5. For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and v is an eigenvector of A corresponding to an eigenvalue λ then 2v is an eigenvector of A corresponding to the eigenvalue 2λ.

b) If V is a subspace of $\mathbb{R}^2$ and w is a vector such that $\text{proj}_V\, \mathbf{w} = -\mathbf{w}$ then w must be the zero vector.

c) If A is a square matrix which is both symmetric and orthogonal then A^2 is the identity matrix.

d) If A and B are 2×2 matrices which are both orthogonally diagonalizable, then the matrix $A + B$ is also orthogonally diagonalizable.

a)

+5

f $\lambda=2$ false: The

d x_2 in $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ eigenvalue would

s a free-root, ? stay the same, as

om $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ v and 2v wouldn't

ould be eigenvectors be linearly independent

taining $\lambda \rightarrow v = 2v \not\Rightarrow 2\lambda$

b) True; This has to

+1 be true because This would be The only way proj V would equal -w. No other combo would produce this besides The trivial solution

?

c)

+1

True; The Identity matrix, which is Both A and A^2 in This case, is The only matrix that could be Both symmetric and orthogonal

d) false; This is not always The case. When

+1 matrixies are added, properties are not always presered, such as diagonizability:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ 2 & 5 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

diagonizable diagonizable not diagonizable