$354 \times 9 = 476$

$480 - 476 + 13$

1. Consider the following vectors in $\mathbb{R}^4$:

$\frac{1}{9}$

$-\frac{1}{9}$

$-\frac{1}{18}$

$\frac{3}{18}$

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

The set $\mathcal{B} = \{v_1, v_2, v_3\}$ is a basis of some subspace V of $\mathbb{R}^4$.

Rough.

$4+4+1+9$

$2+0+1+3$

$$\frac{5}{18}\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} 10/18 \\ -10/18 \\ -5/18 \\ 5/6 \end{bmatrix}$$

a) Find an orthogonal basis $\mathcal{D} = \{w_1, w_2, w_3\}$ of the subspace V.

b) Compute the vector $\text{proj}_V u$, the orthogonal projection of u on V.

Using Gram-Smidth Process we computeD.

$W_1 = V_1$

$$W_2 = V_2 - \frac{W_1 \cdot V_2}{V_2 \cdot V_2} \cdot V_2$$

$$= \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{2+0+1+0}{1+0+1+1} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{\cancel{3}}{\cancel{3}} 1 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cancel{0}\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$W_3 = V_3 - \frac{W_2 \cdot V_3}{\cancel{V_3 \cdot V_3}} \cancel{V_3} - \frac{W_1 \cdot V_3}{V_3 \cdot V_3} V_3$$

$$\boxed{D = \left\{ \cancel{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0\ 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} ? \end{bmatrix} \right\}}$$

$2+0+0-3$

$$\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix}}{18} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} \Rightarrow$$

$$= \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \frac{2+0+0-3}{4+4+1+9} \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} + \frac{1}{18}\begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \\ -2 \\ -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1/9 \\ -1/9 \\ -1/18 \\ 1/6 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 19/9 \\ -19/9 \\ -9/18 \\ 19/6 \end{bmatrix}^2 - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 10/9 \\ -19/9 \\ -9/18 \\ 13/6 \end{bmatrix}$$

$\frac{10}{9} - \frac{9}{8} + \frac{13}{6}$