1. Consider the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \quad w = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$ (you do not need to verify it).

a) Compute $[w]_{\mathcal{B}}$, the coordinate vector of w relative to the basis $\mathcal{B}$.

b) Let $u \in \mathbb{R}^3$ be a vector such that

$$[u]_{\mathcal{B}} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$$

Compute the vector u.

a)

$$\begin{array}{c} \\ (-1) \\ (-2) \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 0 & 0 & 5 \\ 2 & 2 & 2 & 8 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right] \begin{array}{c} \\ (-1) \\ \cdot(\frac{1}{2}) \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right] (-1) \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\boxed{[w]_{\mathcal{B}} = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}}$$

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b) $$u = 4v_1 + 5v_2 + 2v_3 = 4\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 22 \end{bmatrix}$$

Hello!

$$\boxed{u = \begin{bmatrix} 9 \\ 4 \\ 22 \end{bmatrix}}$$

to check boxes.

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