4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$\lambda = 3$

$A - 3I$:

$$\begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \begin{matrix} \\ +(-1)R_1 \\ +R_1 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} -2 & 8 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot -\tfrac{1}{2} \rightarrow \begin{bmatrix} 1 & -4 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$$x_1 = 4x_2 + 2x_3$$

$$\lambda = 3: \left\{ \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$\lambda = 5$ $A - 5I$:

$$\begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \cdot -\tfrac{1}{4}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & -1 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \begin{matrix} \\ +2R_1 \\ +(-2)R_1 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 2 & 2 \\ 0 & -4 & -4 \end{bmatrix} \begin{matrix} \\ \\ +2R_2 \end{matrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \cdot \tfrac{1}{2} \rightarrow \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} +2R_2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

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$x_1 = -x_3$

$x_2 = -x_3$

$$\lambda = 5: \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

Note:

also equal to $[v_1, v_2, v_3]$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

With eigenvalues in D corresponding to v_1, v_2, v_3 respectively