3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A.

a) $\lambda = 0$ b) $\lambda = -1$ c) $\lambda = -2$

$2-4+4\lambda$
$4-8+8\lambda$

$\left(\begin{bmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{bmatrix}\right) = 0$

$-\lambda((1-\lambda)(2-\lambda)) - 1(2-\lambda) + 2[2-4(1-\lambda)] = 0$

$-\lambda^3 + 3\lambda^2 - 2\lambda - 2 + \lambda + 4 - 8 + 8\lambda = 0$

$-\lambda^3 + 3\lambda^2 + 7\lambda - 6 = 0$

a) $\lambda = 0$

$-(0)^3 + 3(0)^2 + 7(0) - 6 \neq 0$

$-6 \neq 0 \rightarrow$ Not an eigenvalue of A

b) $-(-1)^3 + 3(-1)^2 + 7(-1) - 6 \neq 0$

$1 + 3 - 7 - 6 \neq 0 \rightarrow$ not an eigenvalue of A

c) $-(-2)^3 + 3(-2)^2 + 7(-2) - 6 = 0$

$8 + 12 - 14 - 6 = 0$

$0 = 0 \rightarrow \lambda = -2$ is an eigenvalue of A

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