

MTH 309T LINEAR ALGEBRA EXAM 1

October 3, 2019

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Instructions:

- Textbooks, calculators and any other electronic devices are not permitted.
 You may use one sheet of notes.
- For full credit solve each problem fully, showing all relevant work.

1	2	3	4	5	6	7	TOTAL	GRADE

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1	2	3	4	5	6	7	TOTAL	GRADE



1. (20 points) Consider the following vectors in \mathbb{R}^3 :

$$\mathbf{v}_1 = \left[\begin{array}{c} 1 \\ 0 \\ 2 \end{array} \right], \quad \mathbf{v}_2 = \left[\begin{array}{c} -1 \\ 1 \\ -3 \end{array} \right], \quad \mathbf{v}_3 = \left[\begin{array}{c} 1 \\ 2 \\ 0 \end{array} \right], \quad \mathbf{w} = \left[\begin{array}{c} -2 \\ 2 \\ b \end{array} \right]$$

- a) Find all values of b such that $w \in \text{Span}(v_1, v_2, v_3)$.
- b) Is the set $\{v_1, v_2, v_3\}$ linearly independent? Justify your answer.

Sor
$$W \in Span(V_1, V_2)^3$$

 $b + 6 = 0$
 $b = -6$

\$ 1, + 0, 1, + 63 V3 = W



2. (10 points) Consider the following matrix:

owing matrix:
$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

Compute
$$A^{-1}$$
.

$$-2.601-12100$$

7/0	00	-2 3 -1 1 -1 1 2 -2 1
A-1 =	[-2 3 - 1 -1] 2 -2 [



3. (10 points) Let A be the same matrix as in Problem 2, and let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

ATC = B

Find a matrix
$$C$$
 such that $A^TC = B$ (where A^T is the transpose of A).

$$A^T = \begin{bmatrix} -1 & 0 \\ -1 & 0 & 7 \\ 2 & 1 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2 & 12 \\ 3 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$(A^{T})^{-1} = \begin{bmatrix} -2 & 12 \\ 3 - 1 - 2 \\ -1 & 1 \end{bmatrix}$$

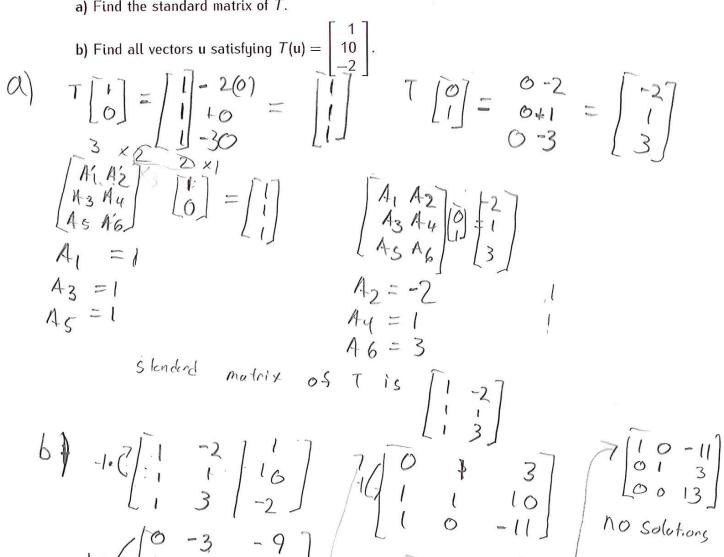
$$2 \cdot \left(\begin{array}{c|c} 1 & 0 - 2 & 0 - 1 & 0 \\ \hline 0 & 1 & 0 & 3 - 1 - 2 \\ \hline 0 & 0 & 1 & - 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{c|c} 1 & 0 & 0 & - 2 & 1 & 2 \\ \hline 0 & 0 & 0 & 3 & - 1 & - 2 \\ \hline 0 & 0 & 1 & - 1 & 1 & 1 \end{array} \right)$$



4. (20 points) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$$

a) Find the standard matrix of T.



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	



5. (20 points) For each matrix A given below determine if the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T_A(\mathbf{v}) = A\mathbf{v}$ is one-to one or not. If T_A is not one-to-one, find two vectors \mathbf{v}_1 and \mathbf{v}_2 such that $T_A(\mathbf{v}_1) = T_A(\mathbf{v}_2)$.

a)
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

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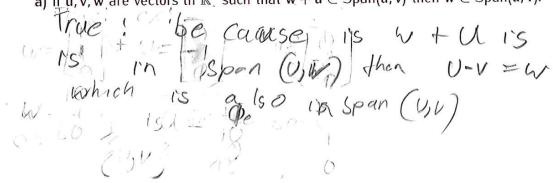
c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 4 \\ 3 & 4 & 2 \end{bmatrix}$

Solution is one to one as every row bead Column has



6. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If u, v, w are vectors in \mathbb{R}^3 such that $w + u \in Span(u, v)$ then $w \in Span(u, v)$.



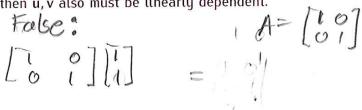
b) If u, v, w are vectors in \mathbb{R}^3 such that the set $\{u, v, w\}$ is linearly independent then the set $\{u, v\}$ must be linearly independent.

True: is Eu, V, W are linearly independent than EU, V & is linearly independent because GUHQHICEW=U would have only one trivial solution, maning that GUHQHICEW=C,UHCEW or GUH + C2 V or GUHCEW i have recombination to sum to 0 where than mollipling by 0, is they did than Eu, V, W & would have a nother solution to GUHCEV + C3 W =0 which have a nother solution to GUHCEV + C3 W =0 which have no combination to O other than 0 they are linearly independent



7. (10 points) For each of the statements given below decide if it is true or false. If it is true explain why. If it is false give a counterexample.

a) If A is a 2×2 matrix and u, v are vectors in \mathbb{R}^2 such that Au, Av are linearly dependent then u, v also must be linearly dependent.



b) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $u, v, w \in \mathbb{R}^2$ are vectors such that u is in Span(v, w) then T(u) must be in Span(T(v), T(w)).