

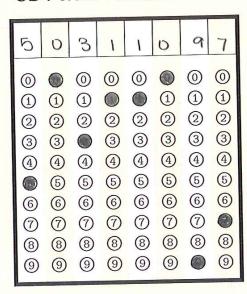
## MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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## **UB Person Number:**



## Instructions:

- Textbooks, calculators and any other electronic devices are not permitted. You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE
						el 1		



1. (20 points) Consider the following vectors in  $\mathbb{R}^3$ :

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

- a) Find all values of b such that  $w \in Span(v_1, v_2, v_3)$ .
- b) Is the set  $\{v_1, v_2, v_3\}$  (linearly independent? Justify your answer.  $c_1 \circ c_2 \circ c_3$

(a) 
$$C_1\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$$

Ret  $C_2\begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_2\begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ b \end{bmatrix}$ 

See iy

each column

has leading one?

 $C_2\begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ -3 \end{bmatrix} + C_3\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -16 \end{bmatrix}$ 

This shows fluiding?

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & -1 & -2 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

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\end{bmatrix}$$

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1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
1 & -1 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

[0 1 2 0] A Vectors V1, V2 & V3 are linearly dependant (NOT Linearly Independent) because, V2 & V3 are

free variables = vectors are

not venables



2. (10 points) Consider the following matrix:

Compute 
$$A^{-1}$$
.

 $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ 
 $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ 
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 $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix}$ 



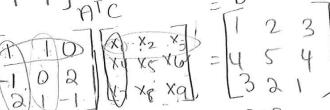
3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Find a matrix C such that  $A^TC = B$  (where  $A^T$  is the transpose of A).

$$A^{T} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \\ 2 & 1 & -1 \end{bmatrix} A^{T}C$$



Come back,



4. (20 points) Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be a linear transformation given by  $\mathcal{T}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$ 

$$\begin{array}{c}
\sqrt{(00)}T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix} \\
\sqrt{(00)}T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

- a) Find the standard matrix of T.
- b) Find all vectors (u) satisfying T(u) =  $\begin{bmatrix} 1 \\ 10 \\ -2 \end{bmatrix}$ . Standard

(a) 
$$T = \begin{bmatrix} 1 & 0 & | 0 \\ 0 & 1 & | 0 \end{bmatrix}$$
?

(b) 
$$U = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} X_1 - 2X_2 & 1 \\ X_1 + X_2 & 210 \\ X_1 - 3X_2 & 2-2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$
  
 $X_1 = 1 + 2X_2$   $\Rightarrow 1 + 2(3) = \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$   
 $1 + 2X_2 + X_2 = (0)$   $\Rightarrow 1 + 3X_2 = 10 \Rightarrow 3X_2 = 9 = \begin{bmatrix} X_2 = 3 \end{bmatrix}$   
 $1 + 2X_2 - 3X_2 = -2 \Rightarrow 1 - X_2 = -2$   
 $1 + 2X_2 - 3X_2 = -2 \Rightarrow 1 - X_2 = -2$ 



5. (20 points) For each matrix A given below determine if the matrix transformation  $T_A: \mathbb{R}^3 \to \mathbb{R}^3$  given by  $T_A(\mathbf{v}) = A\mathbf{v}$  is one-to one or not. If  $T_A$  is not one-to-one, find two

vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  such that  $(T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2))$ . **b)**  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$ a)  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$ Pivot Column because column without a

This matrix is not fully reduced yet

- one - to-one Pivot Columps + Dulla)?

ho pivot here pivot columns?

means: not one-to-one



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u,v,w are vectors in  $\mathbb{R}^3$  such that  $w+u\in Span(u,v)$  then  $w\in Span(u,v)$ .

True because, while consists of was well and when while Span(u,v); w must be in Span(u,v).

why?

b) If u, v, w are vectors in  $\mathbb{R}^3$  such that the set  $\{u,v,w\}$  is linearly independent then the set  $\{u,v\}$  must be linearly independent.

Counterexample:

Let {u,v,w} be set {5,6,8} suspectively. Then,

{5,16,8} doesn't man {6,8} is also linearly
Independent.

These are numbers, not vectors. Unless you think of them as 1-dimensional vectors, but then this set is linearly dependent...



- 7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.
- a) If A is a  $2 \times 2$  matrix and u, v are vectors in  $\mathbb{R}^2$  such that Au, Av are linearly dependent then u, v also must be linearly dependent.

(b) If  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is a linear transformation and  $u, v, w \in \mathbb{R}^2$  are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w))

Come

Dack.  $u \in Span(V, w)$ Then  $T(u) \in Span(T(V, T(w)))$ ?