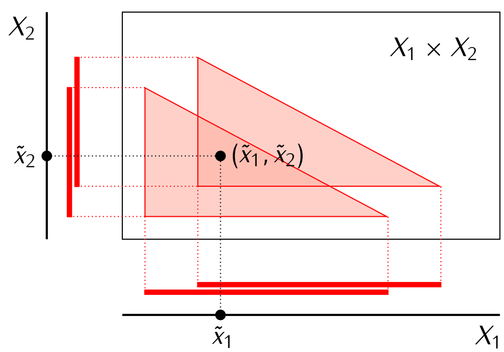
17 | Tychonoff Theorem

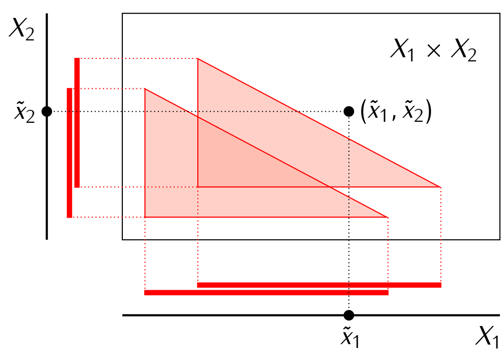
17.1 Tychonoff Theorem. *If $\{X_s\}_{s\in S}$ is a family of topological spaces and X_s is compact for each $s \in S$ then the product space $\prod_{s\in S} X_s$ is compact.*

17.2 Definition. Let $\mathcal{A}$ be a family of subsets of a space X. The family $\mathcal{A}$ is *centered* if for any finite number of sets $A_1, \dots, A_n \in \mathcal{A}$ we have $A_1 \cap \dots \cap A_n \neq \varnothing$

17.5 Lemma. *Let X be a topological space. The following conditions are equivalent:*

1) *The space X is compact.*
2) *For any centered family $\mathcal{A}$ of closed subsets of X we have $\bigcap_{A \in \mathcal{A}} A \neq \varnothing$.*


X_2
$X_1 \times X_2$
$\tilde{x}_2$
$(\tilde{x}_1, \tilde{x}_2)$
$\tilde{x}_1$
X_1


X_2
$X_1 \times X_2$
$\tilde{x}_2$
$(\tilde{x}_1, \tilde{x}_2)$
$\tilde{x}_1$
X_1

17.16 Proposition. *If X_i is a Hausdorff space for each $i \in I$ then the product space $\prod_{i \in I} X_i$ is also Hausdorff.*

Proof. Exercise. $\square$

17.17 Corollary. *If X_i is a compact Hausdorff space space for each $i \in I$ then the product space $\prod_{i \in I} X_i$ is also compact Hausdorff.*