

2. Find the equation $f(x) = ax + b$ of the least square line for the points $(1, 0)$, $(-1, 2)$, $(2, 1)$.

$$\underbrace{\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}}_b \quad (A^T A)x = A^T b$$

$$A^T A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1+4 & 1-1+2 \\ 1-1+2 & 1+1+1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0-2+2 \\ 0+2+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 6 & 2 & 0 \\ 2 & 3 & 3 \end{array} \right] \xrightarrow{R_1 - 3R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 6 & 2 & 0 \\ 0 & -7 & -9 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 6 & 2 & 0 \\ 0 & 1 & 9/7 \end{array} \right] \xrightarrow{-2R_2 + R_1 \rightarrow R_1}$$

$$\left[\begin{array}{cc|c} 6 & 0 & -18/7 \\ 0 & 1 & 9/7 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & -3/7 \\ 0 & 1 & 9/7 \end{array} \right] \rightarrow \begin{matrix} a = -3/7 \\ b = 9/7 \end{matrix}$$

$$\boxed{f(x) = -\frac{3}{7}x + \frac{9}{7}}$$

✓

20/20