Exposure time calculator

1. Relations

1.1. Filter Magnitude and Photon Flux:

AB magnitude in a filter and photon flux relation (in SI system), if we assume that our filter's response function is constant, is like the relation below:

$$mag_{AB} = -2.5\, log\left(\frac{Photon\ flux\ (\#/s\ /m^2)}{\int_{\nu_{min}}^{\nu_{max}} (h\nu)^{-1} d\nu}\right) - 56.1$$

Source: http://faraday.uwyo.edu/~admyers/ASTR5160/handouts/516016.pdf, https://en.wikipedia.org/wiki/AB_magnitude

Which turns to this relation after integration:

$$photon\ flux\ (\#/s\ /m^2)\ = \frac{1}{h}\left(ln\left(\frac{c}{\lambda_{max}}\right) - ln\left(\frac{c}{\lambda_{min}}\right)\right) \times 10^{-0.4(mag_{AB}+56.1)}$$

In the top relation max correlates with maximum frequence and correlates with minimum frequency.

If we don't assume that response function is constant the relation will look like this:

$$mag_{AB} = -2.5\, log\left(\frac{\int_{\nu_{min}}^{\nu_{max}} f(\nu)(W/m^2/nm)(h\nu)^{-1} e(\nu) d\nu}{\int_{\nu_{min}}^{\nu_{max}} (h\nu)^{-1} e(\nu) d\nu}\right) - 56.1$$

In here, we can use two templates. First one assumes that $f(\nu)$ is constant and the second one assumes that our object is a blackbody in redshift z.
For the first template we have:

$$mag_{AB} = -2.5\, log\left(\frac{f(\nu)(W/m^2/nm) \int_{\nu_{min}}^{\nu_{max}} (h\nu)^{-1} e(\nu) d\nu}{\int_{\nu_{min}}^{\nu_{max}} (h\nu)^{-1} e(\nu) d\nu}\right) - 56.1$$

$$photon\ flux\ (\#/m^2/nm) = \ f(\nu)(W/m^2/nm) \times (\nu_{max} - \nu_{min})$$

For the second one, first we know that the blackbody radiation in the redshift z is like the following:

$$f(\nu) = A\frac{2h\nu^3}{(1+z)^3c^2}\frac{1}{e^{\frac{h(1+z)\nu}{kT_0}} - 1}$$

In which T_0 is the temperature in the rest frame and A is the distance coefficient ($\left(\frac{distance}{effective\ radius}\right)^2$) of the object. After finding A using the filter magnitude, we can find the photon flux using the relation bellow:

$$photon\ flux = \int_{\nu_{min}}^{\nu_{max}} f(\nu)(h\nu)^{-1}d\nu$$

Because I didn't have response function, I stuck to the very first relation for finding photon flux.

1.2. AB mag and Vega mag conversion:

The table below has been used for AB and Vega magnitudes conversion:

Vega - AB Magnitude Conversion

Band	λ_{eff}	$m_{AB} - m_{Vega}$	$M_{Sun}(AB)$	$M_{Sun}(Vega)$
U	3571	0.79	6.35	5.55
B	4344	-0.09	5.36	5.45
V	5456	0.02	4.80	4.78
R	6442	0.21	4.61	4.41
I	7994	0.45	4.52	4.07
J	12355	0.91	4.56	3.65
H	16458	1.39	4.71	3.32
K_s	21603	1.85	5.14	3.29
u	3546	0.91	6.38	5.47
g	4670	-0.08	5.12	5.20
r	6156	0.16	4.64	4.49
i	7472	0.37	4.53	4.16
z	8917	0.54	4.51	3.97
Y	10305	0.634		

These data are mostly from Blanton et al. (2007).

Source: https://www.astronomy.ohio-state.edu/martini.10/usefuldata.html

1.3. Total FWHM:

Sky seeing can be calculated as the function of wavelength and airmass using Kolmogorov model:

$$seeing(X, \lambda) = seeing(1, \lambda_0) X^{\frac{3}{5}} \left(\frac{\lambda}{\lambda_0}\right)^{-\frac{1}{5}}$$

Source: https://www.eso.org/observing/etc/doc/helphawki.html

The reference seeing needs to be calculated for each sky contribution using the telescope data. Sky seeing is equal to sky FWHM

The telescope FWHM can be calculated using equation below:

$$FWHM_{tel} = 0.00021 \frac{\lambda(nm)}{D(m)} \; arcsec$$

Source: https://www.eso.org/observing/etc/doc/helphawki.html

Total FWHM can be obtained using relation below:

$$FWHM_{tot} = \sqrt{FWHM_{tel}^2 + FWHM_{sky}^2 + FWHM_{CCD}^2}$$

CCD's FWHM also needs to be given.

1.4. SNR

Signal can be found using the relation below:

$$point: N_{obj} = photon\ flux \times time \times efficiency(filter) \times telescope\ surface$$

$$Extended: N_{obj} = photon\ flux(in\ the\ unit\ of\ solid\ angle) \times time \times efficiency(filter) \times telescope\ surface \times solid\ angle$$

SNR can be calculated using the equation below:

$$\frac{S}{N} = \frac{N_{Obj}}{\sqrt{N_{Obj} + n_{bin} \cdot N_{Sky} + n_{bin} \cdot RON^2 + n_{pix} \cdot DARK \cdot T}}$$

$$n_{pix} = 2 \times \frac{seeing}{pix\_scale}$$

Source: https://www.eso.org/observing/etc/doc/formulabook/node6.html

Where RON is readout noise, DARK is dark noise in the unit of time, N_{sky} is the sky signal.

1.5. Saturation Time

1.5.1. Point Source

Stars point source function (PSF) for photons is:

$$I(x,y) = I_0 e^{-\frac{x^2+y^2}{2\sigma^2}} = I_0 e^{-\frac{r^2}{2\sigma^2}}$$

Where the unit of I is ($\#/m$^2). The total signal can be obtained from the integration:

$$S = \sqrt{2}\pi^{\frac{3}{2}}\sigma I_0$$
$$I_0 = \frac{S}{\sqrt{2}\pi^{\frac{3}{2}}\sigma}$$

So, the PSF become:

$$I(x,y) = \frac{S}{\sqrt{2}\pi^{\frac{3}{2}}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

For the maximum signal in a pixel, we assume that our pixel in the middle of a pixel. The maximum signal in a pixel then becomes:

$$S_{pix} = \int_{-\frac{l}{2}}^{\frac{l}{2}}\int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{S}{\sqrt{2}\pi^{\frac{3}{2}}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} dxdy$$
$$= \frac{S}{\sqrt{2}\pi^{\frac{3}{2}}\sigma} \int_{-\frac{l}{2}}^{\frac{l}{2}} e^{-\frac{x^2}{2\sigma^2}} dx \int_{-\frac{l}{2}}^{\frac{l}{2}} e^{-\frac{y^2}{2\sigma^2}} dy$$
$$= \frac{S}{\sqrt{2}\pi^{\frac{3}{2}}\sigma} \left(\sqrt{2\pi}\,\mathrm{erf}\left(\frac{l}{2^{\frac{3}{2}}\sigma}\right)\sigma\right)^2 = \sqrt{\frac{2}{\pi}} S\sigma\,\mathrm{erf}^2\left(\frac{l}{2^{\frac{3}{2}}\sigma}\right)$$

The signal is:

$$S = F(\#/m^2)tes$$

Also:

$$S_{pix} = S_{max} - F_{sky_{pix}} t$$

So:

$$S_{max} = \sqrt{\frac{2}{\pi}}\sigma F(\#/m^2)tes\,\mathrm{erf}^2\left(\frac{l}{2^{\frac{3}{2}}\sigma}\right) + F_{sky_{pix}}t$$

$$\Longrightarrow t_{saturation} = \frac{S_{max}}{\sqrt{\frac{2}{\pi}}\sigma F(\#/m^2)es\,\mathrm{erf}^2\left(\frac{l}{2^{\frac{3}{2}}\sigma}\right) + F_{sky_{pix}}}$$

Also, Saturation flux for a specific time is:

$$F(\#/m^2) = \frac{S_{max} - F_{sky_{pix}}t}{\sqrt{\frac{2}{\pi}}\sigma tes\,\mathrm{erf}^2\left(\frac{l}{2^{\frac{3}{2}}\sigma}\right)}$$

Where:

$$\sigma = \frac{FWHM}{2\sqrt{2ln2}} = \frac{FWHM}{2.355}$$

1.5.2. Extended Source:

The maximum count fpr an extended source can be found from this equation:

$$S_{max} = \frac{S \times \ pix\ scale^2}{\Omega^2} + F_{sky_{pix}}t$$

$$S = F(\#/m^2)tes\Omega$$

$$t_{saturation} = \frac{S_{max}}{F(\#/m^2)es \times \frac{pix\ scale^2}{\Omega} + F_{sky_{pix}}}$$

And Saturation flux for a specific time is:

$$F(\#/m^2) = \frac{S_{max} - F_{sky_{pix}}t}{tes \times \frac{pix\ scale^2}{\Omega}}$$