

$$\begin{array}{ccccccc}
\pi_1(V_{k \in K} S^1)^{\text{ab}} & \xrightarrow{\psi_*} & \pi_1(V_{i \in I} S^1)^{\text{ab}} & \xrightarrow{i_*} & \pi_1(X)^{\text{ab}} & \longrightarrow & 0 \\
\downarrow \bar{h} \cong & & \downarrow \bar{h} \cong & & \downarrow \bar{h} & & \downarrow \cong \\
H_1(V_{i \in I} S^1) & \xrightarrow{\psi_*} & H_1(V_{i \in I} S^1) & \xrightarrow{i_*} & H_1(X) & \longrightarrow & 0
\end{array}$$