -1}} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right) \xrightarrow{\text{R3} \cdot -1} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right) \xrightarrow{\substack{\text{R2} \cdot 3 \\ \text{R3} \cdot 3}} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 9 & 27 \\ 0 & 1 & 3 \end{array} \right) \\
 = \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array} \right) \Rightarrow \begin{array}{l} u_1 - 2u_2 = 1 \\ u_2 = 3 \\ u_1 - 6 = 1 \end{array} \quad \begin{array}{l} u_1 - 2(3) = 1 \\ u_1 = 7 \end{array}
 \end{array}$$

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1), T(e_2)] \quad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = \begin{bmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{bmatrix} = \begin{bmatrix} 1 - 0 \\ 1 + 0 \\ 1 - 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{bmatrix} = \begin{bmatrix} 0 - 2 \\ 0 + 1 \\ 0 - 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$A = \boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)

$$\rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -1 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{array} \right]$$

$$\boxed{x_1 = 2x_2}$$

$$x_2 = x_2$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix} \quad e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T(e_1) = T\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$T(e_2) = T\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

a)

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $Tu = Au = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{-1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 8 \\ 0 & -1 & -4 \end{array} \right] \xrightarrow{\quad} \rightarrow$$

$$\rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 3 & 8 \end{array} \right] \xrightarrow{-1} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 3 & 8 \end{array} \right] \xrightarrow{-3} \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -4 \end{array} \right]$$

no solutions  
for  $u$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .  $x_1 = 2x_2 + 1$

a)  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

b)

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 & -2 \\ 0 & +1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$\therefore$   $\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

Standard Matrix of  $T = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$

$$\left[ T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1-2(0) \\ 1+0 \\ 1-3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0-2(1) \\ 0+1 \\ 0-3(1) \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

a)  $\boxed{\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}}$

b)  $\begin{pmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 3 & 11 \\ 0 & -1 & -3 \end{pmatrix} \xrightarrow{R_3 + R_2} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$

b) No vectors satisfy that condition



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

- a) Find the standard matrix of  $T$ .  $T$  is  $3 \times 2$   $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$
- b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{aligned} u_1 - 2u_2 &= 1 \\ u_1 + u_2 &= 10 \\ u_1 - 3u_2 &= -2 \end{aligned}$$

$$\begin{aligned} -u_2 &= 3 \\ u_2 &= -3 \\ u_1 &= 7 \end{aligned}$$

$u = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

A)  $T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

Standard Matrix =  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

B)  $x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_1 - R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & -1 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & -3 & -1 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \rightarrow 3R_3 + R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_1 \rightarrow 2R_3 + R_1} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \xleftarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} x_1 & x_2 & 7 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned} \quad u = \boxed{\begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1-0 \\ 1+0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

standard matrix  $A$ :

$$A = \begin{bmatrix} T(e_1) & T(e_2) \end{bmatrix}$$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0-2 \\ 0+1 \\ 0-3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$   $\xrightarrow{(1) \leftrightarrow (2)} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$

$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$   $\xrightarrow{(2) \leftrightarrow (3)} \left( \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 10 \\ 0 & 0 & -2 \end{array} \right)$

$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$   $\xrightarrow{(1) + (-1) \cdot 1} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 0 & -4 & -3 \end{array} \right)$

$\left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 0 & -4 & -3 \end{array} \right)$   $\xrightarrow{(-4) \cdot 1 + 2} \left( \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 0 & 0 & -13 \end{array} \right)$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ -13 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & -2 \\ 0 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 0 & -2 & -6 \\ 0 & 3 & 9 \\ 1 & 0 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cc} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{array} \right] \left[ \begin{array}{c} 7 \\ 3 \\ 7 \end{array} \right] = \left[ \begin{array}{c} 1 \\ 10 \\ -2 \end{array} \right]$$

$$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ 7 \end{bmatrix}$$

Augmented  
matrix

Row  
reduction



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

c)  $T \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$      $T \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$      $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

c)  $\begin{pmatrix} T(e_1) & T(e_2) \end{pmatrix}$        $T(e_1) = T\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Standard Matrix  $= \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \quad | \quad T(e_2) = T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 & -2 \\ 0 & 1 \\ 0 & -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

d)  $x_1 - 2x_2 = 1$        $\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right|$   
 $x_1 + x_2 = 10$   
 $x_1 - 3x_2 = -2$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a) the standard matrix of  $T$

is  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 10 \\ 0 & 3 & 9 \\ 0 & 4 & 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 10 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$u_1 = 7$$

$$u_2 = 3$$

$$u_3 = u_3$$

$$u = \begin{bmatrix} 7 \\ 3 \\ u_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_3.$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T(e_1) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad T(e_2) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

b)  $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 10 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 10 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 10 \\ 0 & 0 & 12 \end{bmatrix} \xrightarrow{2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \left\{ \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array} \right.$

$$T \begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{bmatrix} 7-6 \\ 7+3 \\ 7-9 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\boxed{\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \Rightarrow \text{standard matrix of } T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$(R_1 \times -1) + R_2 \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right]$$

$$(R_1 \times -1) + R_3 \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -3 & -2 \end{array} \right] \xrightarrow{R_3 \times -1}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{R_3 + R_2}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array} \right] \xrightarrow{(R_3 \times 2) + R_1}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array} \right] \xrightarrow{(R_2 \times -1) + R_3}$$

$$\rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 7$$

$$x_2 = 3$$

$$u = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$\mathbf{u} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 7 \\ x_2 &= 3 \end{aligned}$$

$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a.

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b. ①  $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix}$

② -①

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

③ -①

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{bmatrix}$$

7.

③  $\times 2 - ①$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$

$$3x_2 = 9$$

$$x_2 = 3$$

$$x_1 - 2x_2 = 1$$

$$x_1 - 6 = 1$$

$$x_1 = 7$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix} \quad 3 \times 2 \quad 2 \times 2$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

~~(Hand a)~~

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 2 & 10 \\ 1 & -3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 2 & 0 \\ 1 & -3 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{cc|c} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -2 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & -2 & 0 \end{array} \right] \xrightarrow{\text{Row operations}} \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{array} \right]$$

~~(Hand a)~~

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 2 & 10 \\ 1 & -3 & -2 \end{array} \right] = \left[ \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right]$$

~~a~~ ~~be~~

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

a)  $\begin{bmatrix} 1 & -2 \\ 1 & 2 \\ 1 & -3 \end{bmatrix}$

b)  $u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = (T(e_1), T(e_2))$        $T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$      $T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $T(\mathbf{u}) = A\mathbf{u} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$      $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 4 & 12 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 \\ 0 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} x_2 = 3 \\ x_1 = 7 \end{array} \quad \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $\mathbf{u} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$\begin{aligned} x_1 - x_2 &= 1 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 \end{aligned} \quad \begin{aligned} 2x_1 &= 11 \Rightarrow x_1 = 4.5 \text{ or } \frac{9}{2} \\ 9/2 - 3x_2 &= -2 \\ -3x_2 &= -\frac{13}{2} \\ x_2 &= -\frac{13}{6} \end{aligned}$$

But  $\frac{9}{2} - \frac{13}{6} \neq 10$

$\therefore$  There are no vectors  $\mathbf{u}$ , which satisfy  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)

$$\begin{bmatrix} 3 \times 2 \\ 3 \times 1 \end{bmatrix} = \begin{bmatrix} 3 \times 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$3 \times 2 \quad 2 \times 1 \quad 3 \times 1$$

$\text{ref}( \begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{vmatrix} )$   
would solve it

$$U = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$\begin{aligned} c_1 - 2c_2 &= 1 & \left( \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 7 \end{bmatrix} \right) \\ c_1 + c_2 &= 10 & \left( \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix} \right) \\ c_1 - 3c_2 &= -2 & \cancel{\left( \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix} \right)} \\ \cancel{\left( \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix} \right)} & & \cancel{c_1 = 3} \\ & & \cancel{c_2 = 7} \\ & & \cancel{c_1 - 2c_2 = 1} \\ & & \cancel{c_1 = 12} \\ & & \cancel{c_1 = 15} \end{aligned}$$

15-14



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$   $T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$

b)  $\begin{aligned} 1 &= x_1 - 2x_2 \\ 10 &= x_1 + x_2 \\ -2 &= x_1 - 3x_2 \end{aligned}$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 1 & 1 & 10 & 10 \\ 1 & -3 & -2 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 0 & 1 & 3 & 0 \\ 1 & 1 & 10 & 10 \\ 1 & -3 & -2 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 3 & 10 \\ 1 & -3 & -2 & -2 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 3 & 10 \\ 0 & -3 & -2 & -2 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array} \quad \boxed{\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

Ⓐ  $\begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix} \mathbf{u} = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$

Ⓑ  $\begin{pmatrix} 1 & -2 & | & 1 \\ 1 & 1 & | & 10 \\ 1 & -3 & | & -2 \end{pmatrix} \xrightarrow{R_1 + R_3} \begin{pmatrix} 1 & -2 & | & 1 \\ 1 & 1 & | & 10 \\ 0 & 1 & | & -1 \end{pmatrix} \xrightarrow{R_1 + R_2} \begin{pmatrix} 1 & -2 & | & 1 \\ 0 & 3 & | & 10 \\ 0 & 1 & | & -1 \end{pmatrix}$

$\boxed{\begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}}$

$$7 - 2(3) = 1$$

$$7 + 3 = 10$$

$$7 - (3)(3) = -2$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} v$

$$T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$$

b)  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} u = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$u_1 - 2u_2 = 1$$

$$u_1 + u_2 = 10$$

$$u_1 - 3u_2 = -2$$

$$u = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1) \ T(e_2)]$   
 $T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$     $T(e_2) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

$$Au = T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\cdot(-1)}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 1 & -3 & -2 \end{array} \right] \xrightarrow{\cdot(-1)}$$

$$\boxed{\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{\cdot(1/3)}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \xrightarrow{\cdot(1)}$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \xrightarrow{\cdot(2)}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \boxed{\begin{cases} x_1 = 7 \\ x_2 = 3 \end{cases}}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

a)  $u = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad v = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(u) = \begin{pmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{pmatrix} \quad T(v) = \begin{pmatrix} b_1 - 2b_2 \\ b_1 + b_2 \\ b_1 - 3b_2 \end{pmatrix}$$

$$T(e_1) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad T(e_2) = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$$

standard matrix of  $T = \begin{pmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{pmatrix}$

b)  
 $R_3 = R_3 - R_1$   $\left( \begin{array}{ccc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right)$

$+2R_2 \rightarrow \left( \begin{array}{ccc|c} 1 & -2 & 1 \\ 0 & 3 & 10 \\ 0 & 1 & -3 \end{array} \right)$

$\downarrow$   
 $\left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 10 & 1 \\ 0 & 1 & -3 & -2 \end{array} \right) \xrightarrow{\text{Row } 3 - \text{Row } 2} \left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 10 & 1 \\ 0 & 0 & -13 & -3 \end{array} \right)$

$\downarrow$   
 $\left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 10 & 1 \\ 0 & 0 & -13 & -3 \end{array} \right)$

all vectors  $u$  satisfying  
 $T(u) = \begin{pmatrix} 1 \\ 10 \\ -3 \end{pmatrix}$   
 are:  
 $\begin{pmatrix} 1 \\ x_2 \\ 10 \\ -3 \end{pmatrix}$  where  $x_2$  is free

$x_1 = 1$   
 $x_2 = x_2$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

$$\text{a)} \quad T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 2(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + 1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \\ A_5 & A_6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A_1 = 1$$

$$A_3 = 1$$

$$A_5 = 1$$

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \\ A_5 & A_6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$A_2 = -2$$

$$A_4 = 1$$

$$A_6 = 3$$

standard

matrix

$$\text{of } T \text{ is } \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$\text{b)} \quad \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & 3 & -2 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & 3 & 3 \\ 1 & 1 & 10 & 10 \\ 1 & 0 & -11 & -11 \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -11 & 3 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 13 & 13 \end{array} \right]$$

No solutions

$$\left[ \begin{array}{ccc|c} 0 & -3 & -9 & \\ 1 & 1 & 10 & \\ 1 & 3 & -2 & \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & 3 & \\ 1 & 0 & 7 & \\ 1 & 0 & -11 & \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 0 & -3 & -9 & \\ 1 & 1 & 10 & \\ 1 & 0 & -11 & \end{array} \right]$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & 3 & \\ 0 & 0 & 18 & \\ 1 & 0 & -11 & \end{array} \right]$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad T(e_1) \Rightarrow T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$   
 $T(e_2) \Rightarrow T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

standard Matrix of  $T$ :

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 0 & -3 \end{bmatrix}$$

b)  $A\mathbf{v} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} \Rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 0 & -3 & -2 \end{array} \right] \Rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 9 \\ 1 & -3 & -2 \end{array} \right]$

(1/3)  $\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right] \Rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{array} \right] \Rightarrow \left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$

$$\left[ \begin{array}{cc|c} x_1 & x_2 & \\ \hline 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array} \quad \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 4$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 - 2(0) \\ 1 + 0 \\ 1 - 3(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$   $T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 - 2(1) \\ 0 + 1 \\ 0 - 3(1) \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$

Standard matrix of  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)

$$\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right| \quad | \quad \left| \begin{array}{cc|c} 1 & -2 & 0 \\ 1 & 1 & 0 \\ 1 & -3 & 0 \end{array} \right|$$

$$\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & -1 & -3 \end{array} \right| \quad | \quad \left| \begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & -1 & 0 \end{array} \right|$$

$$\left| \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right| \quad | \quad \left| \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right|$$

$$x_1 = 1 + 2x_2$$

$$x_2 = 3 \quad \left| \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right| \quad \text{Null}(A) = \{0\}$$

$$\begin{bmatrix} 7 \\ 3 \end{bmatrix} \quad \left| \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right| \quad \begin{array}{l} x_1 = 7 \\ x_2 = 3 \end{array}$$

$$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

all vectors satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix} : \left\{ \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

(a)  $T(e_1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$T(e_2) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

Standard matrix of  $T$ :  $\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

(b)  $\mathbf{u} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$$T(\mathbf{u}) = T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_1 - 2a_2 \\ a_1 + a_2 \\ a_1 - 3a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$\begin{aligned} a_1 - 2a_2 &= 1 \\ a_1 + a_2 &= 10 \end{aligned}$$

$$\begin{aligned} 3a_2 &= 9 \\ a_2 &= 3 \end{aligned}$$

$$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $u$  satisfying  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a) Let  $T$  be the  $3 \times 2$  matrix

$$T = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \\ c_5 & c_6 \end{bmatrix}$$

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

$$c_1x_1 + c_2x_2 = x_1 - 2x_2$$

$$c_3x_1 + c_4x_2 = x_1 + x_2$$

$$c_5x_1 + c_6x_2 = x_1 - 3x_2$$

b)  $T(u) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$

Let  $u$  be vector  $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$$\therefore \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

Aug. matrix

$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{bmatrix} \xrightarrow{(1)+(3)} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 10 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(2)-(1)} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 3 & 9 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(2)\div 3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix} \xrightarrow{(3)-(2)} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{(1)+(2)\times 2} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{cases} c_1 = 7 \\ c_2 = 3 \end{cases}$$

$$\therefore u = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

Ⓐ 
$$\begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}_{\text{Matrix } A}$$

Ⓑ 
$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] \text{ so } \begin{aligned} x_1 - 2x_2 &= 1 & x_1 &= 1 + 2x_2 \\ x_1 + x_2 &= 10 \\ x_1 - 3x_2 &= -2 & x_1 &= 1 + 6 = 7 \end{aligned}$$

or 
$$\left[ \begin{array}{ccc|c} 1 & -2 & 1 & \\ 1 & 1 & 10 & \\ 0 & -4 & -12 & \end{array} \right] \text{ so } -4x_2 = -12$$

$x_2 = 3$  so  
 $x_1 = 7$   $\left[ \begin{array}{c} 7 \\ 3 \end{array} \right]$   ~~$\left[ \begin{array}{c} 7 \\ 3 \end{array} \right]$~~



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

b)  $\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right] = \left[ \begin{array}{c} \\ \\ \end{array} \right]$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{pmatrix} 1 \\ 10 \\ -2 \end{pmatrix}$ .

Standard Matrix  $T = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}$

$$x_1 - 2x_2 = 1$$

$$T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$$

$$x_1 + x_2 = 10$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 1 & 1 & 10 \\ 1 & -3 & -2 \end{array} \right]$$

$$x_1 - 3x_2 = -2$$

$$-3x_2 = -9$$

$$x_2 = 3$$

$$x_1 + 3 = 10$$

$$x_1 = 7$$

$$\left[ \begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 4 & 9 \\ 0 & 1 & 3 \end{array} \right]$$

$$\mathbf{u} = \begin{bmatrix} 7 \\ 3 \\ 3 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 4 & 9 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & -3 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array} \right]$$



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .



4. (20 points) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{pmatrix}$$

a) Find the standard matrix of  $T$ .

b) Find all vectors  $\mathbf{u}$  satisfying  $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ .

a)  $A = [T(e_1) \ T(e_2)]$  where  $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$   $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -3 \end{bmatrix}}$$

b)