$$\begin{array}{ccccccc}
\pi_n(\bigvee_{k \in K} S^n) & \xrightarrow{(\bigvee_{k \in K} \varphi_k)_*} & \pi_n(X^{(n)}) & \xrightarrow{i_*} & \pi_n(X^{(n+1)}) & \longrightarrow & 0 \\
\downarrow{\scriptstyle h}{\cong} & & \downarrow{\scriptstyle h}{\cong} & & \downarrow{\scriptstyle h} & & \downarrow{\cong} \\
H_n(\bigvee_{k \in K} S^n) & \xrightarrow[(\bigvee_{k \in K} \varphi_k)_*]{} & H_n(X^{(n)}) & \xrightarrow{i_*} & H_n(X^{(n+1)}) & \longrightarrow & H_{n-1}(\bigvee_{k \in K} S^n)
\end{array}$$