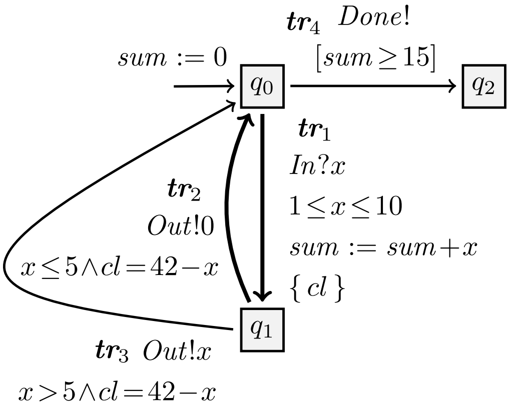
Test case generation

The construction of the test case is obtained by applying dedicated symbolic execution techniques to the reference timed symbolic automaton, in order to derive a symbolic subtree restricted to the test purpose, i.e., a path represented as a sequence of transitions of the reference automaton. In the following, we **first provide an overview of these test-oriented symbolic techniques**, and **then describe the test case generation itself**, obtained by applying transformations to this subtree (mirroring and constraint simplifications).

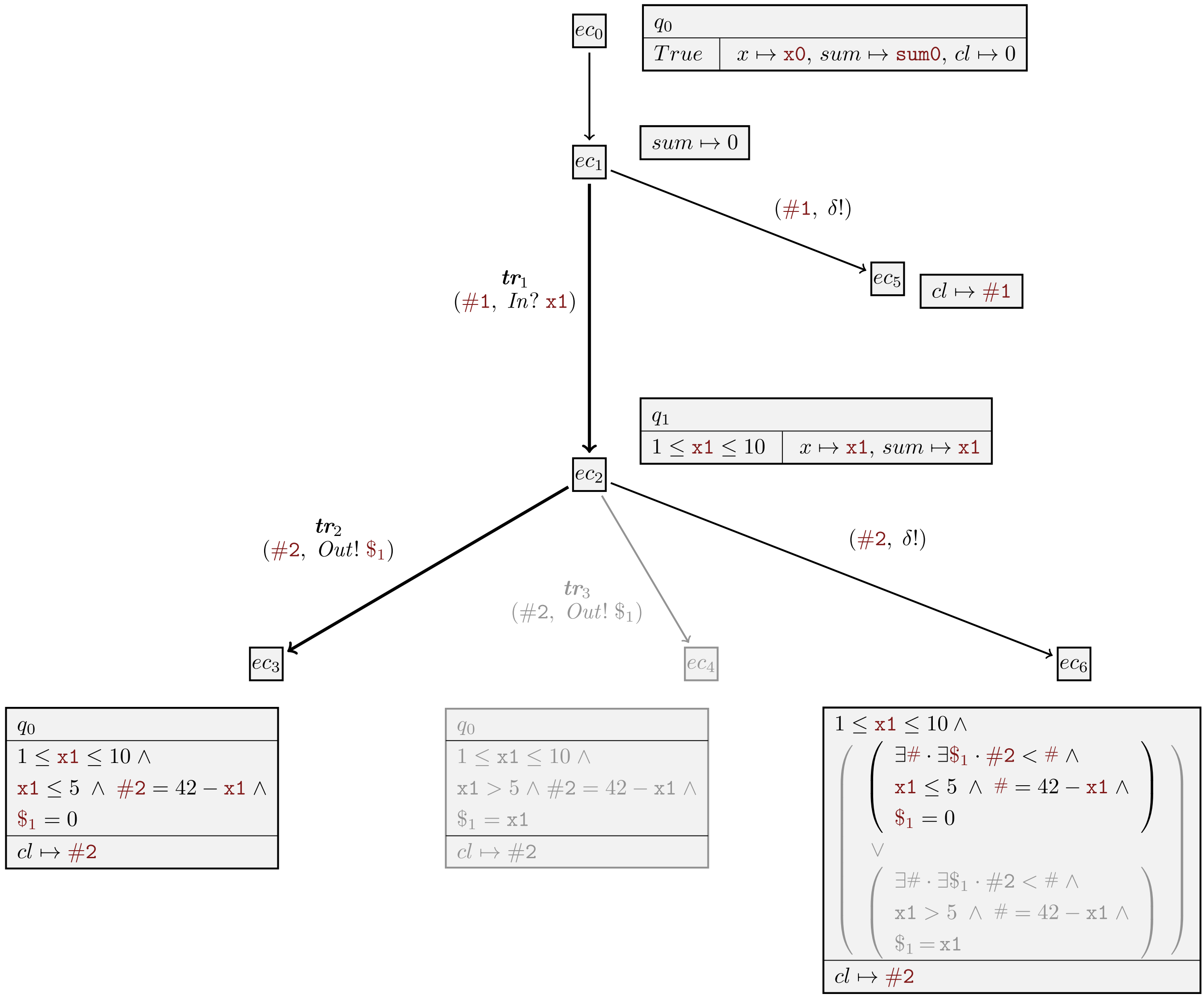
1. Test-oriented Symbolic Execution Techniques

Symbolic execution explores a model by representing both data and time with symbolic variables instead of concrete values. It unfolds the automaton while generating constraints over symbolic variables, producing a **symbolic execution tree** . The tree's nodes are **execution contexts**, and its edges represent symbolic steps such as initialization, transition firing, or **quiescence completion**.

Recall the dummy automaton example (discussed model specification tutorial):



The symbolic execution tree (restricted by test purpose transitions sequence $\mathbf{tr}_1.\mathbf{tr}_2$):


ec_0
q_0
$True$ | $x \mapsto \texttt{x0},\ sum \mapsto \texttt{sum0},\ cl \mapsto 0$
$sum \mapsto 0$
ec_1
$(\#1,\ \delta!)$
ec_5
$cl \mapsto \#1$
tr_1
$(\#1,\ In?\ \texttt{x1})$
q_1
$1 \leq \texttt{x1} \leq 10$ | $x \mapsto \texttt{x1},\ sum \mapsto \texttt{x1}$
ec_2
tr_2
$(\#2,\ Out!\ \$_1)$
tr_3
$(\#2,\ Out!\ \$_1)$
$(\#2,\ \delta!)$
ec_3
ec_4
ec_6
q_0
$1 \leq \texttt{x1} \leq 10\ \wedge$
$\texttt{x1} \leq 5\ \wedge\ \#2 = 42 - \texttt{x1}\ \wedge$
$\$_1 = 0$
$cl \mapsto \#2$
q_0
$1 \leq \texttt{x1} \leq 10\ \wedge$
$\texttt{x1} > 5 \wedge \#2 = 42 - \texttt{x1}\ \wedge$
$\$_1 = \texttt{x1}$
$cl \mapsto \#2$
$1 \leq \texttt{x1} \leq 10\ \wedge$
$\exists\# \cdot \exists\$_1 \cdot \#2 < \#\ \wedge$
$\texttt{x1} \leq 5\ \wedge\ \# = 42 - \texttt{x1}\ \wedge$
$\$_1 = 0$
$\vee$
$\exists\# \cdot \exists\$_1 \cdot \#2 < \#\ \wedge$
$\texttt{x1} > 5\ \wedge\ \# = 42 - \texttt{x1}\ \wedge$
$\$_1 = \texttt{x1}$
$cl \mapsto \#2$

Execution Contexts

An **execution context** $ec = (q, \pi, \lambda, ev, pec)$ consists of:

- The current state q.
- The **path condition** π (accumulated constraints).
- The mapping λ of variables and clocks to symbolic terms.
- The triggering event ev.
- The predecessor context pec.

The **root context** ec_0 starts in q_0, with clocks at zero, variables assigned fresh symbols, $\pi = True$, and ev and pec undefined. Initialization produces the first successor, ec_1.

Symbolic Variables: Fresh symbolic variables are introduced:

`x0`, `x1`, ... represent successive values of a data variable x (with `x0` being the initial value).

`#1`, `#2`, ... denote **symbolic delays**.

`$1`, `$2`, ... denote **emitted values** typed according to their channels.

Symbolic Paths

Contexts ec_2, ec_3, and ec_4 illustrate the symbolic execution of transitions $\mathbf{tr}_1$, $\mathbf{tr}_2$, and $\mathbf{tr}_3$.

1. **Edge from ec_1 to ec_2 ($\mathbf{tr}_1$):**
   - Transition from q_0 to q_1 via input In.
   - x is updated to `x1`. Clock cl is reset to 0.
   - Edge label: symbolic action $In?$`x1` and delay `#1`.
   - **Path condition**: 1 $\leq$ `x1` $\leq$ 10 (from guard $1 \leq x \leq 10$).
   - Update: $sum \mapsto$ `x1`.

2. **Edge from ec_2 to ec_3 ($\mathbf{tr}_2$):**
   - Transition from q_1 to q_0, emitting on channel Out.
   - `#2` is elapsed time, and `$1` is the emitted value. Clock value becomes `#2`.
   - **Path condition**: `x1` $\leq$ 5 and `#2` = 42 - `x1` (from guard $x \leq 5$ and $cl = 42 - x$), and `$1` = 0.

The symbolic path $ec_1.ec_2.ec_3$ corresponds to model path $\mathbf{tr}_1.\mathbf{tr}_2$, yielding the symbolic trace (`#1`, $In?$`x1`).(`#2`, $Out!$`$1`).

The **path condition** for this trace (`#1` is unconstrained) is:

1 $\leq$ `x1` $\leq$ 10 $\land$ `x1` $\leq$ 5 $\land$ `#2` = 42 - `x1` $\land$ `$1` = 0

This is **satisfiable** e.g. with `x1` $\mapsto$ 1, `$1` $\mapsto$ 0, `#1` $\mapsto$ 0, `#2` $\mapsto$ 41, producing the **timed trace** $(0, In?1).(41, Out!0)$. This trace shows the system receives $In?1$ after initialization and emits $Out!0$ 41 time units later.

Completion by Quiescence

Contexts ec_5 and ec_6 model **quiescence** (system silence). Symbolic variables are reused across sibling contexts (e.g., `#1` for ec_2 and ec_5).

- **Quiescence context ec_5**: Derived from ec_1. The edge is labeled with the quiescence event (`#1`, $\delta!$). The system may remain silent indefinitely, reflected by $\pi = True$ and unconstrained delay `#1`.

- **Quiescence context ec_6**: Derived from ec_2's output successors (ec_3 and ec_4). Its path condition is a disjunction of existential constraints (e.g., $\exists$ `#` · $\exists$ `$1` · `#2` < `#` $\land \ldots$), capturing that quiescence persists until an output is possible.

- **Trace-determinism and pruning**: For a chosen Test Path (TP) $ec_1.ec_2.ec_3$ (which implies `x1` $\leq$ 5), context ec_4 (which implies `x1` > 5) **conflicts** and is removed (grayed out). This simplifies ec_6's path condition.

A **witness timed trace** $(0, In?1) \cdot (40, \delta!)$ covers ec_6 (with `x1` $\mapsto$ 1, `#2` $\mapsto$ 40), demonstrating that after $In?1$, the system can remain silent for 40 time units, expecting the next output at 41.

SPTG Workflow

For a model $\mathbb{G}$, the **Symbolic Path-guided Test Generation (SPTG)** workflow restricts symbolic exploration to a **model path**

$p = \mathbf{tr}_1 \cdots \mathbf{tr}_n$, chosen as a **test path (TP)**.

Starting from the initial state q_0, the workflow performs **symbolic execution along** p, using the SMT solver **Z3** to verify:

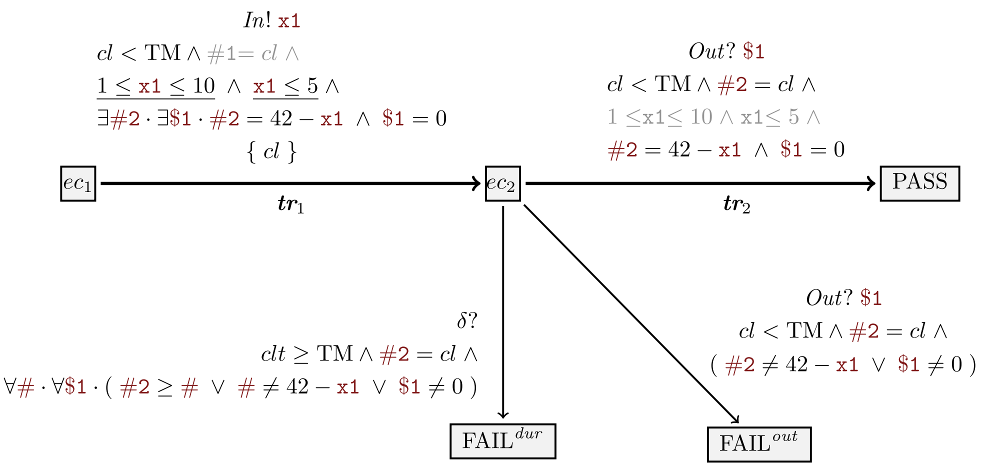
- satisfiability of execution contexts,
- trace-determinism, and
- conflict detection.

The workflow proceeds through the following five main steps:

1. **Symbolic execution along the path**
   - From the current execution context ec_1, all successor contexts are computed (Custom Symbex).
   - For each transition $\mathbf{tr}_i$, the workflow checks whether it can be fired.
   - If the transition is fireable, exploration continues from the successor produced by \$\textbf{tr}*i , exploringtheremainingsuffixp*' = *\textbf{tr}*{i+1} \cdots \textbf{tr}_n\$.
   - Otherwise, the exploration stops.
2. **Conflict removal**
   - Any conflicting contexts detected during symbolic execution are removed.
3. **Trace-determinism verification**
   - The workflow verifies that no two sibling contexts on the same channel could be covered by the same trace.
   - Exploration halts if nondeterminism is detected.
4. **Incorporation of quiescence contexts**
   - Quiescence contexts are added, producing a restricted, deterministic, quiescence-augmented symbolic execution tree $SE(\mathbb{G})^{\delta}_{/p}$, which contains the path p and its immediate trace-deterministic divergences.
5. **Test case synthesis**
   - The final step synthesizes the **timed symbolic test case** \$\mathbb{TC}*p fromSE(\mathbb{G})^{\delta}*{/p}\$.

In the following, we detail the construction of $\mathbb{TC}_p$, illustrated below for our running dummy example, and explain how SPTG generates it from the given model path p, which serves as the test purpose.

The test case which corresponds to the test purpose path $\mathbf{tr}_1.\mathbf{tr}_2$ (partial view):


$In!\,\mathtt{x1}$
$cl < \mathrm{TM} \wedge \#1 = cl \wedge$
$\underline{1 \leq \mathtt{x1} \leq 10} \wedge \underline{\mathtt{x1} \leq 5} \wedge$
$\exists\#2 \cdot \exists\$1 \cdot \#2 = 42 - \mathtt{x1} \wedge \$1 = 0$
$\{ cl \}$
ec_1
tr_1
ec_2
$Out?\,\$1$
$cl < \mathrm{TM} \wedge \#2 = cl \wedge$
$1 \leq \mathtt{x1} \leq 10 \wedge \mathtt{x1} \leq 5 \wedge$
$\#2 = 42 - \mathtt{x1} \wedge \$1 = 0$
PASS
tr_2
$\delta?$
$clt \geq \mathrm{TM} \wedge \#2 = cl \wedge$
$\forall\# \cdot \forall\$1 \cdot (\#2 \geq \# \vee \# \neq 42 - \mathtt{x1} \vee \$1 \neq 0)$
FAIL^{dur}
$Out?\,\$1$
$cl < \mathrm{TM} \wedge \#2 = cl \wedge$
$(\#2 \neq 42 - \mathtt{x1} \vee \$1 \neq 0)$
FAIL^{out}

2. Symbolic path-guided test case

The test case $\mathbb{TC}_p$ is defined as a **timed symbolic transition system** equipped with a **single clock** `cl`, which measures the elapsed time before each action it performs.

The **data variable set** of $\mathbb{TC}_p$ includes all symbolic variables used to produce the execution contexts covering the path p.
These variables represent the information known and manipulated by the test case as execution progresses, including:

- **Input values** to stimulate the SUT with (e.g., `x1`) and their associated **submission durations** (e.g., `#1`).
- **Output values** expected from the SUT (e.g., `$1`) and their corresponding **observation times** (e.g., `#2`).

Clock constraint

- The clock satisfies:

$$cl < \mathrm{TM}$$

  where TM denotes the maximal waiting time before either:
  - applying a stimulation, or
  - observing an output.

This timing mechanism, combined with quiescence detection ($cl \geq \mathrm{TM}$), ensures that the test case can be implemented in a real-time environment.

Relation to symbolic execution

As illustrated in Fig.~???, the test case $\mathbb{TC}*pmirrorsSE(\mathbb{G})^{\delta}*{/p}$ and is used to **check the conformance** of the SUT to $\mathbb{G}$ along the symbolic path p.

Execution structure

- The execution contexts related to path p form the **main branch** leading to the verdict `PASS`.
  The target context is replaced by `PASS`.

- Any deviation from this branch triggers a verdict state:
  - `FAIL` if the behavior violates expectations.
  - `INC` (inconclusive) if no clear verdict can be determined.

Guard derivation

The **guard** of the test-case transition from ec_1 to $ec_2$ is derived from the target of the test path (TP), denoted ec_3.
It guides the selection of the stimulation $In!x_1$ along this path.
The guard is expressed as:

cl < \text{TM} ;\wedge; 1 \leq x_1 \leq 10 ;\wedge; x_1 \leq 5 ;\wedge;
\exists #_2 \cdot \exists $_1 \cdot (#_2 = 42 - x_1 ;\wedge; $_1 = 0)

At this stage:

- x_1 and its duration $\boxed{\#\_1}$ are determined.
- $\boxed{\#\_2}$ and $$_1$ remain undetermined.

The variable x_1 is constrained by the path condition of ec_3 (small inputs), whereas $\boxed{\#\_1}$ is unconstrained and may be omitted (grayed).

Quantifier simplification

Conditions producing ec_3 are, by default, under existential quantifiers:

\exists #_2 ;\exists $_1 ;\big(x_1 \leq 5 ;\wedge; #_2 = 42 - x_1 ;\wedge; $_1 = 0 \big)

Since $\boxed{\#\_2}$ and $$_1$ do not occur freely in $x_1 \leq 5$, this constraint is moved outside the quantifiers, yielding the **final guard**.

Expected observation and transition to PASS

Following the test path, the test case expects an observation $Out?$_1$ on channel `Out`, storing it in $$_1$.
It transitions from $ec_2$ to `PASS` under the following guard:

cl < \text{TM} ;\wedge; #_2 = cl ;\wedge; 1 \leq x_1 \leq 10 ;\wedge;
x_1 \leq 5 ;\wedge; #_2 = 42 - x_1 ;\wedge; $_1 = 0

- The formulas $1 \leq x_1 \leq 10$ and $x_1 \leq 5$ appear *grayed* because they are inherited from earlier transitions.
- The remaining guard ensures that:
  - the observed value $$_1$ matches the expected output $0$ for small inputs ($x_1 \leq 5$), and
  - the measured duration $\boxed{\#\_2}$ recorded by cl equals $42 - x_1$.

Verdict transitions

- **Transition to `FAIL^{out}`**
  Triggered when `#_2` is within the time limit (TM), but either the duration or the observed value $$_1$ violates the guard from ec_2 to `PASS`.

- **Transition to `FAIL^{dur}`**
  Captures **invalid quiescence**, defined by:

```
cl \geq \text{TM} ;\wedge; #_2 = cl ;\wedge;
\forall # \cdot \forall $_1 \cdot (#_2 \geq # ;\vee; # \neq 42 - x_1 ;\vee; $_1 \neq 0)
```

Example verdicts (for $\mathrm{TM} = 60$)

Verdict	Trace	Description
PASS	$(0, In?1).(41, Out!0)$	Valid output and timing
FAIL^{out}	$(0, In?1).(40, Out!0)$ or $(0, In?1).(41, Out!1)$	Output mismatch or incorrect timing
FAIL^{dur}	$(0, In?1).(60, \delta!)$	Quiescence beyond allowed duration

The last trace shows quiescence exceeding the allowed duration, with only $(41, Out!0)$ as a valid output after $(0, In?1)$, resulting in a `FAIL^{dur}` verdict.

Additional material

- Other test case transitions are illustrated in Fig.~$???$, labeled in **Diversity syntax** for readability.
- A 5-depth test case is presented in Fig.~$???$ in the appendix.