1. Consider the following vectors in $\mathbb{R}^4$:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V.

am-Schmidt

$$w_1 = v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \qquad w_2 = v_2 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1 \qquad w_3 = v_3 - \left(\frac{w_1 \cdot v_2}{w_1 \cdot w_1}\right) w_1 - \left(\frac{w_2 \cdot v_3}{w_2 \cdot w_2}\right) w_2$$

$$w_1 \cdot v_2 = 1(2) + 0(1) + (-1)(-1) + 1(0) = 2 + 0 + 1 + 0 = 3$$

$$w_1 \cdot w_1 = 1^2 + 0^2 + (-1)^2 + (1)^2 = 3$$

$$w_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{3}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$w_2 \cdot v_3 = 1(2) + 1(-2) + 0(-1) + (-1)(3)$$
$$= 2 - 2 + 0 - 3 = -3$$

$$w_2 \cdot w_2 = 1^2 + 1^2 + 0^2 + (-1)^2 = 3$$

$$w_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{\cancel{3}\ 6}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \frac{-3}{3}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \cancel{2}\ 1 \\ -1 \\ \cancel{0}\ 1 \\ \cancel{1}\ 0 \end{bmatrix}$$

$$\boxed{\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \cancel{\begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}} \right\}}$$

16/20

$$\text{proj}_V u = \left(\frac{u \cdot w_1}{w_1 \cdot w_1}\right) w_1 + \left(\frac{u \cdot w_2}{w_2 \cdot w_2}\right) w_2 + \left(\frac{u \cdot w_3}{w_3 \cdot w_3}\right) w_3 = \frac{3}{3} w_1 + \frac{3}{3} w_2 + \frac{3}{3} w_3 = w_1 + w_2 + w_3$$

$$w_1 = 3 + 0 - 3 + 3 = 3$$
$$w_2 = 3 + 3 + 0 - 3 = 3$$
$$w_3 = 6 - 3 + 0 + 3 = 6$$
$$w_3 = 4 + 1 + 0 + 1 = 6$$

$$\text{proj}_V u = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix}}$$

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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20