



$$E = -\frac{1}{2} \sum_k (T_k - y_k)^2$$

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial H_j} \frac{\partial H_j}{\partial w_{ji}} \quad \text{where} \quad \frac{\partial E}{\partial H_j} = \sum_k \frac{\partial E}{\partial y_k} \frac{\partial y_k}{\partial H_j}$$

$$\frac{\partial E}{\partial y_k} = -(T_k - y_k)$$

$$\text{let } \text{net}_k = \sum_j (w_{kj} H_j), \text{ then, } \frac{\partial y_k}{\partial H_j} = \frac{\partial f(\text{net}_k)}{\partial H_j} = \frac{\partial f(\text{net}_k)}{\partial \text{net}_k} \frac{\partial \text{net}_k}{\partial H_j} = \frac{\partial f(\text{net}_k)}{\partial \text{net}_k} \frac{\sum_j (w_{kj} H_j)}{\partial H_j} = f'(\text{net}_k) w_{kj}$$

$$\therefore \frac{\partial E}{\partial H_j} = \sum_k \frac{\partial E}{\partial y_k} \frac{\partial y_k}{\partial H_j} = - \sum_k (T_k - y_k) f'(\text{net}_k) w_{kj}$$

$$\text{let } \text{net}_j = \sum_i (w_{ji} x_i), \text{ then, } \frac{\partial H_j}{\partial w_{ji}} = \frac{\partial f(\text{net}_j)}{\partial w_{ji}} = \frac{\partial f(\text{net}_j)}{\partial \text{net}_j} \frac{\partial \text{net}_j}{\partial w_{ji}} = \frac{\partial f(\text{net}_j)}{\partial \text{net}_j} \frac{\sum_i (w_{ji} x_i)}{\partial w_{ji}} = f'(\text{net}_j) x_i$$

$$\therefore \frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial H_j} \frac{\partial H_j}{\partial w_{ji}} = - \sum_k (T_k - y_k) f'(\text{net}_k) w_{kj} \times f'(\text{net}_j) x_i$$

$$= - x_i f'(\text{net}_j) \sum_k \delta_k w_{kj} = - x_i \delta_j$$