

$$\begin{array}{ccc}
[X \vee X, Y]_* & \xrightarrow[\cong]{\varphi_Y} & [X, Y]_* \times [X, Y]_* \\
\downarrow f_* & & \downarrow f_* \times f_* \\
[X \vee X, Y']_* & \xrightarrow[\varphi_{Y'}]{\cong} & [X, Y']_* \times [X, Y']_*
\end{array}$$