

3. Consider the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 4 & 2 & 2 \end{bmatrix}$$

$$\det(A - \lambda I_n)$$

$$\det \begin{pmatrix} -\lambda & 1 & 2 \\ 1 & 1-\lambda & 0 \\ 4 & 2 & 2-\lambda \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 & 2 & -\lambda & 1 \\ 1 & 1-\lambda & 0 & 1 & 1-\lambda \\ 4 & 2 & 2-\lambda & 4 & 2 \end{vmatrix}$$

For each value of λ given below determine if it is an eigenvalue of A. $(-\lambda \cdot 1 - \lambda \cdot 2 - \lambda) + 0 + 4 =$

a) $\lambda = 0$
No

b) $\lambda = -1$
No

c) $\lambda = -2$
Yes

$$-\lambda^3 + 3\lambda^2 + 7\lambda - 6 = 0$$

$$0 + 0 + 0 - 6 \neq 0 \quad \checkmark$$

$$-(-1)^3 + 3(-1)^2 + 7(-1) - 6 = 0 \quad \checkmark$$

$$1 + 3 - 7 - 6 \neq 0$$

$$-(-2)^3 + 3(-2)^2 + 7(-2) - 6 = 0$$

$$8 + 12 - 14 - 6 = 0$$

$$20 - 20 = 0 \quad \checkmark$$

$$(2 \cdot 4 \cdot 1 - \lambda) - 0 - (2 - \lambda)$$

$$8 - 8\lambda$$

$$(1 - \lambda)(2 - \lambda)$$

$$\lambda^2 - \lambda - 2\lambda + 2$$

$$-\lambda(\lambda^2 - 3\lambda + 2)$$

$$-\lambda^3 + 3\lambda^2 - 2\lambda + 4 - (8 - 8\lambda)$$

$$-\lambda^3 + 3\lambda^2 + 16\lambda - 4 - (2 - \lambda)$$

$$-\lambda^3 + 3\lambda^2 + 17\lambda - 6 = 0$$

20/20