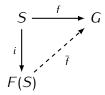
## 14 Presentations of Groups

**14.1 Definition.** Free group generated by a set S

**14.3 Note.** We will say that a group G is free if G is isomorphic to the group F(S) for some set S. Notice that by Theorem 13.13 the fundamental group of any 1-dimensional CW complex is free.

**14.4 Theorem.** Let S be a set and G be a group. For any map of sets  $f: S \to G$  there exists a unique homomorphism of groups  $\bar{f}: F(S) \to G$  such that the following diagram commutes:



**14.5 Definition.** Let S be a set, and let R be a subset of elements of the free group F(S). By  $\langle S \mid R \rangle$  we denote the group given by

$$\langle S \mid R \rangle = F(S)/N$$

where N is the smallest normal subgroup of F(S) such that  $R \subseteq N$ . We say that elements of S are *generators* of  $S \mid R$  and elements of  $S \mid R$  are *relations* in  $S \mid R$ .

**14.9 Definition.** If G is a group and  $G \cong \langle S \mid R \rangle$  for some set S and some  $R \subseteq F(S)$  then we say that  $\langle S \mid R \rangle$  is a *presentation* of G.

**14.10 Definition.** If a group G has a presentation  $\langle S \mid R \rangle$  such that S is a finite set then we say that G is *finitely generated* and if it has a presentations such that both S and R are finite sets then we say that G is *finitely presented*.

**14.11 Proposition**. Every group has a presentation.

14.12 Note. 1) Every group has inifinitely many different presentations. For example

$$\mathbb{Z} \cong \langle a \rangle \cong \langle a,b \mid b \rangle \cong \langle a,b \mid ab^{-1} \rangle \cong \langle a,b \mid b^2,b^3 \rangle$$

2) In general if we know a presentation of a group it may be very difficult to say anything about the properties of the group (even if the group is trivial or not).