4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$$A-\lambda_1 = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & -8 & -4 & 0 \end{array}\right] \cdot \tfrac{1}{2}$$

$$A-\lambda_2 = \left[\begin{array}{ccc|c} -4 & 8 & 4 & 0 \\ -2 & 6 & 4 & 0 \\ 2 & -8 & -6 & 0 \end{array}\right] \cdot 2$$

$$\left[\begin{array}{ccc|c} 0 & -8 & -8 & 0 \\ 0 & -2 & -2 & 0 \\ 2 & -8 & -6 & 0 \end{array}\right] \begin{array}{l} \cdot -4 \\ \cdot -\tfrac{1}{2} \\ \cdot \tfrac{1}{2} \end{array}$$

$$\begin{bmatrix} 1 & -4 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \quad x_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -4 & -3 & 0 \end{array}\right] \cdot 4$$

$$\boxed{P = \begin{bmatrix} -1 & 4 & 2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_3 = 0$

$x_2 + x_3 = 0$

$x_1 = -x_3$

$x_2 = -x_3$

$$\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = x_1$$

20/20