CSCI 5511 Homework 4

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Cryptarithm Discussion A Constraint Satisfaction Problem is given as a cryptarithm puzzle like this:

$$ODD$$
$$+\underline{ODD}$$
$$EVEN$$

Here each letter is a different number from 0 to 9. There are no leading zeros, i.e. $O \neq 0$ and $E \neq 0$. The addition will be correct.

a.) Variables The variables of these CSP are the distinct letters of the problem above. We also add two additional variables to capture the potential to carry for additions where the sums are greater than 10. These additional variables are: C_{10} and C_{100} for the tens place and the hundreds place respectively. All the variables are then given by the list: $\{C_{10}, C_{100}, D, E, N, O, V\}$.

b.) Domains Each of these variables has a domain of possible values associated with it. The possible domain for the carry variables C_{10} and C_{100} is: $\{0, 1\}$. The possible domains for the other variables are the numbers 0 thru 9: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

c.) Constraints There are four constraints that are defined by this problem. The first constraint is that all variables (other than the carry variables) must have a different value. This is the *AllDiff* constraint. The other constraints are given by the additions implied by each column of the cryptarithm problem. All the constraints are given below:

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10 * C_{10}$$
$$3. D + D + C_{10} = E + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * E$$

d.) CSP Solver Rules The homework also mentions that the CSP solver works in a certain way:

1. Variable choice is by **minimum-remaining values**, alphabetical order for ties (and C_{10} before C_{100}).

2. Value assignment is smallest to largest.

3. Inference is performed by enforcing the *AllDiff* constraint and the other constraints only when there are two or fewer unassigned variables in a given constraint.

We can see that since there are no leading zeros and the summation of the largest numbers in a domain give the result: $9 + 9 = 18$ then we know from constraint 4 that $E = 1$. We will initialize this variable in the set up.

Step 0: Initial Setup

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = ?$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0, 1\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$O = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10 * C_{10}$$
$$3. D + D + C_{10} = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments   This is the initial set up.

Step 1: Inference using *AllDiff*

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = ?$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0, 1\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10 * C_{10}$$
$$3. D + D + C_{10} = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments Removes the value 1 from the domains of all variables: {D, N, O, V}.

Step 2: Choose variable C_{10}, Choose value 0

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 0$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N$$
$$3. D + D = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments Value for C_{10} updated.

Step 3: Inference using *AllDiff* and constraint 2

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 0$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{2, 3, 4\}$$
$$E = \{1\}$$
$$N = \{4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N$$
$$3. D + D = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments The variable N is less than 10 so the sum of D + D is less than 10. Also, D can not be 0 because then N would have to be 0 as well and *AllDiff* makes this impossible.

Step 4: Inference using *AllDiff* and constraint 3

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 0$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{\emptyset\}$$
$$E = \{1\}$$
$$N = \{4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N$$
$$3. D + D = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments   Reviewing the domain of D we see that there are no values that give a value of 1 in the ones place. We must back up to a previous state. Restore to right before step 2 and choose a different value for C_{10}.

Step 5: Restore to right before step 2

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = ?$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{0, 1\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10 * C_{10}$$
$$3. D + D + C_{10} = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments Restored to right before step 2.

Step 6: Choose variable C_{10}, Choose value 1

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10$$
$$3. D + D + 1 = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments   Value for C_{10} updated.

Step 7: Inference using *AllDiff* and constraint 2

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{0, 1\}$$
$$D = \{5, 6, 7, 8, 9\}$$
$$E = \{1\}$$
$$N = \{0, 2, 4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10$$
$$3. D + D + 1 = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments   The domains for D and N are updated.

Step 8: Inference using *AllDiff* and constraint 3

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = ?$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0, 2, 4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10$$
$$3. D + D + 1 = 1 + 10 * C_{100}$$
$$4. O + O + C_{100} = V + 10 * 1$$

d.) Comments From the options in the domains for D and C_{100} there is only 1 possible value for D and C_{100}.

Step 9: Set the value for C_{100} based on its single valued domain

$$ODD$$
$$+\underline{ODD}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = ?$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0, 2, 4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. D + D = N + 10$$
$$3. D + D + 1 = 1 + 10$$
$$4. O + O + 1 = V + 10 * 1$$

d.) Comments   The value for variable C_{100} is updated.

Step 10: Set the value for D based on its single valued domain

$$O55$$
$$+\underline{O55}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0, 2, 4, 6, 8\}$$
$$O = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{0, 2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = N + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. O + O + 1 = V + 10 * 1$$

d.) Comments   The value for variable D is updated.

Step 11: Inference using *AllDiff* and constraint 2

$$O55$$
$$+\underline{O55}$$
$$1V1N$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = ?$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = N + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. O + O + 1 = V + 10 * 1$$

d.) Comments   Reduces the domain for variable N to a single value.

Step 12: Set the value for N based on its single value domain

$$O55$$
$$+\underline{O55}$$
$$1V10$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = 0$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{2, 3, 4, 5, 6, 7, 8, 9\}$$
$$V = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = 0 + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. O + O + 1 = V + 10 * 1$$

d.) Comments The value for variable N is updated.

Step 13: Inference using *AllDiff* and constraint 4

$$O55$$
$$+\underline{O55}$$
$$1V10$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = 0$$
$$O = ?$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{6, 8\}$$
$$V = \{3, 7\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = 0 + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. O + O + 1 = V + 10 * 1$$

d.) Comments   The domains for variables O and V are updated.

Step 14: Choose variable O, Choose value 6

$$655$$
$$+\underline{655}$$
$$1V10$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = 0$$
$$O = 6$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{6\}$$
$$V = \{3, 7\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = 0 + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. 6 + 6 + 1 = V + 10 * 1$$

d.) Comments   The value for variable O is updated.

Step 15: Inference using *AllDiff* and constraint 4

$$655$$
$$+\underline{655}$$
$$1V10$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = 0$$
$$O = 6$$
$$V = ?$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{6\}$$
$$V = \{3\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = 0 + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. 6 + 6 + 1 = V + 10 * 1$$

d.) Comments   The domain for variable V is updated.

Step 16: Set the value for V based on its single valued domain

$$655$$
$$+\underline{655}$$
$$1310$$

a.) Variables and assignments

$$C_{10} = 1$$
$$C_{100} = 1$$
$$D = 5$$
$$E = 1$$
$$N = 0$$
$$O = 6$$
$$V = 3$$

b.) Domains

$$C_{10} = \{1\}$$
$$C_{100} = \{1\}$$
$$D = \{5\}$$
$$E = \{1\}$$
$$N = \{0\}$$
$$O = \{6\}$$
$$V = \{3\}$$

c.) Constraints

$$1. AllDiff(D, E, N, O, V)$$
$$2. 5 + 5 = 0 + 10$$
$$3. 5 + 5 + 1 = 1 + 10$$
$$4. 6 + 6 + 1 = 3 + 10 * 1$$

d.) Comments All variables have now been set to single values. All domains contain only one value. All constraints remain valid. The resulting mathematical equation is valid. This is the solution based on the CSP solver.