Branch-and-Bound Algorithm

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1 Branch-and-Bound Algorithm

Algorithm 1 Branch-and-Bound Algorithm

Require:

Number of processors n

Minimum required utilization W_{min}

Processors sorted based on payoff, $\frac{\Phi_1}{U_1} \leq \frac{\Phi_2}{U_2} \leq \ldots \leq \frac{\Phi_n}{U_n}$

Ensure:

Returns the total power consumption ($\hat{z}$) and a list of all processors ($\hat{x}$) as either 0 or 1, where 1 is selected and 0 is not selected.

1: **procedure** BranchAndBound(n, W_{min} Φ, U)
2: **goto** Alg. 2
3: **end procedure**

Algorithm 2 Part 1. Initialize

4: $z \leftarrow 0$
5: $\hat{z} \leftarrow 0$
6: $\hat{j} \leftarrow 0$
7: $\Phi_{n+1} \leftarrow \infty$
8: $\hat{c} \leftarrow W_{min}$
9: $U_{n+1} \leftarrow \infty$
10: $\hat{x}_1 \leftarrow 0, \ldots, \hat{x}_n \leftarrow 0$
11: **goto** Alg. 3

Algorithm 3 Part 2. Computer upper bound

$sum \leftarrow 0$
for $k \leftarrow j, n+1$ **do**
    $sum \leftarrow sum + U_k$
    if $sum > \hat{c}$ **then**
        $r \leftarrow k$
        break
    end if
end for
if $r = n+1$ & $\hat{c} - \sum_{k=j}^{r-1} U_k > 0$ **then**
    goto Alg. 6
end if
$u \leftarrow \sum_{k=j}^{r-1} \Phi_k + \lfloor (\hat{c} - \sum_{k=j}^{r-1} U_k) * \frac{\Phi_r}{U_r} \rfloor$
if $z = 0$ **then**
    goto Alg. 4
end if
if $z \leq \hat{z} + u$ **then**
    goto Alg. 6
end if
goto Alg. 4

Algorithm 4 Part 3. Forward step

while $U_j \leq \hat{c}$ **do**
    $\hat{c} \leftarrow \hat{c} - U_j$
    $\hat{z} \leftarrow \hat{z} + \Phi_j$
    $\hat{x}_j \leftarrow 1$
    $j \leftarrow j + 1$
end while
if $j \leq n$ **then**
    $\hat{x}_j \leftarrow 1$
    $\hat{c} \leftarrow \hat{c} - U_j$
    $\hat{z} \leftarrow \hat{z} + \Phi_j$
    $j \leftarrow j + 1$
end if
if $\hat{c} > 0$ **then**
    goto Alg. 6
end if
goto Alg. 5

Algorithm 5 Part 4. Update best solution

47: **if** $\hat{z} < z$**or**$z = 0$ **then**
48: $z \leftarrow \hat{z}$
49: **for** $k \leftarrow 1, n$ **do**
50: $x_k \leftarrow \hat{x}_k$
51: **end for**
52: **end if**
53: $j \leftarrow n$
54: **if** $\hat{x}_n = 1$ **then**
55: $\hat{c} \leftarrow \hat{c} + U_j$
56: $\hat{z} \leftarrow \hat{z} - \Phi_j$
57: $\hat{x}_j \leftarrow 0$
58: **end if**
59: **goto** Alg. 6

Algorithm 6 Part 5. Backtrack

60: $i \leftarrow -1$
61: **for** $k \leftarrow j - 1, 1$ **do**
62: **if** $\hat{x}_k = 1$ **then**
63: $i \leftarrow k$
64: **break**
65: **end if**
66: **end for**
67: **if** $i = -1$ **then**
68: **return** $\hat{z}$, $\hat{x}$
69: **end if**
70: $\hat{c} \leftarrow \hat{c} + U_j$
71: $\hat{z} \leftarrow \hat{z} - \Phi_j$
72: $\hat{x}_j \leftarrow 0$
73: $j \leftarrow i + 1$
74: **goto** Alg. 3