4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$3 = \begin{bmatrix} 1-3 & 8 & 4 \\ -2 & 11-3 & 4 \\ 2 & -8 & -1-3 \end{bmatrix} = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix}$ $-2x_1 + 8x_2 + 4x_3 = 0$ $\frac{-2x_1}{-2} = \frac{-8x_2}{-2} - \frac{4}{2}x_3$

$x_1 = 4x_2 + 2x_3$

$v_1 = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$ $v_2 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$5 \begin{bmatrix} 1-5 & 8 & 4 \\ -2 & 11-5 & 4 \\ 2 & -8 & -1-5 \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix}$

$-4x_1 + 8x_2 + 4x_3 = 0$ $x_1 = +2x_2 + x_3$

$-2x_1 + 6x_2 + 4x_3 = 0$ $-4x_2 - 2x_3 + 6x_2 + 4x_3$ $2x_2 + 2x_3$ $x_2 = -x_3$

$2x_1 - 8x_2 - 6x_3 = 0$ $v_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$P = \begin{vmatrix} 4 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 4 - 2 + 0 = 2$ $P^{-1} = \frac{1}{2}\begin{bmatrix} & 0 & -1 \\ 0 & & \end{bmatrix}$

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$D = P^{-1}AP =$

$D = P^{-1}\begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$1+8+0$ $2+4$ $9-4$

$-8+11$ $-4+4$ ~~$-2+5$~~ $-2+11+4$

$8-8$ $4-1$

$$\boxed{D = P^{-1}\begin{bmatrix} 9 & 6 & 5 \\ 3 & 0 & 5 \\ 0 & 3 & -5 \end{bmatrix}}$$