Introduction and proofs

A proof is a method for ascertaining the truth

ex) Experimentation & Observing, Sampling & Counter examples, Judge & Jury, Word of god, Conviction

A mathematical proof is a verification of a propositions by chain of logical deductions from a set of axioms

Def: A proposition is a statement that is either T/F

ex) $2+3=5$

natural number: $\{0,1,2,3\cdots\}$

$\forall n \in \mathbb{N}$, n^2+n+41 is a prime number

predicate: proposition whose truth depends on the value of variable(s)

* n	n^2+n+41	Prime
0	0	v
1	43	v
2	47	v
3	53	v
⋮	⋮	⋮
20	461	v
⋮	⋮	⋮
39	1601	v
40	$1681=41^2$	x
41		x

$a^4+b^4+c^4=d^4$ has no positive integer solution

Positive natural number: $\{1,2,3\cdots\}$

* $a=95800$, $b=217519$, $c=414560$, $d=422481 \Rightarrow \exists a,b,c,d \in \mathbb{N}^+$, $a^4+b^4+c^4=d^4$

predicate

$313(x^3+y^3)=z^3$ has no positive integer Solutions

The regions in any maps can be colored in 4 colors so that adjacent regions have different colors

Every even integer but 2 is the sum of 2 primes: $24=13+11$

integers

$\forall n \in \mathbb{Z}$, $n \geq 2 \Rightarrow n^2 \geq 4$

implies

ex) pigs fly ⇒ I'm king

Def: An implication $p \Rightarrow q$ is true if p is F or q is T

Truth Table

p	q	$p\Rightarrow q$	$q\Rightarrow p$	$p\Leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

iff

$\forall n \in \mathbb{Z}$, $n \geq 2 \Leftrightarrow n^2 \geq 4$

: False, $n=-3$

Def: An axiom is a proposition that is "assumed" to be true

ex) If $a=b$ & $b=c$, then $a=c$

Euclidean Geometry: Given a line L & a point p not on L, there is exactly one line through p pararell to L.

Spherical Geometry: Given a line L & a point p not on L, there is no line through p pararell to L.

Hyperbolic Geometry: Given a line L & a point p not on L, there are infinitely many lines through p pararell to L.

Axioms should be 1. Consistent

2. Complete

Def: A set of axioms is consistent if no proposition can be proved T & F

A set of axioms is complete if it can be used to prove every proposition is T or F