4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1}.

$A - 3I = \begin{bmatrix} -2 & 8 & 4 \\ -2 & 8 & 4 \\ 2 & -8 & -4 \end{bmatrix} \to \left[\begin{array}{ccc|c} 2 & -8 & -4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -4 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} 4x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix}$

$x_2 = 1, x_3 = -1 \quad x_2 = -1, x_3 = 1$

$\to \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

$A - 5I = \begin{bmatrix} -4 & 8 & 4 \\ -2 & 6 & 4 \\ 2 & -8 & -6 \end{bmatrix} \to \left[\begin{array}{ccc|c} 2 & -8 & -6 & 0 \\ -2 & 6 & 4 & 0 \\ -4 & 8 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -4 & -3 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -8 & -8 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} \to \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$x_3 = 1$

lin. dependent

$P = \begin{bmatrix} 2 & -2 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ ✓

15/20

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20