



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

TRUE  $\rightarrow$  IF  $u, v, w$  ARE VECTORS IN  $\mathbb{R}^3$

THEN  $\text{SPAN}(u \vee w) = \left\{ \begin{array}{l} \text{SET OF ALL} \\ \text{LINEAR COMBINATIONS} \end{array} \right\}$

?

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

FALSE  $\rightarrow$  ONLY IF

$$u \cdot x_1 + v \cdot x_2 = 0$$

$\{u, v\}$  LINEARLY  
INDEPENDENT

$$u \cdot x_1 + v \cdot x_2 + w \cdot x_3 = 0 \rightarrow \text{LINEARLY INDEPENDENT}$$

THIS EQUATION  
DOES NOT GUARANTEED  
 $u \cdot x_1 + v \cdot x_2 = 0$

?



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False

$$w+u \in \text{Span}(u, v)$$

$$w \in \text{Span}(u, v)$$

$$w = cu + cv$$

$$w+u \in \text{Span}(u, v)$$

$$w+u = cu + cv$$

$$A(v+w) = Av + Aw$$

$$w+u \in \text{Span}(u, v) \neq w \in \text{Span}(u, v)$$

Counter example

$$w \in \text{Span}(u, v) + u \notin \text{Span}(u, v)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

$\{u, v, w\}$  is linearly independent, i.e. every column is a pivot column and there is only one solution

$\{u, v\}$  must also be linearly independent because without  $w$  there will still be a pivot column in every column

ex)

$$\{u, v, w\} = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\}$$

Pivot  
columns

$$\{u, v\} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

Pivot  
columns



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

*false*  
It is ~~true~~. No matter how much you ~~can~~ change it the rule only applies for multiplication

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

*False*- linearly independent means that there ~~is only~~ <sup>must be</sup> one infinite solutions. We do not know which vector has a free variable. So we don't know 100 percent of the time.

Example  
free variable  $v, v, w$   
 $u$  and  $v$   
can be dependent



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True. Combinations of vectors in a span are still within a span and vice versa.

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. In the set of 3, they are all lin. ind. from each other, meaning if one vector was taken away, the remaining two would still be lin. ind.



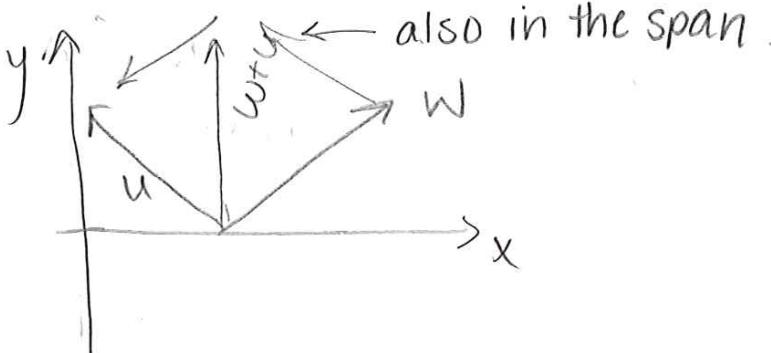
6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True:

$$\text{if } x = w, \text{ then } x_0 = w + u$$

graphical interpretation



- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False

$$A = \begin{bmatrix} u & v & w \\ 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

is linear  
independent

[after row reduction.  
each value of  $u, v, w$   
corresponds to a  
unique solution]

$$A = \begin{bmatrix} 1 & 1 & 9 \\ 0 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}$$

is inconsistent  
after row reduction  
because last row  
is

$$\begin{bmatrix} x & y & z \\ a & b & c \\ 0 & 0 & \# \end{bmatrix}$$

↑  
No solution;  
linearly dependent



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

*true*

$$\text{Span}(u, v) = c_1 u + c_2 v$$

*if  $w+u \in \text{Span}(u, v)$ , then  
w must be lin. combination of  $u, v$*

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

*true - each vector must be linearly independent  
from each other vector for the whole set  
to be linearly independent*



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, linear combination

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, if set of vectors is linearly independent,  
then  $\{u, v\}$  are also linearly independent as long  
as  $u$  and  $v$  are not a scalar multiple of the other



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because,  $w + u$  consists of  $w$  as well and when  $w + u \in \text{Span}(u, v)$ ;  $w$  must be in  $\text{Span}(u, v)$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

Counterexample:

Let  $\{u, v, w\}$  be set  $\{5, 6, 8\}$  respectively Then,  
 $\{5, 6, 8\}$  doesn't mean  $\{5, 8\}$  is also linearly independent.



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, U = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, W = \begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix}$$

True

$V_1 + 0V_2 = 1$	$U + W = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$
$0V_1 + V_2 = 2$ ✓	$\nwarrow$
$0V_1 + 0V_2 = 0$	$V_1 + 0V_2 = 4$
$V_1 + 0V_2 = 4$	$0V_1 + V_2 = 8$
$0V_1 + V_2 = 6$ ✓	$0V_1 + 0V_2 = 0$
$0V_1 + 0V_2 = 0$	

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True because in order for a set to be linearly independent they must have 1 solution (Homogeneous equation) so that means that  $u, v, w$  all only have 1 solution which ~~means~~ means that the set  $\{u, v\}$  is linearly independent.



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False.

$$u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$w+u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \in \text{Span}(v)$$

$$w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\frac{w \notin c u}{w \notin c v}, \text{ so } w \notin \text{Span}(u, v)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. If set  $\{u, v, w\}$  are independent, that means  
 $\underline{u \neq cv}, u \neq cw, v \neq cu, v \neq cw, w \neq cu, w \neq cv$ .  
 Show linear independence between  $u$  and  $v$ .



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False, because

Counter example

$$u = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}, w = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$$u + w = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}\right)$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \notin \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}\right)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

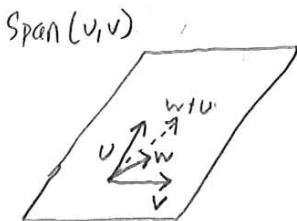
True because in order to be independent there must be no dependency on another vector, so if a vector were to leave the group that vector would still be independent.



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, if  $w$  was not within the  $\text{Span}(u, v)$  then the resultant vector would not be within the plane



- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False, only linearly independent if  $v$  is a scalar multiple of  $u$ , or vice versa.



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

TRUE

If  $w + u = v$ , then  $w = v - u$

$v$  is in the  $\text{span}(u, v)$  and  $(v - u)$  is also in the  $\text{span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

TRUE

The set of  $u, v, w$ , in that each column  $w$  is a pivot column

So, in the set of  $\{u, v\}$  each column will also be a pivot column.



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False. because,  $w + u = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \\ 6 \end{bmatrix} \in \text{Span}(u, v)$ .

not necessarily,

$$\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} \in \text{span}(u, v),$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False.  $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

the set  $\{u, v, w\}$  is linearly independent, be cause,  $x_1 u_1 + x_2 v_2 + x_3 w_3 = 0$  has only one, trivial solution. on the other,  $\{u, v, w\}$  is linearly dependent because  $x_1 u_1 + x_2 v_2 + x_3 w_3 = 0$  has non-trivial solution



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} u \\ v \\ w \end{matrix}$$

~~(1) (2) (3) (4) (5) (6) (7)~~

$$w + v \in \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

True,  
if double plus  
 $u$  kept it the  
Span of  $u, v$ ,  
the  $w$  by itself  
should be in the  
Span.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True,

because if,  $\{u, v, w\}$   
was linearly independent, then  
making  $w$  zero didn't change it  
to be dependent, so remaining  
 $\{u, v\}$  entirely changes nothing.



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

F

$$u \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \quad v \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \quad w \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

linearly independent

F

Linearly independent mean have exactly one solution.

$$\text{if } \{u, v, w\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\{u, v\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \text{have infinite solutions.}$$

∴ It's False



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True.  $\nexists$   $w$  only  $[c_1u + c_2v] \in \text{Span}(u, v)$

if  $w+u \in \text{Span}(u, v)$

$w = c_1u$  or  $w = c_2v$ .

so  $w \in \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False. if  $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, 2u + v + w = 0$ . ~~is linearly independent~~

$u + v + w = 0$ . ~~- linearly independent~~

$u + v + w = 0$ . ~~is linearly independent~~

but  $x_1u + x_2v = 0$  is not linearly independent

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = 0$$

but  $x_1u + x_2v$  is not linear independent

$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$  is linear dependent



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True If  $w + u \in \text{Span}(u, v)$

$$\begin{array}{rcl} w + u & = & x_1 u + x_2 v \\ -u & & -u \\ \hline w & = & (x_1 - 1)u + x_2 v \end{array}$$

$$\text{Let } x_1 - 1 = c_1, \quad x_2 = c_2$$

$$w = c_1 u + c_2 v$$

Therefore  $w \in \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True  $x_1 u + x_2 v + x_3 w = 0$

Linear independence states that  $x_1 = 0, x_2 = 0, x_3 = 0$

Let  $u, v$ , and  $w$  be standard basis vectors.

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$c_1 u + c_2 v = 0$$

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0 \quad \text{also only has the trivial solution}$$

$$c_1 = 0, c_2 = 0$$

Therefore, the set  $\{u, v\}$  must also be linearly independent.



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- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True. Vector  $u$  must be in the  $\text{Span}(u, v)$ , and given that  $w \in \text{Span}(u, v)$  then  $w + u$  must also be in the  $\text{Span}(u, v)$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. In order for  $\{u, v, w\}$  to be linearly independent, it must have a leading one in every column. This means  $\{u, v\}$  also has a leading one in every column, so  $\{u, v\}$  is linearly independent.



$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$u = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad w = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

false,

$$w+u = c_1 u + c_2 v$$

$$w \neq d_1 u + d_2 v$$

$$w+u = \begin{bmatrix} 2+1 \\ 2+1 \\ 0+0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \quad \begin{cases} c_1 = 3 \\ c_2 = 0 \end{cases}$$

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \neq d_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + d_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False,  $u, v, w$  can all be linearly independent  
but if you form the matrix of vectors,  $\text{Nul}(A) \neq \{0\}$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False

$$w+u = x_1u + x_2v$$

$$w+u-u = x_1u + x_2v - u$$

$$w = x_1u + x_2v - u$$

since  $w+u \notin \text{Span}(u, v)$

$$\text{so } w+u = x_1u + x_2v$$

$$\text{and } w = x_1u + x_2v - u$$

does not equal to  $w = x_1u + x_2v$

which  $w$  is not linear combination of  $x_1u + x_2v$

which  $w$  not in  $\text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$\begin{bmatrix} u & v & w \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{True}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{because } u, v \text{ have pivot position at every column}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$



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a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ . *True*

$$U = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, V = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, W = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$W+U = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \therefore W \in \text{Span}(U)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent. *True*. Removing a vector will still make the set linearly independent

$$U = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad V = \begin{bmatrix} 17 \\ 19 \\ 21 \end{bmatrix} \quad W = \begin{bmatrix} 33 \\ 33 \\ 33 \end{bmatrix}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because if  $w+u$  is in the  $\text{Span}(u, v)$

then  $w \in \text{Span}(u, v)$  shows it is a part of  
that span. Graphically it would be on the  
same line because  $w$  is only being added  
by  $u$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False b. you can have  $\left\{ \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 10 \\ 14 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \end{bmatrix} \right\}$   
when it is dependent while these two are  
dependent.



$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False

That means  $w + u$  is linearly dependent for  $w \in \text{Span}(u, v)$ .  $w \notin \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False,  $\{u, v, w\}$  is only linearly independent if it has only one solution to the homogenous equation  $x_1u + x_2v + x_3w = 0$

Counter Example:

$$\left[ \begin{array}{ccc} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & u & 0 \\ 0 & v & 2 \\ 0 & 0 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cc|c} u_1 & 0 & * \\ 0 & v_1 & * \\ 0 & * & * \end{array} \right]$$

$\therefore \{u, v\}$  is not necessarily linearly independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\left[ \begin{array}{c} 1 \\ 1 \end{array} \right] + \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] = \left[ \begin{array}{c} 2 \\ 1 \end{array} \right] \text{ Span}\left( \left[ \begin{array}{c} 1 \\ 0 \end{array} \right], \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] \right)$$

True Because if  $w+u$  is in the  $\text{span}(u, v)$ , then  $w$  must be some linear combination of  $u$  or  $v$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, if the whole set is linearly independent, then its parts must hold the relation.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\begin{pmatrix} u_1 + w_1 \\ u_2 + w_2 \\ u_3 + w_3 \end{pmatrix} \quad \text{True}$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False

$$w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$w + u$  is a linear combination so  
just  $w$  itself may not be in  $\text{Span}$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

L.I. depends on the values  $c_1, c_2, \dots, c_n$

$$\text{Set } \{c_1, c_2, c_3\} \in \mathbb{R}^3$$

$$c_1 = c_2 = c_3 = 0$$

Set  $\{g_1, g_2\} \in \mathbb{R}^2$  Set gets 2nd  
 $c_1 = c_2 = 0$  dimension  
hadn't been dropped by  
one dimension  
solution remains  
the same



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, w = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

TRUE because if  $w + u$  is in  $\text{Span}(u, v)$   
then  $w$  will be in  $\text{Span}(u, v)$  because  
they are linear combinations of one  
another

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$u = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, v = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, w = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

set  $\{u, v\}$

Since the set is linearly independent  
the matrix would have to be

$$\begin{bmatrix} 1 & b_1 \\ a_2 & 1 \\ a_3 & b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & b_1 & c_1 \\ a_2 & 1 & c_2 \\ a_3 & b_3 & 1 \end{bmatrix}$$

this set is still linearly independent  
so the statement is TRUE

in order for every column to  
have a pivot column



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

$$w = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}\right)$$

$$w + u = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$w + u \in \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}\right)$$

False

$$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \notin \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}\right)$$

The statement is true

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

This is false because  
just because

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This is false because  
just because  
 $w + v + w = 0$   
does not mean  
 $u + v = 0$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$w + u$  is a linear combination of  $u, v$ .

$$\Leftrightarrow w + u = c_1 u + c_2 v$$

$w = (c_1 - 1)u + c_2 v$  is a linear combination of  $u, v$ .

$$\therefore w \in \text{Span}(u, v)$$

True

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

①  $x_1 u + x_2 v + x_3 w = 0$  has only one and trivial solution  
 $x_1 = x_2 = x_3 = 0$ .

Then,  $x_1 u + x_2 v = 0$  also has only one and trivial solution

$$x_1 = x_2 = 0.$$

②  $A = [u \ v \ w]$

$A$  has a pivot position in every column.

True

$\hookrightarrow B = [u \ v]$

Then,  $B$  also has a pivot position in every column.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

$\text{Span}(u, v) = \text{all vecs / linear combinations } c_1u + c_2v$   
 or  $c_1w + c_2u$  spanning  $(u, v)$   
 vector  $c_1w$  span  $(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False because if  $\{u, v\}$  are scalar multiples ie  $c_1u_1 + c_2v_2 = 0$  then it is linearly dependent.

$\mathbb{R}^3 \rightarrow 3 = n$  rows

$2u_1 + (-1)v_2 = 0$

~~$u \quad v \quad w$~~   
 ~~$1 \rightarrow 3 \rightarrow 3 = p$  columns~~

~~by definition if  $p > n \rightarrow$  linearly dependent~~

~~but  $p = n$  so it is linearly independent~~

~~but if  $u \quad v$~~   
 ~~$1 \rightarrow 2 \quad p=2$  and  $\mathbb{R}^3 \quad n=3$~~

~~\*does not apply for  $p > 2$~~

~~$p > n$   
 $2 > 3$  so it is linearly independent as well~~



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

This is true. By def. of span,  $w + u = c_1 u + c_2 v$  for some  $c_1, c_2 \in \mathbb{R}$ . Therefore  $w = c_1 u + c_2 v - u$ , and combining gives  $w = (c_1 - 1)u + c_2 v$ . However,  $c_1 - 1$  is just some other constant in  $\mathbb{R}$ , so let  $c_3 = c_1 - 1$ . Therefore  $w = c_3 u + c_2 v$ , meaning  $w \in \text{Span}(u, v)$  by definition.

QED.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

This is true. If  $\{u, v, w\}$  is linearly independent, then  $u \neq c_1 v + c_2 w$  for any  $c_1, c_2 \in \mathbb{R}$   
 $v \neq c_2 u + c_3 w$  for any  $c_2, c_3 \in \mathbb{R}$   
 $w \neq c_3 u + c_4 v$  for any  $c_3, c_4 \in \mathbb{R}$

Since  $u \neq c_1 v + c_2 w$  for any  $c_1, c_2 \in \mathbb{R}$ , then  $\{u, v\}$  is linearly independent by definition.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

$$\text{If } w+u = c_1u + c_2v$$

$$w = c_1u + c_2v - u = c_3u + c_2v$$

Thus  $w \in \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

$$\text{if } u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\{u, v\}$  has to be lin ind for  $\{u, v, w\}$  to be lin ind



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

This statement is false, because  $w + u$  will only span  $(u, v)$  if  $w + u$  is in the null set of  $(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

This statement is true.

If  $\{u, v, w\}$  is linearly independent then if each column in the aug matrix must have a leading one. If you remove one vector (column), each column will still have a leading one.

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True:

$$\begin{aligned} \text{Given: } T(w+u) &= T(w) + T(u) \\ \therefore T(w) &= T(u) + T(v). \end{aligned}$$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True: If  $u, v, w$  are linearly independent, then  $c_1 u + c_2 v + c_3 w = 0$  only if  $c_1 = c_2 = c_3 = 0$ . It means  $\{u, v\}$  is linearly independent. So the set  $\{u, v\}$  has to be linearly independent.

because  $c_1 u + c_2 v = 0$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\text{Span}(u, v) = x_1 u + x_2 v$$

$$\text{for } w \in \text{Span}(u, v), \quad w = x_1 u + x_2 v$$

$$w + u = x_1 u + x_2 v$$

$w = (x_1 - 1)u + x_2 v \therefore w \in \text{Span}(u, v)$  because  $w$  is still a linear combination of  $u$  and  $v$ .

True

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

Lin. Independant: only 1 solution to  $x_1 u + x_2 v + x_3 w = 0$

$$0u + 0v + 0w = 0$$



~~0u + 0v + 0w = 0~~

$$0 + 0 + 0 = 0$$



$$0u + 0v = 0$$



$0 + 0 = 0 \therefore$  No matter which vector is removed from the set, the set of the other two is always linearly independent.

True



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, No tr is in the span  
because if we add u to w we get v ✓  
 $w = 2, u = 1, v = 1$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

false,  $\{u, v\}$  could have a different  
solution with nothing trivial



6) (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False. If  $u, v, w$  are vectors in

$\mathbb{R}^3$  s.t.  $w \notin \text{Span}(u, v)$ ,  $w$  must be a linear combination of  $u+v$  not  $w+u$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. If  ~~$\{u, v, w\} \in \text{Span}(u, v)$~~  If  
 $\{u, v, w\}$  is linearly independent, only have  
 trivial solution as the solution to homogeneous  
 equations  $c_1u + c_2v = 0$  must also  
 only have the trivial solution as the  
 answer.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False

$$w = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$$

$w$  is only in the  $\text{Span}(v, v) - u$  vector

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True if  $u, v, w$  is linearly independent  
 the only solution is zero vector and has a pivot  
 column in every column,  $u, v$  would be independent  
 as well w/ pivot position in each column and only  
 solution would be zero vector.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True; since  $u \in \text{Span}(u, v)$ , then if  $w + u \in \text{Span}(u, v)$  then  $w$  must be a multiple of a vector in  $\text{Span}(u, v)$ . Therefore,  $w \in \text{Span}(u, v)$ .

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

A set is lin. ind. when  $x_1u + x_2v + x_3w = 0$  has only one solution where  $x_1, x_2, x_3 = 0$ . If the set  $\{u, v, w\}$  is lin. ind., then either combination of two of those vectors must also be lin. ind. Therefore, this statement is true.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True  
if  $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  then  $\text{Span}(u, v)$  is all vectors of the form  $\begin{bmatrix} c_1 \\ c_2 \\ 0 \end{bmatrix}$  where  $c_1$  and  $c_2$  are any constants.

If  $w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$  then  $w + u = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$  therefore  $w$  and  $w + u$  are both in the  $\text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True  
If  $x_1u + x_2v + x_3w = 0$  has one solution, then  $x_1u + x_2v$  also has to have one solution, therefore  $\{u, v\}$  is also linearly independent

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$   $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$   $w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  then  $x_1 = x_2 = x_3 = 0$

$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$   $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  then  $x_1 = x_2 = 0$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because  $U$  is not needed for  $w$  to be in the span. If it was given that all 3 were in  $\mathbb{R}^3$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False it would depend on what vectors  $U$  and  $V$  are because the homogeneous solution may be different when using only the 2 vectors out of the 3.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False, because

True, because  $w+u$  is some linear combination of  $u$  that results in the ~~span~~ the new vector being in the span, for that to occur by the definition of a linear combination  $w$  must also be in  $\text{Span}(u, v)$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False, because the row vectors of the vectors in an augmented matrix require  $n \times n$  to be linearly independent so without the 3rd vector in  $\mathbb{R}^3$ , it is impossible to have only 1 solution.

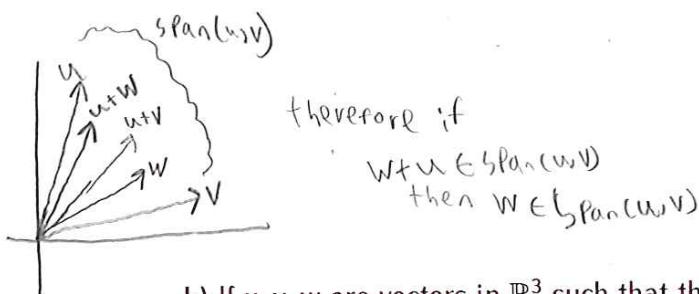


6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because  $\text{Span}(u, v)$  is the "Span" of all vectors

"between" vector  $u$  and vector  $v$ . In  $\mathbb{R}^2$  the graph below explains this concept which still holds true for an extra dimension. To scale this problem up a dimension we could just make the 3D row of values in  $u, v, w = 1$



b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True because in order for  $\{u, v, w\} + 0$  be linear independent every column must be a pivot column

therefore the subset  $\{u, v\}$  must also have every column be a pivot column.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True.

Since, in set  $\{u, v, w\}$  are already indep.

so subset  $\{u, v\}$  will also be indep.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

$$w + u = c_1 u + c_2 v \quad (c_1, c_2 \in \mathbb{R})$$

$$w = (c_1 - 1)u + c_2 v$$

$$w \in \text{Span}(u, v)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

$\{u, v, w\}$  is linearly independent means they're not multiple of each other.

then  $\{u, v\}$ :  $u, v$  won't be multiple of each other

then they're linearly independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False, counter example

$$w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$w + u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ but } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False - Counter example

$$u = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

True, because something is linearly independent

When the vectors are not scalar multiples of each other and taking out one vector of a set wont make it that the other 2 are now scalar multiples of each other.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False take  $w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,  $u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ,  $v = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$w + v = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \in \text{Span}(u, v)$ , but

$w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \notin \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

Let  $x_1 u + x_2 v = 0$

$$x_1 u + x_2 v + x_3 w = 0 \quad x_3 w = 0$$

$$3 + 3 + -6 = 0 \quad x_3 w = 0$$

$$x_1 u + x_2 v = 0$$

$x_1 u = x_2 v$ , so  $x_1 u$  is a scalar multiple

of  
 $x_2 v$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$\begin{aligned} & \left[ \begin{array}{c|cc} u & v \\ \hline 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{array} \right] \xrightarrow{\text{Row } 2 - 2 \cdot \text{Row } 1} \left[ \begin{array}{c|cc} u & v \\ \hline 1 & 2 & 3 \\ 0 & 0 & 0 \\ 3 & 6 & 9 \end{array} \right] \xrightarrow{\text{Row } 3 - 3 \cdot \text{Row } 1} \left[ \begin{array}{c|cc} u & v \\ \hline 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \\ & \text{The matrix is row-reduced to } \left[ \begin{array}{c|cc} u & v \\ \hline 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \text{ which shows } u \text{ and } v \text{ are linearly independent.} \end{aligned}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$w+u \in \text{Span}(u, v) \quad w \in \text{Span}(u, v)$$

$$w = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$w + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$w = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{bmatrix} \xrightarrow{\substack{R_3 \rightarrow -3R_1 + R_3 \\ R_2 \rightarrow -2R_1 + R_2}} \begin{bmatrix} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & -6 & -3 \end{bmatrix} \xrightarrow{R_3 = -2R_2 + R_3} \begin{bmatrix} 1 & 4 & 2 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \rightarrow -4R_2 + R_1} \begin{bmatrix} 1 & 0 & -\frac{2}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad w = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$w + u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + 4v$$

$$w = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \in \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\right)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

**True** because  
if  $w$  is a linear combination  
of  $u$  and  $v$ , then  
it will be in  $\text{span}\{u, v\}$

**True** because if set  $\{u, v, w\}$  is all linearly independent from each other, then  $u$  and  $v$  has to be linearly independent.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because if  $w + l(v)$  is a linear comb of  $\text{span}(v, v)$ , then  $w + 0(v)$  is also a linear comb of  $\text{span}(v, v) \Rightarrow$  therefore  $w$  is an element of  $\text{span}(v, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

$$u = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

because the only way for 3 vectors to be linearly dependent is if they are scalar multiples of one another, but if 3 of the 3 vectors are scalar multiples  $Ax=0$  will have 0 free variables therefore it is not linearly independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True.

$w \in \text{Span}(u, v) \Leftrightarrow \begin{cases} w = c_1 u + c_2 v \\ \text{if } w + u \in \text{Span}(u, v) \\ \text{because } w \text{ can be written as} \\ \text{linear combination} \\ \text{of } u \text{ and } v. \end{cases}$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False.

Let  $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$   
 $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$   
 $\vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$   
 $\vec{u}, \vec{v}$  linearly independent  
 $\Rightarrow \vec{u}, \vec{v}, \vec{w}$  linearly independent  
 $\therefore \vec{u}, \vec{v}, \vec{w}$  linearly independent

$$\begin{bmatrix} u & v & w \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A \vec{x} = \begin{bmatrix} b \end{bmatrix}$   
 $\text{Let } \{\vec{u}, \vec{v}, \vec{w}\} \text{ be exact soltn. Then,}$   
 $\{\vec{u}, \vec{v}\}$  is not a sltn  
 $\rightarrow \vec{u}, \vec{v}$  not linearly independent

True,  $\vec{u}, \vec{v}, \vec{w}$  cannot be written as a linear combination of each other so  
 $\vec{u}, \vec{v}$  cannot be written as a linear combination of each other.  
 $\therefore \{\vec{u}, \vec{v}, \vec{w}\}$  is linearly independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

False  $w + u$  can be in the span of  $u, v$   
but  $w \in$  has no correlation with  
the span of  $u, v$  since  $w$  is added  
to  $u$ .

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True since  $\{u, v, w\}$  in  $\mathbb{R}^3$  are all vectors  
with leading ones and in row reduced form  
to be linearly independent, also to be linearly independent  
 $u, v, w$  cannot be multiples of each other so  
 $\{u, v\}$  must be linearly independent because  
they are not multiples of each other as proven  
by  $\{u, v, w\}$  linear independence



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

Since  $u \in \text{span}(u, v)$

for  $u+w \in \text{span}(u, v)$   $w$  would have to be some combination of  $cu+dv$  where  $c$  &  $d$  are some constants.

Therefore  $w \in \text{Span}(u+v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent. \*Linearly dependent - homogeneous vector equation has more than 1 solution

False

$$\text{let } u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$  is linearly independent

$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$  is linearly dependent

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$  is linearly independent

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$  is linearly dependent

True

For a set of  $p$  vectors ( $2$  in this case) of  $\mathbb{R}^n$  ( $\mathbb{R}^3$  in this case) can only be linearly dependent if  $p > n$ .  
 $2 \not> 3$  so the set  $\{u, v\}$  would be linearly independent.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

*False*

$$w+u$$

$$w = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$\text{Span} \left( \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right)$$

$$u \quad v$$

~~$w+u$~~

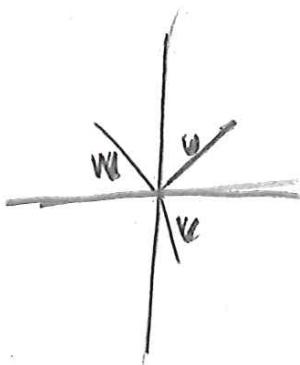
$$w+u = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$\begin{bmatrix} 6 \\ 2 \end{bmatrix}$   
is in  $\text{Span}(u, v)$ , but  $w$  not is  $\text{Span}$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True.  $U, V, W$  are 3 unique vectors, so even if only 2 of them were in the set, it would still be linearly independent





6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, Vectors encompassed by a span are linear combinations of the vectors in that span, so the linear combination of some vector and a vector in some span can only be encompassed by that span if the other vector is also a linear combination of the vectors in the span.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False

$$c_1 U + c_2 V + c_3 W = 0$$

$$c_1 U + c_2 V = 0$$

False

True, because if  $\{u, v\}$  is not linearly dependent then  $\{u, v, w\}$  cannot be linearly independent as multiplying  $w$  by a constant of zero would yield all solutions of  $\{u, v\}$ .

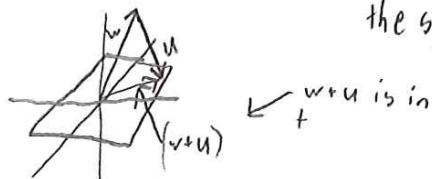


6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

*True; if the linear combination of two vectors with coefficients of 1 lies within the span of those two vectors, then a linear combination with a zero coefficient would also fall within the span.*

*True; since the span of two vectors in  $\mathbb{R}^3$  can be visualized as a plane in 3D space, and  $u$  is in the span of  $(u, v)$ , then  $w$  must also lie in that plane if  $w+u$  is to be in the span as well.*



- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

*True;  $\{u, v\}$  is linearly independent only if  $u$  and  $v$  are scalar multiples, which would not allow  $\{u, v, w\}$  to be linearly independent.*



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

True

$$w + u \in \text{Span}(u, v)$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) \text{ True}$$

$$w \in \text{Span}(u, v)$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) \text{ True}$$

Since  $w + u$  is in the Span of  $u, v$   
their combination of  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  can be made  
from  $c_1u + c_2v = w + u$ . If you let  
 $c_1 = 1$  and  $c_2 = 1$   
 $u + v = w + u$   
 $v = w$

Since  $v = w$   $w$  is in Span of  $v$  which  
makes  $w \in \text{Span}(u, v)$  True

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Lin independent means one trivial solution

To  $AX = b$

TRUE

Since  $u, v$ , and  $w$  are linearly independent that means all columns on  $A$  are pivot columns. So if you were to remove a vector  $w$  for example and solve for independence with  $u$  and  $v$  you are left with a  $3 \times 2$  matrix with pivot positions in every column so they are independent.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, if a vector is in span of other vectors  
when modified, then that vector is in span itself.

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, in  $\mathbb{R}^3$  if  $\{u, v, w\}$  has one particular solution,  
then the vectors are linearly independent with one another  
always.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

true

The span of  $(u, v)$  is the set of all linear combinations of  $u$  and  $v$ . If  $w + u \in \text{Span}(u, v)$ , that means  $w + u$  is a linear combination of  $u$  and  $v$ . If we subtract a multiple of  $u$ , the result should still be obtainable from a combination of  $u$  and  $v$  since we are essentially adding  $-u$  to a combination of  $u$  and  $v$ .

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

true

A linearly independent set means  $u, v$  and  $w$  all cannot be produced from a linear combination of the other elements in the set. This means any combination of any multiples of  $u$  and  $w$  cannot produce  $v$ . If this is the case, we still cannot produce  $v$  from any multiples of just  $u$ . The same logic applies in the opposite case of  $v$  producing  $u$ .



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False,

if  $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

Since  $w + u$  is in the span of  $u, v$ ,

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True.

To be a linearly dependent set, for  $x_1u + x_2v + x_3w = 0$   
 $x_1, x_2, x_3 = 0$  must be the only solution.

If  $\{u, v\}$  was a linearly dependent set,  $\{u, v, w\}$  would also

be linearly dependent, since  $x_3w$  could just set  $x_3 = 0$ .

Therefore, since  $\{u, v, w\}$  is linearly independent,  $\{u, v\}$  MUST be, too.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

~~False~~

set  $u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$w+u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \text{Span}(u, v)$

$w \notin \text{Span}(u, v)$

~~True~~

$w+u \in \text{Span}(u, v)$

then  $w \in \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

~~False~~

~~assume  $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$~~

~~$\{u, v, w\}$  is linearly independent.~~

~~while  $\{u, v\} = \{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\}$  is linearly dependent.~~

~~True.~~

~~$\{u, v, w\}$  is linearly independent.~~

~~then  $x_1u + x_2v + x_3w = 0$  has only one solution  $x_1 = x_2 = x_3 = 0$~~

$x_1u + x_2v = -x_3w$

$x_1u + x_2v = 0$

$\Rightarrow$  every column of the matrix is a pivot column.

since  $\{u, v, w\}$  every column of  $u, v, w$  is pivot column.

$\{u, v\}$  also has pivot column is every column.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_w \in \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

false

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \notin \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, if  $u, v, w$  are all linearly independent,  $u, v$  must be linearly independent

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

all 3 lin. ind.

$u, v \Rightarrow$  lin. ind.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_2 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$\Downarrow$  then ...

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_2 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

w+u can be written  
 as a linear. comb.  
 of u and v

w can be written  
 as a linear comb.  
 of u and v

~~$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = c_1 \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + c_3 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} - \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$~~

o.o true w is in  
 the span(u, v)

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, if  $u, v$ , and  $w$  are already linearly independent and are not linear combinations of each other, then the set  $\{u, v\}$  must also already be linearly independent

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \Rightarrow \text{linearly independent}$$

$u$        $v$        $w$

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \Rightarrow \text{still linearly independent}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

false

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

false

Let set  $\{u, v, w\}$  called  $S = \{u, v, w\}$

$R(S) = 3 = \text{the number of vectors}$

$\Rightarrow$  at least one of  $w = k_1u + k_2v + k_3w \dots$  is right



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

**False**

$$w + u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

but there is no  $x$  value where  $xw = \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

**True** linear independent means vectors are not multiples  
so removing one will not affect linear independence



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True  $u, v$  are in span

$w + u$  is span

$w$  in  $\not\in$  span

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False

$$u \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} v \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} w \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$u v w$  are linearly independent

But  $u v$  are linear dependent.

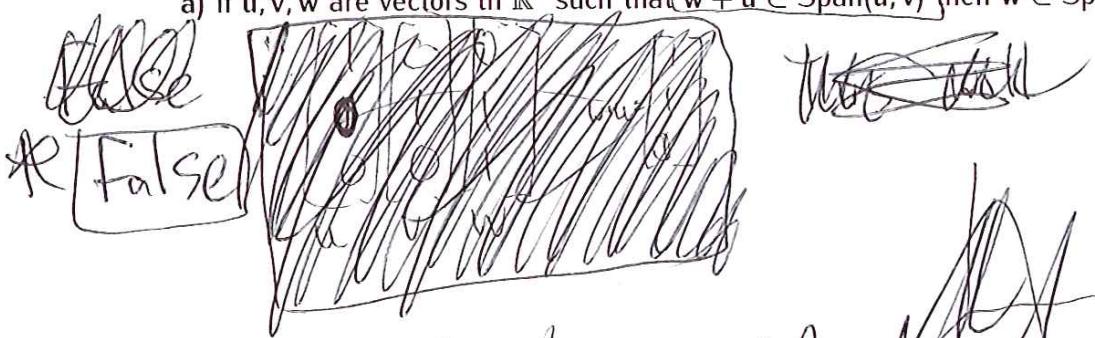
$x_1 u + x_2 v + x_3 w = 0$  has solution of 0

$x_1 u + x_2 v$  will have solution



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .



$w$  could be projected  
outside the span when  
not added to  $u$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True because ~~the~~ ~~three~~ you will  
still have ~~one~~ every column a  
a pivot column



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\begin{bmatrix} u & v & w \\ 0 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix} \quad u+w = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \in \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) \quad \text{True}$$

$w+u \in \text{Span}(u, v)$ . suppose that  $w+u = a.u + b.v$      $a, b$  are scalar.

$$w = (a-1)u + bv$$

$$w \in \text{Span}(u, v)$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  if it is linearly independent.    True.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  they must have pivot in every column.

it also in  $\mathbb{R}^3$ . so the reduce form should be  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

so the set  $\{u, v\}$  must be linearly independent.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True; If there is a linear combination of vectors  $v$  and  $u$  that equals  $w$ ,  $w$  must be in the span of  $u$  and  $v$ . Since addition and subtraction of one vector from another is a linear operator, the problem can be considered as the following:

$$w = (w + u) - u \therefore \text{If } w + u = x_1v + x_2u, \text{ then } w = x_1v + (x_2 - 1)u,$$

which is a linear combination of  $u$  and  $v$   $\therefore w \in \text{Span}(u, v)$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, any subset of a linearly independent set of vectors must also be linearly independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True,  $u$  is in the span of  $v$

So adding it does not take it out

of span.

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

true. there would be no free variables  
when reducing the set to  $\{u, v, w\}$  to  $\{u, v\}$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True

For any equation  $w + u = a_1 u + a_2 v$ ,  $w = a_1 u - u + a_2 v = a_1 u + a_2 v$   
 $w \in \text{Span}(u, v)$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

If  $\{u, v\}$  were dependent, then  $a_1 u + a_2 v + 0w = 0$  would prove  $\{u, v, w\}$  to be dependent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True if  $u$  is in the span as well then  $w+u \in \text{Span}(uv)$

Because

$$w = -u + v$$

$\$ -u, v$  are in the span

$$w+u = v$$

or  $u, v$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

false

$$\begin{array}{ccc} u & v & w \\ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} & \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

$uv$  are dependent

$uvw$  are  
independent



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

TRUE, because  $w + v = w'$  so  
 $w$  has to be in  $\text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

TRUE,  
 $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$   $v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$   $w = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

because  $x_1u_1 + x_2v_2 + x_3w_3 = 0$  has only  
one solution,  $x_1u + x_2v$  will only  
have one solution as well.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True because  $u$  is in the span of itself, making it equal to  $w \in \text{Span}(u, v)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set,  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False  $\rightarrow$  two vectors can be linearly independent but the third can be linearly dependent and vice versa.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \quad v_2 = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 3 \\ -12 \end{bmatrix} \quad v_1, v_2, v_3 \rightarrow \text{linearly dependent}$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -2 & 4 & -12 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

free

However,  $v_1$  and  $v_2$  are linearly independent

$$\left[ \begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ -2 & 4 & 0 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right] \quad \left. \begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array} \right\} \text{trivial solution.}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, because since  $u+w$  is in the span  $(u, v)$  then it must be true that  $w \in \text{Span}(u, v)$  because the sum of the two vectors are in the span  $(u, v)$ .  
 $(u+w)$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True

False, for example

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

then the set  $\{u, v\} \left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$  is also linearly independent because it has only one solution and pivot col in every column. If  $u, v, w$  are independent then it has pivot col. in every column that means with two vectors it still holds true.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True! ~~because~~ cause  $w + u$  is in  $\text{Span}(u, v)$  then  $u - v = w$  which is also in  $\text{Span}(u, v)$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True: if  $\{u, v, w\}$  are linearly independent then  $\{u, v\}$  is linearly independent because  $c_1u + c_2v + c_3w = 0$  would have only one trivial solution meaning that  $c_1u + c_2v + c_3w = 0$ ,  $c_1u + c_3w$  or  $c_1u + c_2v$  or  $c_2v + c_3w$  have no combination to sum to 0 other than multiplying by 0, if they did then  $\{u, v, w\}$  would have another solution to  $c_1u + c_2v + c_3w = 0$  which would make  $\{u, v, w\}$  linearly dependent. Since  $c_1u + c_2v$  have no combination to 0 other than 0 they are linearly independent.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = u \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = v \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w$$

$\left( \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) \right)$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = w + u$$

It is true

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$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow w \in \text{Span}$$

$\left( \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$  ?

$$0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

$$u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow \text{set } \{u, v, w\}$

linearly independent  $\Rightarrow$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\text{set } \{u, v\} \Rightarrow \left[ \begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ 0 = 0 \end{array}$$

doesn't  
matter!

It is true. Since, linear dependence means having a trivial soln. So by decreasing the

$$x_1 = 0, x_2 = 0, x_3 = 0$$

set to  $\{u, v\}$ , it won't change to infinite amount of soln.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$w + u = c_1 u + c_2 v$$

$$w = (c_1 - 1)u + c_2 v$$

TRUE,  $w$  is a linear combination of  $u$  and  $v$

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

By definition,  
 $u, v, w$  are not constant multiples of each other:  $u \neq c_1 v$  for  
any  $c_1$   
 $u, v$  are not constant multiples of each other

$\therefore u, v$  are linearly independent

TRUE



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

~~False~~. True.  $\{u, v, w\}$  is linearly independent  
which means every column of matrix is pivot column.  
If cancel  $w$  vector, ~~the~~ the matrix still ~~has~~ <sup>has</sup> columns  
that are pivot column.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

Yes, since  $w+u \in \text{Span}(u, v)$ , then  $w+u = c_1u+c_2v$  has solutions.  
 $w=c_1u+c_2u-u=(c_1-1)u+c_2u$  has solutions

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$\cancel{w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \quad w = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

False,

$$w+u = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ so}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \text{ is valid}$$

$$\text{but } \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix} \text{ is invalid}$$

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

False,  $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, w = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$

$$u+v = w \text{ so } \{u, v, w\} \text{ is linearly independent}$$

but  $u$  and  $v$  alone  $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$  are

linearly dep. because neither vector can represent the other.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

FALSE

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

FALSE

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

$$w + u = c_1 u + c_2 v$$

$$w = c_1 u + c_2 v - u$$

True because if  $w \in \text{Span}(u, v)$ , according to definition prop 2, the set of vectors is linearly dependent. All the vectors are just linear combinations of each other so  $w$  is in the span of  $u, v$ !

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True because if they are all linearly independent, those vectors can't be changed to be linearly dependent. There is no relationship between  $u, v, w$ , so if you take one out, it would not change the linear independence.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

- a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True, because if we add the vectors together in a vector equation and the resulting vector is in the span, then that means that the vector that was added is in the span as it is a linear combination.

- b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True, because if one of the vectors is linearly dependent then the whole set would have been linearly dependent but it's not.



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that  $w + u \in \text{Span}(u, v)$  then  $w \in \text{Span}(u, v)$ .

True.

b) If  $u, v, w$  are vectors in  $\mathbb{R}^3$  such that the set  $\{u, v, w\}$  is linearly independent then the set  $\{u, v\}$  must be linearly independent.

True. In order to be linearly independent, each column is a pivot column. So if you do set  $\{u, v\}$  they will still have a pivot column in each column.