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Q.1) Quantum mechanics is the branch of physics where we studied about to tiny particle which deals with the wave mechanices.

Postulates:-

(i) ~~It~~ must be differential

(ii) ~~It~~ must be continus

(iii) ~~It~~ must be normalized

(iv) It must have a Singled value

(v) It must flow the norlized ~~Contrual~~ Condition from all space.

(vi) It must have $|\psi|^2$

(vii) the State of System must be defind by ψ

(viii) The probality of finding the particle is given by $|\psi|^2$

(ix) The probality of finding any particle is positive so wave function is given as $|\psi|^2$

(x) It must be Squre Integral.

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(b) $|\psi_0\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ ... (Given

$|\psi_1\rangle = \frac{1}{\sqrt{3}}|\phi_1\rangle + \frac{i}{\sqrt{3}}|\phi_3\rangle$ ... (Given)

Soln:-

$|\psi_0\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ — (i)

$\langle\psi_0| = \frac{1}{\sqrt{2}}|\phi_1\rangle + -\frac{i}{2}|\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ — (ii)

for normalise $\langle\psi_0|\psi_0\rangle = 1$

$\therefore \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{i}{2} \times \frac{i}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2} + \frac{2}{4}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Hence $|\psi_0\rangle$ is normalised

Now,

$|\psi_1\rangle = \frac{1}{\sqrt{3}}|\phi_1\rangle + \frac{i}{\sqrt{3}}|\phi_3\rangle$ ... (from Given) — (i)

$\langle\psi_1| = \frac{1}{\sqrt{3}}|\phi_1\rangle + -\frac{i}{\sqrt{3}}|\phi_3\rangle$ — (ii)

$\therefore$ Now: For normalised condition

$\langle\psi_1|\psi_1\rangle = 1$

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$\therefore \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}} \times \frac{i}{\sqrt{3}}$

$\therefore \frac{1}{3} + \frac{1}{3}$

$\therefore \frac{2}{3} \neq 1$

$\therefore |\psi_1\rangle$ is not normalised

Here; $\langle\psi_0| = \frac{1}{\sqrt{2}} |\phi_1\rangle - \frac{i}{2} |\phi_2\rangle + \frac{1}{2}|\phi_3\rangle$ and ... (from 1)

$|\psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle$ ... (from Given)

$\therefore \langle\psi_0|\psi_1\rangle = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} - \frac{i}{2} + \frac{i}{\sqrt{3}} \times \frac{1}{2}$

$= \frac{1}{\sqrt{6}} - \frac{i}{2} + \frac{i}{2\sqrt{3}}$

$= \frac{1}{\sqrt{6}} - i\left(\frac{1}{2} - \frac{1}{2\sqrt{3}}\right)$

$$\boxed{\langle\psi_0|\psi_1\rangle = \frac{1}{\sqrt{6}} - i\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)},$$

and

$\langle\psi_1| = \frac{1}{\sqrt{3}} |\phi_1\rangle - \frac{i}{\sqrt{3}} |\phi_3\rangle$ $\quad |\psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle \pm \frac{1}{2}|\phi_3\rangle$

$\therefore \langle\psi_1|\psi_0\rangle = \frac{1}{\sqrt{6}} + \frac{i}{2} - \frac{i}{2\sqrt{3}}$

$= \frac{1}{\sqrt{6}} + i\left(\frac{1}{2} - \frac{1}{2\sqrt{3}}\right)$

$$\boxed{\langle\psi_1|\psi_0\rangle = \frac{1}{\sqrt{6}} + i\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)}$$

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(3) $V(x) = \infty \quad x < 0$ and $x > a$

$= -V_0 \quad 0 < x < \frac{a}{2}$

Solution:-

Here $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$

Now 1st order correction is given as

~~$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$~~

$$E_n^{(1)} = \int_{-\infty}^{\infty} \psi_n^{*(0)} H' \psi_n^{(0)} \, dx$$

$$= \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \times (-V_0) \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{2}{a}(-V_0) \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{2}{a}(-V_0) \int_0^a \frac{1 - \cos\left(\frac{2n\pi x}{a}\right)}{2} dx$$

$$= -\frac{V_0}{a} \int_0^a 1 - \cos\left(\frac{2n\pi x}{a}\right) dx$$

$$= -\frac{V_0}{a} \left[\int_0^a 1 \, dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx \right]$$

$$= -\frac{V_0}{a} \left[x \right]_0^a - \left[\sin\left(\frac{2n\pi x}{a}\right) \times \frac{a}{2n\pi} \right]_0^a$$

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$$\therefore \; -\frac{V_0}{a}\left[x\right]_0^a - \left[\sin\left(\frac{2n\pi a}{a}\right)\times\frac{a}{2n\pi} - \sin(0)\times\frac{a}{2n\pi}\right]$$

$$\therefore \; -\frac{V_0}{a}\left[a-0\right] - 0$$

$$= -\frac{V_0}{a}(a)$$

$$= -V_0$$

$$\therefore \; \boxed{E_n^{(1)} = -V_0}$$

(c) Soln:

$$\hat{H}_0 = \begin{bmatrix} E_0+E & 0 \\ 0 & E_0-E \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$$

Soln:

here $|H' - E^n I| = 0$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - E_n^{(1)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - \begin{bmatrix} E_n^{(1)} & 0 \\ 0 & E_n^{(1)} \end{bmatrix} \right| = 0$$

$$\therefore \begin{vmatrix} -E_n^{(1)} & A \\ A & -E_n^{(1)} \end{vmatrix} = 0$$

$$\therefore (E_n^{(1)})^2 - A^2 = 0$$

$$\therefore (E_n^{(1)})^2 = A^2$$

$$\therefore \boxed{E_n^{(1)} = \pm A}$$

1st Order Correction of Energy

For 2nd Order

$$E_n^{(2)} = \frac{\langle \psi_{n(1)} | H' | \psi_n^{(1)} \rangle}{E_n^{(1)} - E_m^{(2)}}$$

$$E_n^{(2)} = \frac{A^2}{2\lambda}$$

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(5)

(a) It is an mathematical term used as the notation in the Quantum mechanics as the wave function Such that

$$\delta(x) = 0 \quad \{x \neq 0$$
$$= \infty \quad \{x = \infty\}$$

on changing the scale

$$\delta(x - x_0) = 0 \quad \{x \neq x_0$$
$$= \infty \quad x = 0\}$$

Here $\delta(x)\,\delta(x-x_0) = 1$

Properties

(i) $\int_{-\infty}^{\infty} \delta(x)\,\delta(x-x_0) = 1$

(ii) $\int_{-\infty}^{\infty} \delta(x)\,\delta(x-x_0) = f(x_0)$

(i) $\int_{-\infty}^{\infty} f(x)\,\delta(x-c) = f(c)$

as c lies between a and b

(ii) $\int_{-\infty}^{\infty} f(x)\,\delta(x-c) = 0$

as c does not lies in between a and b

(iii) $\delta(ax) = \frac{1}{|a|}\delta(x)$

(iv) $\delta(-x) = \delta(x)$

(v) $\frac{d}{dx} f(x) = \delta(x)$

(vi) $x \cdot \delta(x) = 0$

(vii) $\delta(a-x)\,\delta(x-b) = \delta(a-b)$

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	(viii)	$\delta(a-b) = \delta(b-a)$
	(ix)	$\int_{-\infty}^{\infty} \delta(x) = 1$
	(x)	$\int_{-\infty}^{\infty} f(x)\,\delta(x) = 1$
	(c)	Soln:

$$\hat{x} = \left(\frac{\hbar}{2m\omega}\right)^{1/2} (a + a^{+})$$

External perturbation $H' = bx$

Now,

1st order correction to the Energy of the Oscillator is given as

$$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

$$= \langle \psi_n^{(0)} | b\hat{x} | \psi_n^{(0)} \rangle$$

$$= \langle \psi_n^{(0)} | \left(\frac{\hbar}{2m\omega}\right)^{1/2} b \,(a + a^{+}) | \psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{1/2} b \,\langle \psi_n^{(0)} | (a + a^{+}) | \psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{1/2} b \,\langle \psi_n^{(0)} | (a + a^{+}) | \psi_n^{(0)} \rangle$$

$E_n^{(1)} = 0$ //

as $(a + a^{+}) = 0$

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