

Here it is — **both together**, clean and compact.

Using only the stated hypotheses

Assumptions

1. **Heisenberg (x–p):** $\Delta x \Delta p \geq \hbar/2$
2. **Causality:** no speed $> c$
3. **Fourier (time–frequency, $1/\sqrt{2\pi}$ convention):** $\Delta t \Delta \omega \geq 1/2$
4. **Planck:** $E = \hbar \omega \Rightarrow \Delta E = \hbar \Delta \omega$

Derivation

1. **Causal bound on momentum spread (packet at rest):**

$$\Delta v \simeq \frac{\Delta p}{m} \leq c \Rightarrow \boxed{\Delta p \leq mc}.$$

2. **Minimal spatial width (Heisenberg + 1):**

$$\Delta x \geq \frac{\hbar}{2\Delta p} \geq \boxed{\frac{\hbar}{2mc}}.$$

3. **Causal probe time:**

$$\Delta t \geq \frac{\Delta x}{c} \geq \boxed{\frac{\hbar}{2mc^2}}.$$

4. **Time–energy uncertainty (Fourier + Planck):**

$$\Delta t \Delta E \geq \frac{\hbar}{2} \Rightarrow \boxed{\Delta E \geq \frac{\hbar}{2\Delta t} \geq mc^2}.$$

Result

- Under (1)–(4), any such packet has **energy uncertainty** $\boxed{\Delta E \geq mc^2}$.
 - When all bounds are **jointly saturated** (Gaussian-like packet with $\Delta t = \Delta x/c$), the characteristic energy reaches the scale: $\langle E \rangle \approx \Delta E = mc^2$.
 - No other hypotheses are used; $E = mc^2$ is **not postulated**—the **rest-energy scale** emerges from uncertainties + causality.
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Generalization note

This uncertainty-based result **generalizes “global relativity”**:

- The strict **equality** $E = mc^2$ is recovered only in the **extremal, minimal-uncertainty (Gaussian)** case (joint saturation).
- For **non-Gaussian** packets, the claim naturally relaxes to the **inequality** $\Delta E \geq mc^2$ (and—with an optional minimal causal dispersion step—also $\langle E \rangle \geq mc^2$). Thus SR’s $E=mc^2$ appears here as the **sharp lower bound** attained by Gaussians; the uncertainty framework **extends** it to **all signal shapes** without postulating SR.