Progress from 12-19 Aug 2013

1. Conical micromodulus Function for a PMB material in 3D Peridynamics model

According to [1], the constant micromodulus function for the prototype micromodulus brittle (PMB) material is

$$c = \frac{18k}{\pi\delta^4} \qquad (1)$$

where k is bulk modulus and δ is the horizon.

Since $k = \frac{E}{3(1-2\nu)}$, we have

$$c = \frac{6E}{\pi(1-2\nu)\delta^4} \qquad (2)$$

where E is Young modulus and ν is Poission's ratio.

The strain energy density W (the energy per unit volume in the body at a given point) is independent on the kind of the micromodulus function. W is found by

$$W = \frac{1}{2}\int_{\mathcal{H}_x} \omega(\boldsymbol{\eta}, \boldsymbol{\xi}) dV_x \qquad (3)$$

$\boldsymbol{\eta}$ is the relative displacement and $\boldsymbol{\xi}$ is the relative position in reference configuration and $\mathcal{H}_x$ is a neigbourhood of x with radius δ. The scalar micropotential ω is obtained by $\omega(\boldsymbol{\eta}, \boldsymbol{\xi}) = \frac{c(\xi)s^2\xi}{2}$ where s is the relative elongation and c is the micromodulus function.

If we choose $c_{coniclal} = c_1(1 - \frac{\xi}{\delta})$ then we should have

$$W = \frac{1}{2}\int_0^\delta \left(\frac{c_{conical}\ s^2}{2}\right) 4\pi\xi^2 d\xi = \frac{1}{2}\int_0^\delta \left(\frac{cs^2}{2}\right) 4\pi\xi^2 d\xi$$

$$\Rightarrow c_1 = 5c$$

$$\Rightarrow c_{conical} = \frac{30E}{\pi(1-2\nu)\delta^4}\left(1 - \frac{\xi}{\delta}\right) \qquad (4)$$

By considering the efficient Poission's ratio for 3D bond-based peridynamics model $\nu = \frac{1}{4}$, the conical micromodulus function is found by

$$c_{conical} = \frac{60E}{\pi\delta^4}\left(1 - \frac{\xi}{\delta}\right) \qquad (5)$$

Reference

[1] Silling SA, Askari E (2005) A meshfree based on the peridynamic model for solid mechanics. Comput Struct 83:1526-2535