Investments

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# Portfolio Statistics

## Portfolios

-Portfolio are combinations of underlying assets  
-Given return properties of the underlying assets, what are the return properties of their combination?

## Expected Return of Portfolio of Assets

- is the portfolio weight of asset   
- is the expected return of asset   
-The portfolio is fully invested:   
-Notation:

## Variance of Portfolio of Assets

- is the portfolio weight of asset   
- is the covariance between assets and   
-Recall that and   
-Notation: ; ;

## Variance of Portfolio of Assets: A Matrix View

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## Example: Equal-weighted portfolio of two assets

-Expected Return

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-Portfolio Variance

## Variance of Portfolio of Assets: Matrices

-Portfolio weights vector:

-Covariance matrix of returns:

## Covariance and Correlation

-Covariance: absolute degree of co-movement between two assets  
-Correlation: relative degree of co-movement between two assets

-What are the possible values for ?

# Diversification

## Effects of Diversification

Claim: The variance of the return of a portfolio with many securities depends more on the covariances between the individual securities than on the variances of the individual securities.

## Now consider a five-security portfolio

How many variance terms?

How many covariance terms?

## Why does covariance dominate with large ?

-Consider an -asset, equal-weighted portfolio )  
-Assume all assets have the same variance   
-Assume all pairs of assets have the same covariance   
-What is the variance of the portfolio?

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What happens to this as N gets large?

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## Diversification curves

-Diversification eliminates some, but not all, of the risk of individual assets.  
-In large portfolios, ’s effectively diversified away, but not ’s.  
-Diversifiable, non-systematic, idiosyncratic risk vs. non-diversifiable, systematic, market risk

## Diversification curves

INSERT PLOTS OR WEBPAGE OF EMPIRICAL DIVERSIFICATION CURVES?

# Preferences

## Which return series do you prefer?

import numpy as np  
import pandas as pd  
from scipy.stats import norm  
import plotly.express as px  
import plotly.io as pio  
pio.renderers.default='notebook'  
  
# Parameters  
mn1 = 0.15  
sd1 = 0.10  
mn2 = 0.35  
sd2 = 0.10  
T = 50  
  
# Generate data  
rv1 = norm(loc=mn1, scale=sd1).rvs(T)  
rv2 = norm(loc=mn2, scale=sd2).rvs(T)  
time = np.arange(T)  
df = pd.DataFrame(data={'time': time, 'ret1': rv1, 'ret2': rv2})  
  
# Plot data  
fig = px.line(df,x='time', y=['ret1', 'ret2'])  
fig.update\_layout(yaxis\_title='Return',  
 xaxis\_title='',  
 legend\_title\_text='',  
)  
fig.show()

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## Which return series do you prefer?

# Parameters  
mn1 = 0.20  
sd1 = 0.10  
mn2 = 0.20  
sd2 = 0.40  
T = 50  
  
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## Where would your portfolio like to live?

import matplotlib.pyplot as plt  
fig, ax = plt.subplots()  
ax.set\_xlim([0,0.5])  
ax.set\_ylim([0,0.25])  
ax.set\_ylabel(r'$E[r\_p]$')  
ax.set\_xlabel(r'$\[r\_p]$')  
  
# mn = np.arange(0,0.5,0.1)  
# sd = np.arange(0,0.5,0.1)  
  
# ax.plot(sd, sd\*0, color='k')  
# ax.plot(mn\*0,mn, color='k')

Text(0.5, 0, '$\\text{sd}[r\_p]$')

ValueError:   
\text{sd}[r\_p]  
^  
Unknown symbol: \text, found '\' (at char 0), (line:1, col:1)

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