

$$\begin{array}{ccccc}
 H_n(S^n) & \xrightarrow{p_*} & H_n(S^n \vee S^n) & \xrightarrow{(\varphi \vee \psi)_*} & H_n(X) \\
 & \searrow \text{id}_* \oplus \text{id}_* & \downarrow \cong \quad r_{1*} \oplus r_{2*} & \nearrow \varphi_* + \psi_* & \\
 & & H_n(S^n) \oplus H_n(S^n) & & 
 \end{array}$$