

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 8 & 4 \\ -2 & 11 & 4 \\ 2 & -8 & -1 \end{bmatrix}$$

Knowing that eigenvalues of A are $\lambda_1 = 3$ and $\lambda_2 = 5$ diagonalize this matrix; that is, find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}$$

Note: you do not need to compute P^{-1} .

$$\lambda_1 = 3, [A - 3I | 0]$$

$$\begin{bmatrix} -2 & 8 & 4 & | & 0 \\ -2 & 8 & 4 & | & 0 \\ 2 & -8 & -4 & | & 0 \end{bmatrix} \xrightarrow{\substack{R_1 \leftrightarrow R_3 \\ R_2 \leftrightarrow R_1}} \begin{bmatrix} 2 & -8 & -4 & | & 0 \\ -2 & 8 & 4 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & -4 & -2 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{matrix} x_1 = +4x_2 + 2x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{matrix}$$

$$x^2 \begin{bmatrix} 14 \\ 1 \\ 0 \end{bmatrix}, x^3 \begin{bmatrix} +2 \\ 6 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 5, [A - 5I | 0]$$

$$\begin{bmatrix} -4 & 8 & 4 & | & 0 \\ -2 & 6 & 4 & | & 0 \\ 2 & -8 & -6 & | & 0 \end{bmatrix} \xrightarrow{\substack{R_1 \leftrightarrow R_3 \\ R_2 \leftrightarrow R_1}} \begin{bmatrix} 2 & -8 & -6 & | & 0 \\ -2 & 6 & 4 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & -4 & -3 & | & 0 \\ 0 & 2 & 2 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & -4 & -3 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{R_1 + 4R_2} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \begin{matrix} x_1 = -x_3 \\ x_2 = -x_3 \\ x_3 = x_3 \end{matrix} \quad x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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