PP116, $A \in M_{m\times n}$, $x \in M_{n\times 1}$, 열벡터 $(\in \mathbb{R}^n)$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\Rightarrow A \cdot x = \underbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}}_{[m\times n]} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}}_{[n\times 1]} = \underbrace{\begin{bmatrix} a_{11}\cdot x_1 + \cdots + a_{1n}\cdot x_n \\ \vdots \\ a_{m1}\cdot x_1 + \cdots + a_{mn}\cdot x_n \end{bmatrix}}_{[m\times 1]}$$

$$= x_1 \cdot \begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} + \cdots + x_n \cdot \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

Let $Col_i(A) = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix}$, then

$$\Rightarrow x_1 \cdot Col_1(A) + \cdots + x_n \cdot Col_n(A)$$

Theorem

$A \in M_{m\times n}$, $\forall \underbrace{u, v}_{\text{vector}} \in \mathbb{R}^n$, $\forall \underbrace{c}_{\text{scalar}} \in \mathbb{R}$

(i) $A \cdot (c \cdot u) = c\,(A \cdot u)$

(i) $A\cdot(c\cdot u) = c\cdot(A\cdot u)$

(ii) $A\cdot(u+v) = A\cdot u + A\cdot v$

[Proof]

(i) Let

Definition

Let $A \in M_{m\times s}$, $B \in M_{s\times n}$, $B = [\mathbb{b}_1, \mathbb{b}_2, \ldots, \mathbb{b}_n]$

, where $b_i = \begin{pmatrix} b_{2i} \\ \vdots \\ b_{si} \end{pmatrix}_{[s\times 1]}$, then

$$\Rightarrow A\cdot B = \begin{bmatrix} A\cdot b_1 & \cdots & A\cdot b_n \end{bmatrix}_{m\times n}$$

$$\Rightarrow \mathrm{Col}_j(AB) = A\cdot b_j = A\cdot \mathrm{Col}_j(B) \rightsquigarrow \text{Column Rule}$$

$$\Rightarrow \mathrm{Row}_i(AB) = \mathrm{Row}_i(A)\cdot B \rightsquigarrow \text{Row-Rule}$$

Definition in pp 123

$$A \in M_{m\times n} \Rightarrow A^t = A^T \in M_{n\times m} \quad [\text{Transpose}]$$

Definition of Matrix inner Product and Matrix Outer Product

Let $u, v \in \mathbb{R}^n$, where $u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}_{(1\times n)}$ $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}_{(1\times n)}$

[Matrix Inner Product]

$$u^T \cdot v = (u_1 \; u_2 \; \cdots \; u_n) \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = (u_1 v_1 + u_2 v_2 + \cdots + u_n v_n)$$

Matrix Multiplication

$$= \sum_{i=1}^{n} u_i \cdot v_i = u \cdot v$$

$i=1$

Vector Inner Product

[Matrix Outer Product]

$$u \cdot v^T = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix} = \begin{pmatrix} u_1 \cdot v_1 & u_1 \cdot v_2 & \cdots & u_1 \cdot v_n \\ \vdots & & \ddots & \vdots \\ u_1 \cdot v_n & \cdots & & u_n \cdot v_n \end{pmatrix}$$

[n x 1] [1 x n] [n x n]

$\Rightarrow$ Trace of $u \cdot v^T = u_1 \cdot v_1 + \cdots + u_n \cdot v_n$

[Matrix Outer Product] $= \sum_{i=1}^{n} u_i \cdot v_i = u \cdot v$ (OutCome of Matrix inner Pro

PP126 Equation (25), (26), (27)

$$u^T \cdot v = u \circ v = v \circ u = v^T \circ u$$

Matrix Complication

Vector Inner Product

$$u^T \cdot v = \mathrm{Trace}\left[u^T \cdot v\right] = \mathrm{Trace}\left[v^T \cdot u\right]$$