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Q.1) Quantum mechanics is the branch of physics where we studied about tiny particle which deals with the wave mechanics.

Postulates:-

- (i) It must be differential
- (ii) It must be continuous
- (iii) It must be normalized
- (iv) It must have a single value
- (v) It must show the normalized condition from all space.

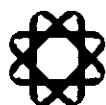
(vi) It must have  $|\psi|^2$

(vii) The state of system must be defined by  $\psi$

(viii) The probability of finding the particle is given by  $|\psi|^2$

(ix) The probability of finding any particle is positive so wave function is given as  $|\psi|^2$

(x) It must be square integral.



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(b)  $|\psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle$  ... (Given)

$|\psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle$  ... (Given)

Soln:

$|\psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle$  — (i)

$\langle\psi_0| = \frac{1}{\sqrt{2}} \langle\phi_1| + -\frac{i}{2} \langle\phi_2| + \frac{1}{2} \langle\phi_3|$  — (ii)

for normalise  $\langle\psi_0|\psi_0\rangle = 1$

$\therefore \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{i}{2} \times \frac{i}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2} + \frac{2}{4}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Hence  $|\psi_0\rangle$  is normalised

Now,

$|\psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle$  ... (from Given) — (i)

$\langle\psi_1| = \frac{1}{\sqrt{3}} \langle\phi_1| + -\frac{i}{\sqrt{3}} \langle\phi_3|$  — (ii)

$\therefore$  Now: for normalised condition

$\langle\psi_1|\psi_1\rangle = 1$



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$$\therefore \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}} \times \frac{i}{\sqrt{3}}$$

$$\therefore \frac{1}{3} + \frac{1}{3}$$

$$\therefore \frac{2}{3} \neq 1$$

$\therefore |\psi_1\rangle$  is not normalised

Here;  $\langle \psi_0 | = \frac{1}{\sqrt{2}} \langle \phi_1 | - \frac{i}{2} \langle \phi_2 | + \frac{1}{2} \langle \phi_3 |$  (from 1)

$$|\psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle \quad \dots \text{(from Given)}$$

$$\therefore \langle \psi_0 | \psi_1 \rangle = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} - \frac{i}{2} + \frac{i}{\sqrt{3}} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{6}} - \frac{i}{2} + \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} - i \left( \frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \psi_0 | \psi_1 \rangle = \frac{1}{\sqrt{6}} - i \left( \frac{\sqrt{3}-1}{2\sqrt{3}} \right)}$$

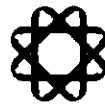
and

$$\langle \psi_1 | = \frac{1}{\sqrt{3}} \langle \phi_1 | - \frac{i}{\sqrt{3}} \langle \phi_3 | \quad |\psi_0\rangle = \frac{1}{\sqrt{2}} \langle \phi_2 | + \frac{i}{2} \langle \phi_2 | + \frac{1}{2} \langle \phi_3 |$$

$$\therefore \langle \psi_1 | \psi_0 \rangle = \frac{1}{\sqrt{6}} + \frac{i}{2} - \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} + i \left( \frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \psi_1 | \psi_0 \rangle = \frac{1}{\sqrt{6}} + i \left( \frac{\sqrt{3}-1}{2\sqrt{3}} \right)}$$



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(3)  $V(x) = \infty \quad x < 0 \quad \text{and} \quad x > a$   
 $= -V_0 \quad 0 < x < \frac{a}{2}$

Solution:-

Here  $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$

Now 1st order correction is given as

~~$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$~~

$E_n^{(1)} = \int_{-\infty}^{\infty} \psi_n^{(0)*} H' \psi_n^{(0)} dx$

$= \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \times (-V_0) \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) dx$

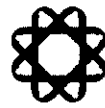
$= \frac{2}{a} (-V_0) \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$

$= \frac{2}{a} (-V_0) \int_0^a \frac{1 - \cos\left(\frac{2n\pi x}{a}\right)}{2} dx$

$= -\frac{V_0}{a} \int_0^a \left(1 - \cos\left(\frac{2n\pi x}{a}\right)\right) dx$

$= -\frac{V_0}{a} \left[ \int_0^a 1 dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx \right]$

$= -\frac{V_0}{a} \left[ x \right]_0^a - \left[ \sin\left(\frac{2n\pi x}{a}\right) \times \frac{a}{2n\pi} \right]_0^a$



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$$\therefore -\frac{V_0}{a} \left[ x \right]_0^a = \left[ \sin\left(\frac{2n\pi x}{a}\right) \times \frac{a}{2n\pi} - \sin(0) \times \frac{a}{2n\pi} \right]$$

$$\therefore -\frac{V_0}{a} [a - 0] = 0$$

$$= -\frac{V_0}{a} (a)$$

$$= -V_0$$

$$\therefore \boxed{E_n^{(1)} = -V_0}$$

(c)

Soln:

$$H_0 = \begin{bmatrix} E_0 + E & 0 \\ 0 & E_0 - E \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$$

Soln:

$$\text{here } |H' - E_n^{(0)} I| = 0$$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - E_n^{(0)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - \begin{bmatrix} E_n^{(0)} & 0 \\ 0 & E_n^{(0)} \end{bmatrix} \right| = 0$$

$$\therefore \begin{vmatrix} -E_n^{(1)} & A \\ A & -E_n^{(1)} \end{vmatrix} = 0$$

$$\therefore (E_n^{(1)})^2 - A^2 = 0$$

$$\therefore (E_n^{(1)})^2 = A^2$$

$$\therefore \boxed{E_n^{(1)} = \pm A}$$

1st Order Correction of Energy

For 2nd Order

$$E_n^{(2)} = \frac{\langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_n^{(1)}}$$

$$E_n^{(2)} = \frac{A^2}{2A}$$



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(5)

(a) It is a mathematical term used as the notation in the Quantum mechanics as the wave function such that

$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

on changing the scale

$$\delta(x-x_0) = \begin{cases} 0 & x \neq x_0 \\ \infty & x = x_0 \end{cases}$$

$$\text{Hence } \delta(x) \delta(x-x_0) = 0$$

Properties

$$(i) \int_{-\infty}^{\infty} \delta(x) \delta(x-x_0) dx = 1$$

$$(ii) \int_{-\infty}^{\infty} \delta(x) \delta(x-x_0) f(x) dx = f(x_0)$$

$$(iii) \int_{-\infty}^{\infty} f(x) \delta(x-c) dx = f(c)$$

as  $c$  lies between  $a$  and  $b$

$$(iv) \int_{-\infty}^{\infty} f(x) \delta(x-c) dx = 0$$

as  $c$  does not lie in between  $a$  and  $b$

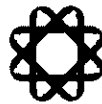
$$(v) \delta(ax) = \frac{1}{|a|} \delta(x)$$

$$(vi) \delta(-x) = \delta(x)$$

$$(vii) \frac{d}{dx} f(x) = \delta(x)$$

$$(viii) x \delta(x) = 0$$

$$(ix) \delta(a-x) \delta(x-b) = \delta(a-b)$$



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$$\textcircled{1} \delta(a-b) = \delta(b-a)$$

$$\textcircled{2} \int_{-\infty}^{\infty} \delta(x) dx = 1$$

$$\textcircled{3} \int_{-\infty}^{\infty} f(x) \delta(x) dx = f(0)$$

(c) Soln:

$$\hat{x} = \left( \frac{\hbar}{2m\omega} \right)^{1/2} (a + a^\dagger)$$

External perturbation  $H' = bx$ Now,

1st order correction to the Energy of the Oscillator is given as

$$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

$$= \langle \psi_n^{(0)} | b \hat{x} | \psi_n^{(0)} \rangle$$

$$= \langle \psi_n^{(0)} | \left( \frac{\hbar}{2m\omega} \right)^{1/2} b | \psi_n^{(0)} \rangle$$

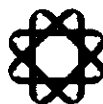
$$= \left( \frac{\hbar}{2m\omega} \right)^{1/2} b \langle \psi_n^{(0)} | (a + a^\dagger) | \psi_n^{(0)} \rangle$$

$$= \left( \frac{\hbar}{2m\omega} \right)^{1/2} b \langle \psi_n^{(0)} | (a + a^\dagger) | \psi_n^{(0)} \rangle = 0$$

$$E_n^{(1)} = 0 //$$

$$\text{as } \langle a + a^\dagger \rangle = 0$$

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