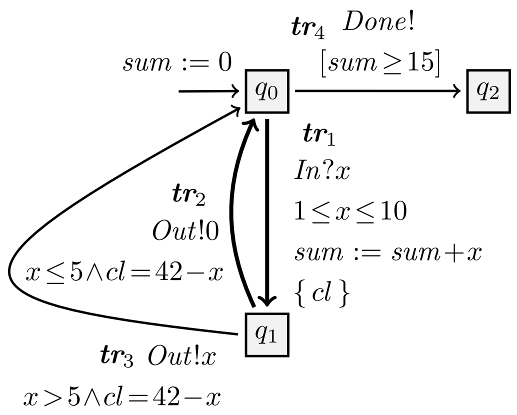
Test case generation

The construction of the test case is obtained by applying dedicated symbolic execution techniques to the reference timed symbolic automaton, in order to derive a symbolic subtree restricted to the test purpose, i.e., a path represented as a sequence of transitions of the reference automaton. In the following, we **first provide an overview of these test-oriented symbolic techniques**, and **then describe the test case generation itself**, obtained by applying transformations to this subtree (mirroring and constraint simplifications).

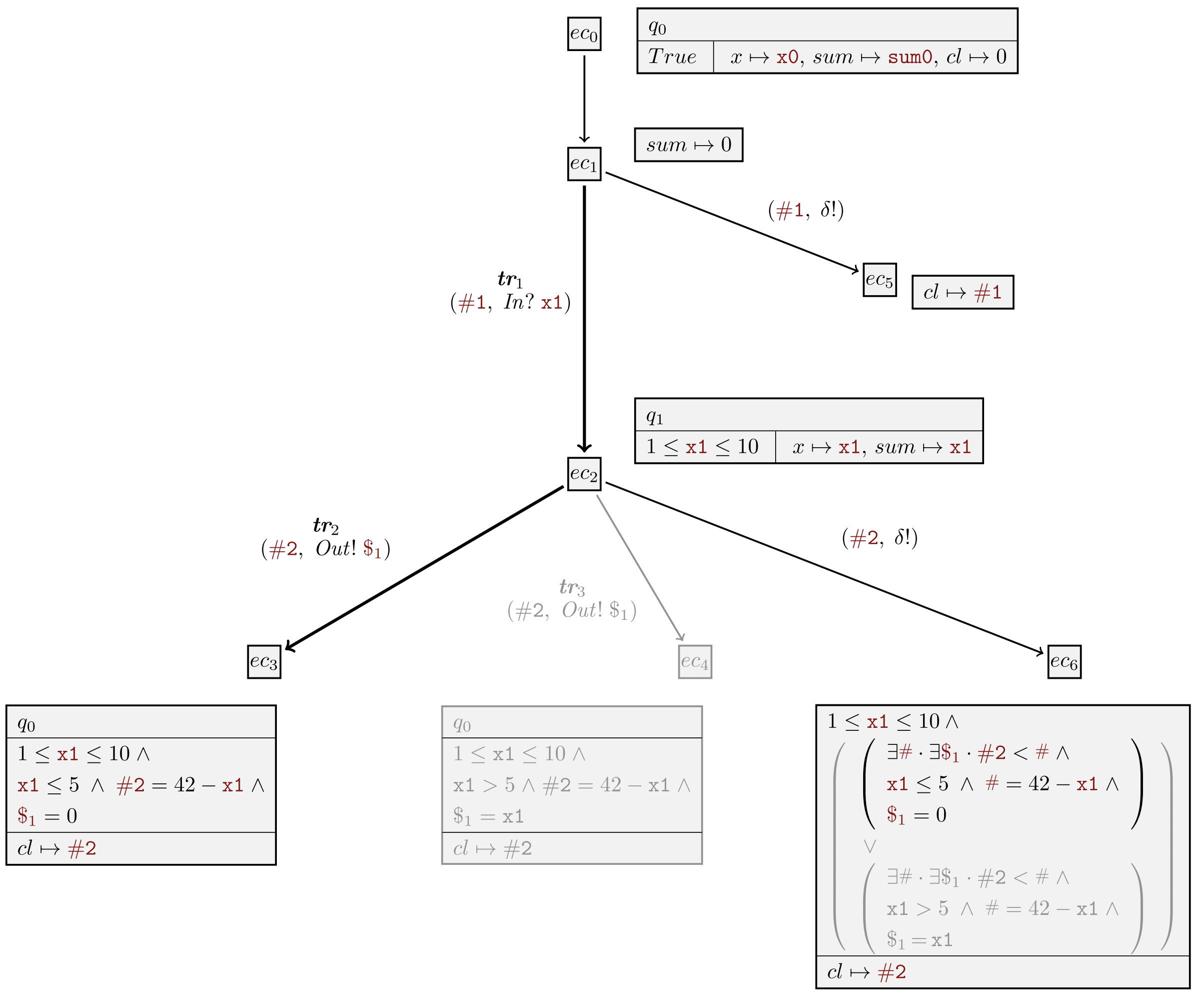
1. Test-oriented Symbolic Execution Techniques

Symbolic execution explores a model by representing both data and time with symbolic variables instead of concrete values. It unfolds the automaton while generating constraints over symbolic variables, producing a **symbolic execution tree** . The tree's nodes are **execution contexts**, and its edges represent symbolic steps such as initialization, transition firing, or **quiescence completion**.

Recall the dummy automaton example (discussed model specification tutorial):



The symbolic execution tree (restricted by test purpose transitions sequence $\mathbf{tr}_1.\mathbf{tr}_2$):



Execution Contexts

An **execution context** $ec = (q, \pi, \lambda, ev, pec)$ consists of:

- The current state q.
- The **path condition** π (accumulated constraints).
- The mapping λ of variables and clocks to symbolic terms.
- The triggering event ev.
- The predecessor context pec.

The **root context** ec_0 starts in q_0, with clocks at zero, variables assigned fresh symbols, $\pi = True$, and ev and pec undefined. Initialization produces the first successor, ec_1.

Symbolic Variables: Fresh symbolic variables are introduced:

`x0`, `x1`, ... represent successive values of a data variable x (with `x0` being the initial value).

`#1`, `#2`, ... denote **symbolic delays**.

`$1`, `$2`, ... denote **emitted values** typed according to their channels.

Symbolic Paths

Contexts ec_2, ec_3, and ec_4 illustrate the symbolic execution of transitions $\mathbf{tr}_1$, $\mathbf{tr}_2$, and $\mathbf{tr}_3$.

1. **Edge from ec_1 to ec_2 ($\mathbf{tr}_1$)**:
   - Transition from q_0 to q_1 via input In.
   - x is updated to x1. Clock cl is reset to 0.
   - Edge label: symbolic action $In?$x1 and delay #1.
   - **Path condition**: 1 $\leq$ x1 $\leq$ 10 (from guard $1 \leq x \leq 10$).
   - Update: $sum \mapsto$ x1.

2. **Edge from ec_2 to ec_3 ($\mathbf{tr}_2$)**:
   - Transition from q_1 to q_0, emitting on channel Out.
   - #2 is elapsed time, and $1 is the emitted value. Clock value becomes #2.
   - **Path condition**: x1 ≤ 5 and #2 = 42 - x1 (from guard $x \leq 5$ and $cl = 42 - x$), and $1 = 0.

The symbolic path $ec_1 . ec_2 . ec_3$ corresponds to model path $\mathbf{tr}_1 . \mathbf{tr}_2$, yielding the symbolic trace (#1, $In?$x1).(#2, $Out!$$1).

The **path condition** for this trace (#1 is unconstrained) is:

1 $\leq$ x1 $\leq$ 10 $\wedge$ x1 $\leq$ 5 $\wedge$ #2 = 42 - x1 $\wedge$ $1 = 0

This is **satisfiable** e.g. with x1 $\mapsto$ 1, $1 $\mapsto$ 0, #1 $\mapsto$ 0, #2 $\mapsto$ 41, producing the **timed trace** $(0, In?1).(41, Out!0)$. This trace shows the system receives $In?1$ after initialization and emits $Out!0$ 41 time units later.

Completion by Quiescence

Contexts ec_5 and ec_6 model **quiescence** (system silence). Symbolic variables are reused across sibling contexts (e.g., #1 for ec_2 and ec_5).

- **Quiescence context** ec_5: Derived from ec_1. The edge is labeled with the quiescence event (#1, $\delta!$). The system may remain silent indefinitely, reflected by $\pi = True$ and unconstrained delay #1.

- **Quiescence context** ec_6: Derived from ec_2's output successors (ec_3 and ec_4). Its path condition is a disjunction of existential constraints (e.g., ∃ # · ∃ $1 · #2 < # ∧ ...), capturing that quiescence persists until an output is possible.

- **Trace-determinism and pruning**: For a chosen Test Path (TP) $ec_1 . ec_2 . ec_3$ (which implies x1 $\leq$ 5), context ec_4 (which implies x1 > 5) **conflicts** and is removed (grayed out). This simplifies ec_6's path condition.

A **witness timed trace** $(0, In?1) \cdot (40, \delta!)$ covers ec_6 (with x1 $\mapsto$ 1, #2 $\mapsto$ 40), demonstrating that after $In?1$, the system can remain silent for 40 time units, expecting the next output at 41.

SPTG Workflow

For a model $\mathbb{G}$, the **Symbolic Path-guided Test Generation (SPTG)** workflow restricts symbolic exploration to a **model path** $p = \mathbf{tr}_1 \cdots \mathbf{tr}_n$, chosen as a **test path (TP)**.

Starting from the initial state q_0, the workflow performs **symbolic execution along** p, using the SMT solver **Z3** to verify:

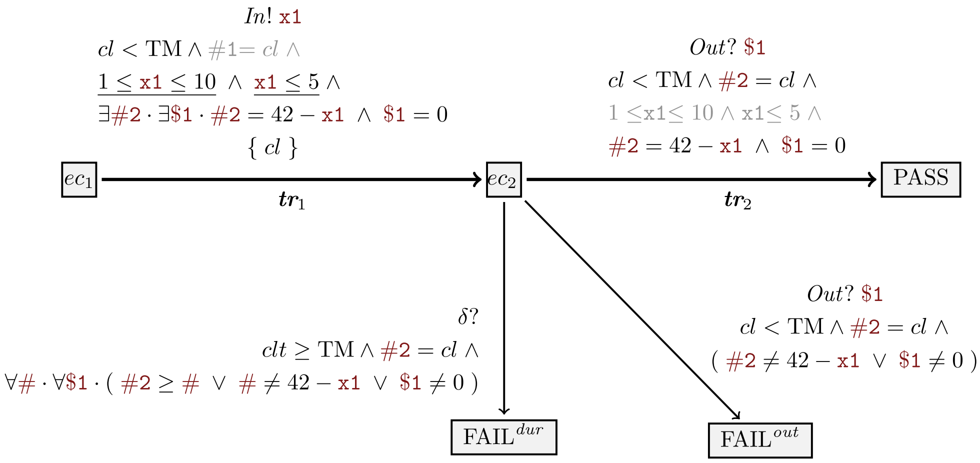
- satisfiability of execution contexts,
- trace-determinism, and
- conflict detection.

The workflow proceeds through the following five main steps:

1. **Symbolic execution along the path**
    - From the current execution context ec_1, all successor contexts are computed (Custom Symbex).
    - For each transition $\mathbf{tr}_i$, the workflow checks whether it can be fired.
    - If the transition is fireable, exploration continues exploring the remaining suffix $p' = \mathbf{tr}_{i+1} \cdots \mathbf{tr}_n$ from the successor produced by $\mathbf{tr}_i$, .
    - Otherwise, the exploration stops.
2. **Conflict removal**
    - Any conflicting contexts detected during symbolic execution are removed.
3. **Trace-determinism verification**
    - The workflow verifies that no two sibling contexts on the same channel could be covered by the same trace.
    - Exploration halts if nondeterminism is detected.
4. **Incorporation of quiescence contexts**
    - Quiescence contexts are added, producing a restricted, deterministic, quiescence-augmented symbolic execution tree $SE(\mathbb{G})^{\delta}_{/p}$, which contains the path and its immediate trace-deterministic divergences.
5. **Test case synthesis**
    - The final step synthesizes from $SE(\mathbb{G})^{\delta}_{/p}$ the timed symbolic test case $\mathbb{TC}_p$.

In the following, we detail the construction of $\mathbb{TC}_p$, illustrated below for our running dummy example, and explain how SPTG generates it from the given model path p, which serves as the test purpose.

The test case $\mathbb{TC}_{\mathbf{tr}_1.\mathbf{tr}_2}$ which corresponds to the test purpose path $\mathbf{tr}_1.\mathbf{tr}_2$ (partial view):



2. Symbolic path-guided test case

The test case $\mathbb{TC}_p$ is defined as a **timed symbolic transition system** equipped with a **single clock** cl, which measures the elapsed time before each action it performs.

The **data variable set** of $\mathbb{TC}_p$ includes all symbolic variables used to produce the execution contexts covering the path p.
These variables represent the information known and manipulated by the test case as execution progresses, including:

- **Input values** to stimulate the SUT with (e.g., x1) and their associated **submission durations** (e.g., #1).
- **Output values** expected from the SUT (e.g., $1) and their corresponding **observation times** (e.g., #2).

Clock constraint

- The clock satisfies:

  $cl < \mathrm{TM}$

  where TM denotes the maximal waiting time before either:

  - applying a stimulation, or
  - observing an output.

This timing mechanism, combined with quiescence detection ($cl \geq \mathrm{TM}$), ensures that the test case can be implemented in a real-time environment.

Test case general structure

The test case mirrors $SE(\mathbb{G})^{\delta}_{/p}$ and is used to **check the conformance** of the SUT to $\mathbb{G}$ along the symbolic path p.

Roughly speaking, test case structure is obtained as follows:

- The execution contexts related to path p form the **main branch** leading to the verdict PASS.
  The target context is replaced by PASS.

- Any deviation from this branch triggers a verdict state:
  - FAIL if the behavior violates expectations.
  - INC (inconclusive) if no clear verdict can be determined.

Test case guard derivation

The **guard** of the test-case transition from ec_1 to ec_2 is derived from the target of the test path (TP), denoted ec_3.
It guides the selection of the stimulation $In!x_1$ along this path.
The guard is expressed as:

cl < TM ∧ 1 ≤ x1 ≤ 10 ∧ x1 ≤ 5 ∧ ∃ #2 · ∃ $1 · (#2 = 42 - x1 ∧ $1 = 0)

At this stage:

- x1 and its duration #1 are determined.
- #2 and $1 remain undetermined.

The variable x1 is constrained by the path condition of ec_3 (corresponding to small input values), whereas #1 is unconstrained and can therefore be omitted (shown as grayed out in the explanatory figure of the test case $\mathbb{TC}_{\mathbf{tr}_1.\mathbf{tr}_2}$).

Conditions producing ec_3 are, by default, under existential quantifiers: ∃ #2 · ∃ $1 · (x1 ≤ 5 ∧ · #2 = 42 - x1 ∧ $1 = 0). Since #2 and $1 do not occur freely in x1 ≤ 5, this constraint is moved outside the quantifiers, yielding the final guard.

Following the test path, the test case expects an observation $Out?$$1 on channel Out, storing it in $1.
It transitions from ec_2 to PASS under the following guard:

cl < TM ∧ #2 = cl ∧ 1 ≤ x1 ≤ 10 ∧ x1 ≤ 5 ∧ #2 = 42 - x1 ∧ $1 = 0

- The formulas 1 ≤ x1 ≤ 10 and x1 ≤ 5 appear *grayed* because they are inherited from earlier transitions.
- The remaining guard ensures that:
  - the observed value $1 matches the expected output 0 for small inputs (x1 ≤ 5), and
  - the measured duration #2 recorded by cl equals 42 - x1.

Transition to FAIL^{out} is triggered when #2 is within the time limit (TM), but either the duration or the observed value $1 violates the guard from ec_2 to PASS:

cl < TM ∧ #2 = cl ∧ 1 ≤ x1 ≤ 10 ∧ x1 ≤ 5 ∧ (#2 ≠ 42 - x1 ∨ $1 ≠ 0)

Transition to FAIL^{dur} captures invalid quiescence, defined by:

cl ≥ TM ∧ #2 = cl ∧ ∀ # · ∀ $1 · (#2 ≥ # ∨ # ≠ 42 - x1 ∨ $1 ≠ 0)

Other test case transitions are shown in (complete) test case image generated by SPTG.

Example verdicts (for $\mathrm{TM} = 60$)

Verdict	Trace	Description

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PASS	$(0, In?1).(41, Out!0)$	Valid output and timing
FAIL^{out}	$(0, In?1).(40, Out!0)$	Incorrect timing
FAIL^{out}	$(0, In?1).(41, Out!1)$	Output mismatch
FAIL^{dur}	$(0, In?1).(60, \delta!)$	Quiescence beyond allowed duration

The last trace shows quiescence exceeding the allowed duration, with only $(41, Out!0)$ as a valid output after $(0, In?1)$, resulting in a FAIL^{dur} verdict.