Pset 1.

1.(a) $\exists x \in X: S(x) \wedge A(x)$

(b) $\forall x \in X: T(x) \wedge S(x) \Rightarrow A(x)$

(c) $\neg \exists x \in X: T(x) \wedge (\neg A(x))$

(d) $\exists x,y,z: (\neg E(x,y) \wedge \neg E(x,z) \wedge \neg E(y,z) \wedge T(x) \wedge \neg S(x) \wedge T(y) \wedge \neg S(y) \wedge T(z) \wedge \neg S(z))$

2. (a)

P	Q	R	$\neg(P \vee (Q \wedge R))$	$(\neg P) \wedge (\neg Q \vee \neg R)$
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	T	T
F	T	F	T	T
F	F	F	T	T

(b)

P	Q	R	$\neg(P \wedge (Q \vee R))$	$(\neg P) \vee (\neg Q \vee \neg R)$
T	T	T	F	F
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	T
F	F	T	T	T
F	T	F	T	T
F	F	F	T	T

3.(a) (i) $\neg$(A nand B) (ii) ($\neg$A) nand ($\neg$B) (iii) A nand ($\neg$B)

(b) (A nand A)

(c) always gives true: A nand ($\neg$A) = A nand (A nand A), always gives false: (A nand (A nand A)) nand (A nand (A nand A))

4. Seperate the coins in 3 groups, A, B, C. First weigh A and B. If the weight differs, the lighter group consists of a fake coin. If A and B weigh the same, C consists of a fake coin. Now seperate the group with fake coins, two by two. The lighter one has the fake coin. now weigh each coins in the lighter side. Now you get a fake coin.

5. pf. (by contradiction) Assume $r^{1/5}$ is rational, then r is rational.

Then, $r^{1/5} = a/b$ (fraction in lowest terms)

$\Rightarrow r = a^5/b^5$, since a/b is rational, a^5/b^5 is rational therefore r is rational. Which implies the contrapositive if r is irrational, then $r^{1/5}$ is irrational is true.

6. w, x, y odd: $(2i+1)^2+(2j+1)^2+(2k+1)^2 = 4i^2+4i+1+4j^2+4j+1+4k^2+4k+1 \Rightarrow$ odd, therefore z is odd

w,x / w,y / x,y odd: $(2i+1)^2+(2j+1)^2+4k^2 = 4i^2+4i+1+4j^2+4j+1+4k^2 = 4(i^2+i+j^2+j+k^2)+2 = 4l^2 \Rightarrow$ impossible therefore z not even

w / x / y odd: $(2i+1)^2+4j^2+4k^2 = 4(i^2+i+j^2+k^2)+1 = 4l^2 \Rightarrow$ impossible therefore z not even

w,x,y even: $4i^2+4j^2+4k^2 = 4(i^2+j^2+k^2) = 4l^2$, z can be even integer $i^2+j^2+k^2$