

func-test

2019 年 3 月 25 日

1 时间序列

```
In [1]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller      # 平稳性检验
import seaborn as sns
import statsmodels.api as sm
from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.tsa.arima_model import ARMA
import warnings
from pywt import wavedec, waverec
warnings.filterwarnings("ignore")
```

1.1 函数

```
In [2]: # 平稳性检验
def adfTest(data):
    dfctest = adfuller(data)
    if dfctest[1] < 0.001:
        return True
    else:
        return False

In [3]: # ACF 和 PACF
def acf_pacf_Test(data,n):
    acf,q,p = sm.tsa.acf(data,nlags=n,qstat=True)
    pacf = sm.tsa.acf(data,nlags=n)
    out = np.c_[range(1,n+1), acf[1:], pacf[1:], q, p]
    output=pd.DataFrame(out, columns=['lag', "AC", "PAC", "Q", "P-value"])
    return output
```

In [4]: # 返回各回归评估参数值 *MSE, RMSE, MAE, R2*

```
def evaluationValue(data_true, data_predict):
    mse = np.sum((data_predict - data_true)**2) / len(data_true)
    rmse = np.sqrt(mse)
    mae = np.sum(np.absolute(data_predict - data_true)) / len(data_true)
    r2 = 1 - mse / np.var(data_true)
    out = np.c_[1, mse, rmse, mae, r2]
    output = pd.DataFrame(out, columns=['index', "MSE", "RMSE", "MAE", "R2"])
    return output
```

In [5]: # 差分处理, 默认最大为 5 阶

```
def bestDiff(df, maxdiff = 6):
    temp = df.copy()
    first_values = []
    for i in range(0, maxdiff):
        if i == 0:
            temp['diff'] = temp[temp.columns[0]]
        else:
            first_values.append(pd.Series([temp['diff'][1]], index=[temp['diff'].index[0]]))
            temp['diff'] = temp['diff'].diff(1)
            temp = temp.dropna() # 差分后, 前几行的数据会变成 nan, 所以删掉
            # print(temp['diff'], '\n')
        if adfTest(temp['diff']):
            bestdiff = i
            return temp['diff'], first_values
        else:
            continue
    return temp['diff'], first_values
```

In [6]: # 差分恢复

```
def recoverDiff(df_diff, first_values):
    df_restored = df_diff
    for first in reversed(first_values):
        df_restored = first.append(df_restored).cumsum()
    return df_restored
```

In [7]: # HP 分解

```
def hpFilter(data, l=1600):
    cycles, trend = sm.tsa.filters.hpfilter(data, l)
    return cycles, trend
```

In [8]: # DW 检验

```
def evaluationDW(resid):
    return sm.stats.durbin_watson(resid)
```

In [9]: # 小波分解

```
def waveletFilter(data, level, func='db4'):
    coeffs = wavedec(data, func, level=level)
    return coeffs
```

In [10]: # 小波恢复

```
def recoverWavelet(coeffs, func='db4'):
    data = waverec(coeffs, func)
    return data
```

In [11]: # 模型评估, 滚动预测

```
def evaluationModle(data, order):
    train_size = int(len(data) * 0.66)
    train, test = data[0:train_size], data[train_size:]
    history = [x for x in train]
    predictions = list()
    for t in range(len(test)):
        model = ARMA(history, order=order)
        model_fit = model.fit(dispatch=0)
        yhat = model_fit.forecast()[0]
        predictions.append(yhat[0])
        history.append(test[t])
    error = evaluationValue(test, predictions)
    # print(predictions)
    sns.lineplot(np.array(list(range(len(predictions)))), np.array(predictions), color='r')
    sns.lineplot(np.array(list(range(len(test)))), test, color='b')
    plt.show()
    return error, predictions
```

In [12]: # 模型 PQ 选择, 方法 1 (使用评估参数)

```
def chooseModels1(data, maxlag=5, method='MSE'):
    best_score, best_cfg = float("inf"), None
    for p in np.arange(maxlag):
        for q in np.arange(maxlag):
            order = (p, q)
            try:
                error, predictions = evaluationModle(data, order)
                print(order, error[method].values)
                if error[method].values < best_score:
```

```

        best_score, best_cfg = error[method].values, order
    except:
        print(order, 'error')
        continue
    return best_score, best_cfg

In [13]: # 模型 PQ 选择, 方法 2 (使用 aic, bic, hqic)
def chooseModels2(data, maxlag=5, method='aic'):
    best_score, best_cfg = float("inf"), None
    for p in np.arange(maxlag):
        for q in np.arange(maxlag):
            order = (p, q)
            model = ARMA(data, order=order)
            try:
                results_ARMA = model.fit(dispatch=0)
                if method == 'aic':
                    score = results_ARMA.aic
                elif method == 'bic':
                    score = results_ARMA.bic
                elif method == 'hqic':
                    score = results_ARMA.hqic
                print(order, score)
                if score < best_score:
                    best_score, best_cfg = score, order
            except:
                print(order, 'error')
                continue
    return best_score, best_cfg

In [14]: # 模型拟合
def fitModel(data, order):
    model = ARMA(data, order=order)
    model_result = model.fit(dispatch=0)
    return model, model_result

In [15]: # 模型检验
def modelBLQ(model_result, n):
    output = acf_pacf_Test(results_ARMA.resid, n)['P-value']
    for i in range(len(output)):
        if output[i] < 0.05:
            return False
    return True

```

In [16]: # 向内模型预测

```
def forecastInModel(model_result):  
    train_predict = model_result.predict()  
    return train_predict
```

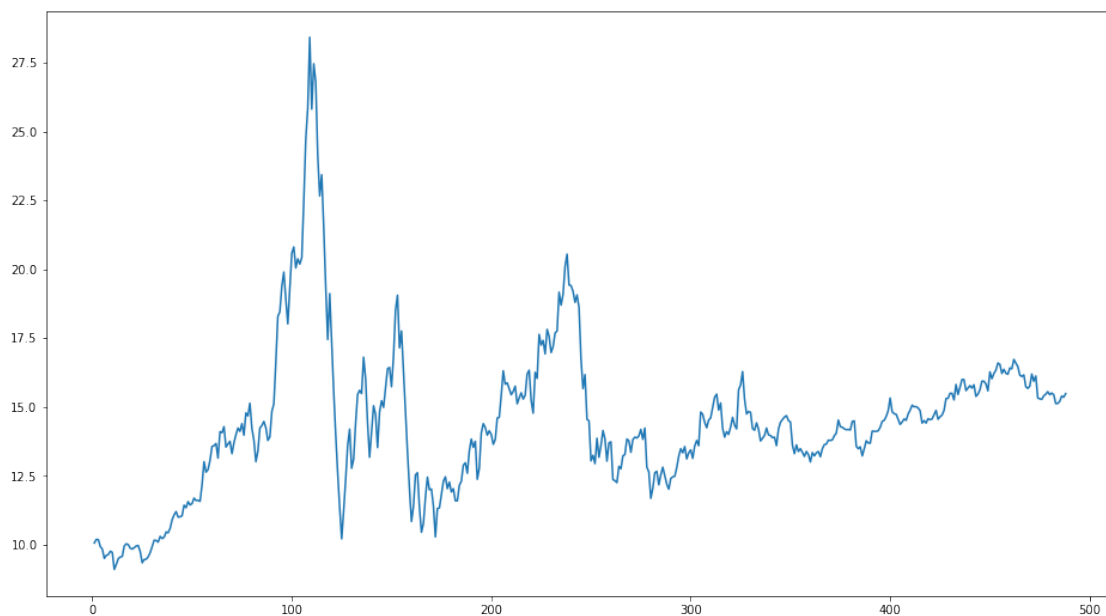
In [17]: # 向外模型预测

```
def forecastOutModel(model_result,step):  
    train_predict,b,train_predict_conf_int = model_result.forecast(step)  
    return train_predict,train_predict_conf_int
```

In [18]: dicstock102419={"1":"10.07","2":"10.2","3":"10.19","4":"9.94","5":"9.85","6":"9.51","7":"

```
x = np.array(list(dicstock102419.keys()),dtype='float64')  
y = np.array(list(dicstock102419.values()),dtype='float64')  
data = pd.DataFrame(y,x)  
plt.figure(figsize=(16,9))  
sns.lineplot(x,y)
```

Out[18]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d01c14a8>



1.2 差分序列

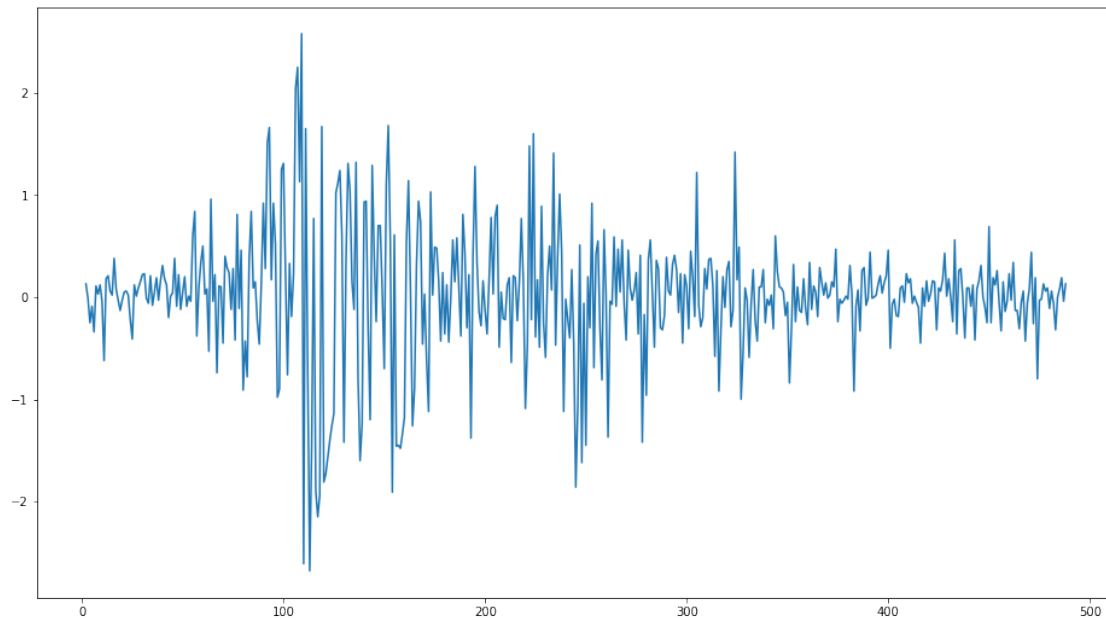
In [19]: adfTest(y) # 不平稳

Out[19]: False

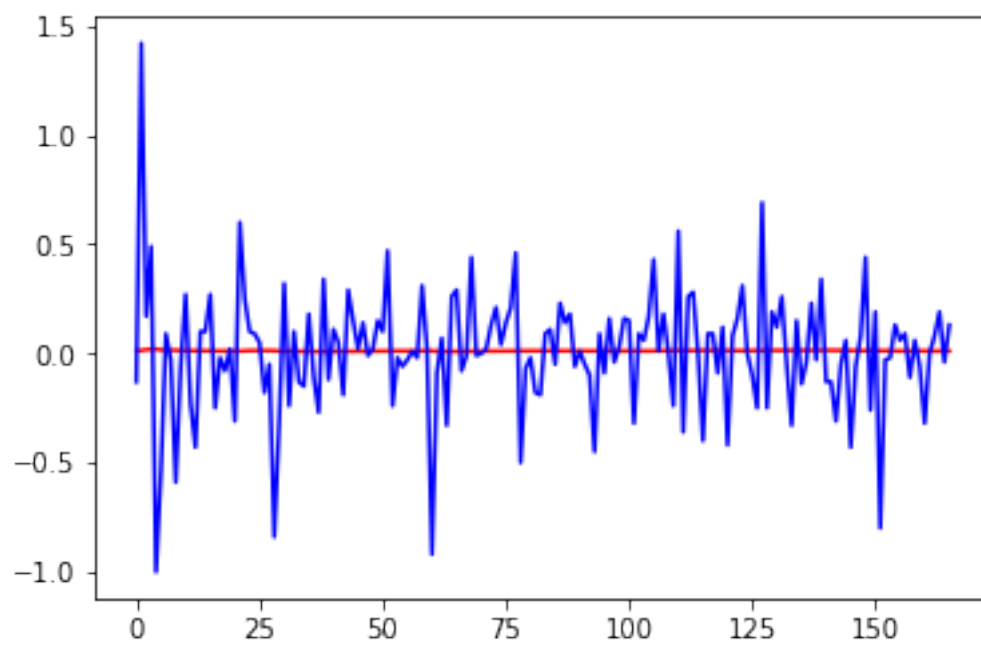
```
In [20]: datad,first_value = bestDiff(data) # 进行差分, 差分阶数为 len(first_value)
```

```
In [21]: yd = datad.values  
plt.figure(figsize=(16,9))  
sns.lineplot(x[1:],yd) # 差分后图像
```

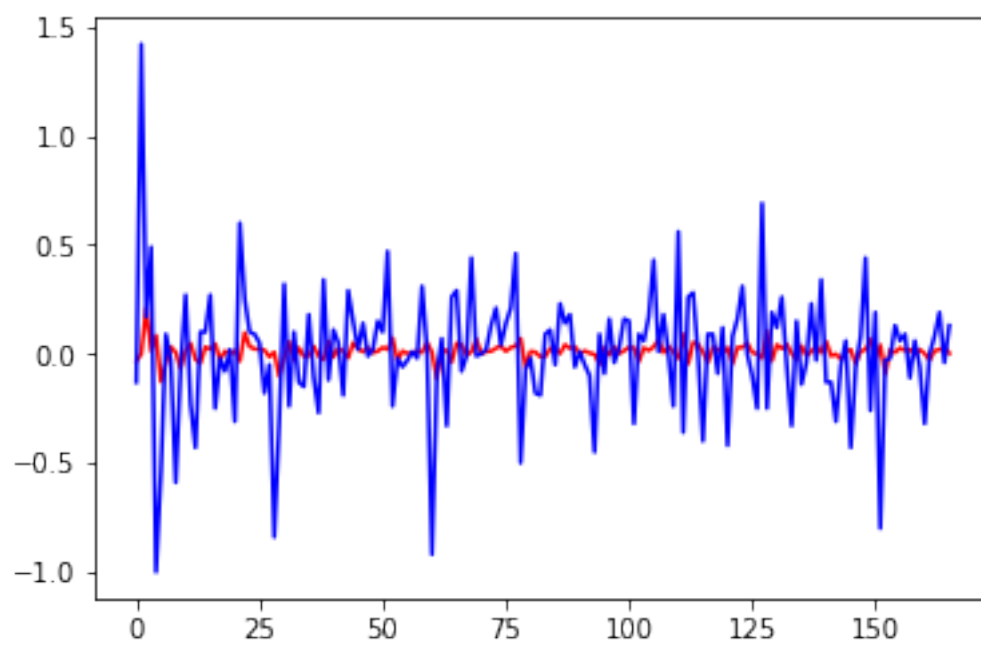
```
Out[21]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d77fb9b0>
```



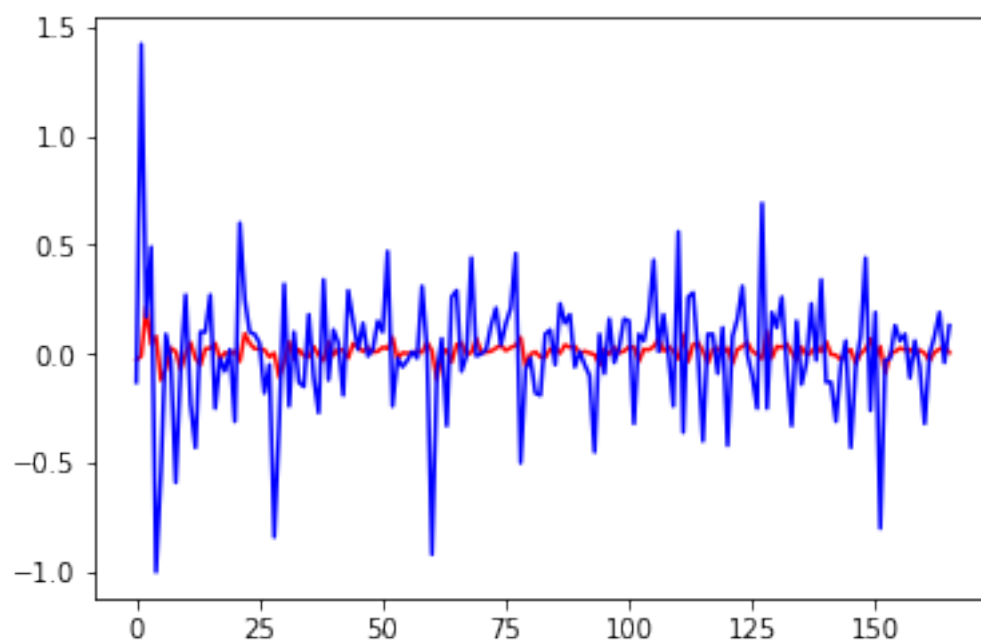
```
In [28]: best_score,best_cfg = chooseModels1(yd, maxlag=6,method='MSE')
```



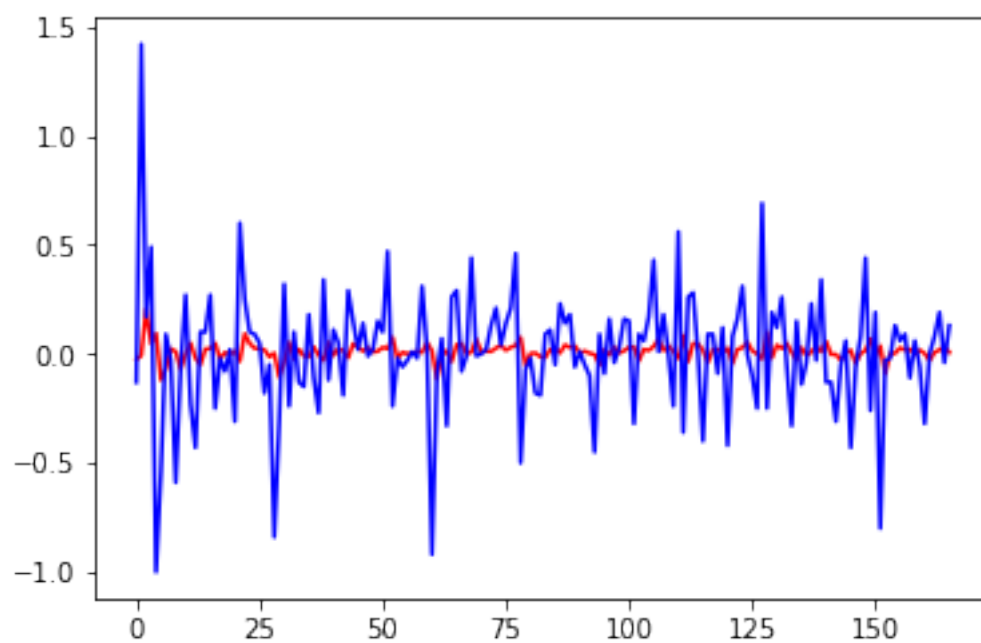
(0, 0) [0.08041686]



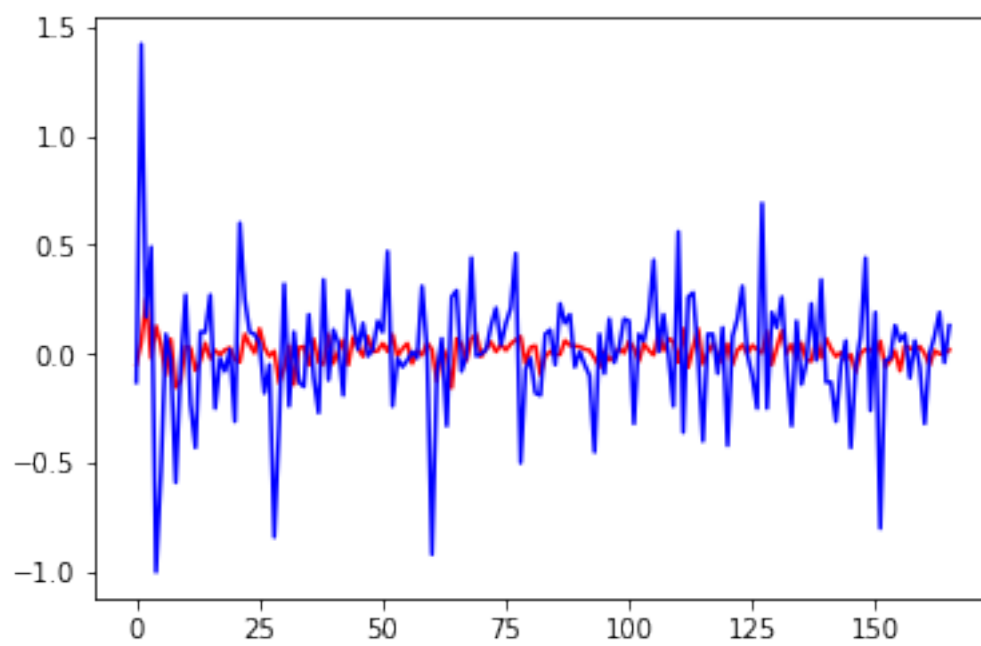
(0, 1) [0.08320272]



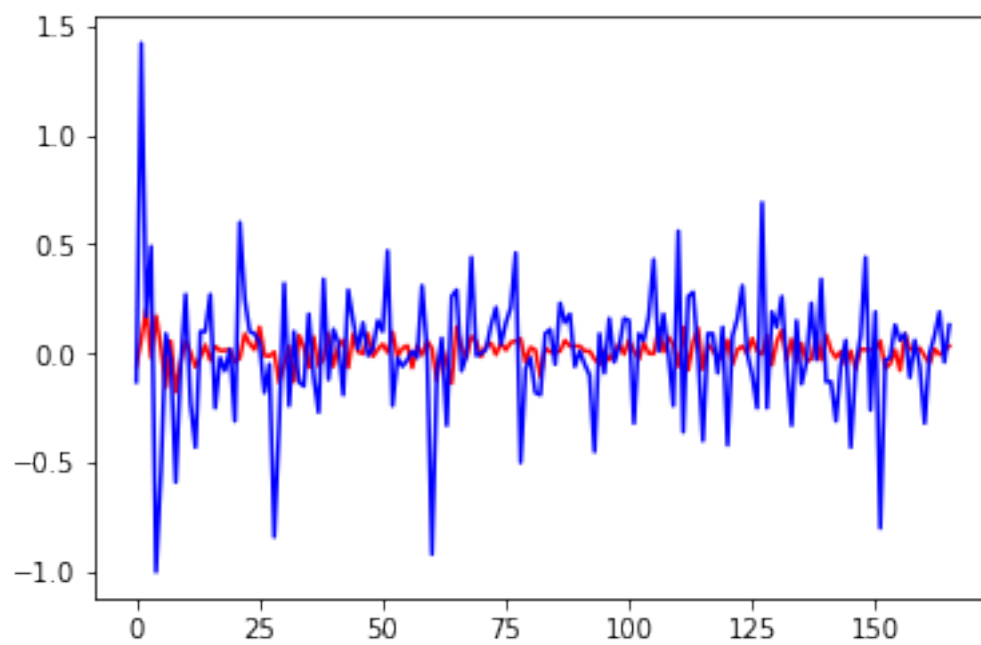
(0, 2) [0.08341979]



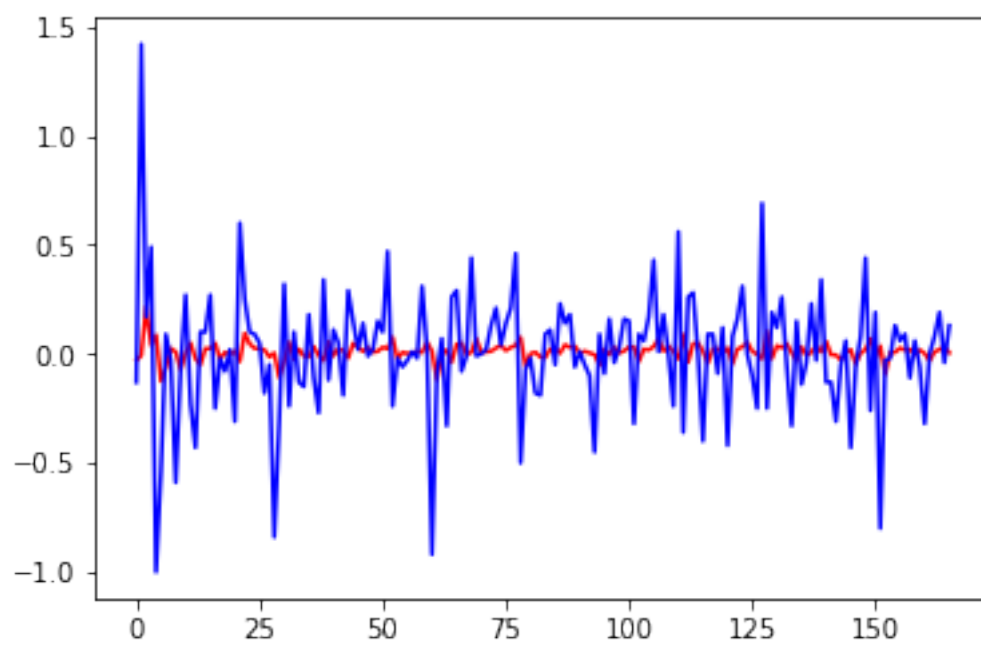
(0, 3) [0.08350787]



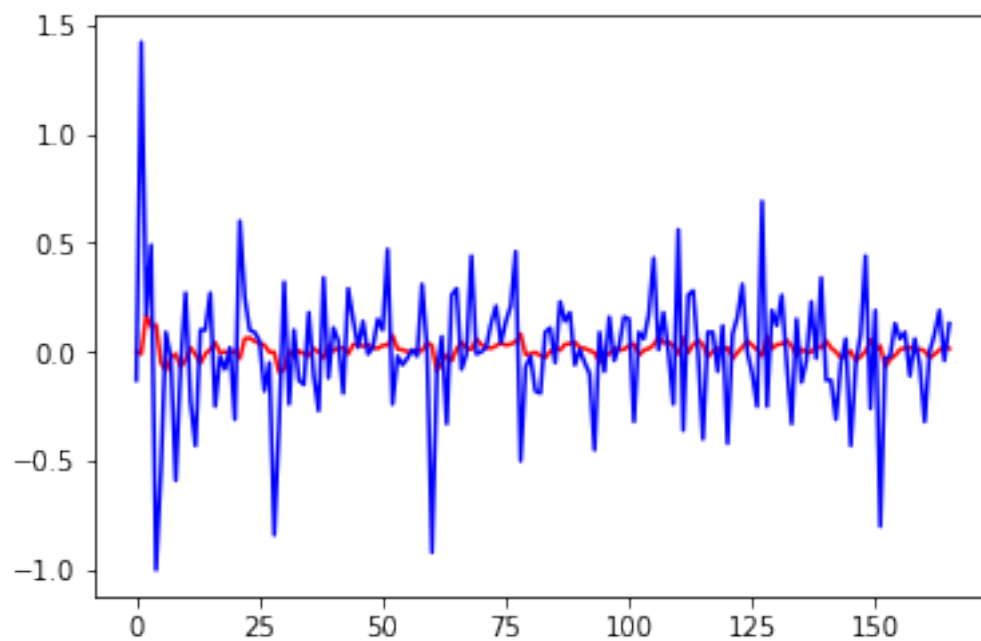
(0, 4) [0.08483243]



(0, 5) [0.08450419]



(1, 0) [0.08344131]



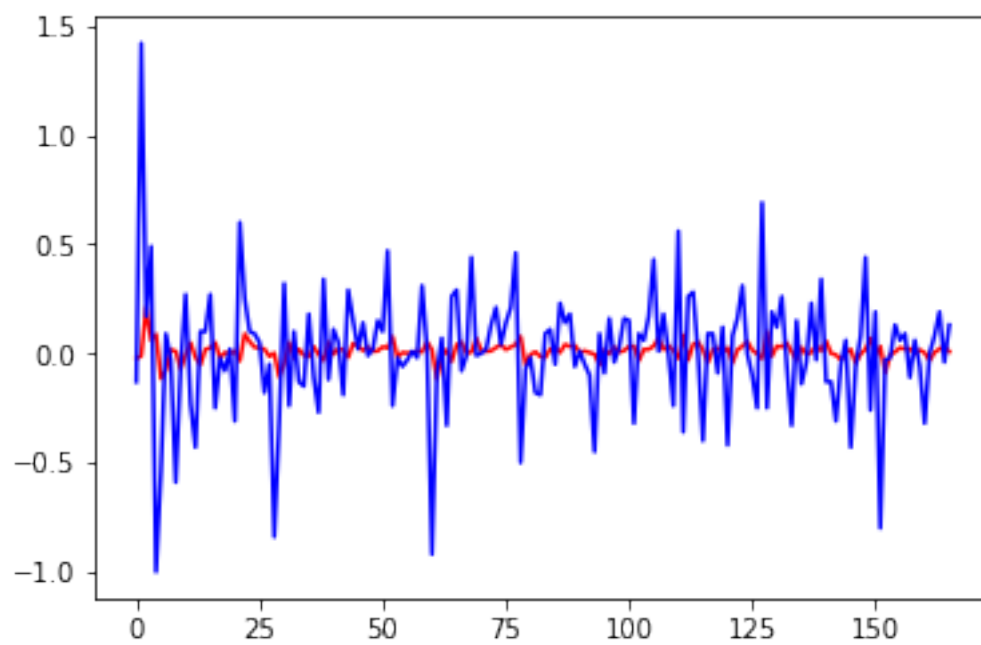
(1, 1) [0.0836965]

(1, 2) error

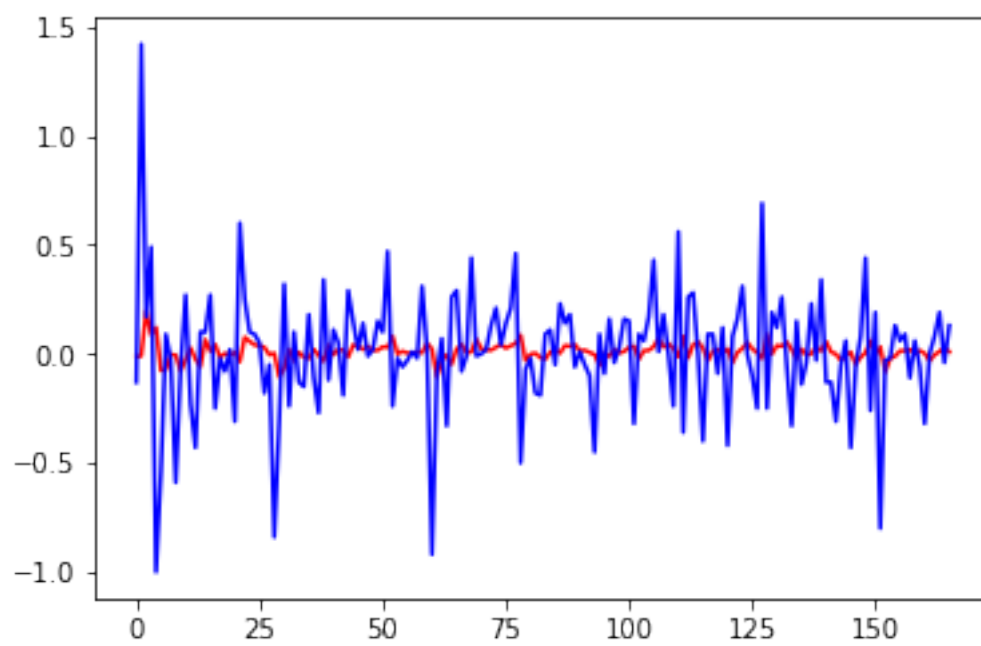
(1, 3) error

(1, 4) error

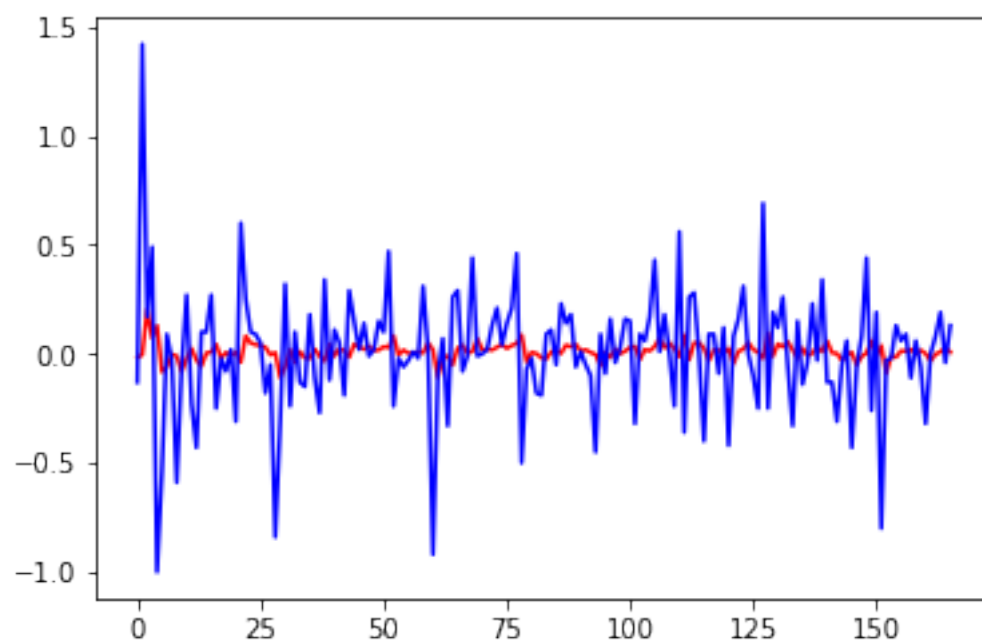
(1, 5) error



(2, 0) [0.08352119]



(2, 1) [0.08383703]

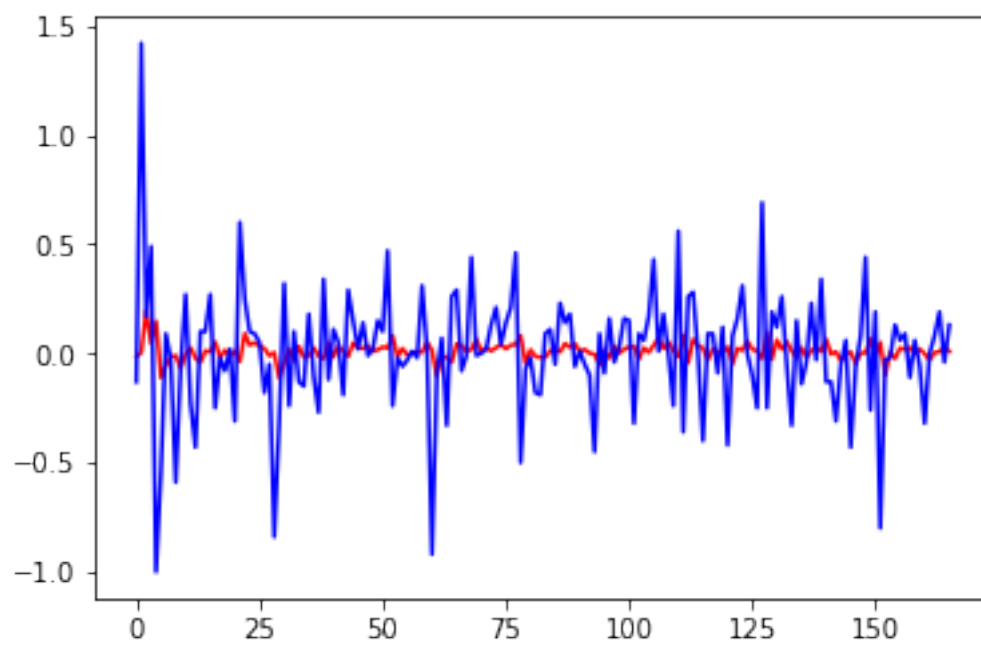


(2, 2) [0.08398973]

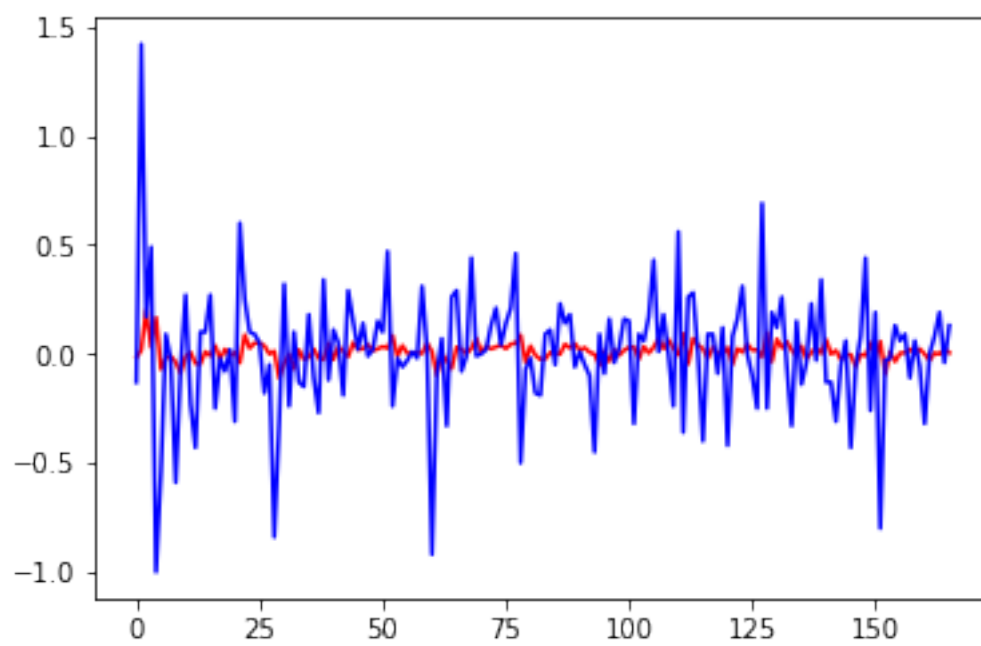
(2, 3) error

(2, 4) error

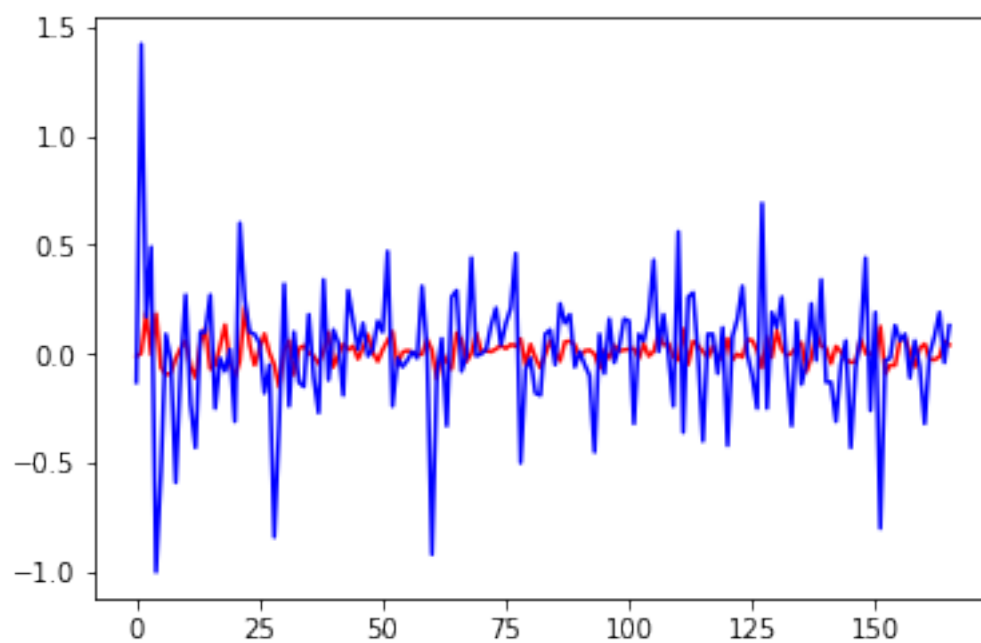
(2, 5) error



(3, 0) [0.08394737]



(3, 1) [0.08423015]

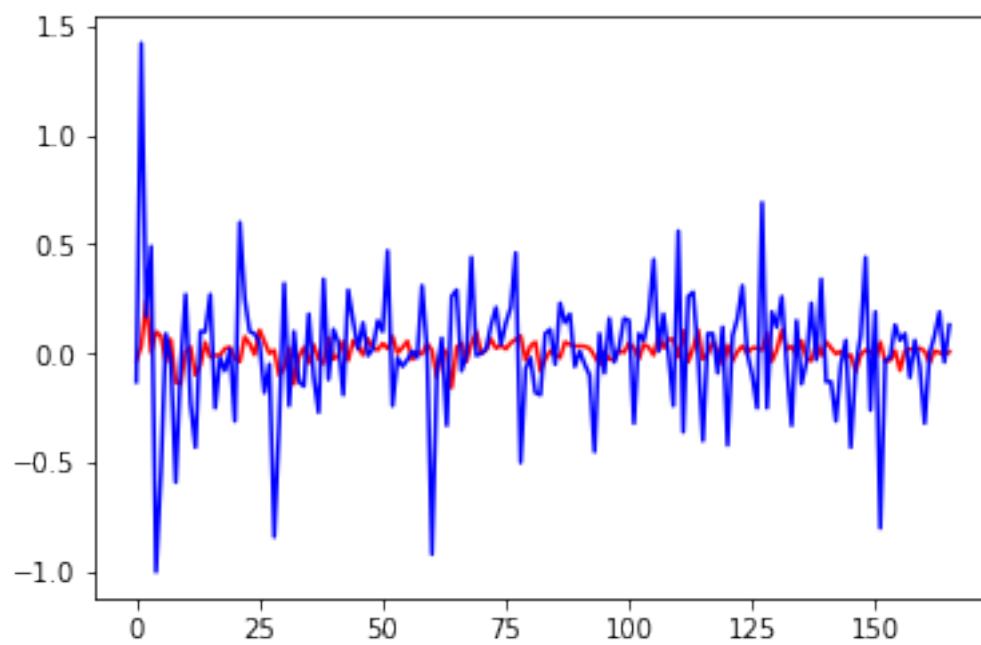


(3, 2) [0.08499342]

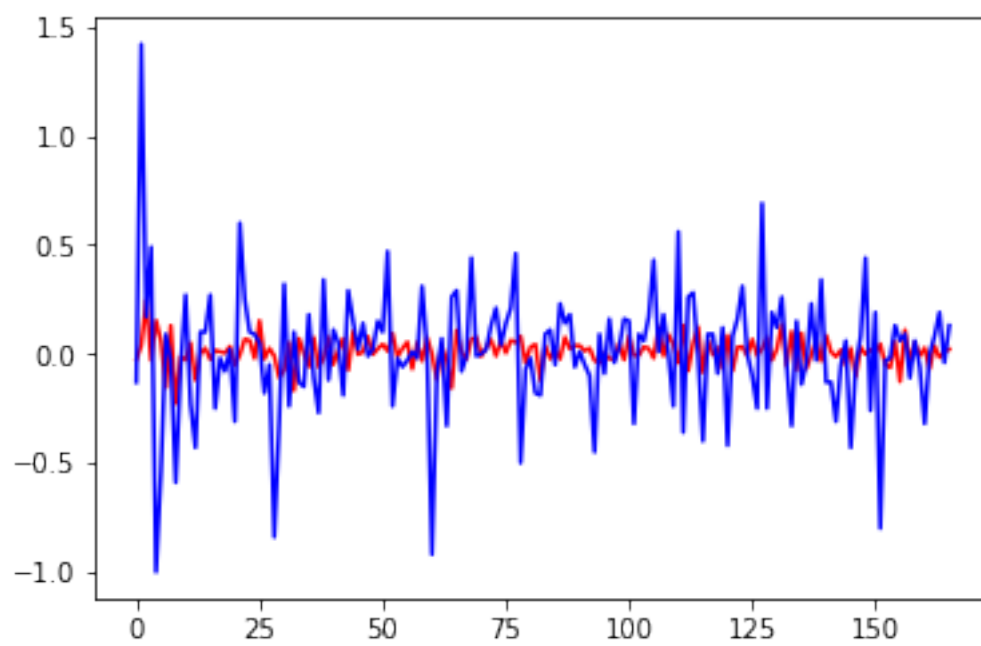
(3, 3) error

(3, 4) error

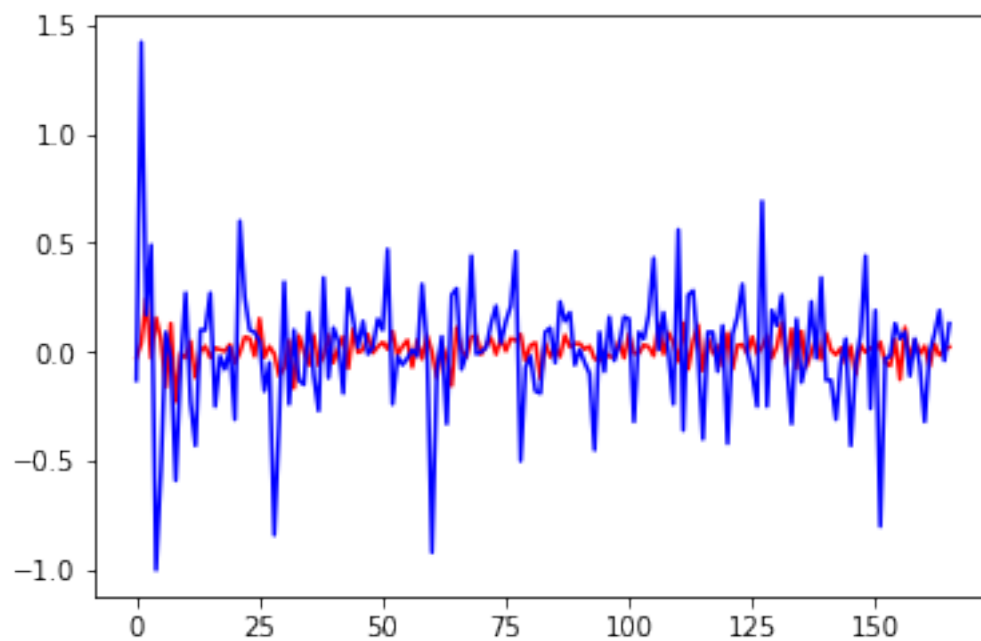
(3, 5) error



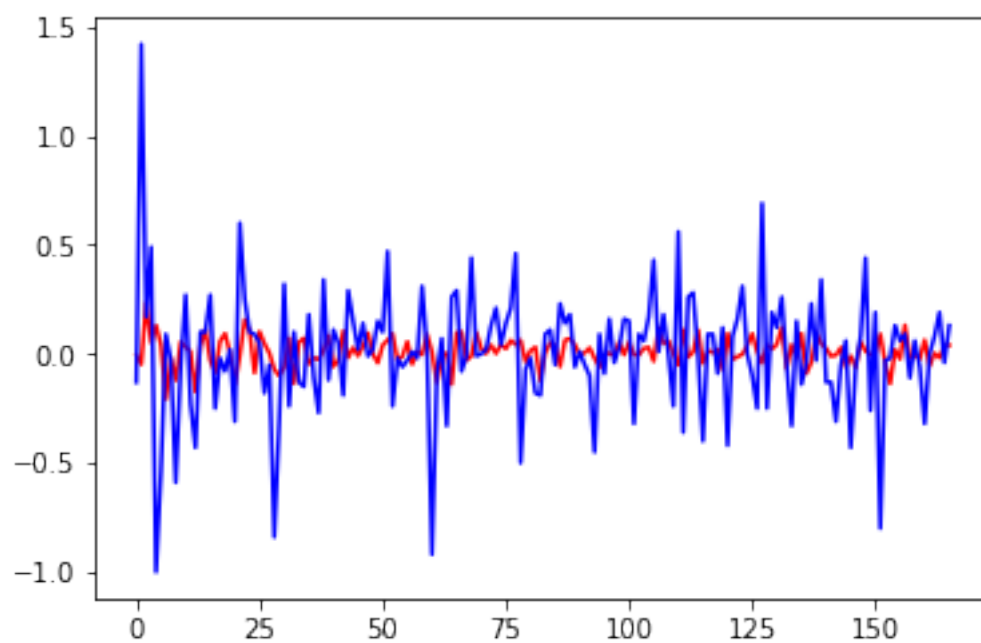
(4, 0) [0.08456955]



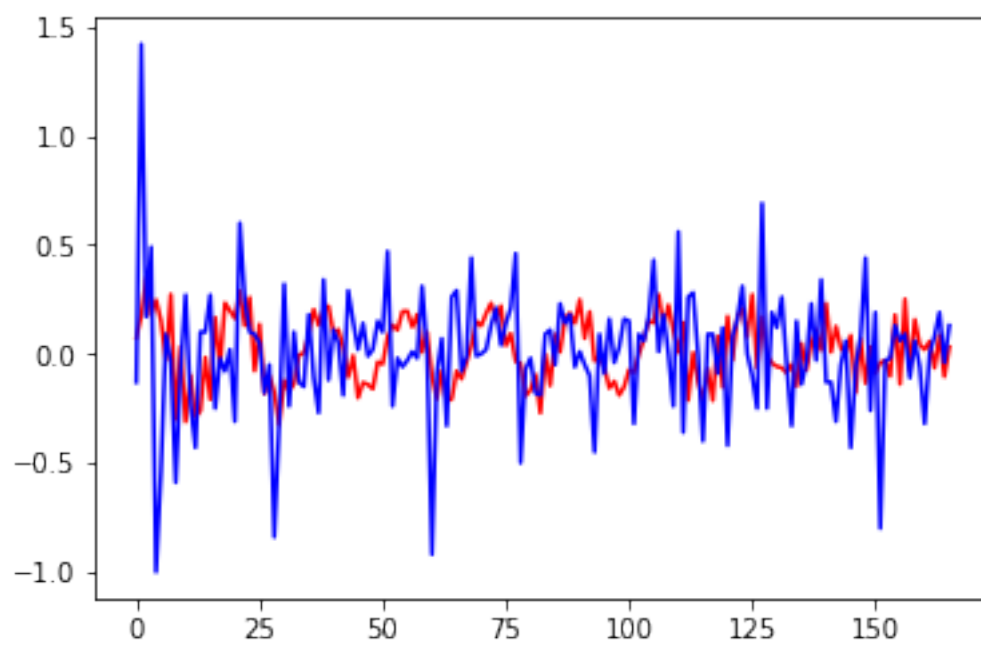
(4, 1) [0.08566541]



(4, 2) [0.085698]

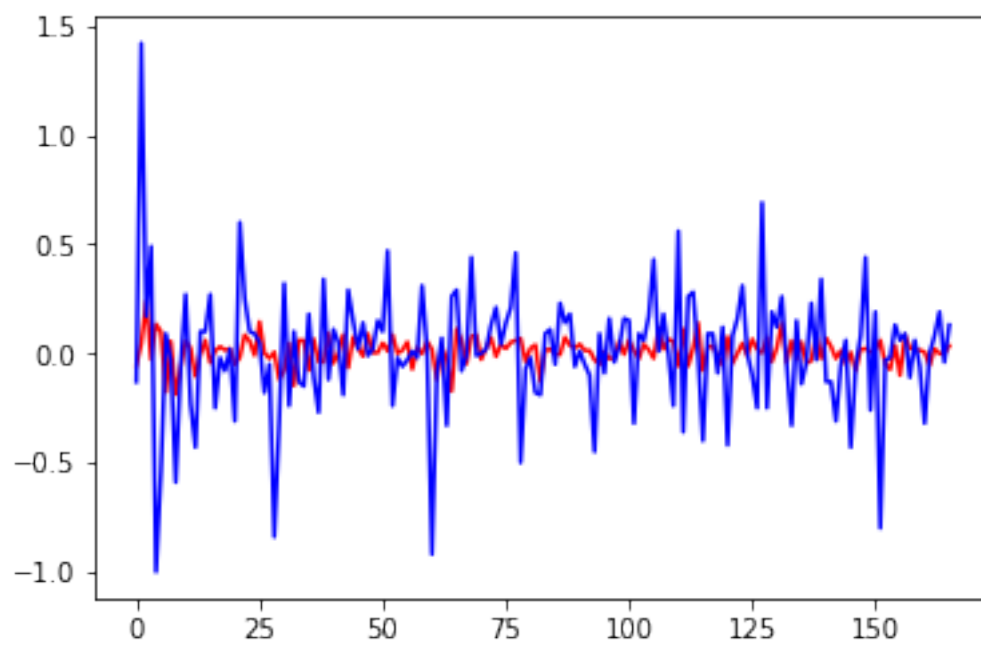


(4, 3) [0.08567986]

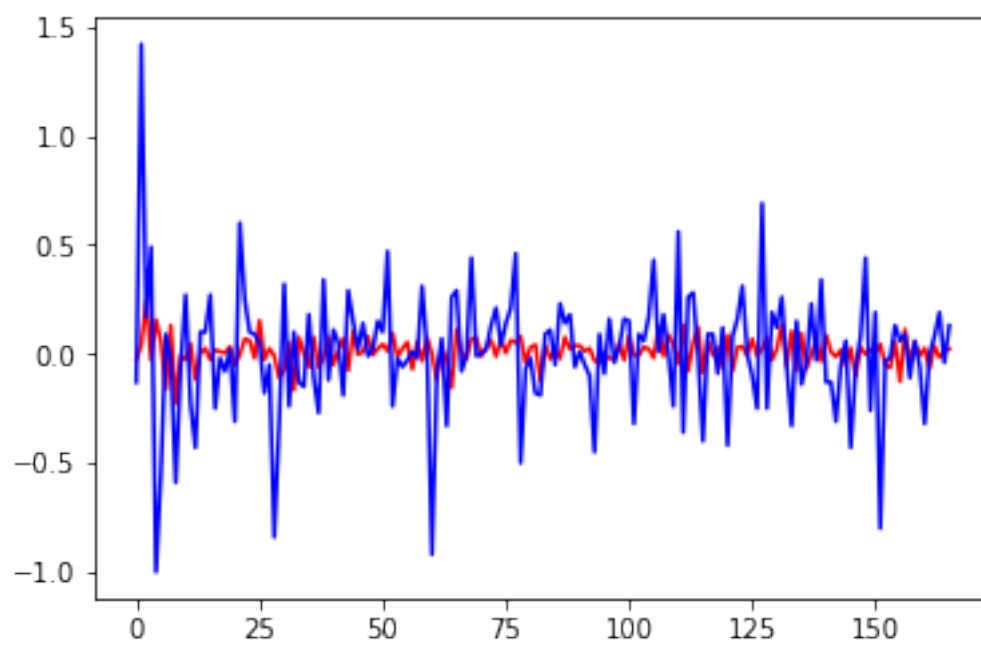


(4, 4) [0.09155374]

(4, 5) error

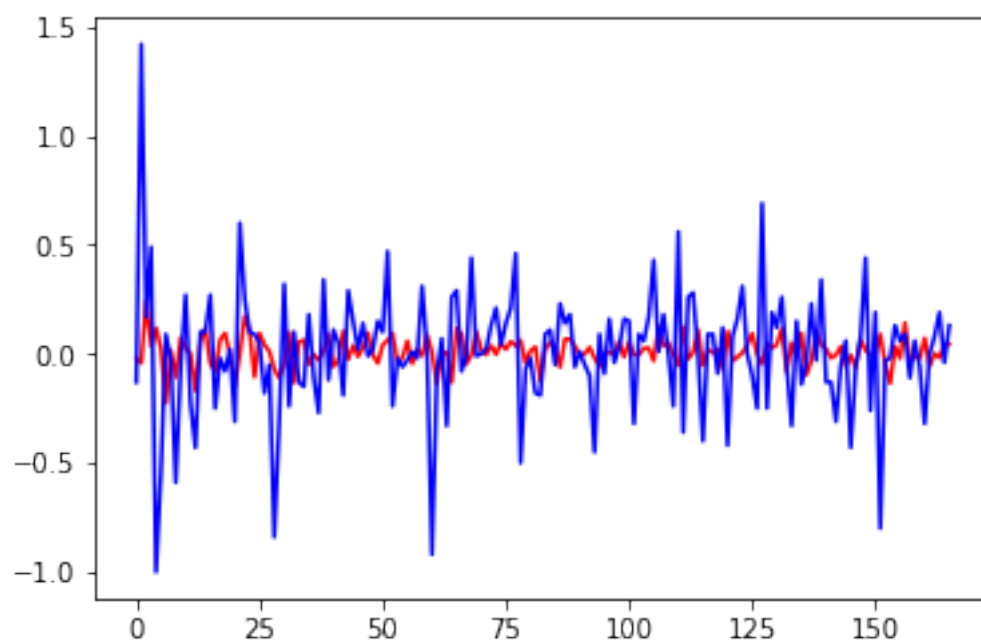


(5, 0) [0.08545094]

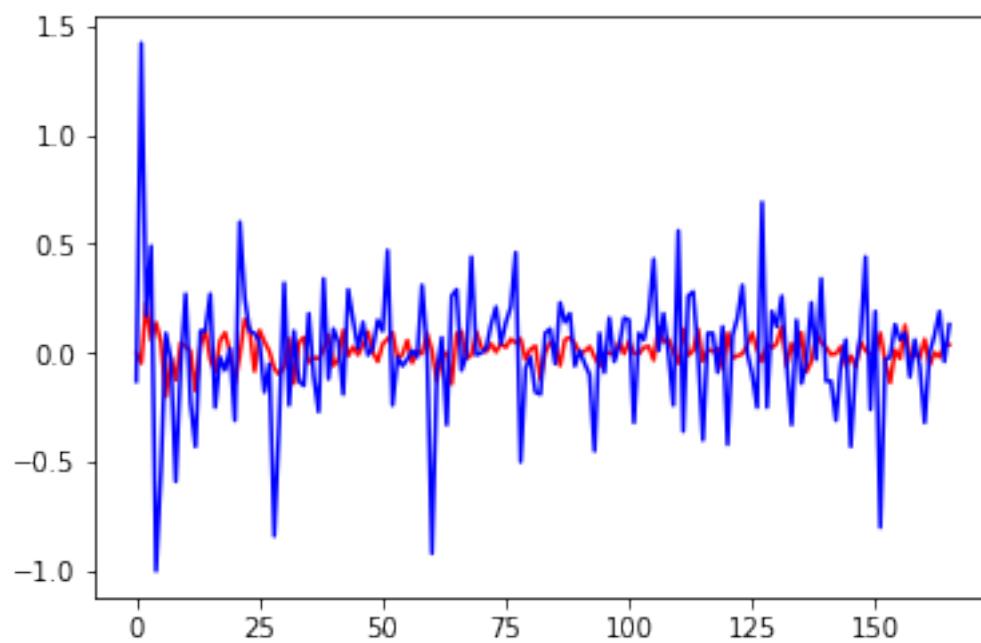


(5, 1) [0.08570125]

(5, 2) error



(5, 3) [0.08585186]



```
(5, 4) [0.08572857]
```

```
(5, 5) error
```

```
In [29]: best_score
```

```
Out[29]: array([0.08041686])
```

```
In [30]: best_cfg
```

```
Out[30]: (0, 0)
```

```
In [22]: best_score,best_cfg = chooseModels2(yd,5,method='bic')
```

```
(0, 0) 959.1172353315972
```

```
(0, 1) 957.4859273902542
```

```
(0, 2) 963.3907405000673
```

```
(0, 3) 969.5482164608977
```

```
(0, 4) 966.5943516648819
```

```
(1, 0) 957.1715060389663
```

```
(1, 1) 962.5312668374183
```

```
(1, 2) error
```

```
(1, 3) error
```

```
(1, 4) error
```

```
(2, 0) 963.2562614753615
```

```
(2, 1) 968.2489674079883
```

```
(2, 2) 974.2159309226545
```

```
(2, 3) error
```

```
(2, 4) error
```

```
(3, 0) 968.8001429399731
```

```
(3, 1) 973.2781191573879
```

```
(3, 2) 972.4588922607925
```

```
(3, 3) error
```

```
(3, 4) error
```

```
(4, 0) 967.2983227765253
```

```
(4, 1) 967.8405427806711
```

```
(4, 2) 974.016051893648
```

```
(4, 3) 977.9998034021537
```

```
(4, 4) 969.5956516007942
```

```
In [23]: best_score
```

```
Out[23]: 957.1715060389663
```

```
In [24]: best_cfg
```

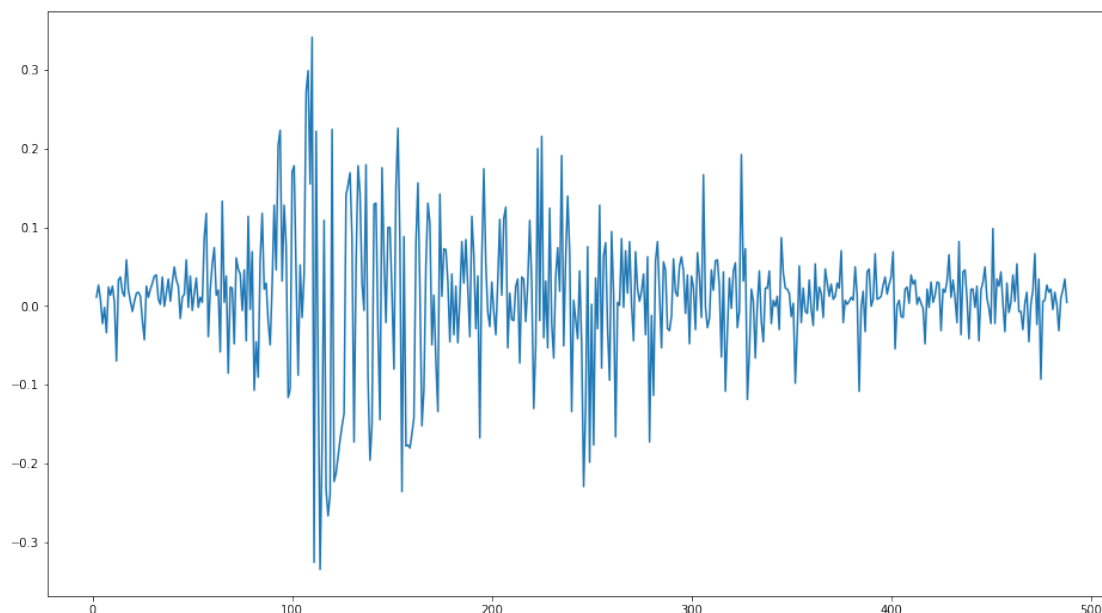
```
Out[24]: (1, 0)
```

```
In [25]: model, model_result = fitModel(yd,best_cfg)
```

```
In [26]: yp = forecastInModel(model_result)
```

```
In [27]: plt.figure(figsize=(16,9))  
sns.lineplot(x[1:],yp)      # 预测后图像
```

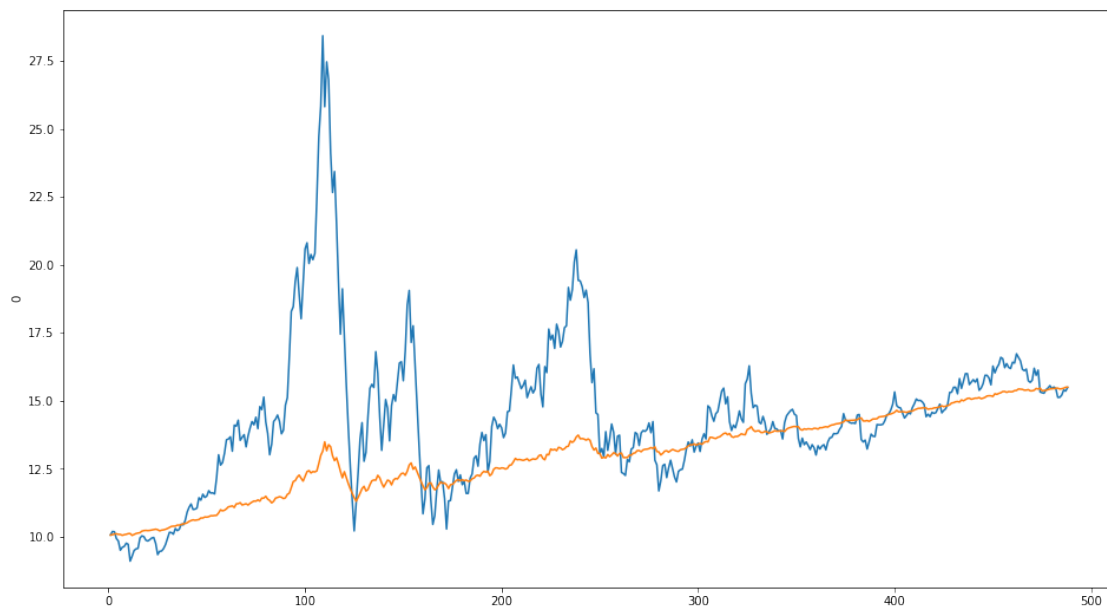
```
Out[27]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d7b7a0f0>
```



```
In [28]: df = pd.DataFrame(yp,index=list(range(len(first_value)+1,len(yp)+len(first_value)+1)))  
ypr = recoverDiff(df, first_value)
```

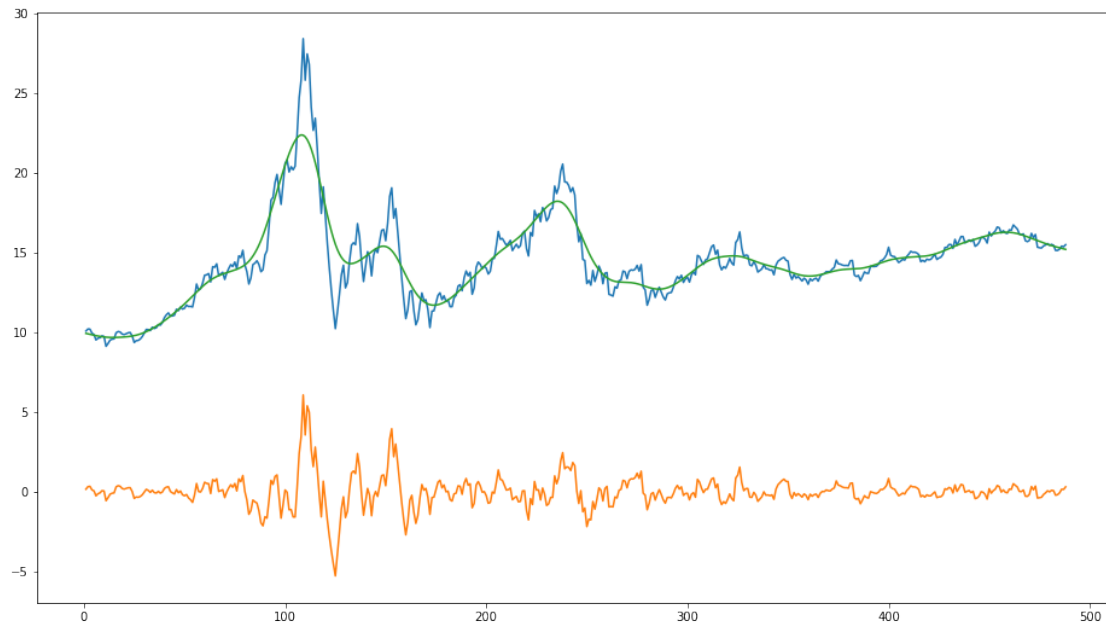
```
In [29]: plt.figure(figsize=(16,9))  
sns.lineplot(x,y)  
sns.lineplot(x,ypr[0])
```

```
Out[29]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d7c00a90>
```



1.3 HP 滤波

```
In [30]: cycles,trend = hpFilter(y)    # 进行 HP 滤波分解
plt.figure(figsize=(16,9))
sns.lineplot(x,y)
sns.lineplot(x,cycles)
sns.lineplot(x,trend)
step = 100;
```



```
In [31]: adfTest(cycles)
```

```
Out[31]: True
```

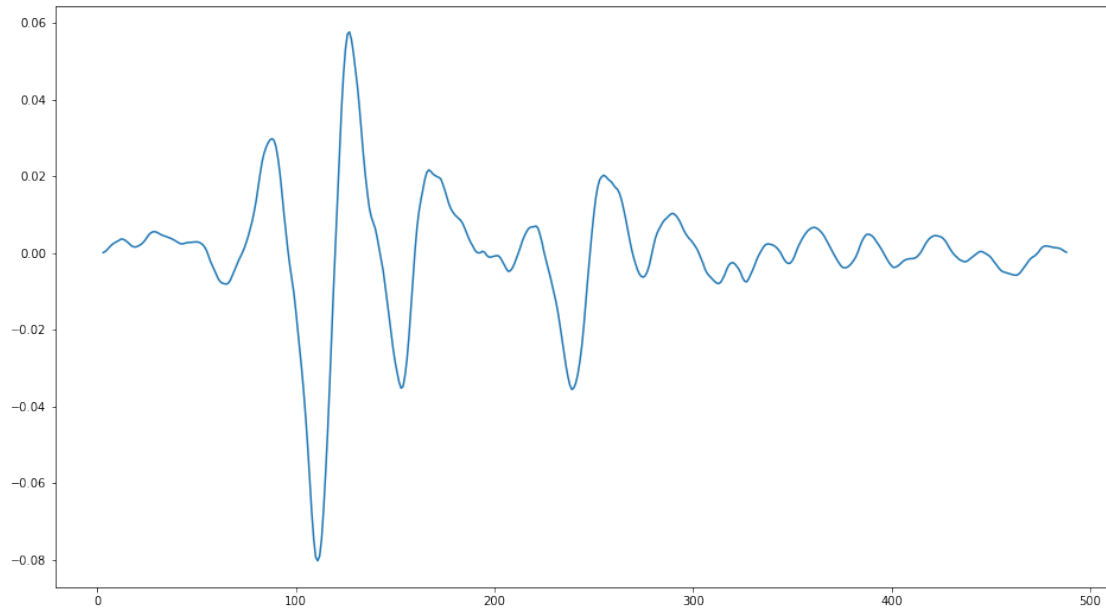
```
In [32]: adfTest(trend)
```

```
Out[32]: False
```

```
In [33]: td,tfv = bestDiff(pd.DataFrame(trend))
```

```
In [34]: td = td.values  
plt.figure(figsize=(16,9))  
sns.lineplot(x[len(tfv):],td)
```

```
Out[34]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d7b9c7b8>
```

```
In [35]: best_score,best_cfg = chooseModels2(td,5,method='bic')
```

```
(0, 0) -2598.043180875577
(0, 1) -3254.759511484198
(0, 2) error
(0, 3) error
(0, 4) error
(1, 0) -4366.111468724495
(1, 1) error
(1, 2) error
(1, 3) error
(1, 4) error
(2, 0) error
(2, 1) -6297.9330007461285
(2, 2) -6367.917773629811
(2, 3) -6367.76080941585
(2, 4) -6384.951528525135
(3, 0) error
(3, 1) -6389.767322626952
(3, 2) -6383.630828513481
(3, 3) -6377.769047619613
(3, 4) -6381.0497443139075
(4, 0) error
```

(4, 1) -6383.586663017474

(4, 2) -6383.510904787156

(4, 3) -6380.21594451777

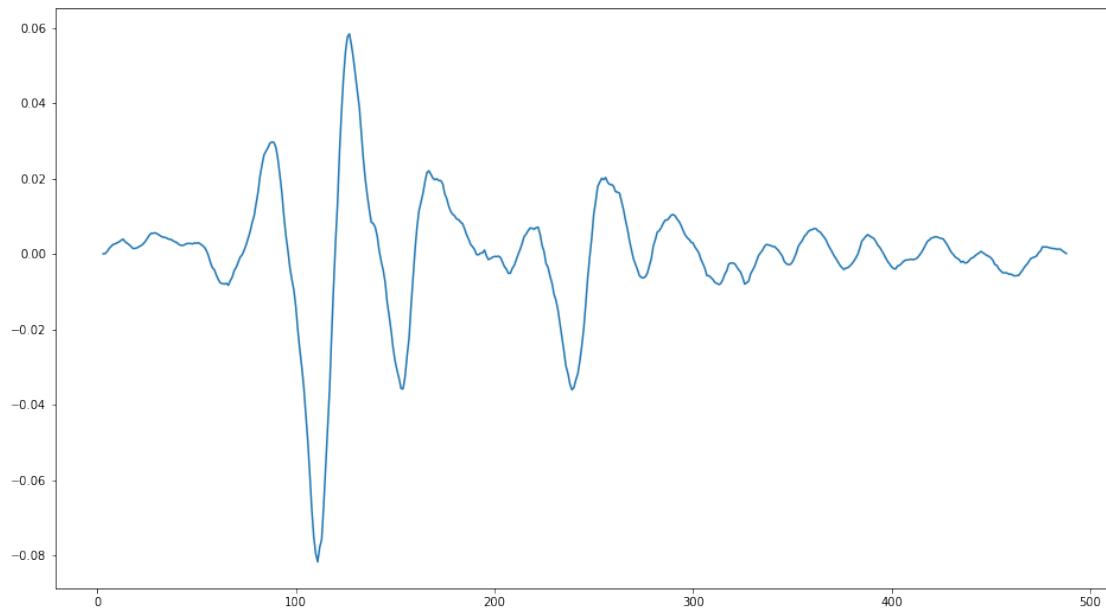
(4, 4) -6378.893412652493

In [36]: best_score,best_cfg

Out[36]: (-6389.767322626952, (3, 1))

```
In [37]: model, model_result = fitModel(td,best_cfg)
         tp = forecastInModel(model_result)
         tpo,tpci = forecastOutModel(model_result,step)
         plt.figure(figsize=(16,9))
         sns.lineplot(x[len(tfv):],tp)
```

Out[37]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d7e997f0>



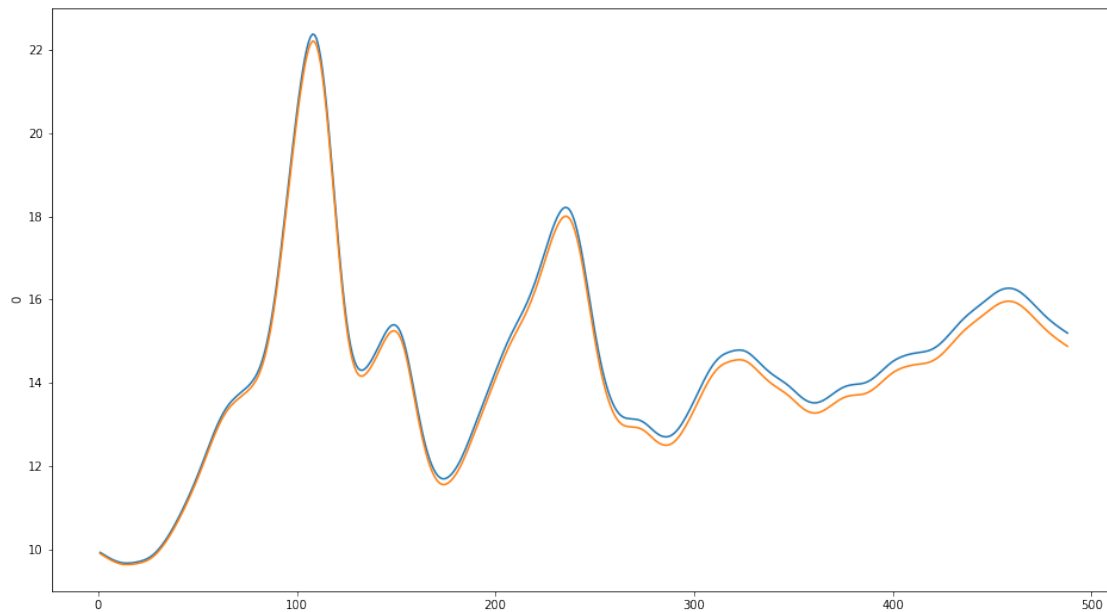
```
In [38]: df = pd.DataFrame(tp,index=list(range(len(tfv)+1,len(tp)+len(tfv)+1)))
         dfp = pd.DataFrame(np.append(tp,tpo),index=list(range(len(tfv)+1,len(tp)+len(tfv)+1+step)
         dfu = pd.DataFrame(np.append(tp,tpci[:,0]),index=list(range(len(tfv)+1,len(tp)+len(tfv)+1
         dfd = pd.DataFrame(np.append(tp,tpci[:,1]),index=list(range(len(tfv)+1,len(tp)+len(tfv)+1
         tpr = recoverDiff(df, tfv)
         tppr = recoverDiff(dfp, tfv)
```

```

tpur = recoverDiff(dfu, tfv)
tpdr = recoverDiff(dfd, tfv)
plt.figure(figsize=(16,9))
sns.lineplot(x,trend)
sns.lineplot(x,tpr[0])

```

Out [38]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3dc2619b0>



```
In [39]: best_score,best_cfg = chooseModels2(cycles,5,method='bic')
```

```

(0, 0) 1389.2153709162344
(0, 1) 1075.8705866617404
(0, 2) error
(0, 3) 918.4695979345088
(0, 4) error
(1, 0) 866.402359403205
(1, 1) 864.4100250597903
(1, 2) 869.7266713362823
(1, 3) 875.8923826627145
(1, 4) 869.1677978497584
(2, 0) 863.5411228112164
(2, 1) 812.4692202890211
(2, 2) 818.6543006755667
(2, 3) 821.2663930238037

```

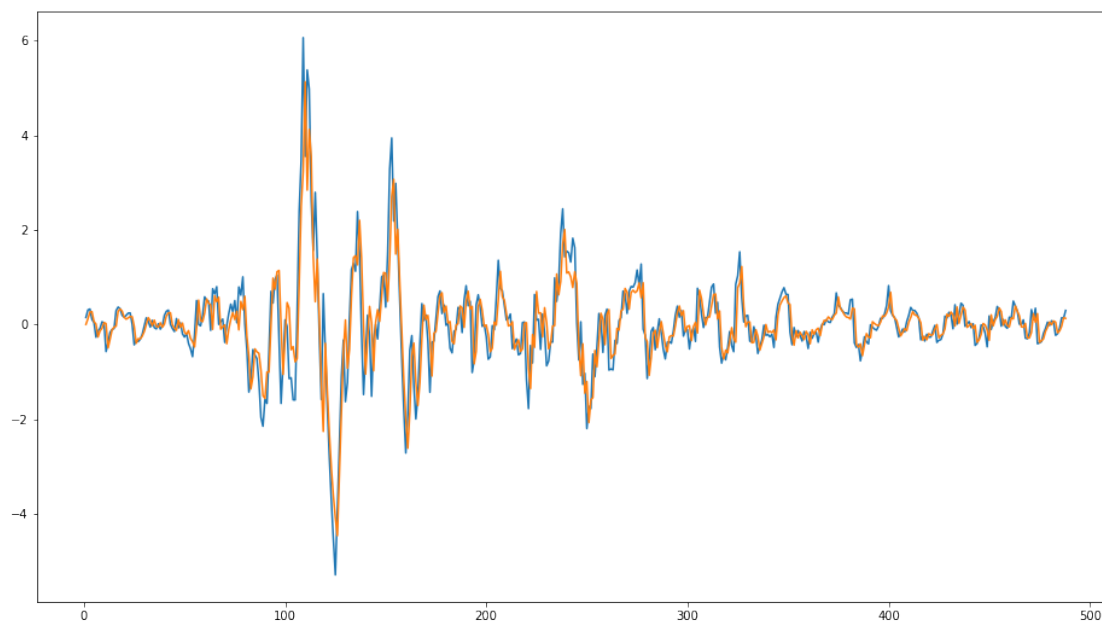
```
(2, 4) 823.6056709598388
(3, 0) 869.0027996275128
(3, 1) 818.6552737713404
(3, 2) 823.5266594353169
(3, 3) 828.8519616678823
(3, 4) 821.2537969513266
(4, 0) 873.2697039404278
(4, 1) 821.0011763720053
(4, 2) 829.2032881198581
(4, 3) 819.1717962916634
(4, 4) 822.7566863330409
```

```
In [40]: best_score,best_cfg
```

```
Out[40]: (812.4692202890211, (2, 1))
```

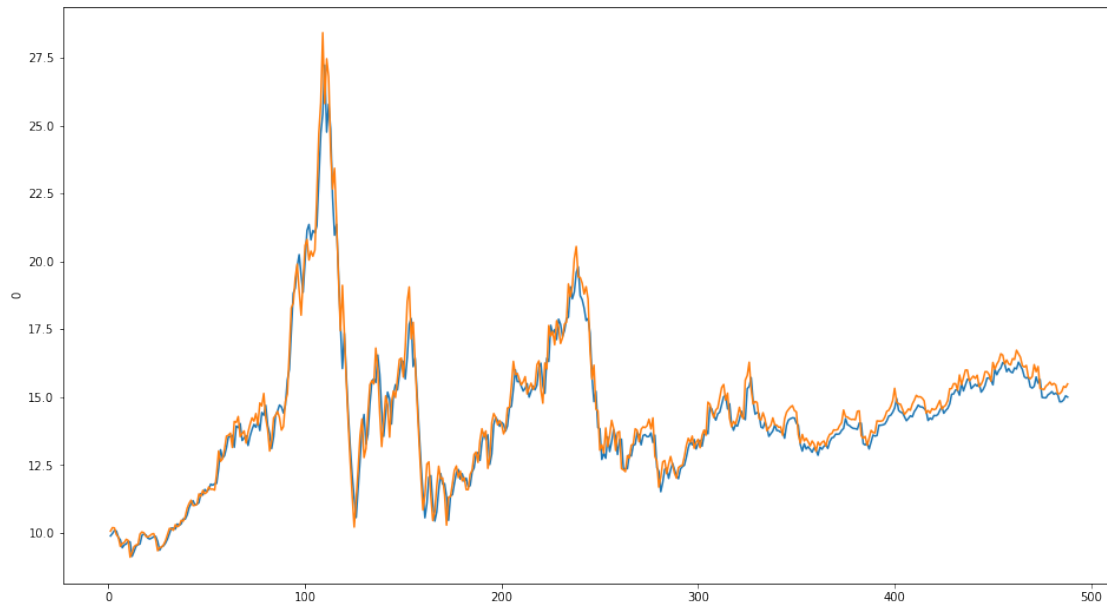
```
In [41]: model, model_result = fitModel(cycles,best_cfg)
         cp = forecastInModel(model_result)
         cpo,cpci = forecastOutModel(model_result,step)
         plt.figure(figsize=(16,9))
         sns.lineplot(x,cycles)
         sns.lineplot(x,cp)
```

```
Out[41]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3dbb4a4a8>
```



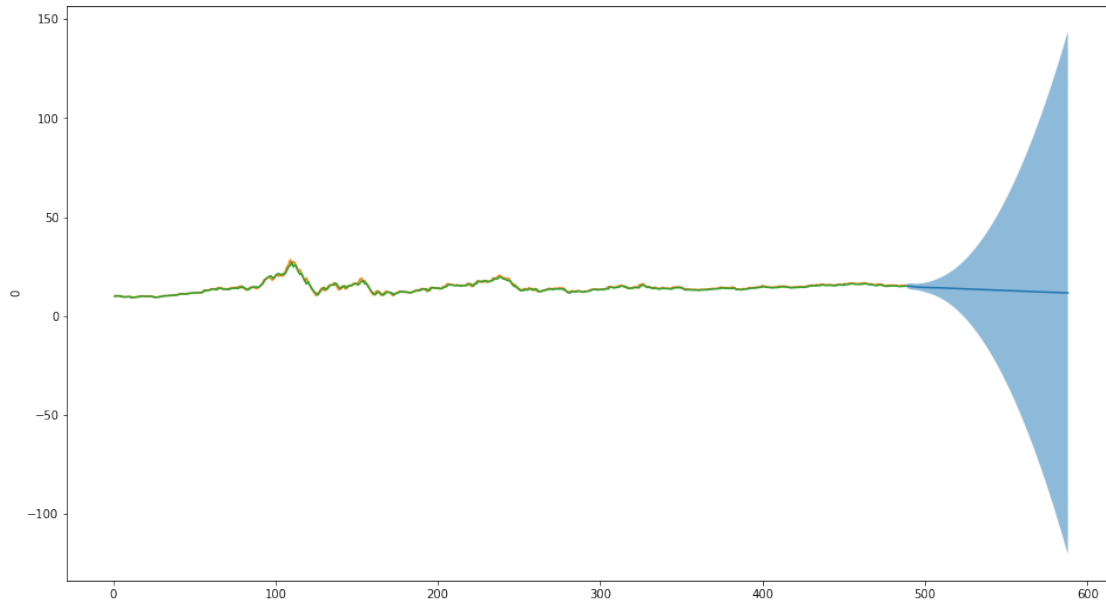
```
In [42]: pdata = cp + tpr[0]
plt.figure(figsize=(16,9))
sns.lineplot(x,pdata)
sns.lineplot(x,y)
```

```
Out [42]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3d4374048>
```



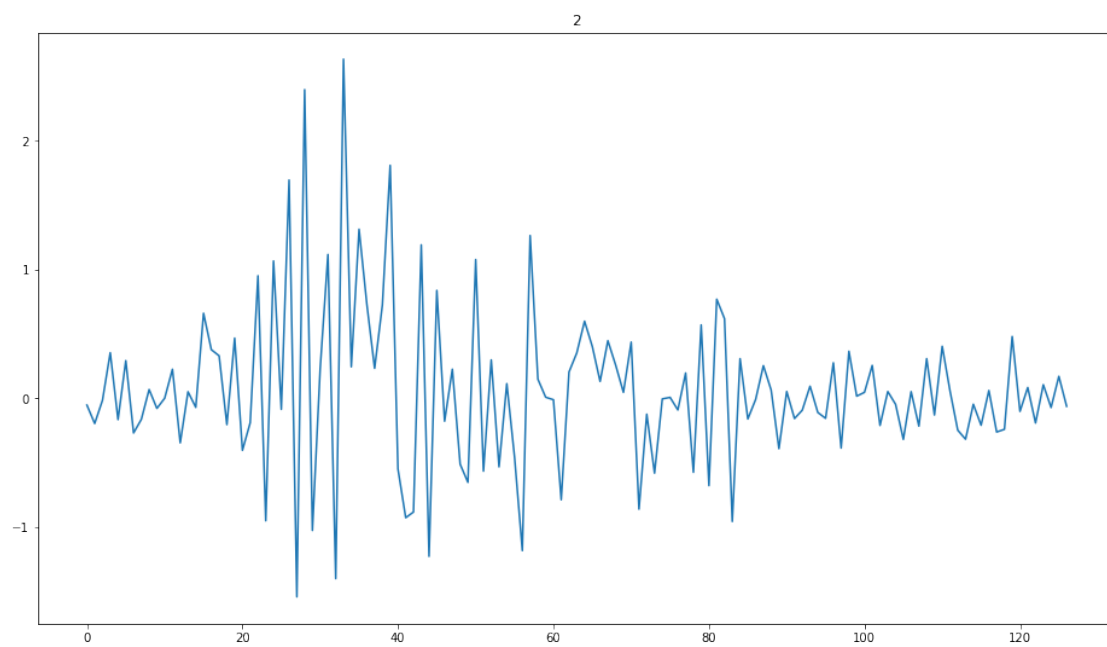
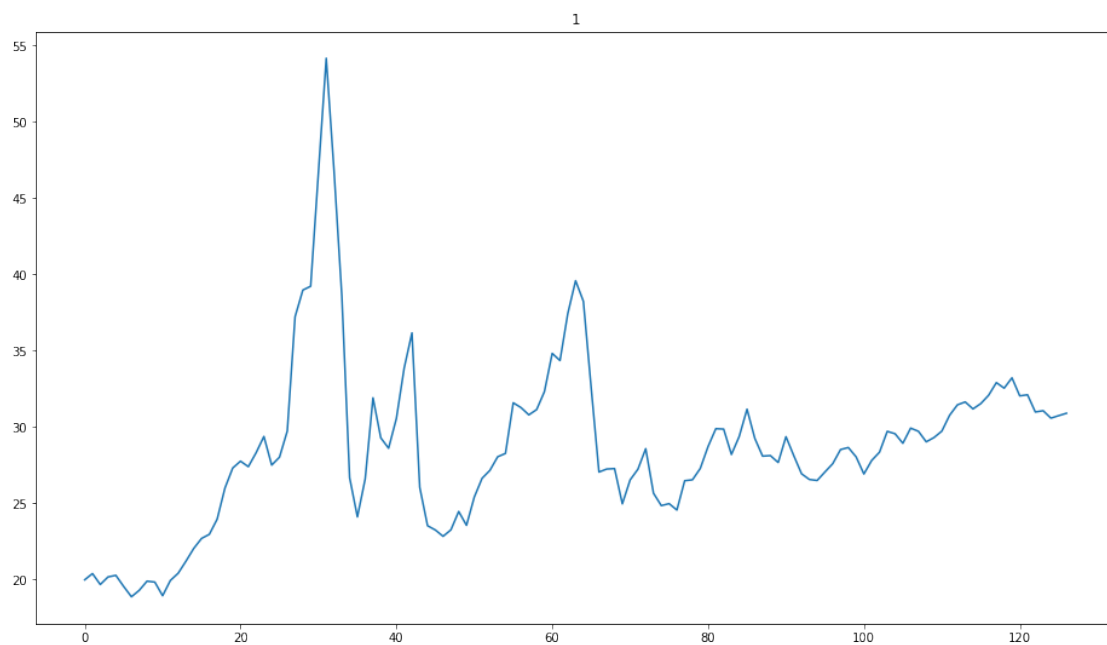
```
In [43]: pdatao = cpo + tppr[0][-step:].values
plt.figure(figsize=(16,9))
sns.lineplot(list(range(489,489+step)),pdatao)
sns.lineplot(x,y)
sns.lineplot(x,pdata)
plt.fill_between(list(range(489,489+step)),cpci[:,0]+tppr[0][-step:].values,cpci[:,1]+tpd
```

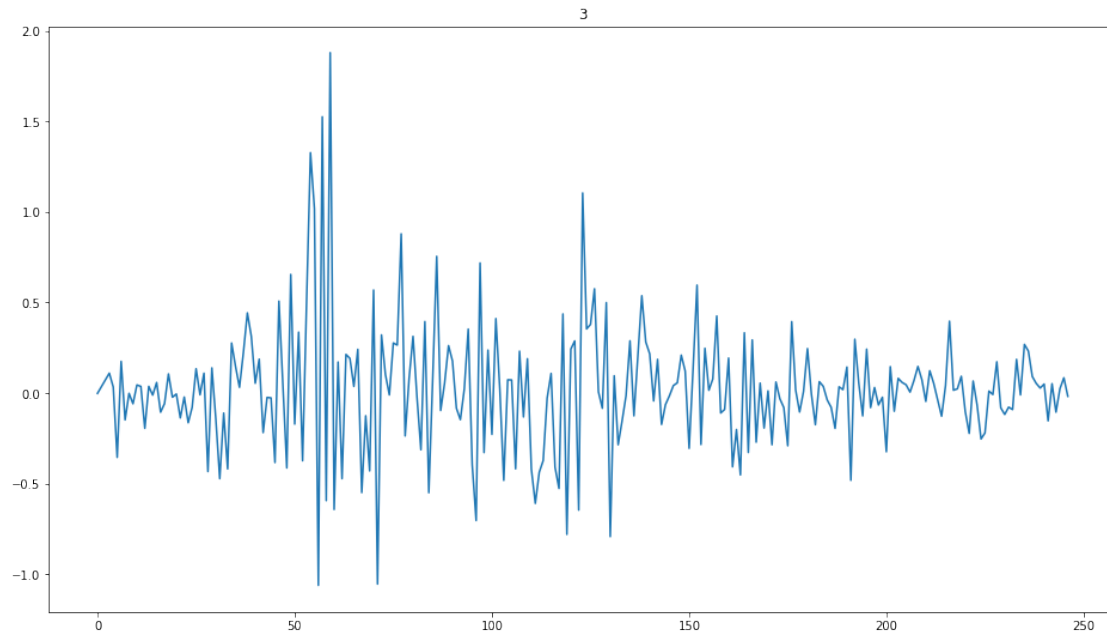
```
Out [43]: <matplotlib.collections.PolyCollection at 0x1c3dca05390>
```



1.4 小波分析

```
In [44]: step = 100;
        coffes = waveletFilter(y,2)    # 进行 HP 滤波分解
        # plt.figure(figsize=(16,9))
        # sns.lineplot(x,y)
        coffes2 = waveletFilter(np.append(y,np.linspace(0,100,100)),2)
        steps = [len(coffes2[0])-len(coffes[0]),len(coffes2[1])-len(coffes[1]),len(coffes2[2])-len(coffes[2])]
        i = 1
        for coffe in coffes:
            # print(coffe)
            plt.figure(figsize=(16,9))
            plt.title(i)
            sns.lineplot(list(range(len(coffe))),coffe)
            i+=1
```





```
In [45]: steps
```

```
Out[45]: [25, 25, 50]
```

```
In [46]: for coffe in coffes:
          print(adfTest(coffe))
```

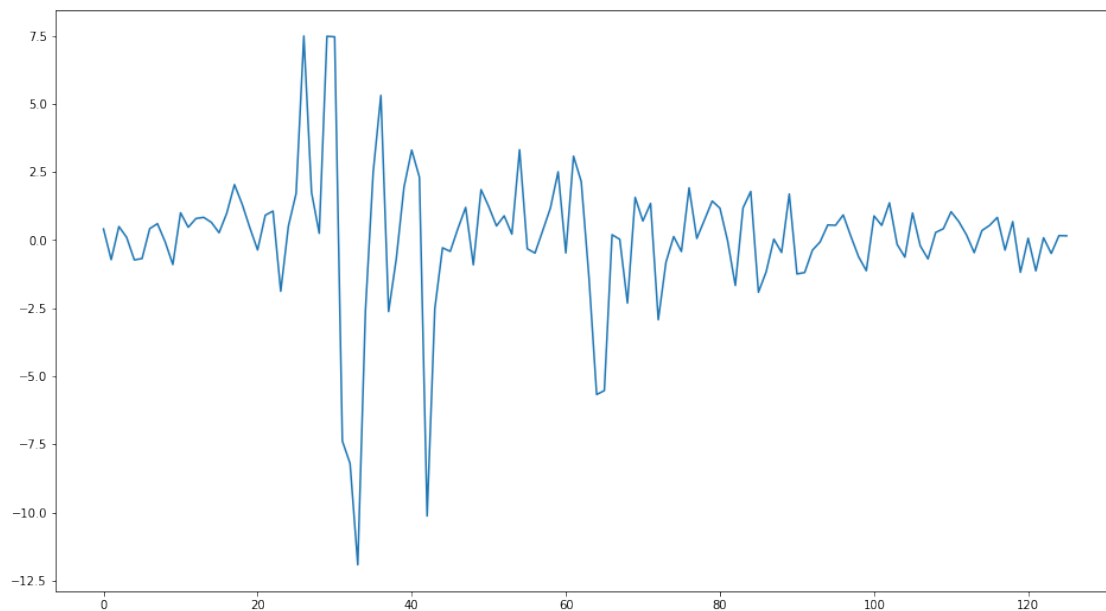
```
False
```

```
True
```

```
True
```

```
In [47]: ad,afv = bestDiff(pd.DataFrame(coffes[0]))
          ad = ad.values
          plt.figure(figsize=(16,9))
          sns.lineplot(list(range(len(ad))),ad)
```

```
Out[47]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3de838780>
```

```
In [48]: best_score,best_cfg = chooseModels2(ad,5,method='bic')
```

```
(0, 0) 600.2690213370469
(0, 1) 585.4940144229992
(0, 2) 589.955112319535
(0, 3) 589.9618183905318
(0, 4) 590.2239513506528
(1, 0) 589.642559131766
(1, 1) 590.1908674016591
(1, 2) 586.4888651264454
(1, 3) 587.6532425232247
(1, 4) error
(2, 0) 586.0944469320939
(2, 1) 586.6458716253658
(2, 2) 592.4930834250305
(2, 3) 592.354181000786
(2, 4) error
(3, 0) 589.0723202526272
(3, 1) 593.6017993920834
(3, 2) 595.5905190833057
(3, 3) 595.1443197782121
(3, 4) error
(4, 0) 593.6773551342563
```

```
(4, 1) 598.4113021914732
```

```
(4, 2) 599.0435780078417
```

```
(4, 3) 595.0999120465101
```

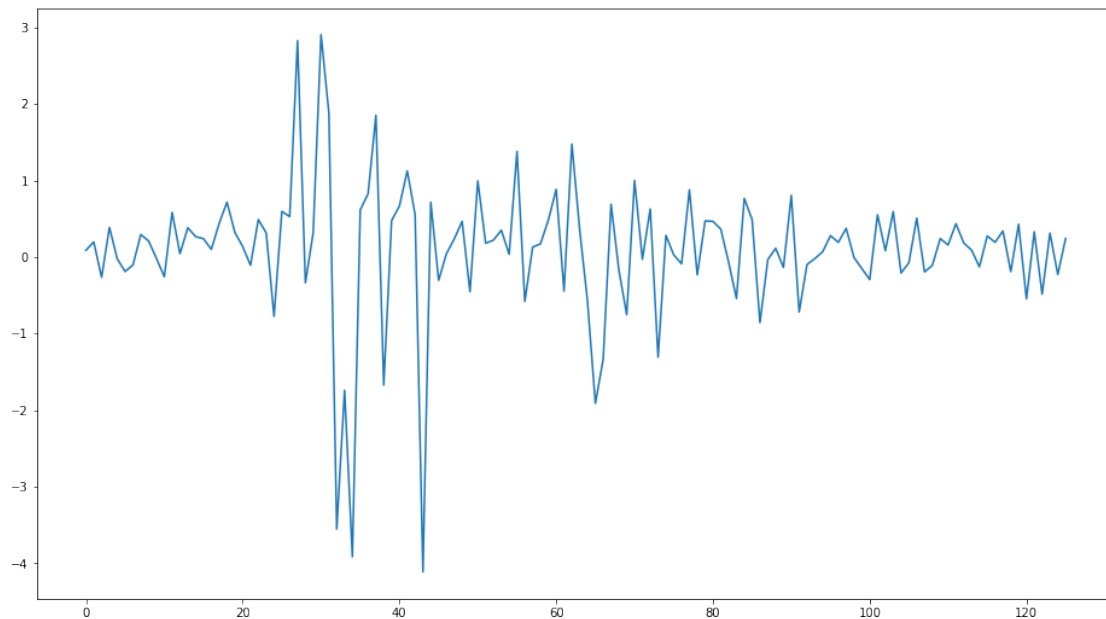
```
(4, 4) error
```

```
In [49]: best_score,best_cfg
```

```
Out[49]: (585.4940144229992, (0, 1))
```

```
In [50]: model, model_result = fitModel(ad,best_cfg)
         ap = forecastInModel(model_result)
         apo,apci = forecastOutModel(model_result,steps[0])
         plt.figure(figsize=(16,9))
         sns.lineplot(list(range(len(ad))),ap)
```

```
Out[50]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3dd1baa20>
```



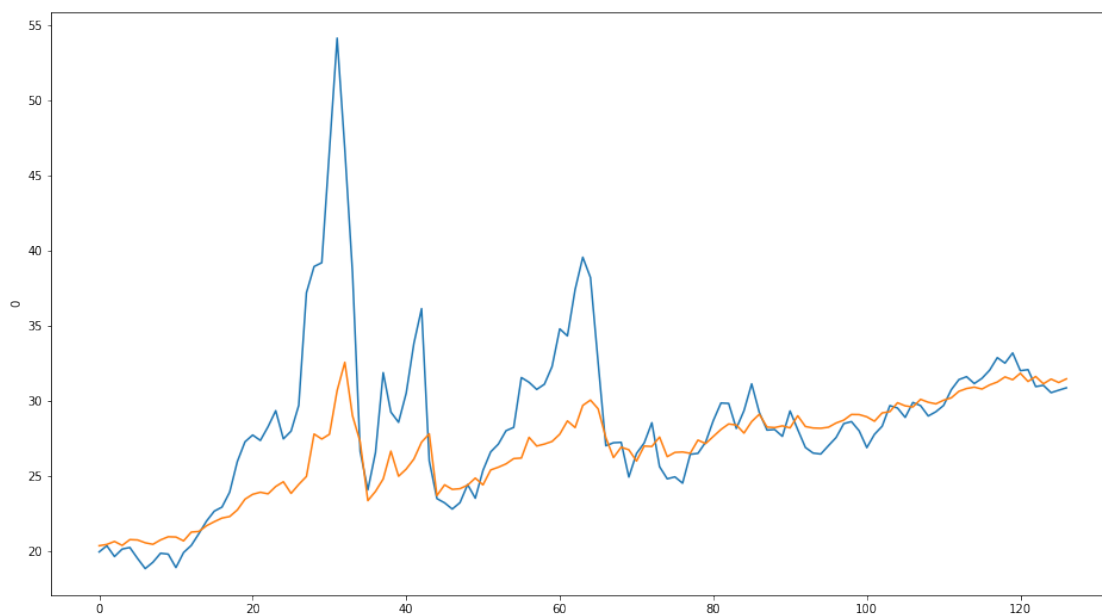
```
In [51]: df = pd.DataFrame(ap,index=list(range(len(afv)+1,len(ap)+len(afv)+1)))
         dfp = pd.DataFrame(np.append(ap,apo),index=list(range(len(afv)+1,len(ap)+len(afv)+1+steps
         dfu = pd.DataFrame(np.append(ap,apci[:,0]),index=list(range(len(afv)+1,len(ap)+len(afv)+1
         dfd = pd.DataFrame(np.append(ap,apci[:,1]),index=list(range(len(afv)+1,len(ap)+len(afv)+1
         apr = recoverDiff(df, afv)
         appr = recoverDiff(dfp, afv)
```

```

apur = recoverDiff(dfv, afv)
apdr = recoverDiff(dfd, afv)
plt.figure(figsize=(16,9))
sns.lineplot(list(range(len(coffes[0]))),coffes[0])
sns.lineplot(list(range(len(coffes[0]))),apr[0])

```

Out [51]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3de841ef0>



```
In [52]: best_score,best_cfg = chooseModels2(coffes[1],5,method='bic')
```

```

(0, 0) 256.8877896759146
(0, 1) 246.60321345988416
(0, 2) 238.54681383591608
(0, 3) 242.4320592760104
(0, 4) 246.40345707547652
(1, 0) 238.75025101238737
(1, 1) 241.8895795959282
(1, 2) 242.24665212356945
(1, 3) 247.92504128502316
(1, 4) error
(2, 0) 240.23701142961139
(2, 1) 242.03960340450195
(2, 2) 246.73250065474633
(2, 3) 250.9756792940887

```

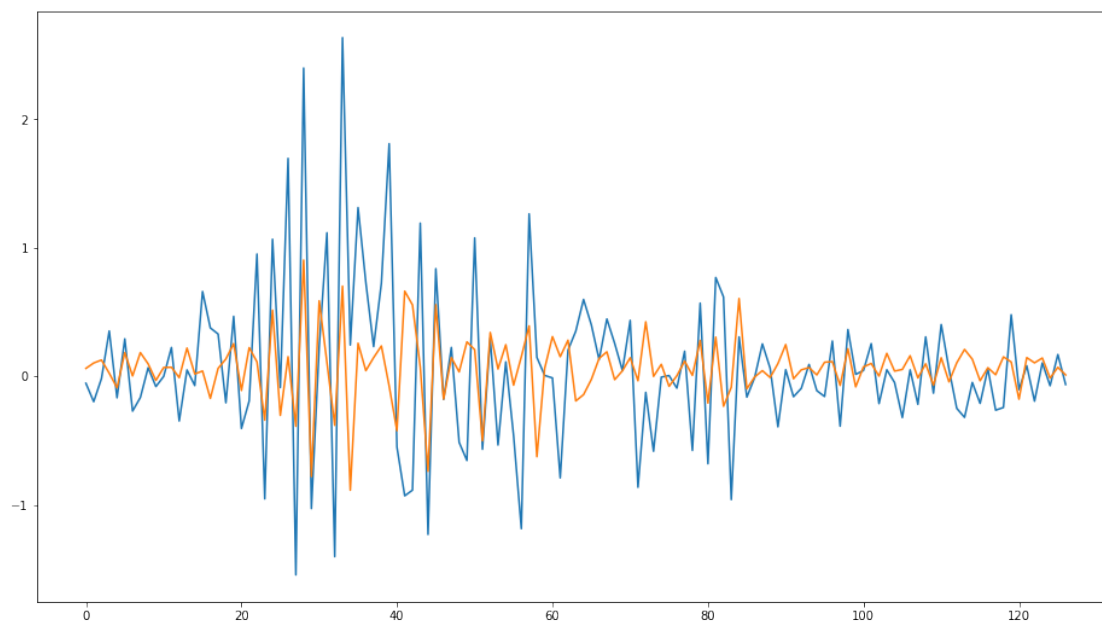
```
(2, 4) error
(3, 0) 241.1703275160015
(3, 1) 239.7856183092954
(3, 2) 243.19634824979374
(3, 3) 247.86181364202332
(3, 4) 255.96318675030494
(4, 0) 245.6051924636973
(4, 1) 243.52734764366147
(4, 2) 247.80261358413503
(4, 3) 252.6311748588194
(4, 4) 256.4190894695959
```

```
In [53]: best_score,best_cfg
```

```
Out[53]: (238.54681383591608, (0, 2))
```

```
In [54]: model, model_result = fitModel(coffes[1],best_cfg)
         dp1 = forecastInModel(model_result)
         dpo1,dpci1 = forecastOutModel(model_result,steps[1])
         plt.figure(figsize=(16,9))
         sns.lineplot(list(range(len(coffes[1]))),coffes[1])
         sns.lineplot(list(range(len(dp1))),dp1)
```

```
Out[54]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3dc95acc0>
```



```
In [55]: best_score,best_cfg = chooseModels2(coffes[2],5,method='bic')
```

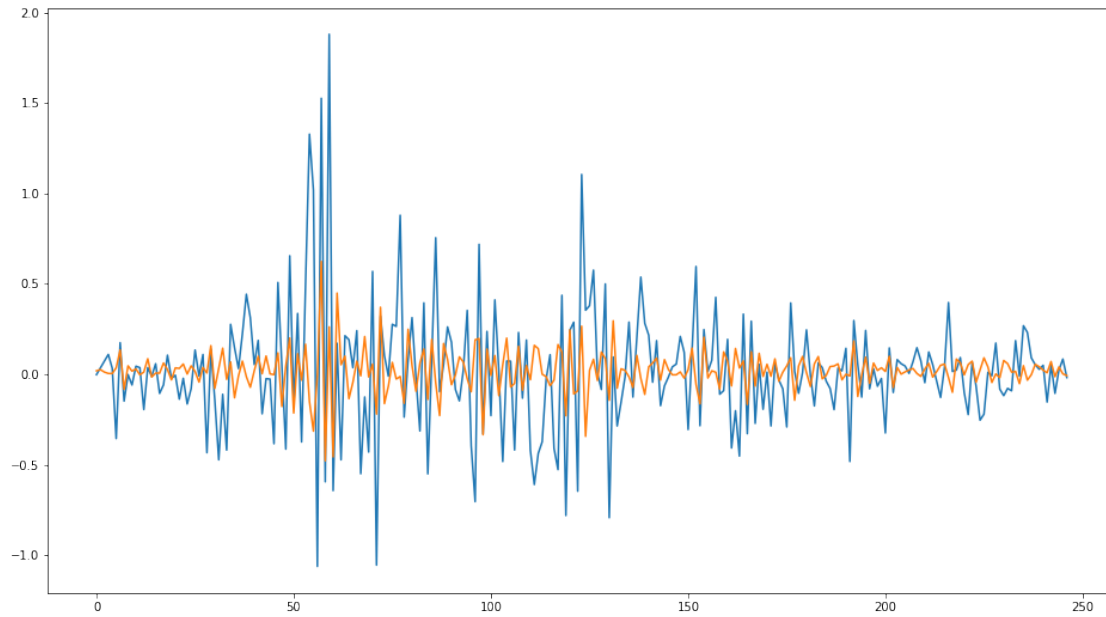
```
(0, 0) 192.80665023518415
(0, 1) 183.66824715354807
(0, 2) 172.94785550694132
(0, 3) 177.43172539946778
(0, 4) 176.220582043445
(1, 0) 177.03889527748385
(1, 1) 178.20279149496992
(1, 2) 174.233164451611
(1, 3) 183.79958962648743
(1, 4) 181.11737921199477
(2, 0) 174.96301675832774
(2, 1) 176.13645538606107
(2, 2) error
(2, 3) error
(2, 4) 183.7705592781964
(3, 0) 176.99509645067036
(3, 1) 181.04908655782322
(3, 2) 179.81491809176617
(3, 3) error
(3, 4) 188.9567476588225
(4, 0) 180.41268832291257
(4, 1) 185.8138812408177
(4, 2) 183.78146079894725
(4, 3) 189.2879747654793
(4, 4) 187.5124519252313
```

```
In [56]: best_score,best_cfg
```

```
Out[56]: (172.94785550694132, (0, 2))
```

```
In [57]: model, model_result = fitModel(coffes[2],best_cfg)
         dp2 = forecastInModel(model_result)
         dpo2,dpci2 = forecastOutModel(model_result,steps[2])
         plt.figure(figsize=(16,9))
         sns.lineplot(list(range(len(coffes[2]))),coffes[2])
         sns.lineplot(list(range(len(dp2))),dp2)
```

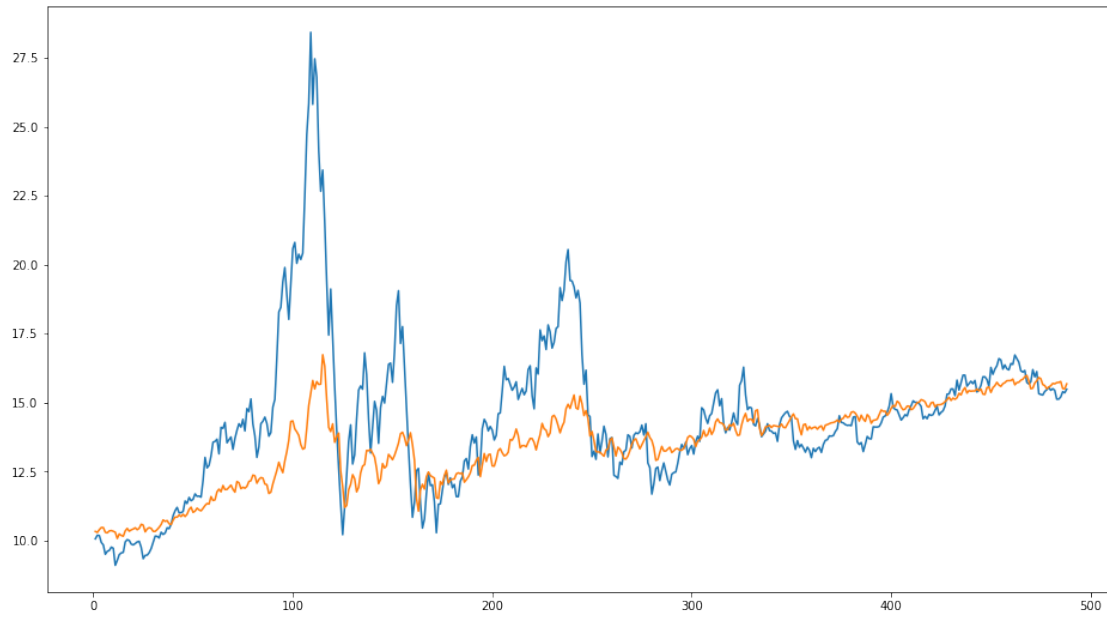
```
Out[57]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3dcf38e80>
```



```
In [58]: coeffnewp = [np.append(apr[0],appr[0][-steps[0]:].values),np.append(dp1,dpo1),np.append(d
coeffnewu = [np.append(apr[0],apur[0][-steps[0]:].values),np.append(dp1,dpci1[:,0]),np.ap
coeffnewd = [np.append(apr[0],apdr[0][-steps[0]:].values),np.append(dp1,dpci1[:,1]),np.ap
dcp = recoverWavelet(coeffnewp)
dcu = recoverWavelet(coeffnewu)
dcd = recoverWavelet(coeffnewd)
```

```
In [60]: plt.figure(figsize=(16,9))
sns.lineplot(x,y)
sns.lineplot(x,dcp[:100])
```

```
Out[60]: <matplotlib.axes._subplots.AxesSubplot at 0x1c3def92940>
```



```
In [61]: plt.figure(figsize=(16,9))
sns.lineplot(list(range(489,489+step)),dcp[-step:])
sns.lineplot(x,y)
sns.lineplot(x,dcp[:-step])
plt.fill_between(list(range(489,489+step)),dcu[-step:],dcd[-step:],alpha=0.5)
```

```
Out[61]: <matplotlib.collections.PolyCollection at 0x1c3decb39e8>
```

