Space for Marks	Question No.	START WRITING HERE
	Q.1)	Quantum mechanics is the branch of physics where we studied about to tiny particle which deals with the wave mechanics.

Postulates:-

(i) ψ must be differential

(ii) ψ must be continus

(iii) ψ must be normalized

(iv) It must have a singled value

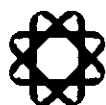(v) It must flow the norlized continual condition from all space.

(vi) It must have $|\psi|^2$

(vii) the state of system must be defind by ψ

(viii) The probality of finding the particle is given by $|\psi|^2$

(ix) The probabuty of finding any particle is positive so wave function is given as $|\psi|^2$

(x) It must be squre integral.

Space for Marks	Question No.	

(b) $|\psi_0\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ ... (Given

$|\psi_1\rangle = \frac{1}{\sqrt{3}}|\phi_1\rangle + \frac{i}{\sqrt{3}}|\phi_3\rangle$ ... (Given)

Soln:-

$|\psi_0\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ — ①

$\langle\psi_0| = \frac{1}{\sqrt{2}}|\phi_1\rangle + -\frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ — ⑪

for normalise $\langle\psi_0|\psi_0\rangle = 1$

$\therefore \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{i}{2} \times \frac{i}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2} + \frac{2}{4}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Hence $|\psi_0\rangle$ is normalised

Now;

$|\psi_1\rangle = \frac{1}{\sqrt{3}}|\phi_1\rangle + \frac{i}{\sqrt{3}}|\phi_3\rangle$ ... (From Given) — ①

$\langle\psi_1| = \frac{1}{\sqrt{3}}|\phi_1\rangle + -\frac{i}{\sqrt{3}}|\phi_3\rangle$ — ⑪

$\therefore$ Now; for normalised condition

$\langle\psi_1|\psi_1\rangle = 1$

$$\therefore \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}} \times \frac{i}{\sqrt{3}}$$

$$\therefore \frac{1}{3} + \frac{1}{3}$$

$$\therefore \frac{2}{3} \neq 1$$

$$\therefore |\psi_1\rangle \text{ is not normalised}$$

Here; $\langle \psi_0| = \frac{1}{\sqrt{2}} |\phi_1\rangle - \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle$ and .... (from 1)

$$|\psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle \qquad \cdots \text{ (from Given)}$$

$$\therefore \langle \psi_0 | \psi_1 \rangle = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} - \frac{i}{2} + \frac{i}{\sqrt{3}} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{6}} - \frac{i}{2} + \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} - i \left(\frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \psi_0 | \psi_1 \rangle = \frac{1}{\sqrt{6}} - i \left(\frac{\sqrt{3} - 1}{2\sqrt{3}} \right)}$$

and

$$\langle \psi_1| = \frac{1}{\sqrt{3}} |\phi_1\rangle - \frac{i}{\sqrt{3}} |\phi_3\rangle \qquad |\psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle$$

$$\therefore \langle \psi_1 | \psi_0 \rangle = \frac{1}{\sqrt{6}} + \frac{i}{2} - \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} + i \left(\frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \psi_1 | \psi_0 \rangle = \frac{1}{\sqrt{6}} + i \left(\frac{\sqrt{3} - 1}{2\sqrt{3}} \right)}$$
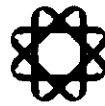
Space for Marks	Question No.	

(3) $V_{(x)} = \infty \qquad x < 0 \qquad \text{and } x > a$

$\qquad = -V_0 \qquad 0 < x < \dfrac{a}{2}$

Solution:-

Here $\psi_{(x)} = \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi x}{a}\right)$

Now 1st order correction is given as

$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$

$E_n^{(1)} = \displaystyle\int_{-\infty}^{\infty} \psi_n^{*(0)} \, H' \, \psi_n^{(0)} \, dx$

$= \displaystyle\int_{0}^{a} \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi x}{a}\right) \times (-V_0) \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi x}{a}\right) dx$

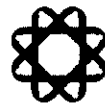$= \dfrac{2}{a}(-V_0) \displaystyle\int_{0}^{a} \sin^2\left(\dfrac{n\pi x}{a}\right) dx$

$= \dfrac{2}{a}(-V_0) \displaystyle\int_{0}^{a} \dfrac{1 - \cos\left(\dfrac{2n\pi x}{a}\right)}{2} dx$

$= \dfrac{-V_0}{a} \displaystyle\int_{0}^{a} 1 - \cos\left(\dfrac{2n\pi x}{a}\right) dx$

$= \dfrac{-V_0}{a} \left[\displaystyle\int_{0}^{a} 1 \, dx - \int_{0}^{a} \cos\left(\dfrac{2n\pi x}{a}\right) dx \right]$

$= \dfrac{-V_0}{a} \left[[x]_{0}^{a} - \left[\sin\left(\dfrac{2n\pi x}{a}\right) \times \dfrac{a}{2n\pi} \right]_{0}^{a} \right.$

Space for Marks	Question No.	

$$\doteq \frac{-U_0}{q}\left[2\right]_0^q - \left[\sin\left(\frac{2n\pi q}{q}\right) \times \frac{q}{2n\pi} - \sin(0)\times\frac{q}{2n\pi}\right]$$

$$\therefore \frac{-U_0}{q}\left[q-0\right] - 0$$

$$= \frac{-U_0}{q}(q)$$

$$= -U_0$$

$$\therefore \boxed{E_n^{(1)} = -U_0}$$

(c) $\underline{Soln:}$

$$\hat{H}_0 = \begin{bmatrix} E_0+E & 0 \\ 0 & E_0-E \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$$

$\underline{Soln:}$

here $\left|H' - E_n^{(1)} I\right| = 0$

$$\therefore \left|\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - E_n^{(1)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\therefore \left|\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - \begin{bmatrix} E_n^{(1)} & 0 \\ 0 & E_n^{(1)} \end{bmatrix}\right| = 0$$

for 2nd Order

$$E_n^{(2)} = \frac{\langle \psi_{a}^{(1)}|H'|\psi_{2}^{a}\rangle}{E_n^{(1)} - E_m^{(2)}}$$

$$\therefore \left|\begin{matrix} -E_n^{(1)} & A \\ A & -E_n^{(1)} \end{matrix}\right| = 0$$

$$E_n^{(2)} \quad \frac{A^2}{2\lambda}$$

$$\therefore (E_n^{(1)})^2 - A^2 = 0$$

$$\therefore (E_n^{(1)})^2 = A^2$$

$$\therefore \boxed{E_n^{(1)} = \pm A}$$

1st Order Correction of Energy
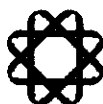
HBSU

Space for Marks	Question No.	START WRITING HERE
	(5)	
	(a)	It is an mathematical term used as the notation in the Quantum mechanics as the wave function Such that

$$\delta(x) = 0 \qquad \left\{ \begin{array}{c} x \neq 0 \\ x \neq \infty \end{array} \right\}$$
$$= \infty$$

on changing the scale
$$\delta(x - x_0) = 0 \qquad \left\{ \begin{array}{c} x \neq x_0 \\ x = 0 \end{array} \right\}$$
$$= \infty$$

Here $\delta(x) \, \delta(x - x_0) = 1$

<u>Properties</u>

(i) $\int_{-\infty}^{\infty} \delta(x) \, \delta(x - x_0) = 1$

(ii) $\int_{-\infty}^{\infty} \delta(x) \, \delta(x - x_0) = f(x_0)$

(iii) $\int_{-\infty}^{\infty} f(x) \, \delta(x - c) = f(c)$
      as c lies between a and b

(iv) $\int_{-\infty}^{\infty} f(x) \, \delta(x - c) = 0$
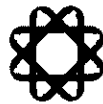   as c does not lies in between a and b

(iv) $\delta(ax) = \dfrac{1}{|a|} \delta(x)$

(v) $\delta(-x) = \delta(x)$

(v) $\dfrac{d}{dx} f(x) = \delta(x)$

(vi) $x \cdot \delta(x) = 0$

(vii) $\delta(a - x) \, \delta(x - b) = \delta(a - b)$

Space for Marks	Question No.	START WRITING HERE

(vii) $\delta(a-b) = \delta(b-a)$

(ix) $\int_{-\infty}^{\infty} \delta(x) = 1$

(x) $\int_{-\infty}^{\infty} f(x)\, \delta(x) = 1$

(c) Soln:

$$\hat{x} = \left(\frac{\hbar}{2m\omega}\right)^{1/2} (a+a^\dagger)$$

External perturbation $H' = bx$

Now,

1st order correction to the Energy of the Oscillator is given as

$$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

$$= \langle \psi_n^{(0)} | b\hat{x} | \psi_n^{(0)} \rangle_{(a+a^\dagger)}$$

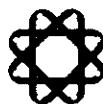$$= \langle \psi_n^{(0)} | \left(\frac{\hbar}{2m\omega}\right)^{1/2} b_x | \psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{1/2} b \, \langle \psi_n^{(0)} | (a+a^\dagger) | \psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{1/2} b \, \langle \psi_n^{(0)} | (a+a^\dagger) | \psi_n^{(0)} \rangle$$

$$E_n^{(1)} = 0 \, //$$

as $(a+a^\dagger) = 0$

$\langle a$

HBSU

Space for Marks	Question No.	