# MATRIX PROJECT - Q33

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# QUESTION:

A circle passes through

$$A = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \tag{1}$$

and touches the $y$-axis at

$$B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}. \tag{2}$$

Which one of the following equations can represent a diameter of this circle?

1. $\begin{pmatrix} 4 & 5 \end{pmatrix} \vec{x} = 6$
2. $\begin{pmatrix} 2 & -3 \end{pmatrix} \vec{x} + 10 = 0$
3. $\begin{pmatrix} 3 & 4 \end{pmatrix} \vec{x} = 3$
4. $\begin{pmatrix} 5 & 2 \end{pmatrix} \vec{x} + 4 = 0$

# SOLUTION :

First, we compute direction vector of line passing through given two points using matrix it comes out to be

$$DirectionVector = T = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{3}$$

Now, we compute perpendicular bisector of this line segment **AB** using its mid point and direction vector of AB =**T**(it is equivalent to normal vector of the perpendicular bisector)

$$MidPoint = M = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{4}$$

# SOLUTION:

Equation of perpendicular bisector:

$$T^T X = p \tag{5}$$

To compute value of p we put X equal to M in the above equation.

$$T^T M = p \tag{6}$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = p \tag{7}$$

$$p = 4 \tag{8}$$

# Solution:

Final Equation of perpendicular bisector,

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \vec{x} = 4 \tag{9}$$

As the circle touches the y-axis at (0,2) then circle should lie on the line

$$y = 2 \tag{10}$$

which can be represented in the following way using matrices

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \vec{x} = 2 \tag{11}$$

# Solution:

The intersection of these two lines (9) and (11) is the center because both these lines pass through center.

$$Center = O = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{12}$$

Now whichever line given in options passes through the center $\mathbf{O}$, is the answer.

# Solution:

**1:** $\begin{pmatrix} 4 & 5 \end{pmatrix} \vec{x} = 6$

$$\begin{pmatrix} 4 & 5 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} \neq 6 \tag{13}$$

**2:** $\begin{pmatrix} 2 & -3 \end{pmatrix} \vec{x} + 10 = 0$

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 10 = 0 \tag{14}$$

As L.H.S is equal to R.H.S.,this answer is correct.

**3:** $\begin{pmatrix} 3 & 4 \end{pmatrix} \vec{x} = 3$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} \neq 3 \tag{15}$$

**4:** $\begin{pmatrix} 5 & 2 \end{pmatrix} \vec{x} + 4 = 0$

$$\begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 4 \neq 0 \tag{16}$$

Line represented in $2^{nd}$ option is one of the diameters of the given circle