5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

+5 False. The eigenvector 2v still corresponds to the eigenvalue λ. Consider (A) below.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

True, see (B) below.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

+5 True by definition to determine if a sym. Matrix is othogonal, you compute A^TA and if you get identity matrix its orthogonal. Because $A^T = A$ then $A^2 = A^TA$ therefore it will be identity matrix

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

+5 By definition a matrix is orthogonally diagonizable is A is symmetric and every symmetric matrix is diagonizable, so the answer is true because any 2×2 symmetric matrix added to another symmetric matrix is still symmetric. TRUE.

for example: $\begin{bmatrix} A & C \\ C & B \end{bmatrix} + \begin{bmatrix} D & F \\ F & E \end{bmatrix} = \begin{bmatrix} A+D & C+F \\ C+F & B+E \end{bmatrix}$

(A) $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ $\lambda^2 - 5\lambda + 6$, $(\lambda-2)(\lambda-3)$, $\lambda = 2, 3$

$\left[\begin{array}{cc|c} -1 & -2 & 0 \\ 1 & 2 & 0 \end{array}\right] \rightarrow x_1 = -2x_2 \rightarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix} \leftarrow$ eigenvector

but $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ is still an eigenvector that corresponds to λ, not 2λ. FALSE

In order for $\text{proj}_V w$ to be $-w$ we would have to have $w_v = -w$. That isn't possible unless w is the zero vector because $(w - \text{proj}_V w)$ must be in $W^\perp$ and not in W. Unless its the zero vector. TRUE.