Peridynamic Numerical Model

For Dynamic Fracture in Unidirectional Fiber Reinforced Composites

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{\mathcal{H}_x} \mathbf{f}(\mathbf{u}(\hat{\mathbf{x}},t) - \mathbf{u}(\mathbf{x},t), \hat{\mathbf{x}} - \mathbf{x})\, dV_{\hat{\mathbf{x}}} + \mathbf{b}(\mathbf{x},t)$$

- $\mathbf{x}, \hat{\mathbf{x}}$: the initial positions of points in the reference configuration.
- **u:** displacement vector field
- **f:** pairwise force function in peridynamic model $\left(\frac{N}{(m^3)^2}\right)$
- **b:** body force acting at $\hat{\mathbf{x}}$ at time t
- $\mathcal{H}_x$: the neigbour centered $\mathbf{x}$ with radius δ “horizon”

- In micro-elastic materials there exits a **<u>micropotential function</u>** ω which:

$$\mathbf{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \frac{\partial \omega(\boldsymbol{\eta}, \boldsymbol{\xi})}{\partial \boldsymbol{\eta}}$$

Where $\boldsymbol{\eta} = \mathbf{u}(\hat{\mathbf{x}}, t) - \mathbf{u}(\mathbf{x}, t)$
and $\boldsymbol{\xi} = \hat{\mathbf{x}} - \mathbf{x}$.

- The **strain energy density** is defined:

$$W = \frac{1}{2} \int_{\mathcal{H}_x} \omega(\boldsymbol{\eta}, \boldsymbol{\xi}) d\, V_\xi$$

Linear Micro-Elastic Potential

$$\omega(\boldsymbol{\eta}, \boldsymbol{\xi}) = \frac{c(\xi) s^2 \xi}{2}$$

where $\xi = \|\boldsymbol{\xi}\|$ and s is the bond relative elongation:

$$s = \frac{\|\boldsymbol{\xi} + \boldsymbol{\eta}\| - \xi}{\xi}$$

Linear relationship between the bondforce **f** and the relative elongation s:

$$\mathbf{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = \frac{\partial \omega(\boldsymbol{\eta}, \boldsymbol{\xi})}{\partial \boldsymbol{\eta}} = c(\xi) s \|\boldsymbol{\xi}\| \frac{\partial \|\boldsymbol{\eta} + \boldsymbol{\xi}\|}{\partial \boldsymbol{\eta}}$$

$$= c(\xi) s \frac{\boldsymbol{\eta} + \boldsymbol{\xi}}{\|\boldsymbol{\eta} + \boldsymbol{\xi}\|} = c(\xi) s \hat{\boldsymbol{e}}$$

where $\hat{\mathbf{e}} = \frac{\boldsymbol{\eta} + \boldsymbol{\xi}}{\|\boldsymbol{\eta} + \boldsymbol{\xi}\|}$.

The Peridynamic discrete model for 2 Dimensions UD FRC (Unidirectional Fiber Reinforced Composites):

- This discretization is restricted to regular square grids with the principal directions aligned with the fiber direction.
- Grid refinement for a fixed horizon size δ (called m-convergence, where m is the ratio between the horizon and the grid spacing and m$\rightarrow +\infty$) the fiber bond area in peridynamic discretization at a node changes. Therefore, the strain energy density will change. So we need a proper scale factor.

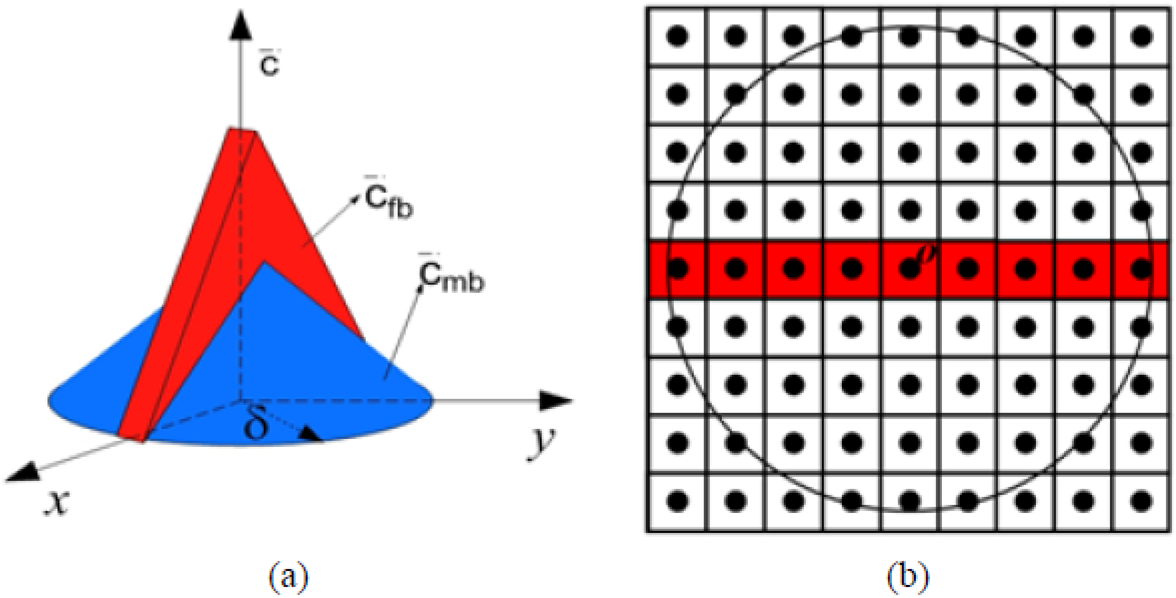


FIG. 3: Conical micromodulus function for the UD composite lamina at a point in the bulk **(a)**. The discrete peridynamic model for unidirectional lamina at a particular node O **(b)**. The circle is the horizon for this node. Fiber direction is horizontal in this example; thus, the peridynamic fiber bonds for the central node exist only with nodes having their areas colored in red (since only these bonds, centered at O, have the same direction as the fiber direction).

Example

Consider a material (M55J/M18 carbon/epoxy) properties in this table:

Property	Unidirectional
Longitudinal Young's modulus E_{11} (GPa)	329
Transverse Young's modulus E_{22} (GPa)	6
Shear modulus E_{12} (GPa)	4.4
Poisson's ratio ν_{12}	0.346
Density ρ (kg/ m^3)	1630
Fracture energy G_0^{11} (KJ/m^2)	15.49
Fracture energy G_0^{22} (KJ/m^2)	0.168

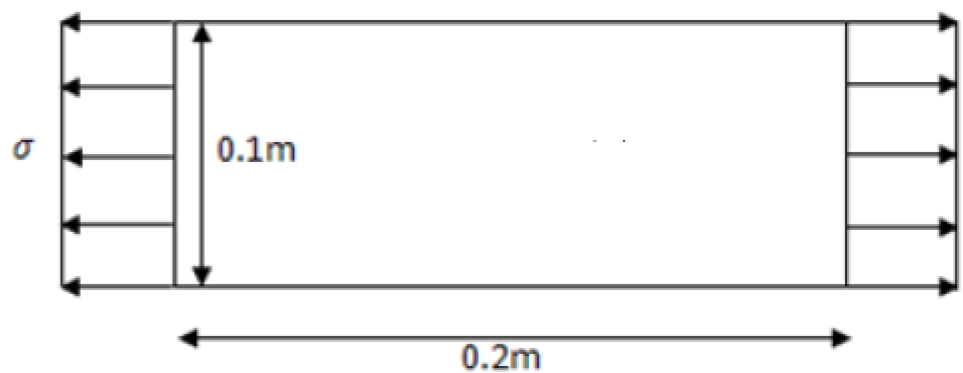
where

E_{11}: longitudinal elastic Young modulus in the principal material axes

E_{22}: transverse elastic Young modulus (perpendicular to the fiber direction)

υ_{12}: longitudinal Poisson's ratio
${G_0}^{11}$: the fracture energy for a UD composite with 0° fiber orientation
${G_0}^{22}$: the fracture energy for a UD composite with 90° fiber orientation

Consider a thin rectangular plate with dimensions $0.2m \times 0.1m$. Along the left and right edges a uniform tensile load $\sigma = 40$ Pa is applied suddenly and maintained constant in time after that.



- In this case we know the initial position of each nodes and the initial velocity of each node is 0.
- After choosing the horizon δ and the uniform spacing grid Δx, for each fixed node $\mathbf{x}_i$, we can firstly find the isotropic micromudulus functions for the longitudinal direction ${c^{iso}}_{11}$ and transverse direction ${c^{iso}}_{22}$, for each bond $\boldsymbol{\xi} = \mathbf{x}_p - \mathbf{x}_i$ (every $\mathbf{x}_p$ which satisfies $\left\|\mathbf{x}_p - \mathbf{x}_i\right\| \le \delta$):

$${c^{iso}}_{11} = \frac{12(E_{11}+\upsilon_{12}E_{22})}{(1-\upsilon_{12}\upsilon_{21})\pi\delta^3}\left(1-\frac{\xi}{\delta}\right) \quad , \quad {c^{iso}}_{22} = \frac{12(E_{22}+\upsilon_{12}E_{22})}{(1-\upsilon_{12}\upsilon_{21})\pi\delta^3}\left(1-\frac{\xi}{\delta}\right)$$

where $\xi = \|\boldsymbol{\xi}\|$.

- In 2D model υ_{12} is considered $\frac{1}{3}$ and the transverse Poisson's ratio υ_{21} is equal to $\frac{E_{22}}{E_{11}}\upsilon_{12}$.
- Now for each time step n we can find the acceleration of displacement ${\ddot{\mathbf{u}}_i}^n$ with using:
(we have the displacement ${\mathbf{u}_i}^n$ and velocity ${\dot{\mathbf{u}}_i}^n$ of all points.)

Numerical Method

$$\rho \ddot{\mathbf{u}}_i{}^n = \sum_p c^d(\xi,\theta) s^n \mu^n(\xi,\theta) V_p \frac{\boldsymbol{\xi} + \boldsymbol{\eta}^n}{\|\boldsymbol{\xi} + \boldsymbol{\eta}^n\|} + \mathbf{b}_i{}^n$$

- n is the time step number and subscripts denote the node number.
- $\boldsymbol{\eta}^n = \mathbf{u}_p{}^n - \mathbf{u}_i{}^n$
- $s^n = \frac{\|\boldsymbol{\xi}+\boldsymbol{\eta}^n\| - \|\boldsymbol{\xi}\|}{\|\boldsymbol{\xi}\|}$
- $\theta = arctan \frac{\xi_2}{\xi_1}$ where $\boldsymbol{\xi} = (\xi_1, \xi_2)$
- $c^d(\xi,\theta) = \begin{cases} \frac{\pi m}{2} c^{iso}{}_{11}(\xi) & if \;\; \theta = 0 \quad or \quad \theta = \pi \\ \frac{\pi m}{\pi m - 2} c^{iso}{}_{22}(\xi) & otherwise \end{cases}$

where $m = \frac{\delta}{\Delta x}$ and Δx is the grid spacing

- $\mu^n(\xi,\theta)=\begin{cases}1 & if \;\; s^m < s_0 \quad for\, all\; 0 \le m \le n \\ 0 & otherwise\end{cases}$

where

$$s_0 = \begin{cases} {s_0}^{fb} = \sqrt{\dfrac{20\; {G_0}^{11}}{{c^{iso}}_{11}\delta^4}} & if\; \theta = 0 \; or\; \theta = \pi \\ {s_0}^{mb} = \sqrt{\dfrac{20\; {G_0}^{22}}{{c^{iso}}_{22}\delta^4}} & otherwise \end{cases}$$

and

$${c^{iso}}_{11} = \frac{12(E_{11}+\upsilon_{12}E_{22})}{(1-\upsilon_{12}\upsilon_{21})\pi\delta^3}\left(1-\frac{\xi}{\delta}\right) \quad , \quad {c^{iso}}_{22} = \frac{12(E_{22}+\upsilon_{12}E_{22})}{(1-\upsilon_{12}\upsilon_{21})\pi\delta^3}\left(1-\frac{\xi}{\delta}\right)$$

- $V_p = (\Delta x)^2 \times 1$
- The $\sum$ is over every point $\mathbf{x}_p$ which $\left\|\mathbf{x}_p - \mathbf{x}_i\right\| \le \delta$ where δ is the horizon

Velocity-Verlet algorithm

- In the time step n, we have the displacement ${\mathbf{u}_i}^n$ and velocity ${\dot{\mathbf{u}}_i}^n$ of all points.
- We have calculated the ${\ddot{\mathbf{u}}_i}^n$. By using the Velocity-Verlet algorithm

$$\dot{u}_{n+\frac{1}{2}} = \dot{u}_n + \frac{\Delta t}{2}\ddot{u}_n$$

$$u_{n+1} = u_n + \Delta t \dot{u}_{n+\frac{1}{2}}$$

$$\dot{u}_{n+1} = \dot{u}_{n+\frac{1}{2}} + \frac{\Delta t}{2}\ddot{u}_{n+1}$$

we can calculate the next displacement ${\mathbf{u}_i}^{n+1}$ and next velocity ${\dot{\mathbf{u}}_i}^{n+1}$. In these formula Δt is the time interval.

- The algorithm will be repeated with the new displacement and velocity vectors.

References

- W. Hu, Y.D. Ha, F. Bobaru, Modeling dynamic fracture and damage in fiber reinforced composites with peridynamics, Int. J. Multiscale Comput. Engrg. 9 (2011) 707–726.
- W. Hu, Y.D. Ha, F. Bobaru, Peridynamic model for dynamic fracture in unidirectional fiber-reinforced composites, Comput. Methods Appl. Mech. Engrag. 217-220 (2012) 247-261.

What I have done last month

- Reading articles to learn and understand what Peridynamic is.

  Silling SA, Askari E (2005) A meshfree method based on the peridynamic model of solid mechanics. Comput Struct 83:1526–2535

  YD. Ha and **F. Bobaru**, “Studies of dynamic crack propagation and crack branching with peridynamics,” *International Journal of Fracture*, **162**(1-2): 229-244 (2010).

  W. Hu, Y.D. Ha, F. Bobaru, Modeling dynamic fracture and damage in fiber reinforced composites with peridynamics, Int. J. Multiscale Comput. Engrg. 9 (2011) 707–726.

  W. Hu, Y.D. Ha, F. Bobaru, Peridynamic model for dynamic fracture in unidirectional fiber-reinforced composites, Comput. Methods Appl. Mech. Engrag 217-220 (2012) 247-261.
- Having weekly and regularly meeting with Florin
- Trying to read and understand Biswajit’s codes about Peridynamic

The plan for next month

1. Continuing to understand and run Biswajit’s programmes in peridynamic (for example Box) and trying to find the probable problems in the code
2. Reading more Peridynamics articles especially 3D models
3. Reading some articles about fracture in wood
4. Trying to find an appropriate peridynamic model for wood

Thank you very much

END