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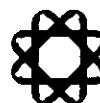
 Question
No.

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Q.1) Quantum mechanics is the branch of physics where we studied about to tiny particle which deals with the wave mechanics.

Postulates:

- (i) It must be differential
- (ii) It must be continuous
- (iii) It must be normalized
- (iv) It must have a Singled Value
- (v) It must follow the normalized continuity condition from all space.
- (vi) It must have $|\psi|^2$
- (vii) the state of System must be defined by ψ
- (viii) The probability of finding the particle is given by $|\psi|^2$
- (ix) The probability of finding any particle is positive so Wave function is given as $|\psi|^2$
- (x) It must be Square Integrable.



START WRITING HERE

Space
for
MarksQuestion
No.

$$(b) |\Psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle \quad \dots \text{ (Given)}$$

$$|\Psi\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle \quad \dots \text{ (Given)}$$

Soln:

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}} |\phi_1\rangle + \frac{i}{2} |\phi_2\rangle + \frac{1}{2} |\phi_3\rangle \quad \dots \text{ (1)}$$

$$\langle \Psi_0 | = \frac{1}{\sqrt{2}} \langle \phi_1 | + -\frac{i}{2} \langle \phi_2 | + \frac{1}{2} \langle \phi_3 | \quad \dots \text{ (1)}$$

$$\text{for normalise } \langle \Psi_0 | \Psi_0 \rangle = 1$$

$$\therefore \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} - \frac{i}{2} \times \frac{i}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} + \frac{2}{4}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Hence $|\Psi_0\rangle$ is normalised

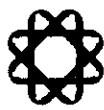
Now,

$$|\Psi_1\rangle = \frac{1}{\sqrt{3}} |\phi_1\rangle + \frac{i}{\sqrt{3}} |\phi_3\rangle \quad \dots \text{ (From Given)} \quad \text{--- (1)}$$

$$\langle \Psi_1 | = \frac{1}{\sqrt{3}} \langle \phi_1 | + -\frac{i}{\sqrt{3}} \langle \phi_3 | \quad \text{--- (1)}$$

\therefore Now: for normalised condition

$$\langle \Psi_1 | \Psi_1 \rangle = 1$$



Space
for
Marks

Question
No.

START WRITING HERE

$$\therefore \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}} \times \frac{i}{\sqrt{3}}$$

$$\therefore \frac{1}{3} + \frac{1}{3}$$

$$\therefore \frac{2}{3} \neq 1$$

$\therefore |\Psi_1\rangle$ is not normalised

$$\text{Hence; } |\Psi_0\rangle = \frac{1}{\sqrt{2}} |\Psi_1\rangle + \frac{i}{2} |\Psi_2\rangle \text{ and } \dots \text{ (from m=1)}$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{3}} (\Psi_1) + \frac{i}{\sqrt{3}} (\Psi_2) \quad \dots \text{ (from given)}$$

$$\therefore \langle \Psi_0 | \Psi_1 \rangle = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}} - \frac{i}{2} + \frac{i}{\sqrt{3}} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{6}} - \frac{i}{2} + \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} - i \left(\frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \Psi_0 | \Psi_1 \rangle = \frac{1}{\sqrt{6}} - i \left(\frac{\sqrt{3}-1}{2\sqrt{3}} \right)}$$

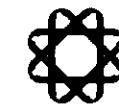
and

$$\langle \Psi_1 | = \frac{1}{\sqrt{3}} (\Psi_1) - \frac{i}{\sqrt{3}} (\Psi_2) \quad |\Psi_0\rangle = \frac{1}{\sqrt{2}} (\Psi_1) + \frac{i}{2} (\Psi_2) + \frac{1}{2} (\Psi_3)$$

$$\therefore \langle \Psi_1 | \Psi_0 \rangle = \frac{1}{\sqrt{6}} + \frac{i}{2} - \frac{i}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}} + i \left(\frac{1}{2} - \frac{1}{2\sqrt{3}} \right)$$

$$\boxed{\langle \Psi_1 | \Psi_0 \rangle = \frac{1}{\sqrt{6}} + i \left(\frac{\sqrt{3}-1}{2\sqrt{3}} \right)}$$



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Space
for
Marks

(3) $V(x) = \infty \quad x < 0 \quad \text{and} \quad x > a$
 $= -V_0 \quad 0 < x < \frac{a}{2}$

Solution:-

$$\text{Here } \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

Now 1st order correction is given as

$$E_n^{(1)} = \langle \psi_n^{(0)} | H' | \psi_n^{(0)} \rangle$$

$$E_n^{(1)} = \int_{-\infty}^{\infty} \psi_n^{(0)*} H' \psi_n^{(0)} dx$$

$$= \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \times (-V_0) \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) dx$$

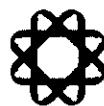
$$= \frac{2}{a} (-V_0) \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{2}{a} (-V_0) \int_0^a 1 - \cos\left(\frac{2n\pi x}{a}\right) dx$$

$$= -\frac{V_0}{a} \int_0^a 1 - \cos\left(\frac{2n\pi x}{a}\right) dx$$

$$= -\frac{V_0}{a} \left[\int_0^a 1 dx - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx \right]$$

$$= -\frac{V_0}{a} \left[[x]_0^a - \left[\sin\left(\frac{2n\pi x}{a}\right) \times \frac{a}{2n\pi} \right]_0^a \right]$$



Space
for
Marks

START WRITING HERE

$$\begin{aligned} \therefore -\frac{v_0}{a} [2]_0 &= \left[\sin\left(\frac{2n\pi a}{a}\right) \times \frac{a}{2nm} - \sin(0) \times \frac{a}{2nm} \right] \\ \therefore -\frac{v_0}{a} [a-0] &= 0 \\ &= -\frac{v_0}{a} (a) \\ &= -v_0 \\ \therefore [E_n^{(1)} = -v_0] & \end{aligned}$$

(c) Soln:

$$\hat{H}_0 = \begin{bmatrix} E_0 + \epsilon & 0 \\ 0 & E_0 - \epsilon \end{bmatrix} \quad \text{and} \quad H' = \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$$

Soln:

$$\text{here } |H' - E_n^{\infty} I| = 0$$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - E_n^{\infty} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\therefore \left| \begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix} - \begin{bmatrix} E_n^{(1)} & 0 \\ 0 & E_n^{(1)} \end{bmatrix} \right| = 0$$

for 2nd Order

$$\therefore \left| \begin{bmatrix} -E_n^{(1)} & A \\ A & -E_n^{(1)} \end{bmatrix} \right| = 0$$

$$E_n^{(2)} = \frac{2 \psi_{n(1)} (H' | H'_0)}{E_n^{(1)} - E_m^{(1)}}$$

$$\therefore (E_n^{(1)})^2 - A^2 = 0$$

$$\therefore (E_n^{(1)})^2 = A^2$$

$$\therefore E_n^{(1)} = \pm A$$

$$E_n^{(2)} = \frac{A^2}{2\lambda}$$

1st Order Correction of Energy



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Space for Marks	Question No.
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- (a) It is a mathematical term used as the notation in the Quantum mechanics as the wave function such that

$$\delta(x) = \begin{cases} 0 & x \neq 0 \\ \infty & x = 0 \end{cases}$$

on changing the scale

$$\delta(x - x_0) = \begin{cases} 0 & x \neq x_0 \\ \infty & x = x_0 \end{cases}$$

Here $\delta(x) \delta(x - x_0) = 1$ Properties

(1) $\int_{-\infty}^{\infty} \delta(x) \delta(x - x_0) = 1$

(2) $\int_a^b \delta(x) \delta(x - x_0) = f(x_0)$

(3) $\int_a^b f(x) \delta(x - c) = f(c)$

as c lies between a and b

(4) $\int_{-\infty}^{\infty} f(x) \delta(x - c) = 0$

as c does not lie in between a and b

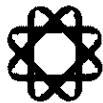
(5) $\delta(a) = \lim_{x \rightarrow a} \delta(x)$

(6) $\delta(-x) = \delta(x)$

(7) $\frac{d}{dx} f(x) = \delta(x)$

(8) $x \delta(x) = 0$

(9) $\delta(a-x) \delta(x-b) = \delta(a-b)$



Space
for
Marks

Question
No.

START WRITING HERE

$$(v) \delta(a-b) = \delta(b-a)$$

$$(vi) \int_{-\infty}^{\infty} \delta(x) = 1$$

$$(vii) \int_{-\infty}^{\infty} f(x) \delta(x) = 1$$

(c) Soln:

$$\hat{x} = \left(\frac{\hbar}{2mw} \right)^{1/2} (a + a^\dagger)$$

external perturbation $\hat{x}' = bx$

Now,

1st order correction to the Energy of the oscillator is given as

$$E_n^{(1)} = \langle \Psi_n^{(0)} | H' | \Psi_n^{(0)} \rangle$$

$$= \langle \Psi_n^{(0)} | b \hat{x}' | \Psi_n^{(0)} \rangle_{a+a^\dagger}$$

$$= \langle \Psi_n^{(0)} | \left(\frac{\hbar}{2mw} \right)^{1/2} b_x | \Psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2mw} \right)^{1/2} b \langle \Psi_n^{(0)} | (a + a^\dagger) | \Psi_n^{(0)} \rangle$$

$$= \left(\frac{\hbar}{2mw} \right)^{1/2} b \langle \Psi_n^{(0)} | (a + a^\dagger) | \Psi_n^{(0)} \rangle$$

$$E_n^{(1)} = 0$$

$$\text{as } (a + a^\dagger) = 0$$

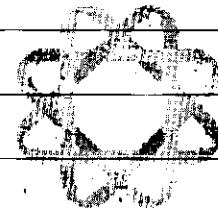
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Space for Marks	Question No.
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