HW2

1.

(a)

$g_1(t) = 2\exp(-\pi(2t^2+4t+2)) = 2\exp(-2\pi(t+1)^2)$

$\exp(-\pi t^2) \to \exp(-\pi f^2)$

$\exp(-\pi(\sqrt{2}t)^2) \to \frac{1}{\sqrt{2}}\exp(-\pi f^2/2)$

$\exp(-2\pi(t+1)^2) \to \frac{1}{\sqrt{2}}\exp(-\pi f^2/2)\exp(j2\pi f)$

$G_1(f) = \sqrt{2}\exp(-\pi f^2/2)\exp(j2\pi f)$

(b)

$FT[\exp(-\pi t^2)g_1(t)]$
$\to FT[\exp(-\pi t^2)] * FT[g_1(t)]$
$\to \exp(-\pi f^2) * \sqrt{2}\exp(-\pi f^2/2)\exp(j2\pi f)$

(c)

已知$\exp(-\pi t^2)$滿足測不准原理的下界

$g_1(t)$可透過$\exp(-\pi t^2)$經過Time Shifting、Scaling獲得

Time Shifting、Scaling不影響測不准原理的下界

因此$g_1(t)$也滿足測不准原理的下界

2.

(a)

complexity: Direct implementation > Chirp-Z transform method > FFT-based method > FFT-based method with recursive formula

(b)

Direct implementation:

$\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$

FFT-based method:

1. $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$
2. $\Delta t \Delta f = \frac{1}{N}$
3. $N \geq 2Q+1$

FFT-based method with recursive formula:

1. $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$
2. $\Delta t \Delta f = \frac{1}{N}$
3. $N \geq 2Q+1$
4. rectangular window

Chirp-Z transform method:

1. $\Delta t < \frac{1}{2(\Omega_x+\Omega_w)}$
2. $N \geq 2Q+1$

(c)

- Direct implementation
- FFT-based method
- Chirp-Z transform method

3.

(a) $\exp(j2\pi t^3)$是三次方，有交錯項的問題。不適合WDF

(b) Music signal是多組訊號組成，有交錯項的問題。不適合WDF

(c) sin function是兩組訊號組成，有交錯項的問題。不適合WDF

(d) $\exp(-\pi t^2)$是二次項，沒有交錯項。適合WDF

4.

(a)

$W_x(t,f)^*$
$= [\int_{-\infty}^{\infty} x(t+0.5\tau)x^*(t-0.5\tau)\exp(-j2\pi f\tau)]^* d\tau$
$= \int_{-\infty}^{\infty} x^*(t+0.5\tau)x(t-0.5\tau)\exp(j2\pi f\tau)d\tau$
$= \int_{\infty}^{-\infty} x^*(t-0.5\omega)x(t+0.5\omega)\exp(-j2\pi f\omega)d\omega$
$= W_x(t,f)$

(b)

$g(\tau)w(\tau) = g^*(-\tau)w^*(-\tau) \rightarrow w(\tau) = w^*(-\tau)$

(c)

$C_x^*(t,f) = \iint A_x^*(\tau,\eta)\phi^*(\tau,\eta)\exp(j2\pi(-\eta t+\tau f))d\eta d\tau$
$= \iint A_x^*(-\tau_1,-\eta_1)\phi^*(-\tau_1,-\eta_1)\exp(j2\pi(\eta_1 t-\tau_1 f))(-1)^2 d\eta d\tau$
$C_x(t,f) = \iint A_x(\tau,\eta)\phi(\tau,\eta)\exp(j2\pi(\eta t-\tau f))d\eta d\tau$

$C_x^*(t,f) = C_x^*(t,f)$ iff $A_x(\tau,\eta)\phi(\tau,\eta) = A_x^*(-\tau,-\eta)\phi^*(-\tau,\eta)$

由(a)已知$A_x(\tau,\eta)$是實數

所以推算出$\phi(\tau,\eta) = \phi^*(-\tau,-\eta)$

5.

(a) Cohen’s class distribution

柯恩分布的auto term大多位於（0, 0）。因此可以用low-pass filter當作window funtion。

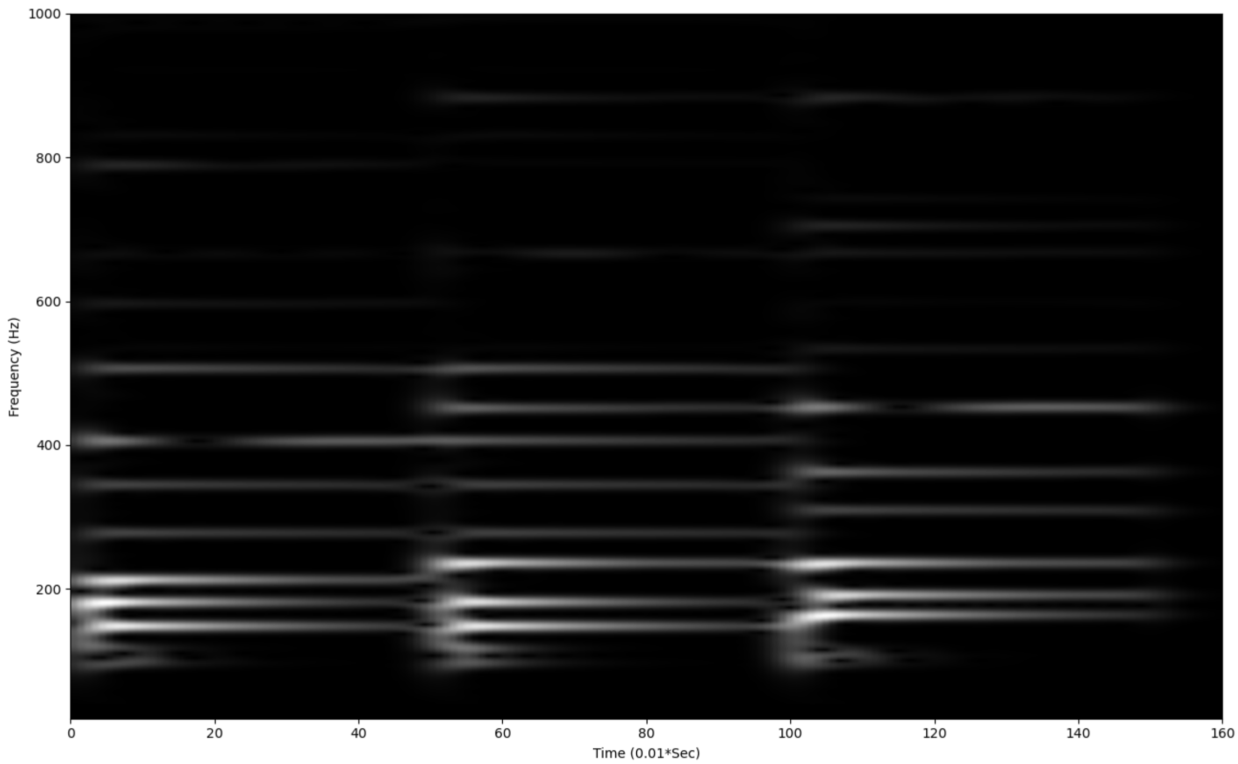
(b) the polynomial WDF

The polynomial WDF can avoid the cross term when the order of phase of the exponential
function is no larger than q/2+1 with appropriate d_l

6.

tau = 0:1/44100;1.6

running time: 0.13286328315734863



學號尾數1&6

If $x(t) = \exp(j\phi(t))$ and $\phi(t)$ is a polynomial with order $\geq$ 3, the cross term problem appears.