$0\,1\,0\,1 \quad 0+1 \quad 0+1 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{+} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{+} = \begin{matrix} a & c \\ b & d \end{matrix}$

5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

$A^T = A \qquad B^+B = I$

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ then $\begin{bmatrix} 1-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$ gives $\lambda = 1, 2$ for $\lambda = 1$ Eigenvector $= \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$2(1) = 2$ $\begin{bmatrix} 1-2 & 1 \\ 0 & 2-2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$ Eigenvector $= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $2 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\boxed{\therefore \text{False}}$

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~~True, the formula for~~ ~~$\text{Proj}_V w = \left(\frac{w \cdot v}{v \cdot v}\right) v$ if $w =$ zero vector $\left(\frac{w \cdot v}{v \cdot v}\right) = 0$~~

~~$\therefore \left(\frac{w \cdot v}{v \cdot v}\right)$ will $= 0$ thus being true.~~

True because the matrix is symmetric $A^T = A$, and orthogonal $A^+ \cdot A = I$. A^2 will be equivalent to $A \cdot A^+$ as $A = A^T$ therefore equating to the identity matrix.

True

~~True~~, only symetric 2x2 matrices and a symmetric matrix plus another symetric matrix will be symmetric.

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~~False~~, if $V = -w$ then $\text{Proj}_V W = \left(\frac{-w^2}{-w^2}\right) \cdot w = 1(-w) = -w$

therefore w can be a non-zero vector